Transverse Beam Profile after BC3 and in Undulator
(for Flash with 3rd harmonic rf)

BC3 $\rightarrow$ collimator

1. Observations for the 1nC case (see 30th Nov.)

2. Slice Analysis – “Methods”

3. Slice Emittance – “good and bad particles”

Undulator

4. SC Effects

5. Transverse Profile (without SC Effects)

both

6. Summary
6. Summary

**BC3 → collimator**

- rms beam properties underestimate real particle density
- slice emittance is better than expected
- therefore: SC effects are stronger
- to be done: optics with SC effects

**Undulator**

- rms beam properties underestimate real particle density
- SC effects at 1GeV nearly negligible
- peak current density larger than for Gaussian beam
- pseudo energy spread (from emittance) larger than real energy spread
- pseudo energy spread than for Gaussian beam
1. Observations for the 1nC case
(see meeting 30th Nov.)

Transverse Dynamics

![Diagram showing transverse dynamics with various beta functions and current levels.]
Transverse Dynamics
slice emittance

BC3 → collimator

undulator-start
\( \gamma \varepsilon_{x/y}^{(0.8)} = 0.5 \ldots 1.5 \mu m \)

design optic (2+)
2. Slice Analysis – “Methods”

- BC3 → collimator
- exit BC3
- entrance collimator

![Graphs showing horizontal and vertical beta profiles](image)

- design optic (2+)
- slice model

Selected slice

- Astra, full bunch
- centroids extracted!

![Graph showing centroids extracted](image)
slice model
(see meeting 28th Sept.)

run Astra with 7 particles without self effects (map many steps)

\[ X_i^{(a)} = \left( \begin{array}{c} x_i^{(a)} - x_0^{(a)} \\ x_i^{(a)} - x_0^{(a)} \\ \vdots \end{array} \right), \quad i = 1, 2, \ldots, 6 \]

calculate linear transport matrices

\[ T^{(b\leftarrow a)} = (X_1^{(a)} X_2^{(a)} \cdots X_6^{(a)})^{-1} (X_1^{(b)} X_2^{(b)} \cdots X_6^{(b)}) \]

select slice particles from initial distribution \( X_p \)

\[ \{X_p^{(\text{start})}\} = \text{slice} \{X_p^{(\text{start})}\} \]

track from a to b with self effect

\[ \{X_p^{(a)}\} = \text{slice} \left\{ 0.5\delta X_p^{(a)} + T^{(b\leftarrow a)} \left[ 0.5\delta X_p^{(a)} + X_p^{(a)} \right] \right\} \]

transverse self forces at “a” and “b”
slice model comparison with Astra

design optic (2+)

slice model (rz)

selected slice

Astra (rz), slice
slice model
comparison rz, xyz

f.i. exit BC3

BC3 → collimator

\[ q(r) \]
\[ E_r(r) \]
\[ E_x(x,0) \]
\[ E_y(0,y) \]
comparison slice model:
rz ↔ xyz ↔ design

red = rz (50 lines in r)

blue = xyz (25 lines in x,y)

black = design

→ rz approach and slice model are roughly ok
comparison slice model:
rz ↔ xyz ↔ design
red = rz (50 lines in r)
blue = xyz (40 lines in x,y)
black = design
3. Slice Emittance

Transverse Dynamics

slice emittance

slice model
“slice” = 28um .. 32um

design optic (2+)

solid = rz model
dashed = xyz model
cross coupling?  

“slice” = 28um .. 32um

xx – correlation

yy – correlation

xy – correlation

spatial correlation

momentum correlation

spatial-momentum corr.

no xy correlation!
“good” and “bad” particles

“slice” = 28um .. 32um

xy space (+- 1mm)

movie 1

xy space (+- 1mm)

movie 2

xx’ and yy’ space

movie 3
slice model (xyz)  

“slice” = 28um .. 32um

BC3 → collimator

4087 “red” particles

1012 “blue” particles
slice model (xyz)  

“slice” = 28um .. 32um

[Diagram showing particle beam path through BC3 and collimator with graphs of energy, beta functions, and slice models]
4. SC Effects in Undulator

**no match at all!**

**slice** = -2um .. +2um

**strong effect due to initial mismatch compared to that:**
**weak effect due to space charge**
match = -10um .. -6um
slice    = -10um .. -6um

perfect initial match:
$\Delta \beta$ at end < 0.3m
weak difference between r and xy model
match = -10um .. -6um
slice = -10um .. -6um

perfect initial match:
$\Delta \beta$ at end < 0.1m
difference between r and xy model very small

= gaussian distribution with same rms properties as initial distribution (slice)
match = -20um .. +7um
slice    =   -2um .. +2um

initial mismatch: $\Delta \beta(z_0) \approx 0.6m$
$\Delta \beta$ along undulator $\approx 0.6m$
weak difference between r and xy model
match = -20um .. +7um
slice = -2um .. +2um

initial mismatch: $\Delta \beta(z_0) \approx 0.6m$
$\Delta \beta$ along undulator $\approx 0.6m$
very weak difference between r and xy model
5. Transverse Profile (without SC Effects)

for matching:

bunch = 1 .. 200000
core = 68510 .. 111196
slice = 95000 .. 105000
Gaussian replica, slice match

\( n_a = 95000 \quad n_b = 105000 \quad N = 200000 \)

slice:

- beta functions (real & design)
- inverse size of rms ellipse
- particle density in rms ellipses
- particles in rms ellipse
slice:

- design $x,y$
- real $x,y$

---

bunch match

- na = 95000
- nb = 105000
- n1 = 68510
- n2 = 111196
- N = 200000

---

- beta functions (real & design)
- inverse size of rms ellipse
- particle density in rms ellipses
- particles in rms ellipse
transverse profile – core match – averaged along undulator

tracked particles

Gauss replica

red: \( \text{den}(x,0) \) blue: \( \text{den}(0,y) \) black: gaussian, \( \text{den}(i) \)
tracked particles

Gauss replica

transverse profile – slice match – averaged along undulator

red: \text{den}(x,0) \hspace{0.5cm} \text{blue: den}(0,y) \hspace{0.5cm} \text{black: gaussian, den}(r)
effective energy spread

\[ \lambda_{ph} = \frac{\lambda_u}{(\gamma_0 + \delta\gamma)^2} \left( 1 + \frac{K^2}{2} \right) + \frac{\lambda_u}{2} \left( x'^2 + y'^2 \right) \]

\[ \left( \frac{\delta\gamma}{\gamma_0} \right)_{eff} = \frac{\delta\gamma}{\gamma_0} - \frac{\lambda_u}{4\lambda_{ph}} \left( x'^2 + y'^2 \right) \]

pseudo spread: \[ \text{rms}\left\{ \left( \frac{\delta\gamma_{pseu}}{\gamma_0} \right) \right\} = \frac{\lambda_u}{4\lambda_{ph}} \text{rms}\{ x'^2 + y'^2 \} \]

Gaussian

\[ \text{rms}\left\{ \left( \frac{\delta\gamma}{\gamma_0} \right)_{eff} \right\} = \sqrt{\text{rms}\left\{ \left( \frac{\delta\gamma}{\gamma_0} \right) \right\}^2 + \left( \frac{\lambda_u}{4\lambda_{ph}} \left( \varepsilon_x \gamma_x + \varepsilon_y \gamma_y \right) \right)^2} \approx \sqrt{\text{rms}\left\{ \left( \frac{\delta\gamma}{\gamma_0} \right) \right\}^2 + \left( \frac{\lambda_u}{2\lambda_{ph}} \frac{\varepsilon}{\text{min}\{\beta\}} \right)^2} \]
bunch match

\[ \text{rms} \{ \delta E_{\text{pseu}} \} = 984 \text{ keV} \]

core match

\[ \text{rms} \{ \delta E_{\text{pseu}} \} = 799 \text{ keV} \]

\[ 1020 | 578 \text{ keV} \]

slice match

Gaussian bunch

\[ \text{rms} \{ \delta E_{\text{pseu}} \} = 459 \text{ keV} \]

\[ E_0 \frac{\lambda_{\text{in}}}{2\lambda_{\text{ph}}} \frac{\varepsilon}{\min \{ \beta \}} = 437 \text{ keV} \]

\[ \varepsilon_n = 1.1 \mu \text{m} \]

\[ \min \{ \beta \} = 2.7 \text{ m} \]
6. Summary

**BC3 → collimator**

rms beam properties underestimate real particle density
slice emittance is better than expected
therefore: SC effects are stronger
to be done: optics with SC effects

**Undulator**

rms beam properties underestimate real particle density
SC effects at 1GeV nearly negligible
**peak current density larger** than for Gaussian beam
pseudo energy spread (from emittance) larger than real energy spread
**pseudo energy spread** than for Gaussian beam