Gauged supergravity and U-duality

Hamburg, 02/2007

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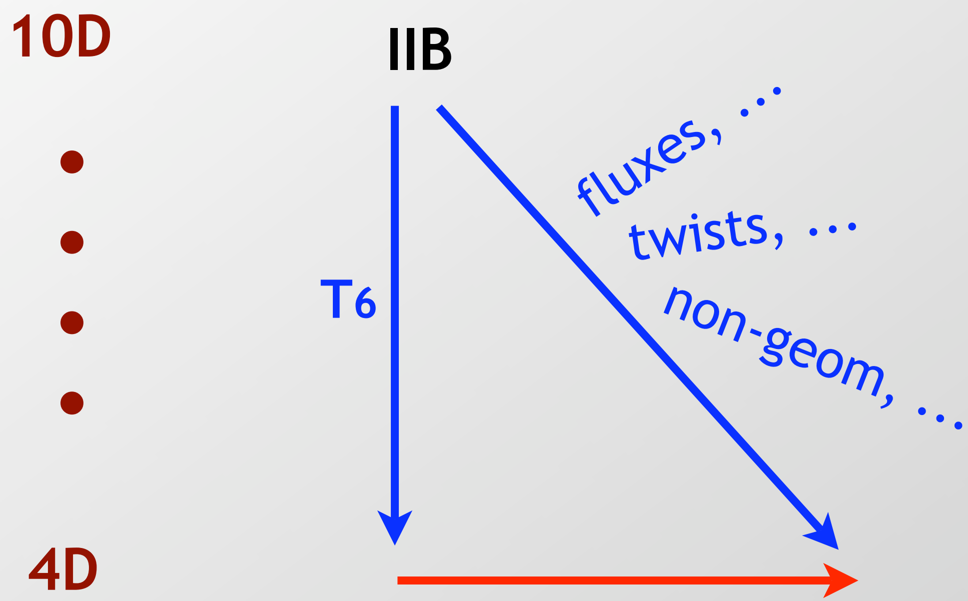
plan

- gauged supergravity and flux compactifications
- duality covariant formulation
- 4D: electric magnetic duality
- 2D: integrability and affine duality groups

H.S., Martin Weidner: - to appear

Bernard de Wit, H.S., Mario Trigiante: - hep-th/0507289
- to appear very soon

gauged supergravity and flux compactifications



gauged supergravity description

vector fields

$$D_\mu = \partial_\mu - A_\mu{}^M \Theta_M{}^\alpha t_\alpha$$

generators

embedding tensor **Θ = (H_{MNK}, ...)**

gauged supergravity

- **general**

  action completely determined in terms of Θ
  in particular scalar potential $V(\Phi, \Theta)$

- **consistency**

  encoded in a set of algebraic constraints on Θ
  linear & quadratic $$f_{\alpha\beta}{}^{\gamma}\,\Theta_M{}^{\alpha}\,\Theta_N{}^{\beta} + (t_\alpha)_N{}^P\,\Theta_M{}^{\alpha}\Theta_P{}^{\gamma} \;=\; 0$$

- **duality**

  gaugings employ rank p antisymmetric tensors together with their dual rank (D-p-2) tensors

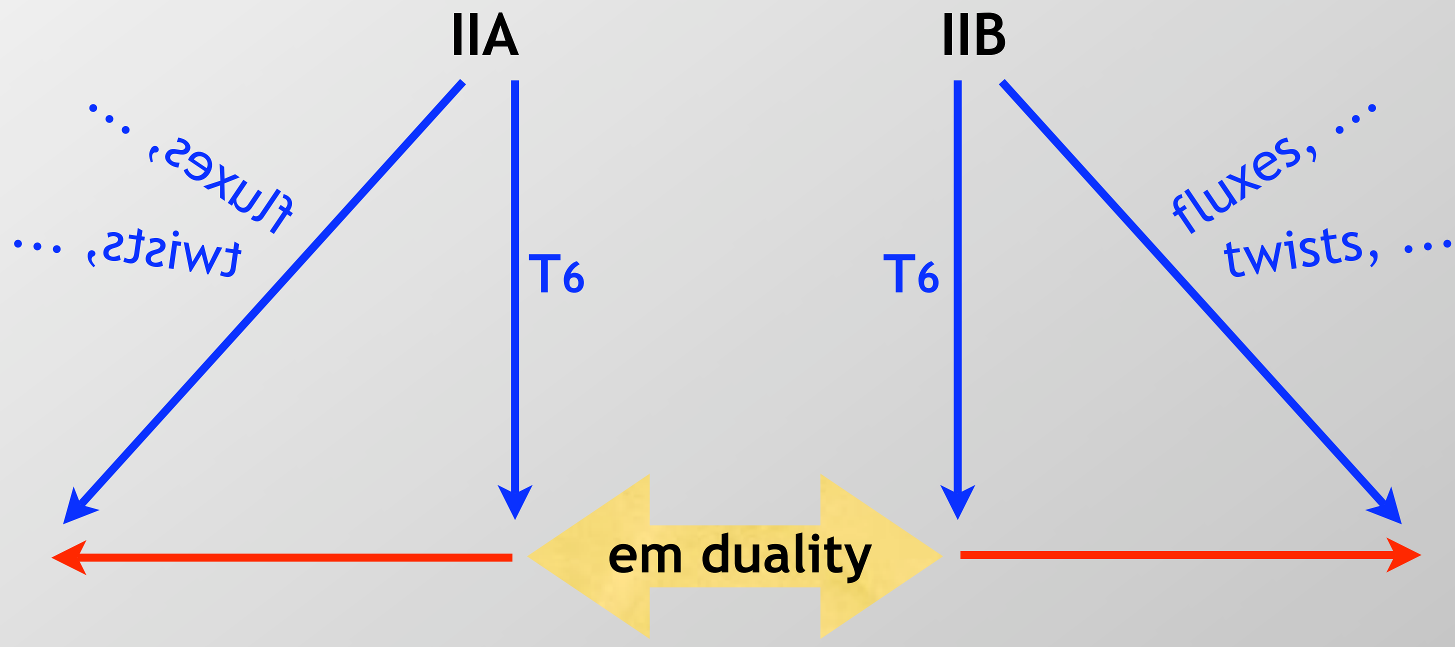
  $$dA \;=\; *\,dB + \cdots$$ **on-shell**

D=4: scalars $\rightleftarrows$ two-forms
vectors $\rightleftarrows$ vectors **em - duality**

D=2: scalars $\rightleftarrows$ scalars **integrability**

duality covariant formulation

dualities



different field content (vectors)
different off-shell symmetries

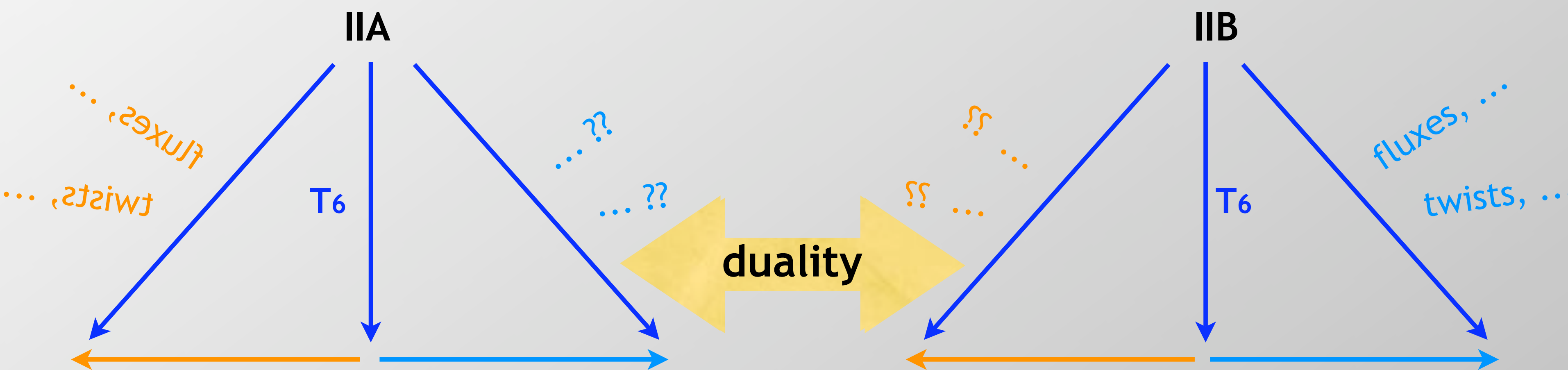
$$D_\mu \;=\; \partial_\mu - A_\mu{}^M \Theta_M{}^\alpha \, t_\alpha \qquad\qquad D_\mu \;=\; \partial_\mu - B_\mu{}^K \Theta_K{}^\beta \, t_\beta$$

$$dA \;=\; * \, dB \;+\; \cdots$$

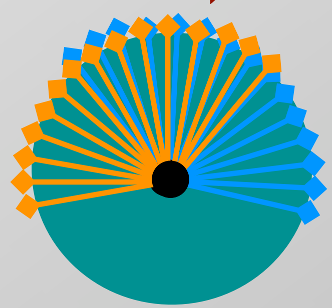
a priori different deformations

duality covariant formulation

- **standard construction depends on symplectic frame**



- **universal (duality covariant) formulation**



electric gauging ("standard")

$$D_\mu = \partial_\mu - A_\mu{}^M \Theta_M{}^\alpha t_\alpha = \partial_\mu - A_\mu{}^\Lambda \Theta_\Lambda{}^\alpha t_\alpha - A_{\mu\,\Lambda} \Theta^{\Lambda\,\alpha} t_\alpha$$

magnetic gauging ("non-standard")

4D: electric magnetic duality

covariant formulation:

electric gauging ("standard")

$$D_\mu = \partial_\mu - A_\mu{}^M \Theta_M{}^\alpha t_\alpha = \partial_\mu - A_\mu{}^\Lambda \Theta_\Lambda{}^\alpha t_\alpha - A_{\mu\,\Lambda} \Theta^{\Lambda\,\alpha} t_\alpha$$

magnetic gauging ("non-standard")

➜ consistency requires further coupling to 2-form tensors!

- modified field strengths

$$\mathcal{H}_{\mu\nu}{}^M \;=\; 2\partial_{[\mu} A_{\nu]}{}^M + X_{[NP]}{}^M \, A_\mu{}^N A_\nu{}^P + \tfrac{1}{2}\,\Theta^{M,\alpha}\, B_{\mu\nu\,\alpha}$$

- and additional topological term

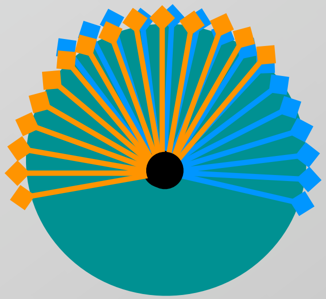
$$\mathcal{L}_{\rm top} \;=\; -\tfrac{1}{8}\Theta^{\Lambda\alpha}\, B_\alpha \wedge \Big(2\,\partial A_\Lambda + X_{MN\Lambda}\, A^M \wedge A^N - \tfrac{1}{4}\Theta_\Lambda{}^\beta B_\beta\Big) \;+\; \cdots$$

in order to yield a gauge invariant Lagrangian.

4D: duality covariant formulation

- Universal (duality covariant) gauged Lagrangian
  - in particular scalar potential

$$V(\Phi, \Theta)$$



- Various ways of gauge fixing
  - e.g. integrate out 2-forms $\rightarrow$ symplectic rotation
  - e.g. integrate out scalars $\rightarrow$ massive 2-forms
  - “non-standard” gauged supergravities

examples: N=8 supergravity

Invariance group G = E7 – embedding tensor

$$D_\mu = \partial_\mu - A_\mu{}^M \Theta_M{}^\alpha t_\alpha$$

➜ linear constraint Θ : $56 \times 133 = \cancel{56} + \boxed{912} + \cancel{6480}$

28 el. + 28 magn. vector fields ↗ ↖ generators

➜ quadratic constraint

$\Theta\,\Theta$: $912 \times_s 912 = \cancel{133} + \boxed{1463} + \cancel{8645} + \boxed{152152} + \boxed{253935}$

flux gaugings can be constructed and analyzed
using representation theory and branchings of **E7**

carry (up to) **56** vectors, **133** 2-forms, **912** 3-forms

examples: N=8 supergravity

➜ gaugings from M-theory fluxes and torsion: SL(7) ⊂ E7

embedding tensor **912** ⋯➝ 1 + 35’ + 7 + 140 + ...

912

1 ⟶	$\langle \tilde{F}_{klmnpqr} \rangle$	7-form flux	[Aurilia, Nicolai, Townsend, 1980]
35’ ⟶	$\langle F_{klmn} \rangle$	4-form flux	[Dall’Agata, D’Auria, Ferrara, Trigiante, 2005]
7+140 ⟶	$\langle T_{mn}{}^{q} \rangle$	twist	

⋮

▹ grading: torus size

▹ quadratic constraint

▹ couplings
vector fields & generators

$$D_\mu = \partial_\mu - A_\mu{}^M \Theta_M{}^\alpha t_\alpha$$

	7	21’	21	7’
7’	1	35’	7+140	
35	35’	140		
1	7			
48	7+140			

examples: N=8 supergravity

➜ gaugings from IIB fluxes: SL(6) x SL(2) ⊂ E7

embedding tensor **912** $\dashrightarrow$ (6,1) + (20,2) + ...

<u>**912**</u>

(6,1) $\longrightarrow$ $\langle F_{klmpq} \rangle$ 5-form flux

(20,2) $\longrightarrow$ $\langle H^{\alpha}_{klm} \rangle$ 3-form flux

(84,1) $\longrightarrow$ ${f_{kl}}^{m}$ twist

⋮

duality chains $\underset{\mathbf{20}}{H_{abc}} \longrightarrow \underset{\mathbf{84}}{{f_{ab}}^{c}} \longrightarrow \underset{\mathbf{84'}}{{Q_{a}}^{bc}} \longrightarrow \underset{\mathbf{20'}}{R^{abc}}$

couplings
vector fields & generators

	(6',1)	(6,2)	(20,1)	(6',2)	(6,1)
(1,2)		(6,1)	(20,2)		
(15,1)	(6,1)	(20,2)			
(15',2)	(20,2)				
(1,1)					

duality covariant formulation

4D

- duality group E_7
- 912–dimensional orbit of gaugings
- contains fluxes, twists, non-geometric compactifications

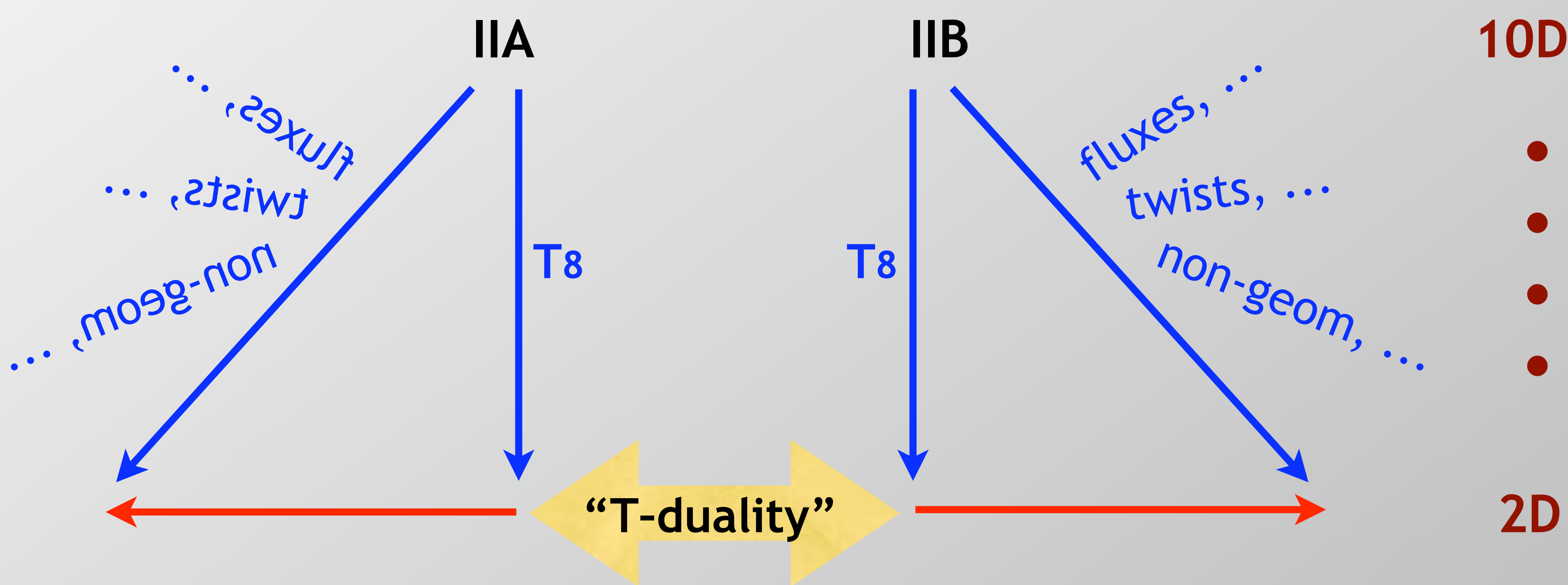
descending in dimension — increasing the duality group

2D

- infinite dimensional duality group E_9
- integrability
- infinite dimensional parameter space of deformations
- contains ....

2D: duality covariant formulation

dualities



different field content (scalars)
different off-shell symmetries

$$D_\mu \;=\; \partial_\mu - A_\mu{}^M \Theta_M{}^\alpha\, t_\alpha \qquad\qquad D_\mu \;=\; \partial_\mu - B_\mu{}^K \Theta_K{}^\beta\, t_\beta$$

no vector fields $\rightarrow$ only "non-standard" gaugings

gauging of non-target-space isometries!

2D: supergravity and integrability

- **Lagrangian** $\mathcal{L} \;=\; -\tfrac{1}{4}\sqrt{-g}\,\rho\Big(-R + \mathrm{tr}\,[P^\mu P_\mu]\Big) \;+\; \mathcal{L}_{\mathrm{ferm}}$

  coset space sigma model coupled to dilaton gravity

- **field equations**

$$g_{\mu\nu} \;=\; e^{2\sigma}\,\eta_{\mu\nu} \qquad\qquad \Box\rho \;=\; 0 \qquad\qquad \partial^\mu\Big(\rho\mathcal{V}P_\mu\mathcal{V}^{-1}\Big) \;=\; 0$$

metric dilaton scalars

$$\mathcal{V} \in \mathbf{E}8\,/\,\mathbf{SO}(16) \qquad\qquad \mathcal{V}^{-1}\partial_\mu\mathcal{V} \;=\; Q_\mu + P_\mu$$

so(16)

- **duality**

  $\partial_\mu Y_1 \;=\; \epsilon_{\mu\nu}\,\rho\mathcal{V}P^\nu\mathcal{V}^{-1}$ defines a set of dual scalars

  continues to an infinite tower ➜ **integrability**

2D: supergravity and integrability

linear system

the equations of motion can be encoded as integrability conditions of a *linear system* [Belinskii,Zakharov / Maison] (light-cone-coord. $x^{\pm}$)

$$\hat{\mathcal{V}}^{-1}\partial_{\pm}\hat{\mathcal{V}} \;=\; Q_{\pm} + \frac{1\mp\gamma}{1\pm\gamma}\,P_{\pm}$$

for a function $\hat{\mathcal{V}}(\gamma)$ and the *spectral parameter*

$$\gamma = \frac{1}{\rho}\left(w+\tilde{\rho}-\sqrt{(w+\tilde{\rho})^2-\rho^2}\right) \qquad\qquad \partial_{\pm}\rho = \pm\partial_{\pm}\tilde{\rho}$$

expansion in w gives rise to the infinite series of dual scalars

$$\hat{\mathcal{V}} \;=\; \ldots e^{Y_3 w^{-3}} e^{Y_2 w^{-2}} e^{Y_1 w^{-1}}\mathcal{V}$$

$$\begin{aligned}
\partial_{\pm}Y_1 &= \pm\rho\mathcal{V}P_{\pm}\mathcal{V}^{-1}\\
\partial_{\pm}Y_2 &= -(\pm\rho\tilde{\rho}+\tfrac{1}{2}\rho^2)\mathcal{V}P_{\pm}\mathcal{V}^{-1}+\tfrac{1}{2}[Y_1,\partial_{\pm}Y_1]\\
\partial_{\pm}Y_3 &= \ldots
\end{aligned}$$

conserved charges, integrability, ...

2D: supergravity and integrability

affine symmetry algebra E9

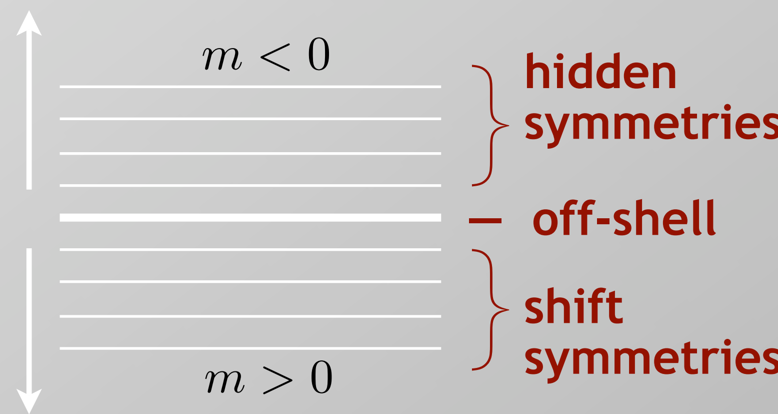
action parametrized by a meromorphic function $\Lambda(w)$

$$\delta\sigma = \kappa - \mathrm{tr}\left\langle \Lambda(w)\,\partial_w\hat{\mathcal{V}}(w)\hat{\mathcal{V}}^{-1}(w)\right\rangle_w \qquad \hat{\mathcal{V}}^{-1}\Lambda(w)\hat{\mathcal{V}} \;=\; \tilde{\Lambda}_{\mathfrak{h}} + \tilde{\Lambda}_{\mathfrak{k}}$$

$$\delta\rho = 0 \qquad \langle f(w)\rangle_w \;\equiv\; \oint \frac{dw}{2\pi i}\,f(w)$$

$$\mathcal{V}^{-1}\,\delta\mathcal{V} = \left\langle \frac{2\gamma(w)}{\rho\,(1-\gamma(w)^2)}\,\tilde{\Lambda}_{\mathfrak{k}}(w)\right\rangle_w$$

algebra spanned by $\{t^\alpha_m,\, L_1,\, k\}$



extends to the set of dual scalars

$$\delta\tilde{\rho} = \lambda$$

$$\hat{\mathcal{V}}^{-1}\delta\hat{\mathcal{V}}(w) = \lambda\,\hat{\mathcal{V}}^{-1}\partial_w\hat{\mathcal{V}}(w) + \tilde{\Lambda}(w) - \left\langle \frac{1}{v-w}\left(\tilde{\Lambda}_{\mathfrak{h}}(v) + \frac{\gamma(v)\,(1-\gamma^2(w))}{\gamma(w)\,(1-\gamma^2(v))}\tilde{\Lambda}_{\mathfrak{k}}(v)\right)\right\rangle_v$$

→ gauge part of this nonlinear, nonlocal, on-shell symmetry

gauging D=2 maximal supergravity

Lagrangian with vector fields

$$\mathcal{L} \;=\; \partial^\mu \rho \, D_\mu \sigma - \tfrac{1}{2} \rho \, \mathrm{tr}(\mathcal{P}_\mu \mathcal{P}^\mu) \;+\; \mathcal{L}_{\rm top}$$

with covariantized derivatives and a “topological” term

$$\begin{aligned} \mathcal{L}_{\rm top} \;=\; & + \epsilon^{\mu\nu} \mathrm{tr} \Big\langle A_\mu(w) \, (\partial_\nu \hat{\mathcal{V}} - \hat{\mathcal{V}} Q_\nu) \, \hat{\mathcal{V}}^{-1} - \frac{1+\gamma^2}{1-\gamma^2} \, A_\mu(w) \hat{\mathcal{V}} P_\nu \hat{\mathcal{V}}^{-1} \Big\rangle_w \; + \epsilon^{\mu\nu} \Big(C_\mu - \mathrm{tr} \Big\langle A_\mu(w) \, \partial_w \hat{\mathcal{V}}(w) \hat{\mathcal{V}}^{-1}(w) \Big\rangle_w \Big) \partial_\nu \tilde{\rho} \\ & - \tfrac{1}{2} \epsilon^{\mu\nu} C_\mu B_\nu + \tfrac{1}{2} \, \epsilon^{\mu\nu} \mathrm{tr} \Big\langle \Big\langle \frac{1}{v-w} [\tilde{A}_\mu(w)]_{\mathfrak{h}} [\tilde{A}_\nu(v)]_{\mathfrak{h}} \; + \frac{(\gamma(v)-\gamma(w))^2 + (1-\gamma(v)\gamma(w))^2}{(v-w)(1-\gamma(v))^2(1-\gamma(w))^2} [\tilde{A}_\mu(w)]_{\mathfrak{k}} [\tilde{A}_\nu(v)]_{\mathfrak{k}} \Big\rangle_v \Big\rangle_w \end{aligned}$$

the Lagrangian carries dual scalars and vector fields (topological) such that variation w.r.t. the vector fields yields the duality equations

$$\delta \mathcal{L} \;=\; \delta C^{\pm} \, (\partial_\pm \rho \mp \partial_\pm \tilde{\rho}) + \mathrm{tr} \Big\langle \delta A^{\pm} \, (\partial_\pm \hat{\mathcal{V}} \hat{\mathcal{V}}^{-1} - \hat{\mathcal{V}} (Q_\pm + \frac{1 \mp \gamma}{1 \pm \gamma} \, P_\pm) \hat{\mathcal{V}}^{-1}) \Big\rangle_w$$

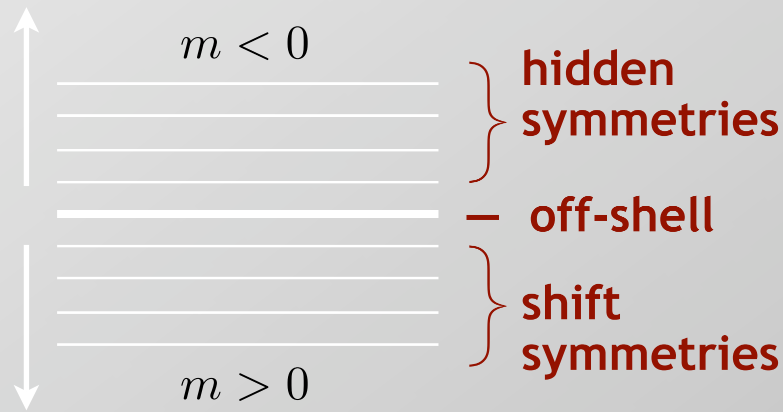
and part of the former on-shell symmetry (non-isometry!) is gauged

➜ fermionic mass terms and scalar potential

gauging D=2 maximal supergravity

group theory

affine global symmetry $\{t^\alpha_m,\, L_1,\, k\}$



- **vector fields** (nonpropagating in D=2)
  restore by embedding known examples: basic representation of **E9**

$$\chi_{\omega 0} \;=\; 1 + 248q + 4124q^2 + 34752q^3 + 213126q^4 + 1057504q^5 + 4530744q^6 + \ldots$$

McKay-Thompson series of class 3C for the monster

- **embedding tensor** linear constraint: *basic* x *adjoint* = *basic* + ...
  transforms in the **dual** representation

$$D_\mu \;=\; \partial_\mu - A_\mu^{\mathcal{M}}\Theta_{\mathcal{M}}{}^{\mathcal{A}}\,t_{\mathcal{A}} \;=\; A_\mu^{\mathcal{M}}\Theta_{\mathcal{N}}\,\eta^{\mathcal{AB}}\,t_{\mathcal{A}\,\mathcal{M}}{}^{\mathcal{N}}\,t_{\mathcal{B}}$$

➜ infinite-dimensional parameter space of deformations!

gauged D=2 maximal supergravity

embedding tensor — basic representation

branching under E8 identifies gaugings of 3d origin

$\chi_{\omega 0}$	
1	▷ **flux of 3d Kaluza-Klein vector field**
248	▷ **Scherk-Schwarz reductions from 3d**
4124	▷ **torus reduction of 3d gaugings**
34752	▷ ...?
213126	▷ ...??
1057504	▷ ...???
$\vdots$	

▷ grading: torus size

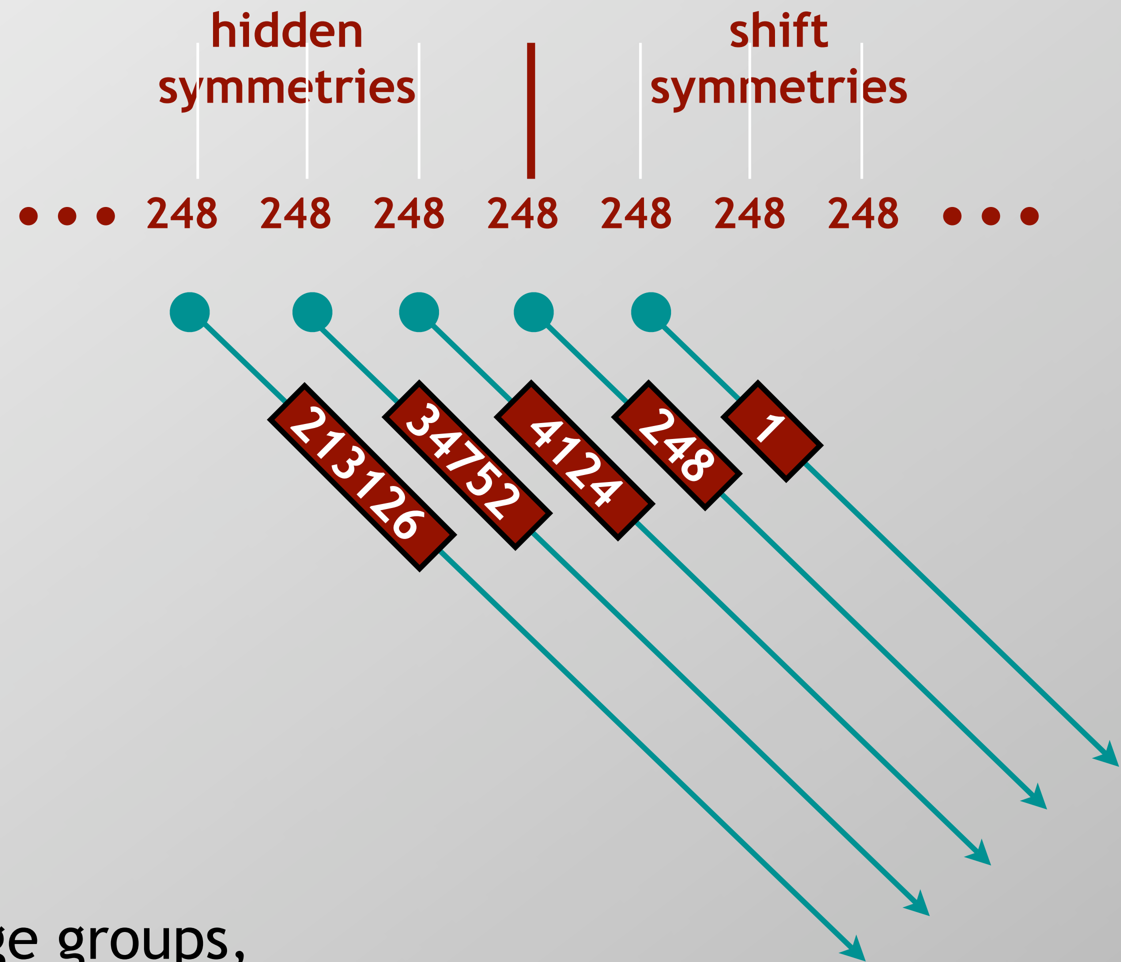
▷ quadratic constraint

▷ couplings
vector fields & generators ...

gauged D=2 maximal supergravity

couplings

$$D_\mu \;=\; \partial_\mu - A_\mu^{\mathcal{M}}\,\Theta_{\mathcal{M}}{}^{\mathcal{A}}\,t_{\mathcal{A}}$$



- infinite dimensional gauge groups, only a finite part acts on the physical fields
- similarly: only finitely many vector fields appear in the Lagrangian
- still: infinitely dimensional parameter space!

gauged D=2 maximal supergravity

embedding tensor — basic representation

branching under SL(9) identifies gaugings of 11d origin

$\chi_{\omega 0}$

- **9** ▷ **flux of 11d Kaluza-Klein vector fields**
- **36** ▷ **11d four-form flux**
- **126** ▷ **11d seven-form flux**
- **324** ▷ ...?
- **801** ▷ ...?? ← **SO(9) gauging — S8 compactification**
  ➜ AdS/CFT ...
- $\vdots$ ▷ ...???

gauged D=2 maximal supergravity

embedding tensor — basic representation

branching under SL(8) x SL(2) identifies gaugings of 10d IIB origin

$\chi_{\omega 0}$

(8,1)	▷ **flux of 10d Kaluza-Klein vector fields**
(8,2)	▷ **10d three-form flux**
(56,1)	▷ **10d five-form flux**
(56,2)	▷ **10d seven-form flux**
(224,1) + (8,3)	▷ ...??
⋮	▷ ...???

conclusions / outlook

- **maximal D=4 supergravities**
  - electric/magnetic vector fields and 2-form tensor fields
  - duality covariant description: 912-dimensional orbit
- **maximal D=2 supergravities**
  - infinite-dimensional parameter space of deformations (**E_9**)
  - universal Lagrangian (encodes linear system)
  - new examples of effective actions (IIA on S^8)
- **scalar potential**
  - on an infinite-dimensional moduli space
- **T-duality**
  - gauging non-target space isometries
- **higher dimensional origin ?**
  - infinite-dimensional parameter space of deformations!