“FEL” using pipe with Surface Impedance

pipes with (loss free) surface impedance

comparison with FEL

derivation of FEL gain using wakefield approach (Stupakov)

using pipe with corrugated walls for FEL (Stupakov)

space charge effects (plasma oscillations)

pipes with (loss free) surface impedance

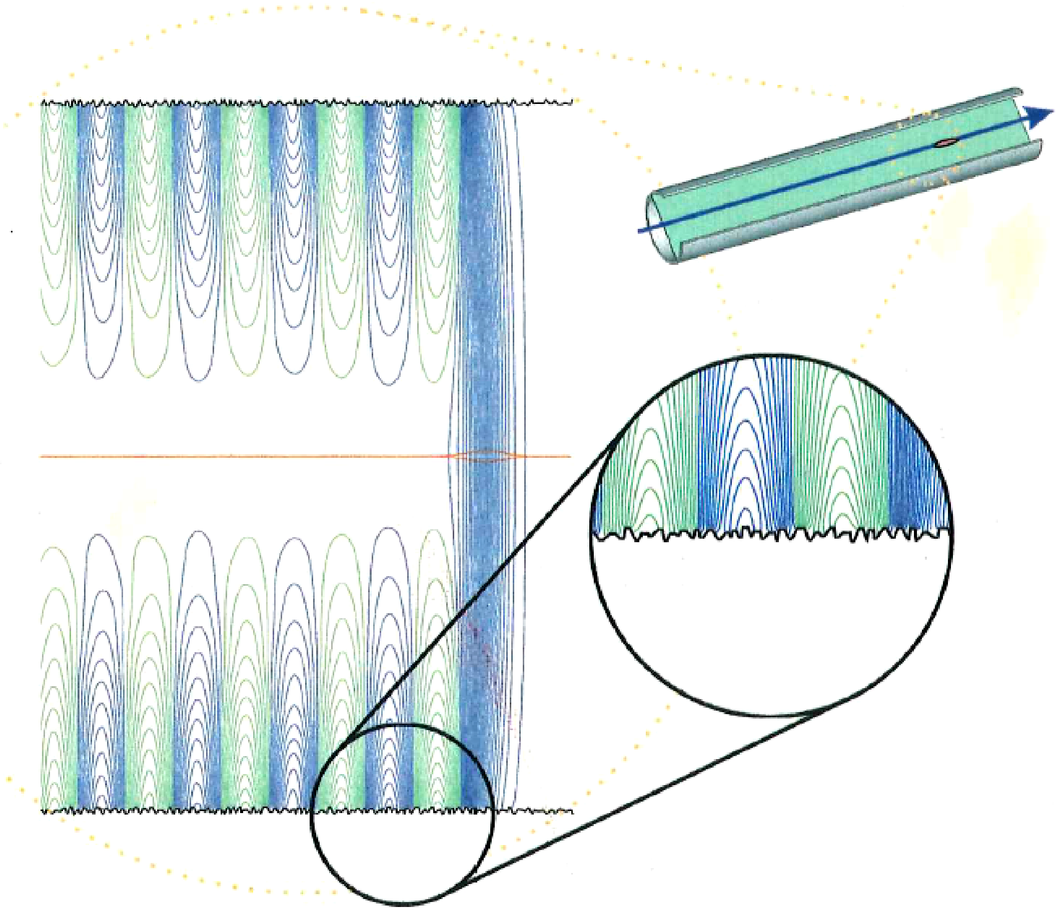
M. Timm, S. Nokhatski and T. Weiland
pipes with surface roughness $\rightarrow$ dielectric layer model
pipes with thin dielectric surface layer

G. Stupakov and K. Bane
pipes with small corrugations

Surface impedance formalism for a metallic beam pipe with small corrugations
PhysRevSTAB 15, 124401 (2012)

PhD Thesis (2000) **Martin Brüne Timm**

Wake Fields of Short Ultra-Relativistic Electron Bunches



wave

$$H_\varphi = E_0 \frac{\varepsilon_0 \omega}{i\alpha} I_0'(\alpha r) \exp\left(ik_p\left(z - v_p t\right)\right)$$

$$E_z = E_0 I_0(\alpha r) \exp\left(ik_p\left(z - v_p t\right)\right)$$

$$E_r = E_0 \frac{k_p}{i\varepsilon_0} I_0'(\alpha r) \exp\left(ik_p\left(z - v_p t\right)\right)$$

with

$$v = v_p = \beta_p c$$

$$k_p = \frac{\omega}{v_p}$$

$$\alpha = k_p / \gamma$$

asymptotic behaviour for

$$r \le r_b$$

$$\alpha r_b << 1$$

using

$$I_0(x) \approx 1 + \left(\frac{x}{2}\right)^2$$

$$I_0'(x) \approx \frac{x}{2} + \frac{x^3}{16}$$

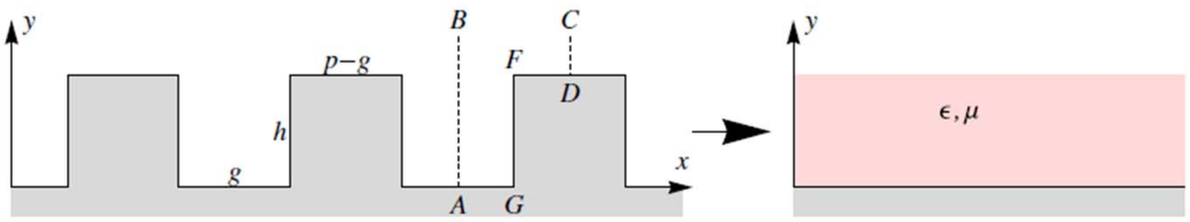
$$H_\varphi = E_0 \frac{\varepsilon_0 \omega}{i\alpha} \left(\frac{\alpha r}{2} + \frac{(\alpha r)^3}{16}\right) \exp\left(ik_p\left(z - v_p t\right)\right)$$

$$E_z = E_0 \left(1 + \frac{(\alpha r)^2}{4}\right) \exp\left(ik_p\left(z - v_p t\right)\right)$$

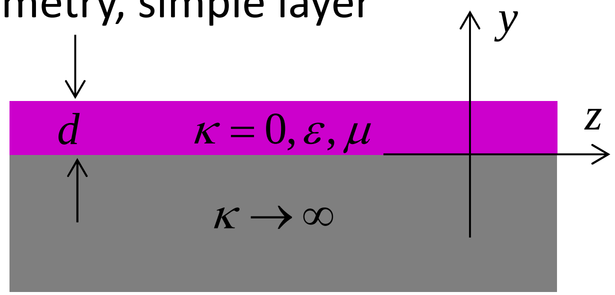
$$E_r = E_0 \frac{k_p}{i\varepsilon_0} \left(\frac{\alpha r}{2} + \frac{(\alpha r)^3}{16}\right) \exp\left(ik_p\left(z - v_p t\right)\right)$$

boundary condition $Z_b(\omega) = -\left.\frac{E_z}{H_\varphi}\right|_{r=r_b}$ radius of pipe r_b

G. Stupakov and K. Bane



planar geometry, simple layer



$$Z_b = -\frac{E_z}{H_x} \rightarrow i\omega d\left(\frac{k_p^2}{\omega^2\varepsilon} - \mu\right)$$

$Z_b(\omega) \approx -i\omega L$ with $L = L(g, h, p) \approx \mu_0 \frac{gh}{p}$

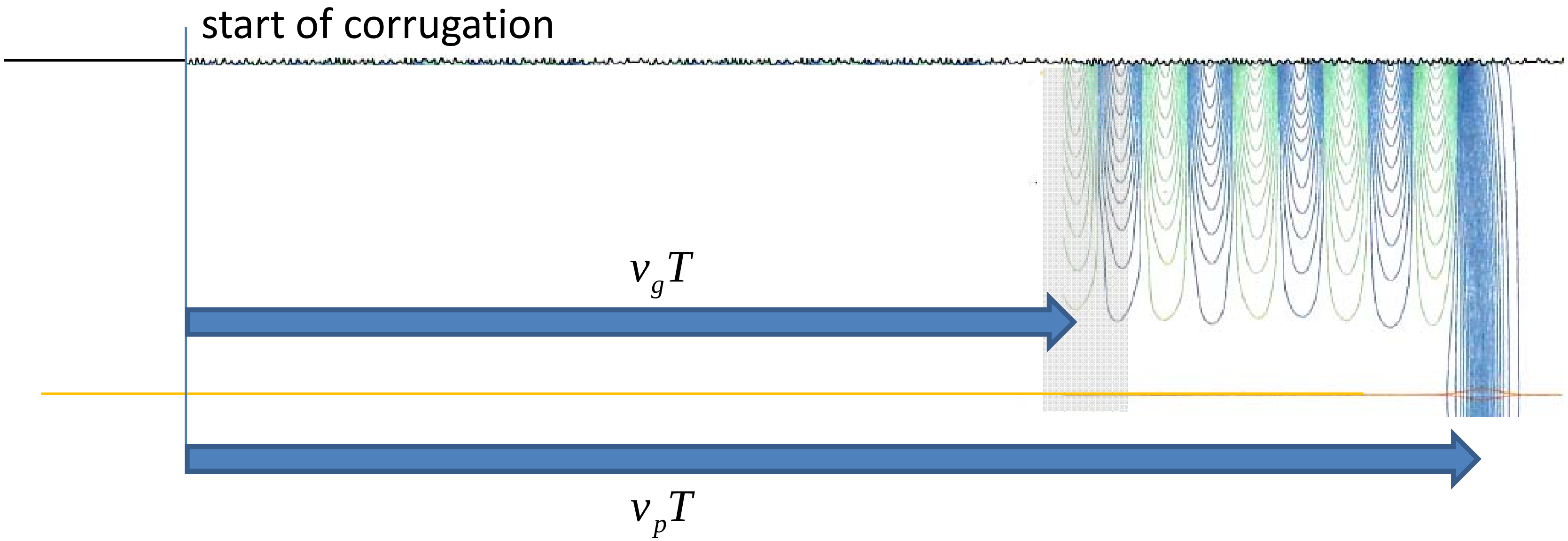
notation $f(t) = \mathrm{Re}\{\tilde{f}\exp(-i\omega t)\}$

phase velocity $\beta_p = \frac{1}{\sqrt{1+u(yx)^2}}$ group velocity $\beta_g = \frac{1}{1+u\frac{yy'}{x}}\beta_p^{-1}$

from normalized eigenmode equation $\frac{yI_0(y)}{I_1(y)} = 2x^2$

with $u = \frac{L}{2\mu_0 r_b}$, $\omega_0 = \frac{c}{r_b\sqrt{u}}$ and $x = \frac{\omega}{\omega_0}$ and $y' = \frac{4xy}{y^2 + 4(x^2 - x^4)}$

simplified wake



$$W(Z,z) = \begin{cases} 2\kappa \cos(k_0 \beta_p z) & (\beta_g / \beta_p - 1) Z < z < 0 \\ \kappa & z = 0 \\ 0 & \text{otherwise} \end{cases}$$

κ loss-parameter (energy loss per length) $\kappa = \dfrac{c}{4\left(1/\beta_g - 1/\beta_p\right)\left\langle P/E_0^2 \right\rangle}$

Z (upper case) = beamline coordinate

z (lower case) = bunch coordinate

comparison with FEL

pipe with surface impedance

$$W(Z,z)=\begin{cases} 2\kappa\cos(k_0\beta_p z) & \overbrace{(\beta_g/\beta_p-1)Z<z<0}^{\text{wave behind bunch}} \\ \kappa & z=0 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{\partial z}{\partial Z}=\frac{1}{\gamma^2}\left(\frac{\Delta\gamma}{\gamma}\right)$$ longitudinal dispersion

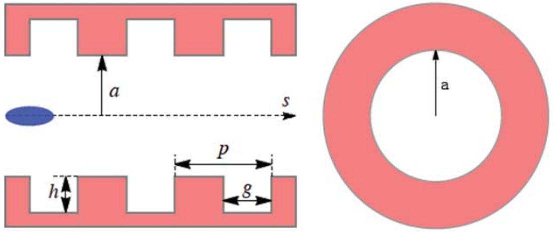
undulator

$$W(Z,z)=\begin{cases} 2\kappa\cos(k_{ph}\bar{\beta}_z z) & \overbrace{0<z<(\bar{\beta}_z^{-1}-1)Z}^{\text{wave before bunch}} \\ \kappa & z=0 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{\partial z}{\partial Z}=\frac{2k_u}{k_{ph}}\left(\frac{\Delta\gamma}{\gamma}\right) \quad \text{or} \quad \frac{\partial\psi}{\partial Z}=2k_u\left(\frac{\Delta\gamma}{\gamma}\right)$$

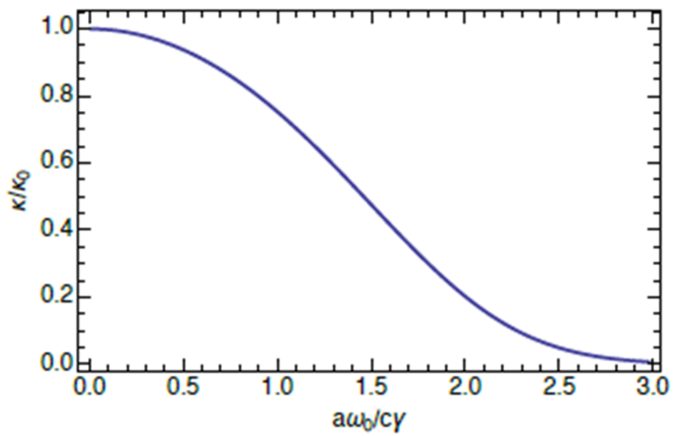
using pipe with corrugated walls for FEL

Using pipe with corrugated walls for a sub-terahertz FEL
G. Stupakov, SLAC-PUB-16171, December 2014



power gain length (cold beam, on resonance)

$$L_g = \frac{\gamma}{\sqrt{3}} \sqrt[3]{\frac{I_A}{I k_u \kappa}} \quad \text{with} \quad k_u = \frac{2\pi}{\lambda_u} \qquad k_u = \frac{2\pi}{\lambda_u} = \frac{\omega}{v_p}\left(1 - \frac{v_g}{v_p}\right)$$



Using pipe with corrugated walls for a sub-terahertz FEL
G. Stupakov, SLAC-PUB-16171, December 2014

IV. NUMERICAL EXAMPLE

TABLE I. Corrugation and beam parameters

Pipe radius, mm	2
Depth h, μm	50
Period p, μm	40
Gap g, μm ?	10
Bunch charge, nC	1
Energy, MeV	5
Bunch length, ps	10

$$I = 100\ \mathrm{A}$$

$$f = 0.34\ \mathrm{THz}$$

$$1-\beta_{\mathrm{p}} = 0.0052$$

$$1-\beta_g = 0.053$$

gain length $L_g \approx 7\ \mathrm{cm}$

saturation power $P_{sat} \approx 6.7\ \mathrm{MW}$

slip condition $\frac{\lambda_u}{v_p}\left|v_p - v_g\right| = \lambda_p \rightarrow \quad \lambda_p \approx 0.88\ \mathrm{mm}$

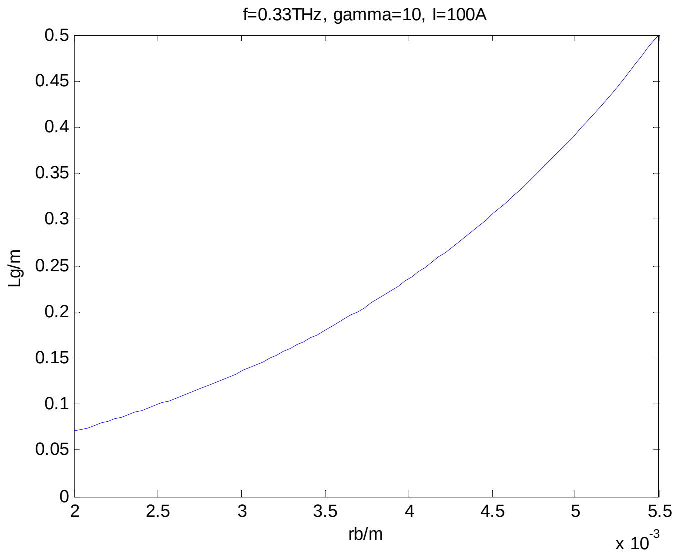
$$\lambda_u \approx 1.8\ \mathrm{cm}$$

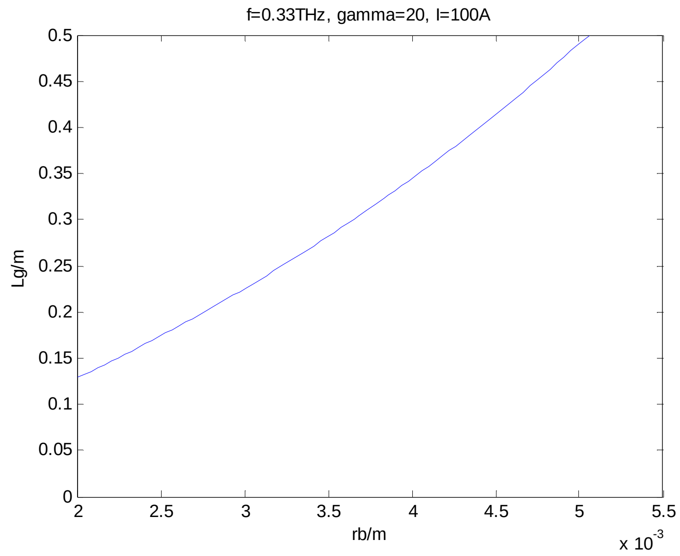
note: $\frac{\partial \psi}{\partial Z} = k_p \frac{\partial z}{\partial Z} \neq 2k_u\left(\frac{\Delta\gamma}{\gamma}\right)$

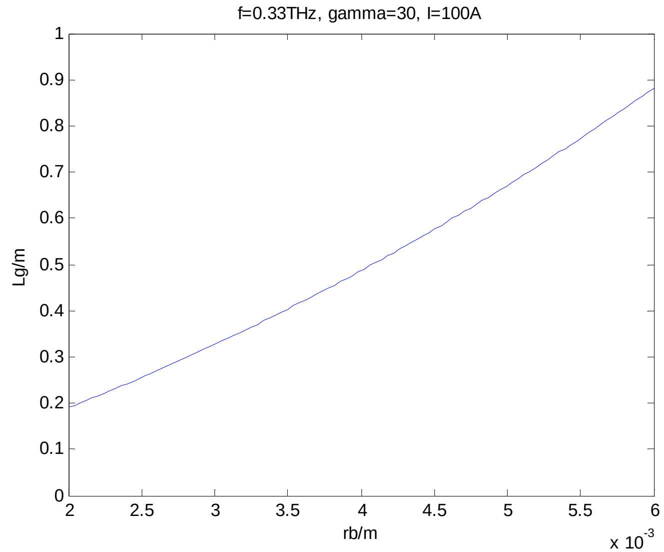
argument of modified Bessel function $\alpha r_b = \frac{k_p r_b}{\gamma} = 2\pi \frac{r_b}{\gamma\lambda_p} \approx 1.46$

gain length for other parameters

different energy and pipe radius
still current=100 A and 0.33 THz







for plasma oscillations:
emittance= 1um
beta_av=2m

gamma=10 --> Sp =	2.56 m
20	10.49 m
30	25.30 m

Stupakov's derivation of FEL gain using wakefield approach

Derivation of FEL gain using wakefield approach
G. Stupakov and S. Krinsky, PAC 2003

longitudinal charge density $\lambda(Z,z)=\int f(Z,z,\gamma)d\gamma$

instantaneous longitudinal wake $E_{\|}(Z,z)=\int W(Z,u)\lambda(Z,z-u)du$

trick: use wake with retarded source distribution

$$E_{\|}(Z,z)=\int W(Z,u)\lambda\left(Z-\frac{\bar{v}_z u}{c-\bar{v}_z},z-u\right)du$$

$\rightarrow$ power gain length, etc $\quad L_g=\frac{\gamma}{\sqrt{3}}\sqrt[3]{\frac{I_A}{Ik_u\kappa}}$

derivation per analogy

$$\frac{d\psi}{dZ} = a\eta$$

$$\frac{d\eta}{dZ} = -\frac{e}{\mathsf{E}}\mathrm{Re}\left\{\hat{E}\exp(i\psi)\right\}$$

$$\tilde{i} = I_0 \int F_1(\eta, Z)\, d\eta$$

$$\frac{d\left(\hat{E} + \tilde{i}\,\tilde{Z}\right)}{dZ} = -b\tilde{i}$$

FEL:

$$a = 2k_u$$

$$b = \frac{\mu c\hat{K}}{4\gamma}\frac{\hat{K}}{2\gamma}$$

$$\tilde{Z} = \frac{1}{-i\omega\varepsilon}$$

$$\tilde{i} = J_0 = -ecn_e < 0$$

pipe with surface impedance:

$$a = k_p / \gamma^2$$

$$b = \kappa\left(1/v_g - 1/v_p\right)$$

$$\tilde{Z} = \text{space charge impedance}$$

$$\tilde{i} = \text{beam current} < 0$$

3rd order equation for mono-energetic beam (energy η_0)

$$\hat{E}''' + 2ia\eta_0 \hat{E}'' - \left((a\eta_0)^2 - k_{\text{plasma}}^2 \right) \hat{E}' - i\Gamma^3 \hat{E} = 0$$

with $\Gamma = \sqrt[3]{ab\left|\tilde{i}\right| \frac{e}{\mathsf{E}}}$ $\boxed{\rightarrow L_{g,0} = 1/\left(\Gamma\sqrt{3}\right)}$

$$k_{\text{plasma}} = \sqrt{-i\tilde{Z}a\left|\tilde{i}\right| \frac{e}{\mathsf{E}}}$$

FEL: $\Gamma = \sqrt[3]{\frac{k_u \mu \hat{K}^2 e^2 n_e}{4m_0\gamma^3}}$

$$k_{\text{plasma}} = \sqrt{\frac{\mu 2 k_u e^2 n_e c}{\omega m_0 \gamma}}$$

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pipe with surface impedance: $\Gamma = \sqrt[3]{\frac{4\pi\varepsilon}{\beta_g} \frac{\kappa k_u}{\gamma^3} \frac{|I|}{I_a}}$ (= Stupakov's result)

$$k_{\text{plasma}} = \sqrt{4\pi \frac{-ik_p\tilde{Z}}{Z_0\gamma^3} \frac{|I|}{I_a}}$$