Status of Resistive Wall Wakefield Calculations with PBCI

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General Requirements on Wakefield Solver

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Solver Capabilites

- Fully Time Domain
- 3D Complex Structures
- Ultra-short bunches

- Resistive Walls with Freq. Dep. Conductivity
- Surface Roughness & Metal Oxidation Effects

Numerical Method

- 3D Numerical Scheme
- Dispersion Free in longitudinal direction

- Time Domain SIBC Model and Special Implementation to Preserve Dispersion Properties of 3D Method
- No Time & Mesh refinement

- Moving Window
- Parallel computing

Dispersion-Free Numerical Method

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Staggered Finite Volume Time Domain Method

Volume Integral Form of Maxwell's Equations

$$\oint_{\partial V} \vec{E} \times d\vec{A} = -\frac{d}{dt}\int_V \mu\vec{H}\, dV$$

$$\oint_{\partial V} \vec{H} \times d\vec{A} = \int_V \left[\vec{J} + \frac{d}{dt}\varepsilon\vec{E} \right] dV$$

$$\oint_{\partial V} \varepsilon\vec{E} \cdot d\vec{A} = \int_V \rho\, dV$$

$$\oint_{\partial V} \mu\vec{H} \cdot d\vec{A} = 0$$

Space Discretization

Time Continuous MGE

$$\frac{d}{dt}h = -M_{\mu^{-1}} C \cdot e$$

$$\frac{d}{dt}e = M_{\varepsilon^{-1}} C \cdot h - M_{\varepsilon^{-1}} \cdot j$$

Time Integration

No Splitting

$$\begin{pmatrix} e \\ h \end{pmatrix}^{n+1} = G(\Delta t)\begin{pmatrix} e \\ h \end{pmatrix}^{n}$$

Dispersion-free in longitudinal direction

Conclusion on SFVTD Method for Wakefield simulations

- Dispersion free at Courant limit along all three coordinate directions simultaneously.
- Successful TD-SIBC implementation & good agreement with power-loss method.
- Successful convergence study on TD-SIBC order & mesh resolution.
- Initialization of fields & according current for ultra relativistic bunch – not successful.

Dispersion-Free Numerical Method

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Maxwell's Integral Equations

$$\oint_{\partial S} \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\iint_S \mu\vec{H}\cdot d\vec{A}$$

$$\oint_{\partial S} \vec{H}\cdot d\vec{l} = \iint_S \left[\vec{J} + \frac{\partial}{\partial t}\varepsilon\vec{E} \right]\cdot d\vec{A}$$

$$\iint_S \varepsilon\vec{E}\cdot d\vec{A} = \iiint_V \rho\, dV$$

$$\iint_S \mu\vec{H}\cdot d\vec{A} = 0$$

FIT

Finite Integration Technique

Time Continuous MGE

$$M_\mu \frac{d}{dt}\widehat{h} = -C\,\widehat{e}$$

$$M_\varepsilon \frac{d}{dt}\widehat{e} = C^T\,\widehat{h} - \widehat{\widehat{j}}$$

$$S\,M_\varepsilon \widehat{e} = q$$

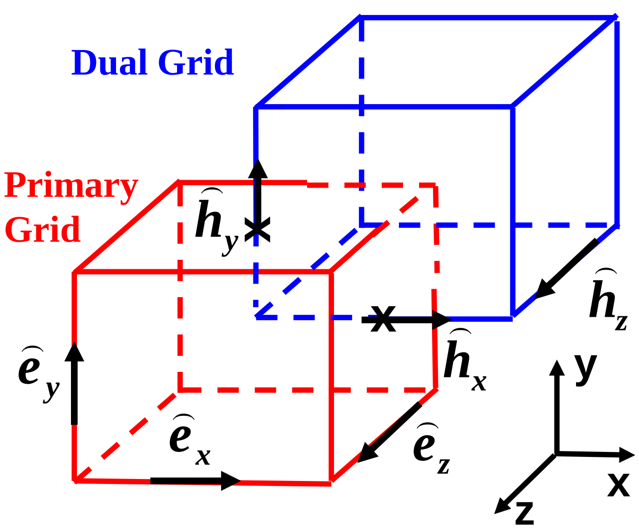
$$S^T\,M_\mu \widehat{h} = 0$$

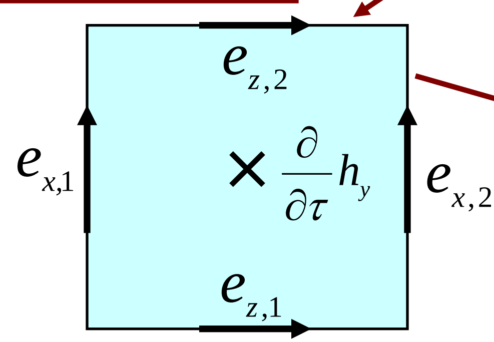
C ~ Curl , S ~ Div

Basic properties of continuous operators are retained

$$S^T C = S C^T = 0 \quad \leftrightarrow \quad div\,curl = 0$$

$$\oint_{\partial S_i} \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\iint_{S_i} \mu\vec{H}\cdot d\vec{A}$$





$$\frac{e_{x,2}-e_{x,1}}{\Delta z} - \frac{e_{z,2}-e_{z,1}}{\Delta x} = -\frac{\partial}{\partial t}\mu h_y$$

$$\frac{\partial}{\partial z}e_x - \frac{\partial}{\partial x}e_z = -\frac{\partial}{\partial t}\mu h_y$$

Finite Integration Technique - T. Weiland (1977)

Dispersion-Free Numerical Method

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Maxwell's Grid Equation written as ODE system

$$y = \begin{pmatrix} h \\ e \end{pmatrix} \qquad A = \begin{pmatrix} 0 & -C \\ C^T & 0 \end{pmatrix}$$

$$\frac{\partial}{\partial t} y = A \cdot y$$

$$R_{L,T}(\Delta t) \equiv \left(1 + \frac{\Delta t}{2} U_{L,T}\right)\left(1 + \Delta t\, L_{L,T}\right)\left(1 + \frac{\Delta t}{2} U_{L,T}\right)$$

$$U_{L,T} \equiv \begin{pmatrix} 0 & -C_{L,T} \\ 0 & 0 \end{pmatrix}, \quad L_{L,T} \equiv \begin{pmatrix} 0 & 0 \\ C_{L,T}^T & 0 \end{pmatrix}$$

LT Splitting

LTs-1

$$e^{A \cdot \Delta t} = R_T\left(\frac{\Delta t}{2}\right) R_L(\Delta t) R_T\left(\frac{\Delta t}{2}\right)$$

LTs-2

$$e^{A \cdot \Delta t} = R_L\left(\frac{\Delta t}{2}\right) R_T(\Delta t) R_L\left(\frac{\Delta t}{2}\right)$$

Updates Equations

LTs-1:

$$\begin{cases} H\_XY(\Delta t/4) \\ E\_XY(\Delta t/2) \leftarrow J_z \\ H\_XY(\Delta t/4) \end{cases} \quad \begin{cases} H\_Z(\Delta t/2) \\ E\_Z(\Delta t) \\ H\_Z(\Delta t/2) \end{cases} \quad \begin{cases} H\_XY(\Delta t/4) \\ E\_XY(\Delta t/2) \leftarrow J_z \\ H\_XY(\Delta t/4) \end{cases}$$

LTs-2:

$$\begin{cases} H\_Z(\Delta t/4) \\ E\_Z(\Delta t/2) \\ H\_Z(\Delta t/4) \end{cases} \quad \begin{cases} H\_XY(\Delta t/2) \\ E\_XY(\Delta t) \leftarrow J_z \\ H\_XY(\Delta t/2) \end{cases} \quad \begin{cases} H\_Z(\Delta t/4) \\ E\_Z(\Delta t/2) \\ H\_Z(\Delta t/4) \end{cases}$$

NO=30 **Number of Operations** ***NO=24***

Dispersion Free in Longitudinal Direction

Stability

$$c\,\Delta t \le \min(\Delta_x, \Delta_y, \Delta_z)$$

LTs-2, TE/TM

$$\begin{cases} c\,\Delta t \le \left(\dfrac{1}{\Delta_x^2} + \dfrac{1}{\Delta_y^2}\right)^{-1/2} \\ c\,\Delta t \le \Delta_z \end{cases}$$

SIBC Time Domain Model

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SIBC in Frequency Domain

$$\vec{E}_\tau\,(\omega) = Z_s(\omega)\,[\vec{n}\times\vec{H}_\tau(\omega)]$$

Transformation to TD

SIBC in Time Domain

$$\vec{E}_\tau\,(t) = L\cdot\frac{d}{dt}[\,\vec{n}\times\vec{H}_\tau\,] + \sum_{i=0}^{Np}\vec{G}_i(t)$$

$$Z_s(\omega) \cong j\omega L + \alpha_0 + \sum_{i=1}^{Np}\frac{\alpha_i}{j\omega+\beta_i}$$

Rational Function Approximation (RFA)

Auxiliary Differential Equations (ADE)

$$\vec{G}_0 = \alpha_0\,[\,\vec{n}\times\vec{H}_\tau\,]$$
$$\frac{d}{dt}\vec{G}_i + \beta_i\vec{G}_i = \alpha_i\,[\,\vec{n}\times\vec{H}_\tau\,]$$

- B. Gustavsen, Improving the pole relocating properties of vector fitting, *IEEE Trans. on Power Delivery*, vol. 21, pp. 1587–1592, 2006

- J. Woyna, E. Gjonaj, and T. Weiland, Broadband surface impedance boundary conditions for higher order time domain discontinuous galerkin method, COMPEL, vol. 33, no. 4, pp. 1082–1096, 2014.

SIBC Time Domain Model

Surface Impedance of Good Conductors

$$Z_s(\omega) \cong \sqrt{\frac{j\omega\mu_0}{\sigma(\omega) + j\omega\varepsilon_0}}$$

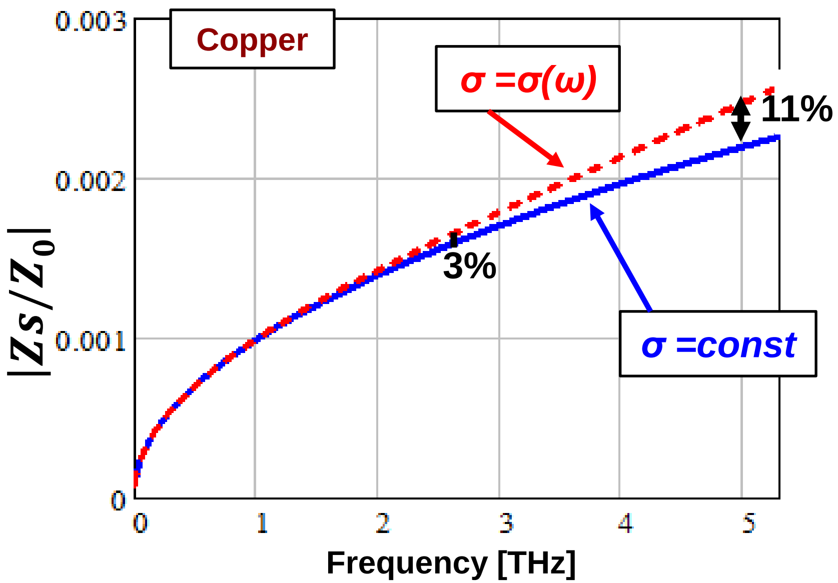
$$\sigma(\omega) \approx \frac{\sigma_0}{1 - j\omega\tau}$$

Metal Type	*Conductivity [MS/m]*	*Relaxation Time [fs]*
Cu	58	24.6
Al	36.6	7.1
SS 316	1.34	2.4
Ti-6Al-4V	0.5	1.04 (?)

Example : ***Short bunch***

$$\sigma_{Bunch} \approx 10\mu m$$

$$f \sim \frac{c}{\sigma_{Bunch}} \approx 5\,THz$$



SIBC Time Domain Model

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Boundary Effects

$$Z_s(\omega) = Z_s^{\sigma}(\omega) + Z_s^{L}(\omega)$$

- ***Finite Resistivity***

$$Z_s^{\sigma}(\omega) \approx \sqrt{\frac{j\omega\mu}{\sigma(\omega) + j\omega\varepsilon}}$$

$$\sigma(\omega) \approx \frac{\sigma_0}{1 - j\omega\tau}$$

- ***Surface Roughness***
- ***Metal Oxidation***

$$Z_s^{L}(\omega) \approx j\omega L$$

$$L \approx \mu_0 \left[\frac{\varepsilon_r - 1}{\varepsilon_r} \cdot \Delta_{oxide} + 0.01 \cdot \Delta_{rough} \right]$$

$$\boxed{\begin{array}{l} \varepsilon_r \sim 10 \\ \Delta_{oxide} \sim 7\,nm \end{array}}$$

$$\boxed{\Delta_{rough} \sim 500\,nm}$$

- M. Dohlus. TESLA report 2001-26, 2001
- K. Bane, G. Stupakov, SLAC-PUB-10707, 2004
- A. Tsakanian, M. Dohlus, I. Zagorodnov, TESLA-FEL-2009-05, 2009

SIBC Time Domain Model

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Accuracy of Vector-Fitting technique

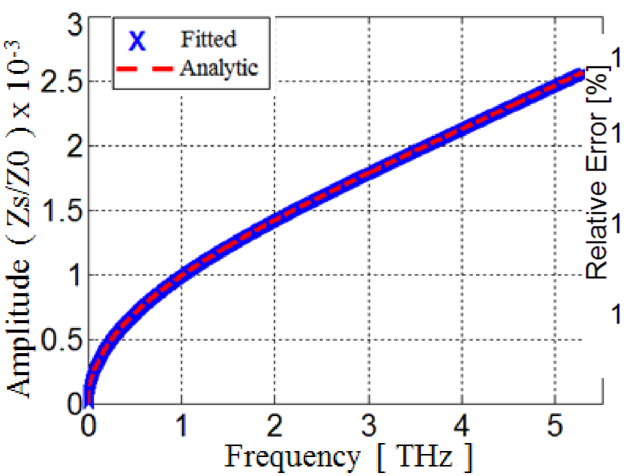
Good Conductors

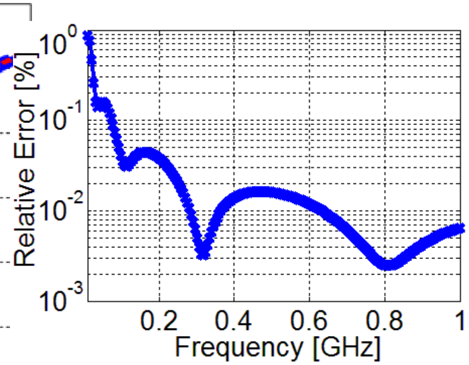
$$Z_s(\omega) \cong \sqrt{\frac{j\omega\mu}{\sigma(\omega) + j\omega\varepsilon}} \xrightarrow{\text{VF}} Z_s(\omega) \cong j\omega L + \alpha_0 + \sum_{i=1}^{Np} \frac{\alpha_i}{j\omega + \beta_i}$$

Pole-residue representation applies to any impedance function.

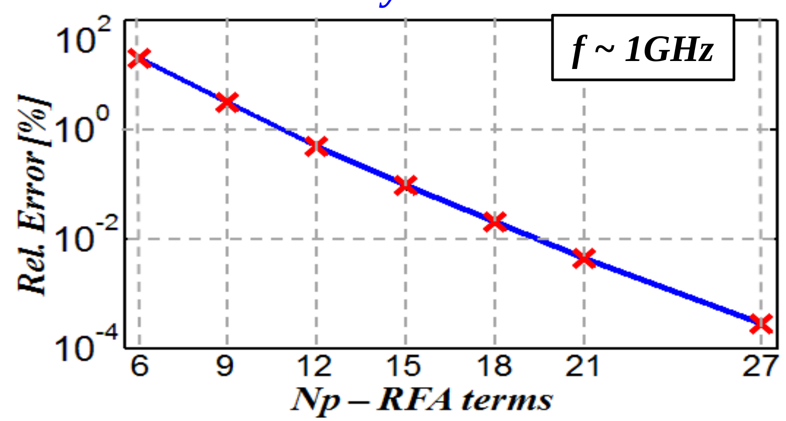
Example : Cu – Np=21,

Frequency range ~ 10MHz-5THz, Δf~5MHz





Sensitivity on RFA terms



TD-SIBC Implementation

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Faraday's Law with SIBC - TD

$$\left(M_{\mu} + L\cdot l_{c}\right)\frac{d}{d\,t}\hat{h} = -C\cdot\hat{e} - l_{c}\cdot\sum_{i=0}^{Np} G_{i}$$

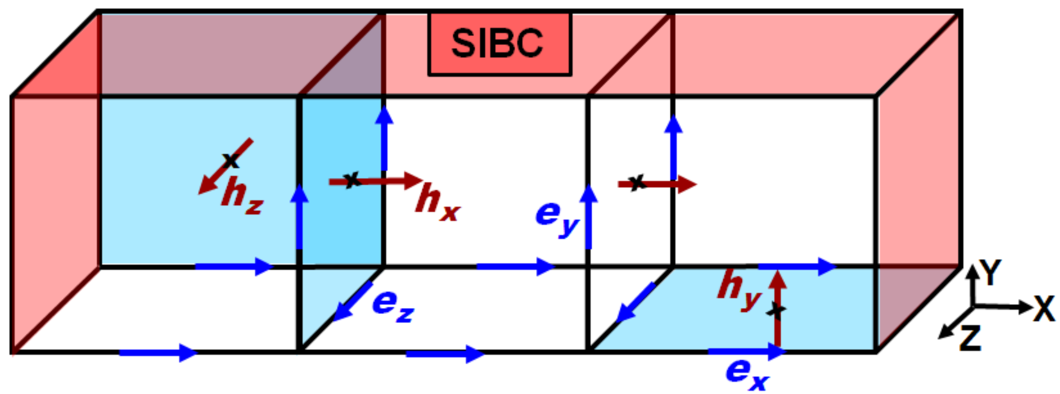
Auxiliary Differential Equations (ADE)

$$\vec{G}_0 = \alpha_0\,[\,\vec{n}\times\vec{H}_\tau\,]$$
$$\frac{d}{dt}\vec{G}_i + \beta_i\vec{G}_i = \alpha_i\,[\,\vec{n}\times\vec{H}_\tau\,]$$

Ampere's Law with PEC

$$M_{\varepsilon}\frac{\partial}{\partial t}\hat{e} = C^{T}\cdot\hat{h} + \hat{\hat{j}}_{s}$$

Boundary Cells with SIBC Surfaces



Semi-Discrete Maxwell's Equations with TD-SIBC

$$\frac{d}{dt}\begin{pmatrix}\hat{e}\\ \hat{h}\\ 0\\ G_1\\ \vdots\\ G_N\end{pmatrix} = \begin{pmatrix} 0 & M_\varepsilon^{-1}C^T & 0 & 0 & \cdots & 0\\ -M_\mu^{-1}C & 0 & C_B & C_B & \cdots & C_B\\ 0 & \alpha_0 & 1 & 0 & \cdots & 0\\ 0 & -\alpha_1 & 0 & \beta_1 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & -\alpha_N & 0 & 0 & \cdots & \beta_N\end{pmatrix}\begin{pmatrix}\hat{e}\\ \hat{h}\\ G_0\\ G_1\\ \vdots\\ G_N\end{pmatrix}$$

TD-SIBC Implementation

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MGE with TD-SIBC as ODE system

$$y = \begin{pmatrix} \hat{e} \\ \hat{h} \\ G_0 \\ G_1 \\ \vdots \\ G_N \end{pmatrix} \qquad \boxed{\frac{\partial}{\partial t} y = A \cdot y}$$

A_c $\quad$ A_{SIBC}

$$A = \begin{pmatrix} 0 & M_\varepsilon^{-1} C^T & 0 & 0 & \cdots & 0 \\ -M_\mu^{-1} C & 0 & C_B & C_B & \cdots & C_B \\ 0 & \alpha_0 & 1 & 0 & \cdots & 0 \\ 0 & -\alpha_1 & 0 & \beta_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & -\alpha_N & 0 & 0 & \cdots & \beta_N \end{pmatrix}$$

Strang Splitting - Second Order

$$\boxed{e^{A*\Delta t} = e^{A_{SIBC}*\Delta t/2} * e^{A_c*\Delta t} * e^{A_{SIBC}*\Delta t/2}}$$

- **TD-SIBC update [Δt/2]**
- **Add Loss to each h-comp.**

- **Main 3D Num. Scheme update (Δt)**

- **TD-SIBC update [Δt/2]**
- **Add Loss to each h-comp.**

Applicable for FIT based methods with different time integrations

For example: LT, TE/TM, LF

TD-SIBC Implementation

- TD-SIBC update [Δt/2]
- Add Loss to each h-comp.

- Main 3D Num. Scheme update (Δt)

- TD-SIBC update [Δt/2]
- Add Loss to each h-comp.

Applicable for FIT based methods with different time integrations

For example: LT, TE/TM, LF

$$G_i^n \ \mathit{update}\left(\Delta t / 4\right)$$

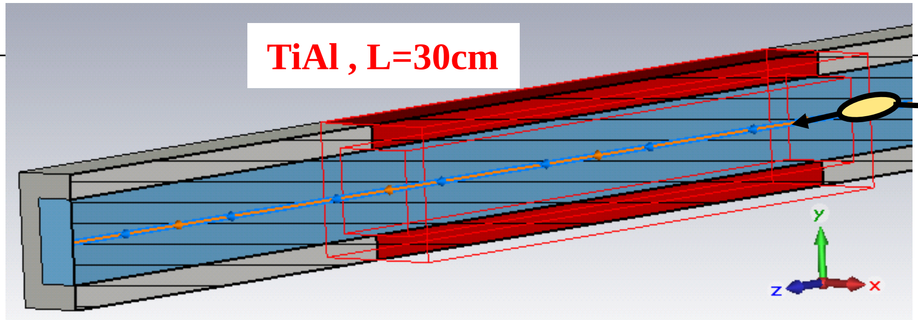
$$\hat{h} = \hat{h} - \frac{\Delta t}{2} M_{\mu}^{-1} l_c \cdot \sum_{i=0}^{Np} G_i$$

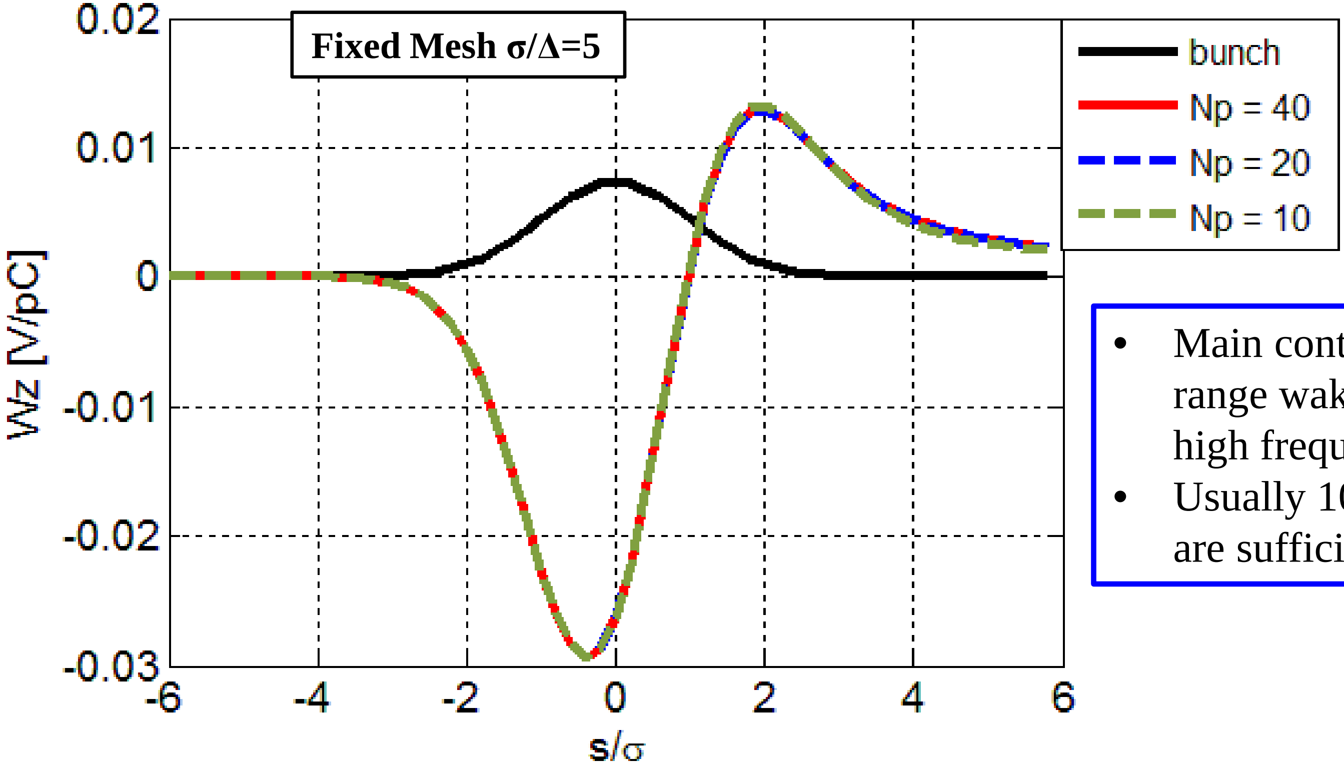
$$G_i^n \ \mathit{update}\left(\Delta t / 4\right)$$

$$\begin{cases} G_0^n = \alpha_0 \cdot h^{n-0.5} \\ G_i^n = G_i^{n-1} \cdot e^{-\beta_i \Delta t} + \dfrac{\alpha_i}{\beta_i}\left(1 - e^{-\beta_i \Delta t}\right) \cdot h^{n-0.5} \end{cases}$$

Exponential Time Integrator for SIBC-ADEs provides better accuracy, stability & independence on pole magnitude of surface impedance expansion.

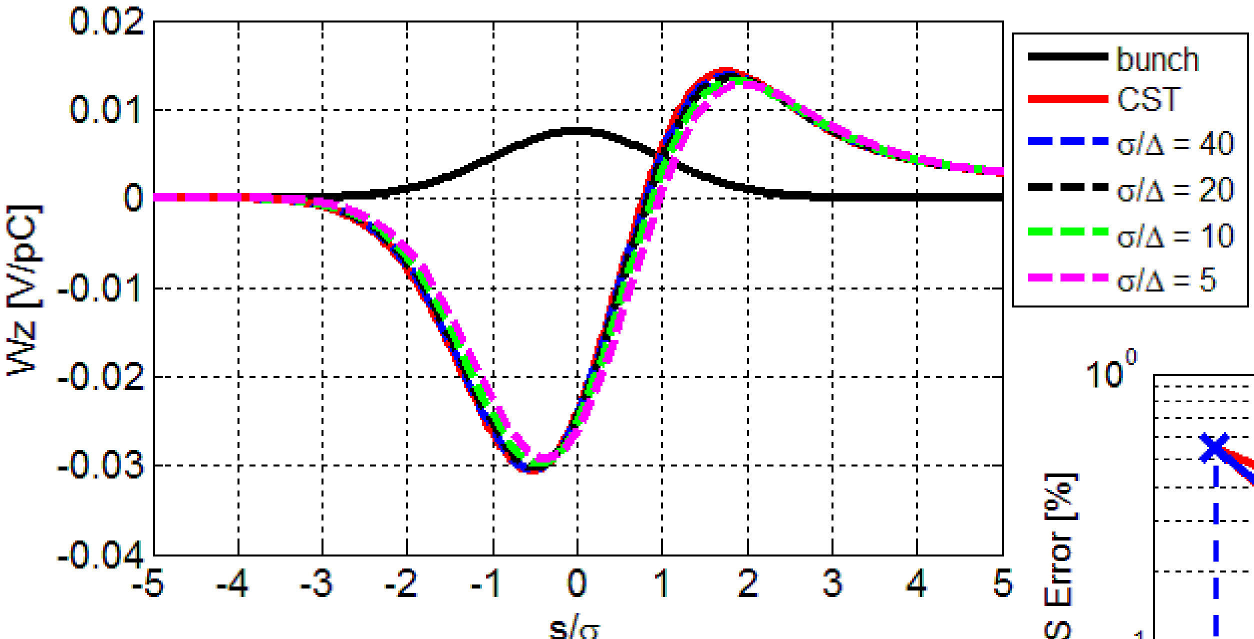
Resistive Wake Test on TD-SIBC order

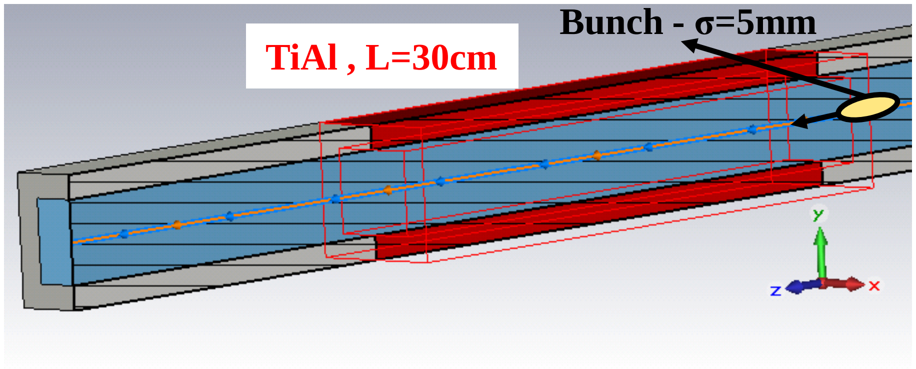


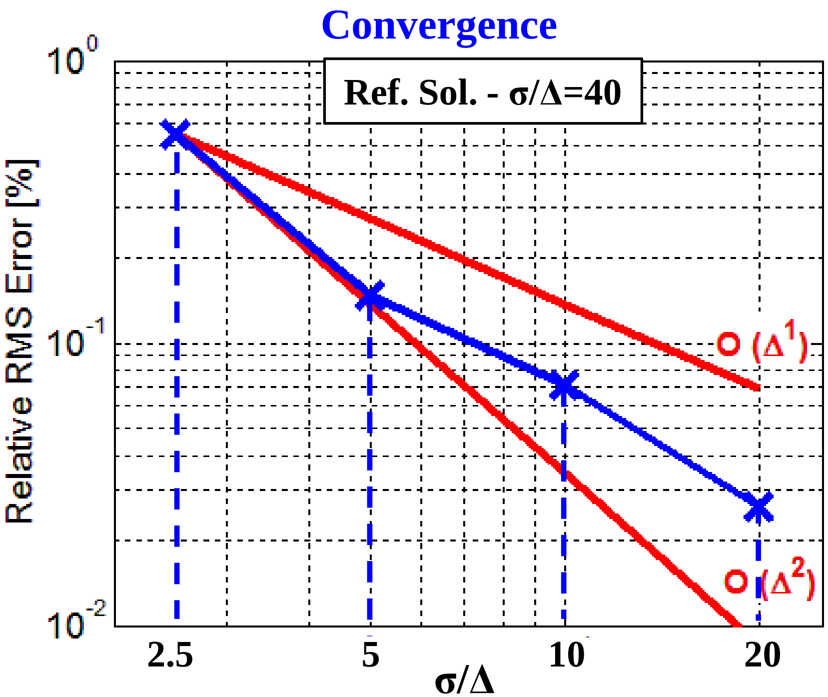


- Main contribution in short range wakes is from SIBC high frequency part.
- Usually 10-20 terms in VF are sufficient.

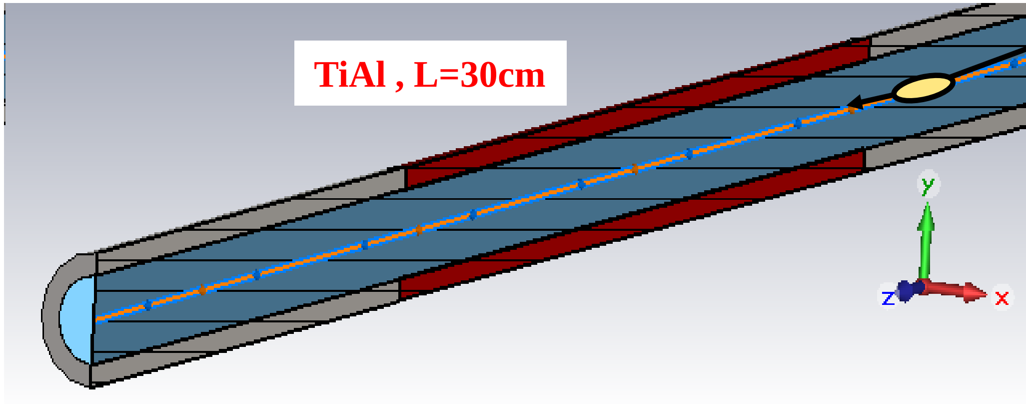
Comparison with CST

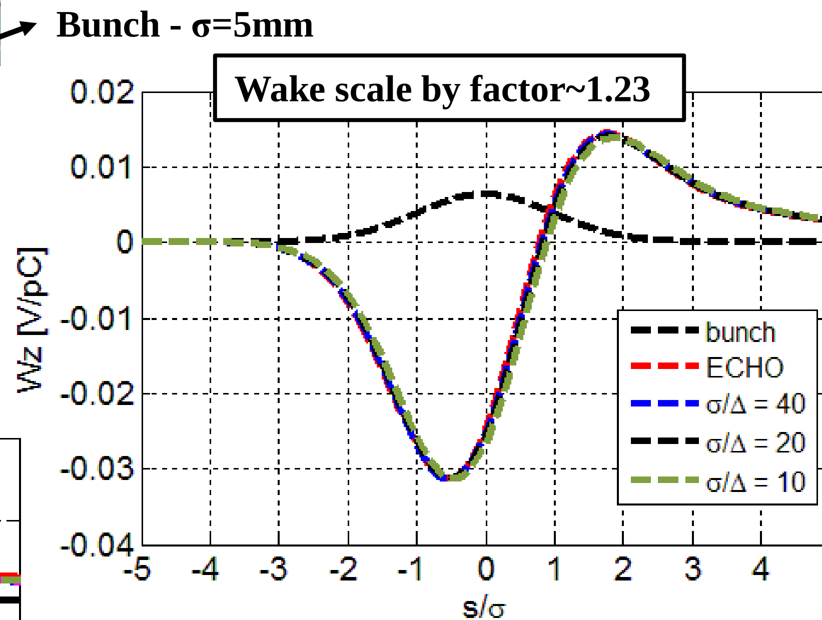


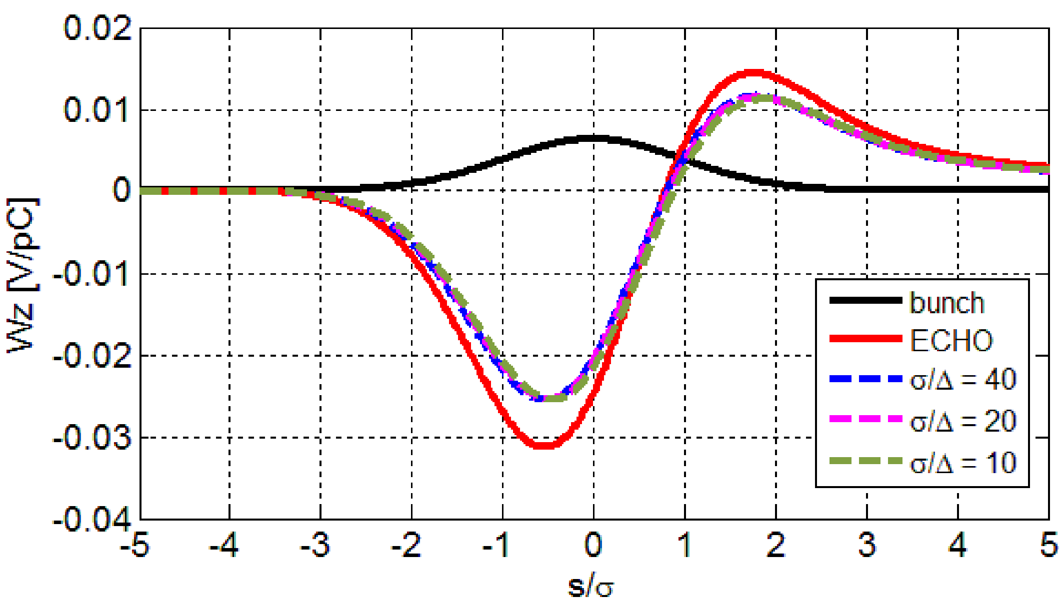




Comparison with ECHO







- Same behavior is observed for different beampipe radius and materials.
- Possible reason is staircase mesh, i.e. boundary cell areas with lossy edges are overestimated.

Comparison with ECHO

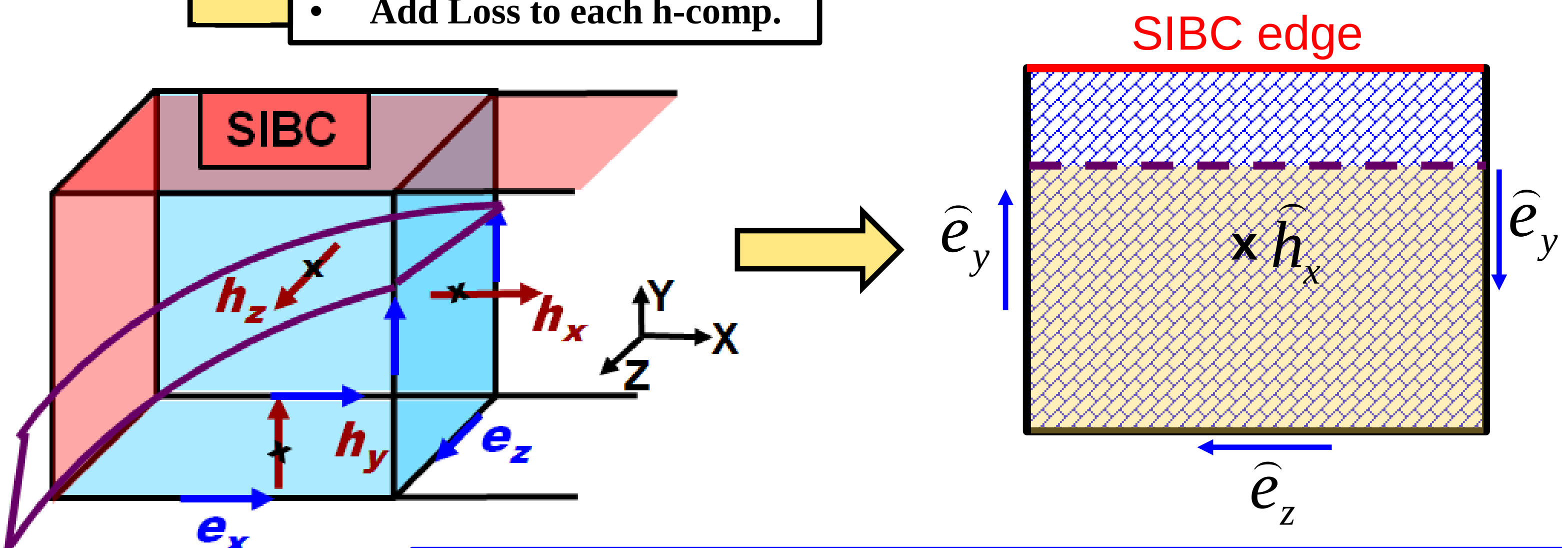
- SIBC–ADE update [Δt/2]
- Add Loss to each h-comp.

Main 3D Num. Scheme update (Δt)

- SIBC–ADE update [Δt/2]
- Add Loss to each h-comp.

$$\hat{h} = \hat{h} - \frac{\Delta t}{2} M_{\mu}^{-1} l_c \cdot \sum_{i=0}^{Np} G_i$$

$$M_{\mu}^{-1} \sim \frac{1}{Cell\_Area}$$



- ➢ Semi-conformal or conformal method implementation needed

Summary

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Achievements

- Time Domain SIBC model & RFA Accuracy
- Successful TD-SIBC Implementation in FIT Based 3D methods
- Stability & Convergence Analyses
- Verification for Rectangular Beampipe.
- Moving Window & Parallelized for PEC Boundaries.

Further Steps

- Application Semi-Conformal or Conformal Boundary Approximation
- Verification for Cylindrical Beampipe & Convergence Study.
- TD-SIBC Part Parallelization & Performance Optimization.
- Application to Realistic Structures with Complex Geometries.

Thank You for Your Attention!