

RELATIVISTIC DESCRIPTION OF HEAVY TETRAQUARKS

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Ebert, Faustov, Galkin (EFG) — Phys. Lett. B (2006) 214-219 [hep-ph/0512230]

Ebert, Faustov, Galkin, Lucha (EFGL) — arXiv:0706.3853 [hep-ph]

New Charmonium-like Mesons

Belle, BaBar, CDF, D0, CLEO

X(3872)

X(3876)

Y(3940)

X(4160)

Y(4260)

Y(4360)

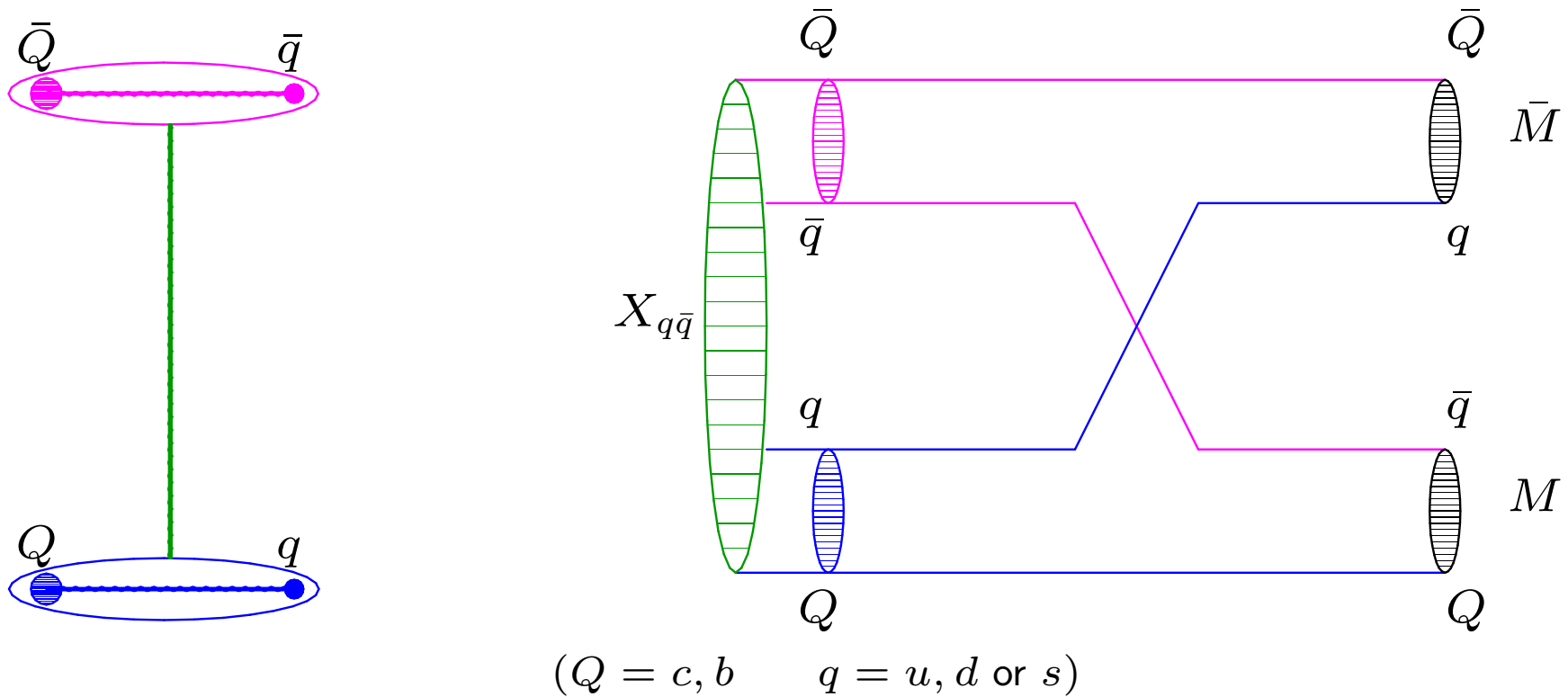
Z(4430)

Y(4660)

TETRAQUARKS in diquark-antidiquark picture

Tetraquarks — diquark and antidiquark in colour $\bar{3}$ and 3 configurations bound by colour forces \implies

- ★ typical hadronic size
- ★ X should be split into two states $[cu][\bar{c}\bar{u}]$ and $[cd][\bar{c}\bar{d}]$ with $\Delta M \sim 7$ MeV
- ★ existence of charged partners $X^+ = [cu][\bar{c}\bar{d}]$, $X^- = [cd][\bar{c}\bar{u}]$
- ★ existence of tetraquarks with open $X_{s\bar{q}} = [cs][\bar{c}\bar{q}]$ and hidden $X_{s\bar{s}} = [cs][\bar{c}\bar{s}]$ strangeness
- ★ rich spectroscopy — radial and orbital excitations between diquarks



Both scalar (asymmetric in flavour $[cq]_{S=0} = [cq]$) and axial vector (symmetric in flavour $[cq]_{S=1} = \{cq\}$) diquarks are considered

The $[cq][\bar{c}\bar{q}']$ ground states:

- ★ Two states with $J^{PC} = 0^{++}$:
(C is defined for $q = q'$)

$$X(0^{++}) = [cq]_{S=0}[\bar{c}\bar{q}']_{S=0}$$

$$X(0^{++'}) = [cq]_{S=1}[\bar{c}\bar{q}']_{S=1}$$

- ★ Three states with $J = 1$:

$$X(1^{++}) = \frac{1}{\sqrt{2}}([cq]_{S=1}[\bar{c}\bar{q}']_{S=0} + [cq]_{S=0}[\bar{c}\bar{q}']_{S=1})$$

$$X(1^{+-}) = \frac{1}{\sqrt{2}}([cq]_{S=0}[\bar{c}\bar{q}']_{S=1} - [cq]_{S=1}[\bar{c}\bar{q}']_{S=0})$$

$$X(1^{+-'}) = [cq]_{S=1}[\bar{c}\bar{q}']_{S=1}$$

- ★ One state with $J^{PC} = 2^{++}$:

$$X(2^{++}) = [cq]_{S=1}[\bar{c}\bar{q}']_{S=1}$$

- Doubly heavy tetraquarks $(QQ')(\bar{q}\bar{q}')$ with open charm and bottom
- explicitly exotic states with heavy flavour number equal to 2
 \implies their observation would be a direct proof of existence of multiquark states
- estimates of the production rates of such tetraquarks indicate that they could be produced and detected at present (SELEX, Tevatron, RHIC) and future facilities (LHC, LHCb, ALICE).
- we considered the doubly heavy $(QQ')(\bar{q}\bar{q}')$ tetraquark ($Q = b, c$ and $q = u, d, s$) as the bound system of the heavy diquark (QQ') and light antidiquark $(\bar{q}\bar{q}')$
- Heavy tetraquarks $(cq)(\bar{b}\bar{q}')$ with open charm and bottom
- we considered heavy $(cq)(\bar{b}\bar{q}')$ tetraquark ($q = u, d, s$) as the bound system of the heavy-light diquark (cq) and heavy-light antidiquark $(\bar{b}\bar{q}')$

RELATIVISTIC QUARK MODEL

Quasipotential equation of Schrödinger type:

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R} \right) \Psi_M(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}; M) \Psi_M(\mathbf{q})$$

\mathbf{p} - relative momentum of quarks

M - bound state mass ($M = E_1 + E_2$)

μ_R - relativistic reduced mass:

$$\mu_R = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}$$

$b(M)$ - on-mass-shell relative momentum in cms:

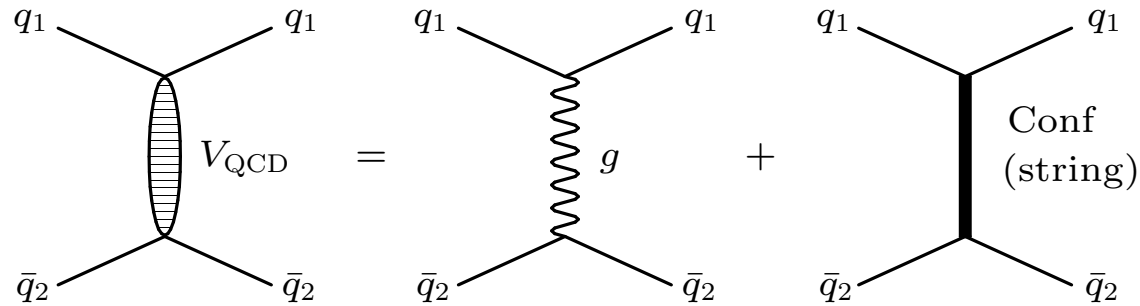
$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}$$

$E_{1,2}$ - center of mass energies:

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}$$

- Parameters of the model fixed from meson sector

- $q\bar{q}$ quasipotential**



$$V(\mathbf{p}, \mathbf{q}; M) = \bar{u}_1(p)\bar{u}_2(-p) \left\{ \frac{4}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma_1^\mu\gamma_2^\nu + V_{\text{conf}}^V(\mathbf{k})\Gamma_1^\mu\Gamma_{2;\mu} + V_{\text{conf}}^S(\mathbf{k}) \right\} u_1(q)u_2(-q)$$

$$\mathbf{k} = \mathbf{p} - \mathbf{q}$$

$D_{\mu\nu}(\mathbf{k})$ - (perturbative) gluon propagator

$\Gamma_\mu(\mathbf{k})$ - effective long-range vertex with **Pauli term**:

$$\Gamma_\mu(\mathbf{k}) = \gamma_\mu + \frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu,$$

κ - anomalous chromomagnetic moment of quark,

$$u^\lambda(p) = \sqrt{\frac{\epsilon(p) + m}{2\epsilon(p)}} \begin{pmatrix} 1 \\ \frac{\boldsymbol{\sigma}\mathbf{p}}{\epsilon(p) + m} \end{pmatrix} \chi^\lambda,$$

with $\epsilon(p) = \sqrt{\mathbf{p}^2 + m^2}$.

- Lorentz structure of $V_{\text{conf}} = V_{\text{conf}}^V + V_{\text{conf}}^S$

In nonrelativistic limit

$$\left. \begin{aligned} V_{\text{conf}}^V &= (1 - \varepsilon)(Ar + B) \\ V_{\text{conf}}^S &= \varepsilon(Ar + B) \end{aligned} \right\} \text{Sum : } (Ar + B)$$

ε - mixing parameter

Parameters A , B , κ , ε and quark masses fixed from analysis of meson masses and radiative decays:

$\varepsilon = -1$ from heavy quarkonium radiative decays ($J/\psi \rightarrow \eta_c + \gamma$) and HQET

$\kappa = -1$ from fine splitting of heavy quarkonium 3P_J states and HQET

$(1 + \kappa) = 0 \implies$ vanishing long-range chromomagnetic interaction !

Freezing of α_s for light quarks

(Simonov, Badalyan)

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln \frac{\mu^2 + M_0^2}{\Lambda^2}}, \quad \beta_0 = 11 - \frac{2}{3}n_f, \quad \mu = \frac{2m_1 m_2}{m_1 + m_2},$$

$$M_0 = 2.24\sqrt{A} = 0.95 \text{ GeV}$$

Quasipotential parameters:

$$A = 0.18 \text{ GeV}^2, \quad B = -0.30 \text{ GeV},$$

$$\Lambda = 0.413 \text{ GeV (from } M_\rho)$$

Quark masses:

$$m_b = 4.88 \text{ GeV}$$

$$m_s = 0.50 \text{ GeV}$$

$$m_c = 1.55 \text{ GeV}$$

$$m_{u,d} = 0.33 \text{ GeV}$$

- Heavy tetraquarks in diquark-antidiquark picture

(Qq) -interaction: $V_{Qq} = \frac{1}{2}V_{Q\bar{q}}$

$$V(\mathbf{p}, \mathbf{q}; M) = \bar{u}_1(p)\bar{u}_2(-p)\mathcal{V}(\mathbf{p}, \mathbf{q}; M)u_1(q)u_2(-q),$$

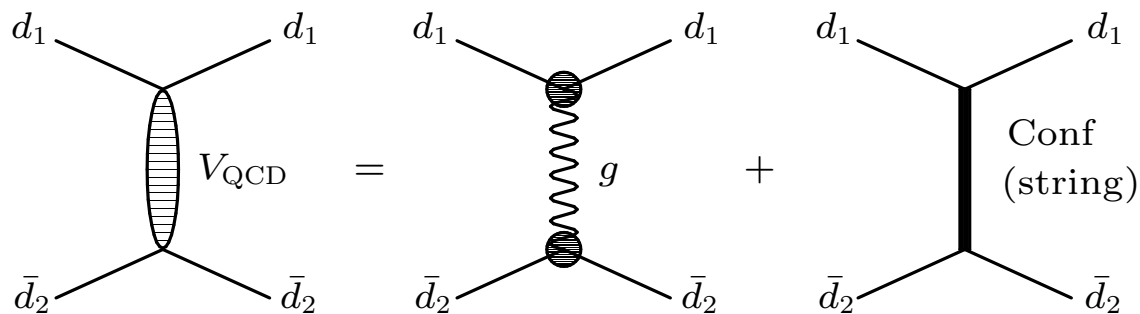
where

$$\mathcal{V}(\mathbf{p}, \mathbf{q}; M) = \frac{2}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma_1^\mu\gamma_2^\nu + \frac{1}{2}V_{\text{conf}}^V(\mathbf{k})\Gamma_1^\mu\Gamma_{2;\mu} + \frac{1}{2}V_{\text{conf}}^S(\mathbf{k})$$

$(d_1\bar{d}_2)$ -interaction:

$$d = (Qq)$$

$$V(\mathbf{p}, \mathbf{q}; M) = \frac{\langle d_1(P)|J_\mu|d_1(Q)\rangle}{2\sqrt{E_{d_1}E_{d_1}}}\frac{4}{3}\alpha_S D^{\mu\nu}(\mathbf{k})\frac{\langle d_2(P')|J_\nu|d_2(Q')\rangle}{2\sqrt{E_{d_2}E_{d_2}}} + \psi_{d_1}^*(P)\psi_{d_2}^*(P')\left[J_{d_1;\mu}J_{d_2}^\mu V_{\text{conf}}^V(\mathbf{k}) + V_{\text{conf}}^S(\mathbf{k})\right]\psi_{d_1}(Q)\psi_{d_2}(Q'),$$



$J_{d,\mu}$ – effective long-range vector vertex of diquark:

$$J_{d;\mu} = \begin{cases} \frac{(P+Q)_\mu}{2\sqrt{E_d E_d}} & \text{for scalar diquark} \\ -\frac{(P+Q)_\mu}{2\sqrt{E_d E_d}} + \frac{i\mu_d}{2M_d} \Sigma_\mu^\nu k_\nu & \text{for axial vector diquark} \\ & (\mu_d = 0) \end{cases}$$

μ_d - total chromomagnetic moment of axial vector diquark

diquark spin matrix: $(\Sigma_{\rho\sigma})_\mu^\nu = -i(g_{\mu\rho}\delta_\sigma^\nu - g_{\mu\sigma}\delta_\rho^\nu)$

\mathbf{S}_d - axial vector diquark spin: $(S_{d;k})_{il} = -i\varepsilon_{kil}$

$\psi_d(P)$ – diquark wave function:

$$\psi_d(p) = \begin{cases} 1 & \text{for scalar diquark} \\ \varepsilon_d(p) & \text{for axial vector diquark} \end{cases}$$

$\varepsilon_d(p)$ – polarization vector of axial vector diquark

$\langle d(P) | J_\mu | d(Q) \rangle$ – vertex of diquark-gluon interaction:

$$\langle d(P) | J_\mu(0) | d(Q) \rangle = \int \frac{d^3 p d^3 q}{(2\pi)^6} \bar{\Psi}_P^d(\mathbf{p}) \Gamma_\mu(\mathbf{p}, \mathbf{q}) \Psi_Q^d(\mathbf{q}) \Rightarrow F(k^2)$$

Γ_μ – two-particle vertex function of the diquark-gluon interaction:

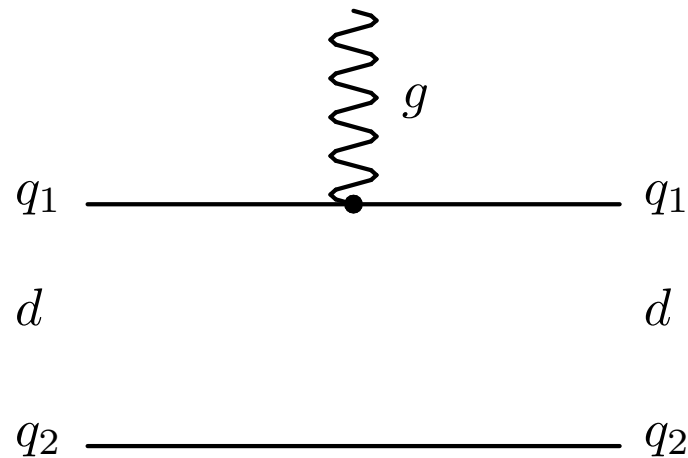


Figure 1: The vertex function Γ of the diquark-gluon interaction in the impulse approximation. The gluon interaction only with one quark is shown.

DIQUARKS

Table 1: Masses of light ground state diquarks (in MeV). S and A denotes scalar and axial vector diquarks antisymmetric $[q, q']$ and symmetric $\{q, q'\}$ in flavour, respectively.

Quark content	Diquark type	Mass				
		our RQM	Ebert et al. NJL	Burden et al. BSE	Maris BSE	Hess et al. Lattice
$[u, d]$	S	710	705	737	820	694(22)
$\{u, d\}$	A	909	875	949	1020	806(50)
$[u, s]$	S	948	895	882	1100	
$\{u, s\}$	A	1069	1050	1050	1300	
$\{s, s\}$	A	1203	1215	1130	1440	

Table 2: Masses of heavy-light and doubly heavy diquarks (MeV).

Quark content	Diquark type	Mass	
		$Q = c$	$Q = b$
$[Q, q]$	S	1973	5359
$\{Q, q\}$	A	2036	5381
$[Q, s]$	S	2091	5462
$\{Q, s\}$	A	2158	5482
$[Q, c]$	S		6519
$\{Q, c\}$	A	3226	6526
$\{Q, b\}$	A	6526	9778

Table 3: Masses of the ground state heavy baryons (in MeV).

Baryon	$I(J^P)$	Theory					Experiment PDG	
		our (2005)	Capstick Isgur	Roncaglia et al.	Savage	Jenkins		Mathur* et al.
Λ_c	$0(\frac{1}{2}^+)$	2297	2265	2285			2290	2286.46(14)
Σ_c	$1(\frac{1}{2}^+)$	2439	2440	2453			2452	2453.76(18)
Σ_c^*	$1(\frac{3}{2}^+)$	2518	2495	2520	2518		2538	2518.0(5)
Ξ_c	$\frac{1}{2}(\frac{1}{2}^+)$	2481		2468			2473	2471.0(4)
Ξ_c'	$\frac{1}{2}(\frac{1}{2}^+)$	2578		2580	2579	2580.8(2.1)	2599	2578.0(2.9)
Ξ_c^*	$\frac{1}{2}(\frac{3}{2}^+)$	2654		2650			2680	2646.1(1.2)
Ω_c	$0(\frac{1}{2}^+)$	2698		2710			2678	2697.5(2.6)
Ω_c^*	$0(\frac{3}{2}^+)$	2768		2770	2768	2760.5(4.9)	2752	2768.3(3.0) [†]
Λ_b	$0(\frac{1}{2}^+)$	5622	5585	5620			5672	5620.2(1.6)
Σ_b	$1(\frac{1}{2}^+)$	5805	5795	5820		5824.2(9.0)	5847	5807.5(3.6) [‡]
Σ_b^*	$1(\frac{3}{2}^+)$	5834	5805	5850		5840.0(8.8)	5871	5829.0(3.3) [‡]
Ξ_b	$\frac{1}{2}(\frac{1}{2}^+)$	5812		5810		5805.7(8.1)	5788	5792.9(3.0) [*]
Ξ_b'	$\frac{1}{2}(\frac{1}{2}^+)$	5937		5950		5950.9(8.5)	5936	
Ξ_b^*	$\frac{1}{2}(\frac{3}{2}^+)$	5963		5980		5966.1(8.3)	5959	
Ω_b	$0(\frac{1}{2}^+)$	6065		6060		6068.7(11.1)	6040	
Ω_b^*	$0(\frac{3}{2}^+)$	6088		6090		6083.2(11.0)	6060	

* error estimates of lattice calculations — ~ 50 MeV for charmed, ~ 100 MeV for bottom baryons

[†] BaBar 2006; [‡] CDF 2006; ^{*} CDF 2007

HEAVY TETRAQUARKS

The potential of the heavy diquark-antidiquark interaction

$$\begin{aligned}
 V(r) = & V_{\text{Coul}}(r) + V_{\text{conf}}(r) + \frac{1}{E_1 E_2} \left\{ \mathbf{p} \left[V_{\text{Coul}}(r) + V_{\text{conf}}^V(r) \right] \mathbf{p} - \frac{1}{4} \Delta V_{\text{conf}}^V(r) + V_{\text{Coul}}'(r) \frac{\mathbf{L}^2}{2r} \right. \\
 & + \frac{1}{r} \left[V_{\text{Coul}}'(r) + \frac{\mu_d}{4} \left(\frac{E_1}{M_1} + \frac{E_2}{M_2} \right) V_{\text{conf}}'^V(r) \right] \mathbf{L}(\mathbf{S}_1 + \mathbf{S}_2) \\
 & + \frac{\mu_d}{4} \left(\frac{E_1}{M_1} - \frac{E_2}{M_2} \right) \frac{V_{\text{conf}}'^V(r)}{r} \mathbf{L}(\mathbf{S}_1 - \mathbf{S}_2) \\
 & + \frac{1}{3} \left[\frac{1}{r} V_{\text{Coul}}'(r) - V_{\text{Coul}}''(r) + \frac{\mu_d^2}{4} \frac{E_1 E_2}{M_1 M_2} \left(\frac{1}{r} V_{\text{conf}}'^V(r) - V_{\text{conf}}''^V(r) \right) \right] \left[\frac{3}{r^2} (\mathbf{S}_1 \mathbf{r})(\mathbf{S}_2 \mathbf{r}) - \mathbf{S}_1 \mathbf{S}_2 \right] \\
 & \left. + \frac{2}{3} \left[\Delta V_{\text{Coul}}(r) + \frac{\mu_d^2}{4} \frac{E_1 E_2}{M_1 M_2} \Delta V_{\text{conf}}^V(r) \right] \mathbf{S}_1 \mathbf{S}_2 \right\},
 \end{aligned}$$

where

$$V_{\text{Coul}}(r) = -\frac{4}{3} \alpha_s \frac{F_1(r) F_2(r)}{r}$$

$\mathbf{S}_{1,2}$ – spins of diquark and antidiquark; \mathbf{L} – orbital momentum.

The diquark-antidiquark model of heavy tetraquarks predicts existence of the $SU(3)$ nonet of states with hidden charm or beauty ($Q = c, b$):

- four tetraquarks ($[Qq][\bar{Q}\bar{q}]$, $q = u, d$) with neither open nor hidden strangeness, which have electric charges 0 or ± 1 and isospin 0 or 1
- four tetraquarks ($[Qs][\bar{Q}\bar{q}]$ and $[Qq][\bar{Q}\bar{s}]$, $q = u, d$) with open strangeness ($S = \pm 1$), which have electric charges 0 or ± 1 and isospin $\frac{1}{2}$
- one tetraquark ($[Qs][\bar{Q}\bar{s}]$) with hidden strangeness and zero electric charge.

In our model we neglect the mass difference of u and d quarks and electromagnetic interactions – thus corresponding tetraquarks will be degenerate in mass. More detailed analysis predicts that such mass differences can be of few MeV.

The (non)observation of such states will be a crucial test of the tetraquark model.

Table 4: Masses of hidden charm tetraquark states (in MeV).

State J^{PC}	Diquark content	Tetraquark mass		
		$cq\bar{c}\bar{q}$	$cs\bar{c}\bar{s}$	$csc\bar{q}/cq\bar{c}\bar{s}$
$1S$				
0^{++}	$S\bar{S}$	3812	4051	3922
$1^{+\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	3871	4113	3982
0^{++}	$A\bar{A}$	3852	4110	3967
1^{+-}	$A\bar{A}$	3890	4143	4004
2^{++}	$A\bar{A}$	3968	4209	4080
$1P$				
1^{--}	$S\bar{S}$	4244	4466	4350

Table 5: Thresholds for open charm decays and nearby hidden-charm thresholds.

Channel	Threshold (MeV)	Channel	Threshold (MeV)	Channel	Threshold (MeV)
$D^0\bar{D}^0$	3729.4	$D_s^+D_s^-$	3936.2	$D^0D_s^\pm$	3832.9
D^+D^-	3738.8	$\eta'J/\psi$	4054.7	$D^\pm D_s^\mp$	3837.7
$D^0\bar{D}^{*0}$	3871.3	$D_s^\pm D_s^{*\mp}$	4080.0	$D^{*0}D_s^\pm$	3975.0
$\rho J/\psi$	3872.7	$\phi J/\psi$	4116.4	$D^0D_s^{*\pm}$	3976.7
$D^\pm D^{*\mp}$	3879.5	$D_s^{*+}D_s^{*-}$	4223.8	$K^{*\pm}J/\psi$	3988.6
$\omega J/\psi$	3879.6			$K^{*0}J/\psi$	3993.0
$D^{*0}\bar{D}^{*0}$	4013.6			$D^{*0}D_s^{*\pm}$	4118.8

Table 6: Masses of hidden bottom tetraquark states (in MeV).

State J^{PC}	Diquark content	Tetraquark mass		
		$bq\bar{b}\bar{q}$	$bs\bar{b}\bar{s}$	$bs\bar{b}\bar{q}/bq\bar{b}\bar{s}$
$1S$				
0^{++}	$S\bar{S}$	10471	10662	10572
$1^{+\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	10492	10682	10593
0^{++}	$A\bar{A}$	10473	10671	10584
1^{+-}	$A\bar{A}$	10494	10686	10599
2^{++}	$A\bar{A}$	10534	10716	10628
$1P$				
1^{--}	$S\bar{S}$	10807	11002	10907

Table 7: Thresholds for open bottom decays.

Channel	Threshold (MeV)	Channel	Threshold (MeV)	Channel	Threshold (MeV)
$B\bar{B}$	10558	$B_s^+ B_s^-$	10739	BB_s	10649
$B\bar{B}^*$	10604	$B_s^\pm B_s^{*\mp}$	10786	$B^* B_s$	10695
$B^* \bar{B}^*$	10650	$B_s^{*+} B_s^{*-}$	10833	$B^* B_s^*$	10742

Table 8: Comparison of theoretical predictions for the masses of charm diquark-antidiquark states $cq\bar{c}\bar{q}$ (in MeV) and possible experimental candidates.

State J^{PC}	Diquark content	Theory			Experiment	
		EFG	Maiani et al.	Maiani et al. ($cs\bar{c}\bar{s}$)	state	mass
$1S$						
0^{++}	$S\bar{S}$	3812	3723			
1^{++}	$(S\bar{A} - \bar{S}A)/\sqrt{2}$	3871	3872 [†]		X(3872)	3871.9±0.5
1^{+-}	$A\bar{A}$	3871	3754			
0^{++}	$A\bar{A}$	3852	3832			
1^{+-}	$A\bar{A}$	3890	3882			
2^{++}	$A\bar{A}$	3968	3952		Y(3943)	3943±11±13
$1P$						
1^{--}	$S\bar{S}$	4244		4330±70	Y(4260)	$\begin{cases} 4259 \pm 8_{-6}^{+2} \\ 4247 \pm 12_{-32}^{+17} \end{cases}$
1^{--}	$(S\bar{A} - \bar{S}A)/\sqrt{2}$	4284			Y(4260)	
1^{--}	$A\bar{A}$	4278				
1^{--}	$A\bar{A}$	4350			Y(4360)	4361±9±9
$2S$						
$1^{+\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	4431			Z(4433)	4433±4±1
0^{++}	$A\bar{A}$	4434				
1^{+-}	$A\bar{A}$	4461	~ 4470			
$2P$						
1^{--}	$S\bar{S}$	4666			Y(4660)	4664±11±5

[†] input

Table 9: Masses M of heavy-diquark (QQ') -light-antidiquark $(\bar{q}\bar{q})$ states. T is the lowest threshold for decays into two heavy-light $(Q\bar{q})$ mesons and $\Delta = M - T$. All values are given in MeV.

System	State $I(J^P)$	$Q = Q' = c$			$Q = Q' = b$			$Q = c, Q' = b$		
		M	T	Δ	M	T	Δ	M	T	Δ
$(QQ')(\bar{u}\bar{d})$										
	$0(0^+)$							7239	7144	95
	$0(1^+)$	3935	3871	64	10502	10604	-102	7246	7190	56
	$1(1^+)$							7403	7190	213
	$1(0^+)$	4056	3729	327	10648	10558	90	7383	7144	239
	$1(1^+)$	4079	3871	208	10657	10604	53	7396	7190	206
	$1(2^+)$	4118	4014	104	10673	10650	23	7422	7332	90
$(QQ')(\bar{u}\bar{s})$										
	$\frac{1}{2}(0^+)$							7444	7232	212
	$\frac{1}{2}(1^+)$	4143	3975	168	10706	10693	13	7451	7277	174
	$\frac{1}{2}(1^+)$							7555	7277	278
	$\frac{1}{2}(0^+)$	4221	3833	388	10802	10649	153	7540	7232	308
	$\frac{1}{2}(1^+)$	4239	3975	264	10809	10693	116	7552	7277	275
	$\frac{1}{2}(2^+)$	4271	4119	152	10823	10742	81	7572	7420	152
$(QQ')(\bar{s}\bar{s})$										
	$0(1^+)$							7684	7381	303
	$0(0^+)$	4359	3936	423	10932	10739	193	7673	7336	337
	$0(1^+)$	4375	4080	295	10939	10786	153	7683	7381	302
	$0(2^+)$	4402	4224	178	10950	10833	117	7701	7525	176

Table 10: Masses M of diquark (cq') –antidiquark $(\bar{b}\bar{q})$ states. T is the lowest threshold for decays into two heavy-light $(Q\bar{q})$ mesons $B^{(*)}$, $D^{(*)}$ and $\Delta = M - T$; T' is the threshold for decays into the $B_c^{(*)}$ and a light meson $(q'\bar{q})$, and $\Delta' = M - T'$. All values are given in MeV.

System	State J^P	$q' = u$					$q' = s$				
		M	T	Δ	T'	Δ'	M	T	Δ	T'	Δ'
$(cq')(\bar{b}\bar{u})$											
	0^+	7177	7144	33	6818	359	7294	7232	62	6768	526
	1^+	7198	7190	8	6880	318	7317	7277	40	6820	497
	1^+	7242	7190	52	6880	362	7362	7277	85	6820	542
	0^+	7221	7144	77	6818	403	7343	7232	111	6768	575
	1^+	7242	7190	52	6880	362	7364	7277	87	6820	544
	2^+	7288	7332	-44	7125	163	7406	7420	-14	7228	178
$(cq')(\bar{b}\bar{s})$											
	0^+	7282	7247	35	6768	514	7398	7336	62	6818	580
	1^+	7302	7293	9	6820	482	7418	7381	37	6880	538
	1^+	7346	7293	53	6820	526	7465	7381	84	6880	585
	0^+	7325	7247	78	6768	557	7445	7336	109	6818	627
	1^+	7345	7293	52	6820	525	7465	7381	84	6880	585
	2^+	7389	7437	-48	7228	161	7506	7525	-19	7352	154

SUMMARY

- Masses of heavy tetraquarks with hidden and open charm and bottom are calculated in the diquark-antidiquark picture.
- Dynamical approach based on the relativistic quark model is used, where both diquark and tetraquark masses are obtained by numerical solution of the quasipotential equation with the corresponding relativistic potentials.
- The diquark size is taken into account with the help of the diquark-gluon form factor in terms of diquark wave functions.
- No free adjustable parameters are introduced.
- $X(3872)$ can be the 1^{++} neutral charm tetraquark state. If it is really a tetraquark, one more neutral and two charged tetraquark states should exist with close masses.
- $Y(4260)$, $Y(4360)$ and $Y(4660)$ can be the 1^{--} P -wave tetraquark states.
- Charged $Z(4433)$ can be the 1^{+-} $2S$ -wave tetraquark state.
- The ground states of tetraquarks with hidden bottom are predicted to have masses below the open bottom threshold and thus should be narrow.
- All the $(cc)(\bar{q}\bar{q}')$ tetraquarks are predicted to be above the decay threshold into the open charm mesons.
- Only the $I(J^P) = 0(1^+)$ state of $(bb)(\bar{u}\bar{d})$ is found to lie below the BB^* threshold.