# Relativistic Corrections to $e^+e^- \longrightarrow J/\psi + \eta_c$

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# The Conflict Between Theory and Experiment

- Experiment
  Belle (2004): $\sigma[e^+e^- \to J/\psi + \eta_c] \times B_{>2} = 25.6 \pm 2.8 \pm 3.4$ fb.
  BaBar (2005): $\sigma[e^+e^- \to J/\psi + \eta_c] \times B_{>2} = 17.6 \pm 2.8^{+1.5}_{-2.1}$ fb.

- NRQCD at LO in $\alpha_s$ and $v$
  Braaten, Lee (2003): $\sigma[e^+e^- \to J/\psi + \eta_c] = 3.78 \pm 1.26$ fb.
  Liu, He, Chao (2003): $\sigma[e^+e^- \to J/\psi + \eta_c] = 5.5$ fb.
  The two calculations employ different choices of $m_c$, NRQCD matrix elements, and $\alpha_s$.
  Braaten and Lee include QED effects.

- Initially, the disagreement was worse:
  - The Belle cross section has moved down from the 2002 value
    $\sigma[e^+e^- \to J/\psi + \eta_c] \times B_{\geq 4} = 33^{+7}_{-6} \pm 9$ fb.
  - Braaten and Lee found a sign error in the QED interference term that raised the prediction from $2.31 \pm 1.09$ fb.

- An important recent development:
  A calculation of corrections at NLO in $\alpha_s$ by Zhang, Gao, and Chao (2005) shows that the $K$ factor is approximately 1.96.

## Matrix Elements for $e^+e^- \to J/\psi + \eta_c$

- The matrix elements at leading order in the heavy-quark velocity $v$:

$$\langle \mathcal{O}_1 \rangle_{J/\psi} = \left| \langle J/\psi(\lambda) | \psi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}(\lambda) \chi | 0 \rangle \right|^2 ,$$
$$\langle \mathcal{O}_1 \rangle_{\eta_c} = \left| \langle \eta_c | \psi^\dagger \chi | 0 \rangle \right|^2 .$$

  - $\psi$ annihilates a heavy quark;
    $\chi^\dagger$ annihilates a heavy antiquark.

- Ratios of matrix elements of higher orders in $v$ to the leading-order matrix elements:

$$\langle \boldsymbol{q}^{2n} \rangle_{J/\psi} = \frac{\langle J/\psi(\lambda) | \psi^\dagger (-\frac{i}{2} \overleftrightarrow{\boldsymbol{D}})^{2n} \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}(\lambda) \chi | 0 \rangle}{\langle J/\psi(\lambda) | \psi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}(\lambda) \chi | 0 \rangle},$$
$$\langle \boldsymbol{q}^{2n} \rangle_{\eta_c} = \frac{\langle \eta_c | \psi^\dagger (-\frac{i}{2} \overleftrightarrow{\boldsymbol{D}})^{2n} \chi | 0 \rangle}{\langle \eta_c | \psi^\dagger \chi | 0 \rangle}.$$

  These are the source of the relativistic corrections.

- $\langle \mathcal{O}_1 \rangle_{J/\psi}$ and $\langle \boldsymbol{q}^{2n} \rangle_{J/\psi}$ appear in $\Gamma[J/\psi \to e^+e^-]$.
  $\langle \mathcal{O}_1 \rangle_{\eta_c}$ and $\langle \boldsymbol{q}^{2n} \rangle_{\eta_c}$ appear in $\Gamma[\eta_c \to \gamma\gamma]$.

# Previous Work on Relativistic Corrections

## Braaten, Lee (2003)

- Showed that the order-$v^2$ corrections to $\sigma(e^+e^- \longrightarrow J/\psi + \eta_c)$ could be large: $\sigma_0 \times 2.0^{+2.9}_{-1.1}$.
- The large uncertainties arise from uncertainties in the NRQCD matrix elements of higher order in $v$.
- In this work, $\langle \boldsymbol{q}^2 \rangle_{J/\psi}$ and $\langle \boldsymbol{q}^2 \rangle_{\eta_c}$ were determined from the Gremm-Kapustin relation:

$$\langle \boldsymbol{q}^2 \rangle \approx \epsilon_{\mathrm{B}} m_c = (M_H - 2m_c)m_c.$$

$\epsilon_{\mathrm{B}}$ is the binding energy.

  - Large uncertainties in $m_c$ make the method unreliable.
  - For $m_c = 1.4 \pm 0.2$ GeV,

$$-0.35 < \langle q^2 \rangle < 0.84.$$

  - Even the sign of $\psi^{(2)}(0)$ is not known with great confidence.
- The large nonrelativistic correction casts doubt on the reliability of the $v$ expansion.

## Bodwin, Kang, Lee (2006)

### Potential Model

- Calculated $\langle \boldsymbol{q}^2 \rangle_{J/\psi}$ by determining $\epsilon_{\rm B}$ directly in a potential model.
- Greatly reduces the uncertainties.
- The potential model describes QCD, up to corrections of relative order $v^2$, provided that the static $Q\bar{Q}$ potential is known exactly.
- Used the Cornell (linear plus Coulomb) potential, which fits the lattice static potential well.
- Parameters fixed using
  - lattice value of the string tension,
  - $m_{\psi(2S)} - m_{J/\psi(1S)}$,
  - $\langle \mathcal{O}_1 \rangle_{J/\psi}$, from comparison of theory and experiment for $\Gamma[J/\psi \to e^+e^-]$.
- Result: $\langle \boldsymbol{q}^2 \rangle_{J\psi} = 0.50 \pm 0.09 \pm 0.15$ GeV$^2$.
- First error bar: uncertainty in the input potential-model parameters and the wave function at the origin.
  Second error bar: neglected relative-order-$v^2$ corrections.

## Resummation

- Proved a generalized Gremm-Kapustin relation:

$$\langle \boldsymbol{q}^{2n} \rangle \approx (m_c \epsilon_{\rm B})^n \approx \langle \boldsymbol{q}^2 \rangle^n .$$

  - Follows from dimensional regularization and pure power behavior of individual terms in the static potential.
  - Accurate up to corrections of relative order $v^2$.
  - Allows one to resum a class of relativistic corrections.

- Suppose that the NRQCD expansion of an amplitude $A$ is of the form

$$A = \sum_n \underbrace{\left[ \frac{1}{n!} \left( \frac{\partial}{\partial \boldsymbol{q}^2} \right)^n H(\boldsymbol{q}^2) \right] \Bigg|_{\boldsymbol{q}^2=0}}_{\text{short-distance coeff.}} \langle \boldsymbol{q}^{2n} \rangle \langle \mathcal{O}_1 \rangle^{1/2},$$

  where $H(\boldsymbol{q}^2)$ is the hard-scattering amplitude.

- Using the generalized Gremm-Kapustin relation, one has

$$A = H(\boldsymbol{q}^2)|_{\boldsymbol{q}^2 = \langle \boldsymbol{q}^2 \rangle} \langle \mathcal{O}_1 \rangle^{1/2} .$$

## Interpretation of the Resummation

- The NRQCD matrix elements are related to the $Q\bar{Q}$ color-singlet wave function in the Coulomb gauge:

$$\langle \boldsymbol{q}^{2n} \rangle \langle \mathcal{O}_1 \rangle^{1/2} = \sqrt{2N_c} \int^{\Lambda} \frac{d^3q}{(2\pi)^3} \boldsymbol{q}^{2n} \psi(\boldsymbol{q}^2).$$

- Therefore

$$\begin{aligned} A &= \sqrt{2N_c} \sum_n \underbrace{\left[ \frac{1}{n!} \left( \frac{\partial}{\partial \boldsymbol{q}^2} \right)^n H(\boldsymbol{q}^2) \right] \Bigg|_{\boldsymbol{q}^2=0}}_{\text{short-distance coeff.}} \int^{\Lambda} \frac{d^3q}{(2\pi)^3} \boldsymbol{q}^{2n} \psi(\boldsymbol{q}^2) \\ &= \sqrt{2N_c} \int^{\Lambda} \frac{d^3q}{(2\pi)^3} H(\boldsymbol{q}^2) \psi(\boldsymbol{q}^2). \end{aligned}$$

  - The NRQCD expansion is the Taylor expansion of the convolution of the hard-scattering amplitude with the wave function.
  - The resummation is equivalent to including all relativistic corrections from the $Q\bar{Q}$ wave function, up to the UV cutoff $\Lambda$ of the NRQCD matrix elements.

## Bodwin, Kang, Kim, Lee, Yu (2006) (BKKLY)

- Nonrelativistic corrections $\sigma[e^+e^- \to J/\psi + \eta_c]$ can come from two sources:
  - Direct corrections to the process $e^+e^- \to J/\psi + \eta_c$ itself,
  - Indirect corrections that enter through $\langle \mathcal{O}_1 \rangle_{J/\psi}$.
    Appear when $\Gamma[J/\psi \to e^+e^-]$ is used to determine $\langle \mathcal{O}_1 \rangle_{J/\psi}$ because of relativistic corrections to the theoretical expression for $\Gamma[J/\psi \to e^+e^-]$.
- The calculation assumes that $\langle \mathcal{O}_1 \rangle_{\eta_c} = \langle \mathcal{O}_1 \rangle_{J/\psi}$ and $\langle \boldsymbol{q}^2 \rangle_{\eta_c} = \langle \boldsymbol{q}^2 \rangle_{J/\psi}$.
  - Heavy-quark spin symmetry.
  - Accurate up to corrections of relative order $v^2$.
- In determining $\langle \boldsymbol{q}^2 \rangle_{J/\psi}$ from the potential model, the effect of relativistic corrections to $\Gamma[J/\psi \to e^+e^-]$ on $\langle \mathcal{O}_1 \rangle_{J/\psi}$ is not taken into account.
- The calculation applies the potential-model value of $\langle \boldsymbol{q}^2 \rangle_{J/\psi}$ to both the direct and indirect corrections to $\sigma[e^+e^- \to J/\psi + \eta_c]$.
- Includes resummation of a class of relativistic corrections.
- Direct correction: $34\%$.
  Indirect correction: $74\%$.

- Nonrelativistic corrections and corrections of NLO in $\alpha_s$ together give

$$\sigma[e^+e^- \to J/\psi + \eta_c] = 17.5 \pm 5.7 \text{ fb.}$$

- Includes pure QED corrections.
- Takes into account only uncertainties from $m_c$, $\langle \boldsymbol{q}^2 \rangle_{J/\psi}$, and $\langle \boldsymbol{q}^2 \rangle_{\eta_c}$.
- Effects of resummation are small: about $10\%$ of both the direct and indirect relativistic corrections.

## He, Fan, Chao (2007)

- $\langle \mathcal{O}_1 \rangle_{J/\psi}$, $\langle \mathcal{O}_1 \rangle_{\eta_c}$, and $\langle \boldsymbol{q}^2 \rangle_{J/\psi} = \langle \boldsymbol{q}^2 \rangle_{\eta_c}$ determined from
  - $\Gamma[J/\psi \to e^+ e^-]$,
  - $\Gamma[\eta_c \to \gamma\gamma]$,
  - $\Gamma[J/\psi \to \text{light hadrons}]$.

  Yields rather different values of the matrix elements from those in BKKLY.

- Nonrelativistic corrections and corrections of NLO in $\alpha_s$ together result in

$$\sigma[e^+ e^- \to J/\psi + \eta_c] = 20.04 \text{ fb}.$$

- Does not include pure QED corrections.

# New Calculation of Relativistic Corrections

## New Calculation of Matrix Elements

(Bodwin, Chung, Kang, Lee, Yu)

- Makes use of $\Gamma[J/\psi \to e^+e^-]$ and $\Gamma[\eta_c \to \gamma\gamma]$ to compute $\langle \mathcal{O}_1 \rangle_{J/\psi}$ and $\langle \mathcal{O}_1 \rangle_{\eta_c}$.
  - A class of relativistic corrections is resummed.
- Makes use of the potential-model method to compute $\langle \boldsymbol{q}^2 \rangle_{J/\psi}$ and $\langle \boldsymbol{q}^2 \rangle_{\eta_c}$.
- The computations of $\langle \mathcal{O}_1 \rangle_{J/\psi}$ and $\langle \boldsymbol{q}^2 \rangle_{J/\psi}$ are actually coupled, as are the computations of $\langle \mathcal{O}_1 \rangle_{\eta_c}$ and $\langle \boldsymbol{q}^2 \rangle_{\eta_c}$.
  - The theoretical expression for $\Gamma[J/\psi \to e^+e^-]$ yields $\langle \mathcal{O}_1 \rangle_{J/\psi}$, but it depends on $\langle \boldsymbol{q}^2 \rangle_{J/\psi}$.
  - $\langle \boldsymbol{q}^2 \rangle_{J/\psi}$ is computed in the potential model, which uses $\langle \mathcal{O}_1 \rangle_{J/\psi}$ as an input.
- In computing $\langle \boldsymbol{q}^2 \rangle_{J/\psi}$, BKKLY ignored the dependence of $\langle \mathcal{O}_1 \rangle_{J/\psi}$ on $\langle \boldsymbol{q}^2 \rangle_{J/\psi}$.
- We improve on this approach by solving numerically two sets of two coupled nonlinear equations:
  - one set for $\langle \mathcal{O}_1 \rangle_{J/\psi}$ and $\langle \boldsymbol{q}^2 \rangle_{J/\psi}$,
  - one set for $\langle \mathcal{O}_1 \rangle_{\eta_c}$ and $\langle \boldsymbol{q}^2 \rangle_{\eta_c}$.

- Other refinements:
  - More precise analysis of the effect of the string tension on the input parameters.
  - Effects of the running of $\alpha_{\rm EM}$ are taken into account.
  - $\langle \mathcal{O}_1 \rangle_{\eta_c}$ is determined by averaging values from $\Gamma[\eta_c \to \gamma\gamma]$ and $\Gamma[J/\psi \to e^+e^-]$. (Uncertainties from the use of the heavy-quark spin symmetry are taken into account in the averaging.)
- Includes a detailed analysis of uncertainties, some of which are highly correlated among the various matrix elements.
- Results:

$$
\begin{aligned}
\langle \mathcal{O}_1 \rangle_{J/\psi} &= 0.440^{+0.067}_{-0.055}\ \text{GeV}^3, \\
\langle \mathcal{O}_1 \rangle_{\eta_c} &= 0.437^{+0.111}_{-0.105}\ \text{GeV}^3, \\
\langle \boldsymbol{q}^2 \rangle_{J/\psi} &= 0.441^{+0.140}_{-0.140}\ \text{GeV}^2, \\
\langle \boldsymbol{q}^2 \rangle_{\eta_c} &= 0.442^{+0.143}_{-0.143}\ \text{GeV}^2.
\end{aligned}
$$

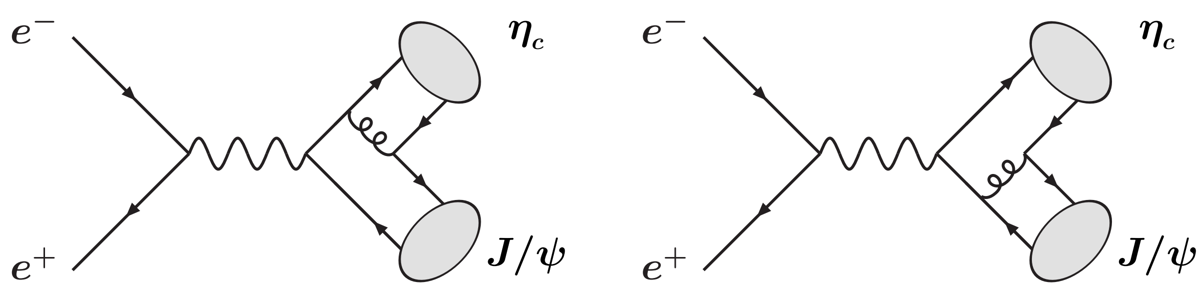
- In comparison with the values in Bodwin, Kang, Lee (2006), $\langle \boldsymbol{q}^2 \rangle_{J/\psi}$ and $\langle \boldsymbol{q}^2 \rangle_{J/\psi}$ are about $12\%$ smaller.
- In comparison with the values in BKKLY, $\langle \mathcal{O}_1 \rangle_{J/\psi}$ and $\langle \mathcal{O}_1 \rangle_{\eta_c}$ are about $1\%$ smaller.

## New Calculation of Relativistic Corrections to $e^+e^- \to J/\psi + \eta_c$

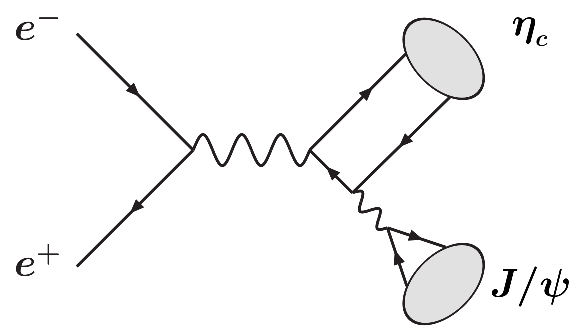
(Bodwin, Lee, Yu)

### Calculational Strategy

- Compute in the NRQCD factorization method.
- QCD/QED diagrams (plus six additional diagrams):




- QED fragmentation diagram (plus one additional diagram):



- Choose a particular frame and coordinate system to compute the helicity amplitudes for $\gamma^* \to (Q\bar{Q})_1(^3S_1) + (Q\bar{Q})_1(^1S_0)$
  - Tensor analysis shows that there is only one independent helicity amplitude.
  - Compute the helicity amplitudes for $\gamma^* \to (Q\bar{Q})_1(\text{spin triplet}) + (Q\bar{Q})_1(\text{spin singlet})$ analytically.
  - Integrate numerically over the angles of the relative $Q\bar{Q}$ momenta to project out the $S$-wave component.
- Form the cross section: Square the helicity amplitudes, multiply by the lepton factor, average over lepton spins, integrate over the phase space, and multiply by the flux.

## Features

- Makes use of the matrix-element calculation of Bodwin, Chung, Kang, Lee, Yu (2007).
- A class of relativistic corrections is resummed.
- QED corrections are included.
- VMD is used to compute the fragmentation part of the QED amplitude, reducing theoretical uncertainties.
- The running of $\alpha_{\mathrm{EM}}$ is taken into account.
- Uses the results of Zhang, Gao, and Chao (2005) for the corrections of NLO in $\alpha_s$.
- The interference between the relativistic corrections and the NLO in $\alpha_s$ corrections is computed.
- A detailed error analysis takes into account the correlations between uncertainties in the NRQCD matrix elements and in the hard-scattering cross section.

## Results

| case | $\langle \mathcal{O}_1 \rangle_{J/\psi}$ | $\langle \boldsymbol{q}^2 \rangle_{J/\psi}$ | $\langle \mathcal{O}_1 \rangle_{\eta_c}$ | $\langle \boldsymbol{q}^2 \rangle_{\eta_c}$ | $\sigma_0$ | $\sigma_v$ | $\sigma_{\rm tot}$ |
|---|---|---|---|---|---|---|---|
| central | 0.440 | 0.441 | 0.437 | 0.442 | 6.4 | 9.3 | 17.6 |
| $+\Delta Q_1 = +\Delta \langle \boldsymbol{q}^2 \rangle_{J/\psi}$ | 0.450 | 0.573 | 0.437 | 0.442 | 6.5 | 9.8 | 18.4 |
| $-\Delta Q_1 = -\Delta \langle \boldsymbol{q}^2 \rangle_{J/\psi}$ | 0.430 | 0.308 | 0.437 | 0.442 | 6.3 | 8.8 | 16.7 |
| $+\Delta Q_2 = +\Delta m_c$ | 0.433 | 0.443 | 0.470 | 0.430 | 6.0 | 7.6 | 13.9 |
| $-\Delta Q_2 = -\Delta m_c$ | 0.451 | 0.437 | 0.413 | 0.450 | 6.9 | 11.8 | 22.8 |
| $+\Delta Q_3 = +\Delta \sigma$ | 0.443 | 0.482 | 0.444 | 0.482 | 6.6 | 9.7 | 18.3 |
| $-\Delta Q_3 = -\Delta \sigma$ | 0.437 | 0.400 | 0.431 | 0.403 | 6.3 | 8.9 | 16.9 |
| $+\Delta Q_4 = +\Delta\, \mathrm{NNLO}_{J/\psi}$ | 0.504 | 0.419 | 0.473 | 0.429 | 7.9 | 11.3 | 21.5 |
| $-\Delta Q_4 = -\Delta\, \mathrm{NNLO}_{J/\psi}$ | 0.387 | 0.459 | 0.408 | 0.452 | 5.3 | 7.8 | 14.6 |
| $+\Delta Q_5 = +\Delta \Gamma_{J/\psi}$ | 0.451 | 0.437 | 0.443 | 0.440 | 6.7 | 9.6 | 18.2 |
| $-\Delta Q_5 = -\Delta \Gamma_{J/\psi}$ | 0.429 | 0.444 | 0.431 | 0.444 | 6.2 | 9.0 | 16.9 |
| $+\Delta Q_6 = +\Delta v^2$ | 0.440 | 0.441 | 0.511 | 0.417 | 7.5 | 10.8 | 20.4 |
| $-\Delta Q_6 = -\Delta v^2$ | 0.440 | 0.441 | 0.364 | 0.467 | 5.3 | 7.8 | 14.7 |
| $+\Delta Q_7 = +\Delta \langle \boldsymbol{q}^2 \rangle_{\eta_c}$ | 0.440 | 0.441 | 0.461 | 0.574 | 6.8 | 10.2 | 19.1 |
| $-\Delta Q_7 = -\Delta \langle \boldsymbol{q}^2 \rangle_{\eta_c}$ | 0.440 | 0.441 | 0.414 | 0.309 | 6.1 | 8.4 | 16.1 |
| $+\Delta Q_8 = +\Delta\, \mathrm{NNLO}_{\eta_c}$ | 0.440 | 0.441 | 0.474 | 0.429 | 7.0 | 10.0 | 19.0 |
| $-\Delta Q_8 = -\Delta\, \mathrm{NNLO}_{\eta_c}$ | 0.440 | 0.441 | 0.408 | 0.452 | 6.0 | 8.7 | 16.4 |
| $+\Delta Q_9 = +\Delta \Gamma_{\eta_c}$ | 0.440 | 0.441 | 0.487 | 0.425 | [7.2 | 10.3 | 19.5 |
| $-\Delta Q_9 = -\Delta \Gamma_{\eta_c}$ | 0.440 | 0.441 | 0.385 | 0.460 | [5.6 | 8.2 | 15.5 |
| $+\Delta Q_{10} = +\Delta \mu$ | 0.440 | 0.441 | 0.437 | 0.442 | 4.4 | 6.3 | 12.3 |
| $-\Delta Q_{10} = -\Delta \mu$ | 0.440 | 0.441 | 0.437 | 0.442 | 9.5 | 13.9 | 25.0 |

$$\sigma_{\rm tot} = 17.6^{+0.8+5.3+0.7+3.9+0.7+2.8+1.6+1.4+1.9}_{-0.9-3.7-0.7-3.0-0.7-2.9-1.5-1.1-2.0}\ \mathrm{fb} = 17.6^{+7.8}_{-6.3}\ \mathrm{fb}$$

- Can estimate uncalculated terms of relative order $\alpha_s^2$ and $\alpha_s v^2$ either by varying the renormalization scale by a factor of two:

$$\sigma_{\rm tot} = 17.6^{+10.7}_{-8.3} \text{ fb}.$$

  or by taking $\alpha_s$ or $v^2$ times the NLO contribution to $\sigma$:

$$\sigma_{\rm tot} = 17.6^{+8.1}_{-6.7} \text{ fb}.$$

- Uncertainty in the NRQCD factorization formula: $\sim m_H^2/(s/4) \approx 34\%$.
- $\sigma_{\rm tot}$ consists of

| | |
|---|---|
| $5.4$ fb | Leading order in $\alpha_s$ and $v$ (including indir. rel. corr., but without QED contribution) |
| $1.0$ fb | QED contribution |
| $2.9$ fb | Direct relativistic corrections |
| $6.9$ fb | Corrections of NLO in $\alpha_s$ |
| $1.4$ fb | Interference between rel. corr. and corr. of NLO in $\alpha_s$ |
| $17.6$ fb | Total |

- The indirect relativistic corrections account for a change of $72\%$.
- The direct relativistic corrections are smaller: $40\%$.
  - Effects from the finite width of the $Q\bar{Q}$ wave function are modest, once one excludes contributions from the high-momentum tails (part of the corrections of NLO in $\alpha_s$).
  - The effect of resummation is small: $-12\%$ of the direct relativistic correction.
    The $v$ expansion appears to be converging well.

## Comparison with BKKLY

- $\sigma^{\mathrm{BKKLY}}[e^+e^- \to J/\psi + \eta_c] = 17.5 \pm 5.7$ fb.
- The effects of the various refinements cancel approximately:
  The central value of our new cross section is essentially the same as in BKKLY.
- The error bars are larger in our new cross section because BKKLY considered only uncertainties from $m_c$, $\langle \boldsymbol{q}^2 \rangle_{J/\psi}$, and $\langle \boldsymbol{q}^2 \rangle_{\eta_c}$.

## Comparison with He, Fan, Chao (2007) (HFC)

- Central value is $\sigma_{\rm tot}^{\rm HFC} = 20.04$ fb.
  - Does not include QED contribution, interference contribution, resummation.
  - Should be compared with $14.7$ fb in our calculation.
  - $\sigma_{\rm tot}^{\rm HFC}$ is $37\%$ larger.
- Main differences relative to our calculation:

| Change | Source |
|---|---|
| $+30\%$ | Use of a larger value of $\langle \mathcal{O}_1 \rangle$ |
| $+47\%$ | Use of a larger value of $\alpha_s$ ($0.2592$ vs. $0.21$) |
| $-\ 9\%$ | Use of a smaller value of $\alpha$ ($1/137$ vs. $1/130.9$) |
| $-12\%$ | Use of a larger value of $m_c$ ($1.5$ GeV vs. $1.4$ GeV) at fixed values of the M.E.'s |
| $-9\%$ | Use of smaller values of $\langle \boldsymbol{q}^2 \rangle_{J/\psi}$ and $\langle \boldsymbol{q}^2 \rangle_{\eta_c}$ |

# Summary

- We have carried out a new calculation in the NRQCD framework of the relativistic corrections to $\sigma[e^+e^- \to J/\psi + \eta_c]$.
- The calculation uses $\Gamma[J/\psi \to e^+e^-]$, $\Gamma[\eta_c \to \gamma\gamma]$, and a potential model to determine the relevant NRQCD matrix elements.
- The calculation contains a number refinements, including
  - a more accurate determination of the matrix elements,
  - use of $m_{J/\psi}$ instead of $2m_c$ to reduce uncertainties,
  - use of the VMD method for calculating fragmentation amplitudes,
  - resummation of a class of corrections to all orders in $v$,
  - a detailed analysis of the uncertainties.
- Our result agrees with experiment, within uncertainties:
  - Theory: $\sigma[e^+e^- \to J/\psi + \eta_c] = 17.6^{+8.1}_{-6.7}\ \mathrm{fb}$
  - Belle: $\sigma[e^+e^- \to J/\psi + \eta_c] \times B_{>2} = 25.6 \pm 2.8 \pm 3.4$ fb.
  - BaBar: $\sigma[e^+e^- \to J/\psi + \eta_c] \times B_{>2} = 17.6 \pm 2.8^{+1.5}_{-2.1}$ fb.
- It would be desirable to reduce the theoretical and experimental uncertainties. Elimination of the $m_c$ uncertainty would decrease the theory error bars by $^{+1.9}_{-1.1}$ fb.
- It seems fair to say that the discrepancy between theory and experiment has been resolved.