DEPENDENCE OF DEEP INELASTIC STRUCTURE FUNCTIONS ON QUARK MASSES

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Abstract

We argue that the difference between the structure functions corresponding to deep inelastic scattering with and without heavy quarks in the current fragmentation region scales at high Q^2 and fixed (low) x_{Bj}. The lower bound on charm contribution to the total structure function, $F_2^c(Q^2, x)$, is calculated and compared with the recent data on $F_2^c(Q^2, x)$ from H1 Collaboration.

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1 Introduction

Quite often mass effects in high–energy collisions are considered as some not very spectacular corrections which finally die off. Nonetheless, it appears that in e^+e^- annihilation even such overall characteristics as hadron multiplicities are quite sensitive to the value of masses of the primary $q\bar{q}$ pairs [1].

Recent considerations have shown that calculations based on QCD agree well with the data at high enough energy [2] and that they yield an asymptotically constant difference between multiplicities of hadrons induced by the primary quarks of different masses.

In this paper we study a similar effect in a deeply inelastic process [3], [4]. As a by-product, we estimate heavy quark contributions to the total structure function.

2 Calculation of quark mass dependence

Let us consider, for definiteness, deep inelastic scattering of the electron (muon) off the proton. The hadronic tensor (an imaginary part of the virtual photon–proton amplitude) is defined via the electromagnetic current J_μ:

$$W_{\mu\nu}(p,q) = \frac{1}{2}(2\pi)^2 \int d^4z \exp(iqz) \langle p | [J_\mu(z), J_\nu(0)] | p \rangle, \tag{1}$$

where p is the momentum of the proton, $p^2 = M^2$, and q is the momentum of virtual photon, $q^2 = -Q^2 < 0$.

A symmetric part of $W_{\mu\nu}$ has two Lorentz structures:

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\nu q_\nu}{q^2}\right) F_1(Q^2,x) + \frac{1}{pq}\left(p_\mu - q_\mu \frac{pq}{q^2}\right)\left(p_\nu - q_\nu \frac{pq}{q^2}\right) F_2(Q^2,x), \tag{2}$$

where the structure functions F_1 and F_2 depend on Q^2 and on the variable

$$x = \frac{Q^2}{pq + \sqrt{(pq)^2 + Q^2M^2}}. \tag{3}$$

In what follows we will analyse the structure function F_2 of deep inelastic scattering with open charm (beauty) production at small x. In this section we consider the case of one single quark loop with mass m_q and electric charge e_q. A general case will be discussed in Section 3.

At small x a leading contribution to F_2 comes from one photon–gluon fusion subprocess [5]:

$$W_{\mu\nu} = \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^4}C_{\mu\nu}^{\alpha\beta}(q,k;m_q)d_{\alpha\alpha'}(k)d_{\beta\beta'}(k)\Gamma^{\alpha'\beta'}(k,p), \tag{4}$$

where k is the momentum of the virtual gluon, $k^2 < 0$. The tensor $C_{\mu\nu}^{\alpha\beta}$ denotes an imaginary two gluon irreducible part of the photon–gluon amplitude, while $\Gamma^{\alpha\beta}$ describes a distribution of the gluon inside the proton. A quantity $d_{\alpha\beta}$ is a tensor part of the gluonic propagator.

Let us choose an infinite momentum frame

$$p_\mu = \Big(P + \frac{M^2}{4P}, 0, 0, P - \frac{M^2}{4P}\Big). \tag{5}$$

Then the gluon distribution $\Gamma^{\alpha\beta}$ has to be calculated in the axial gauge $nA = 0$ with a gauge vector $n_\mu = (1,0,0,-1)$ [5]. One can take, for instance,

$$n_\mu = q_\mu + xp_\mu \tag{6}$$

with x defined by Eq. (3).

From Eq. (2) we get

$$\frac{1}{x}F_2 = \Big[-g_{\mu\nu} + p_\mu p_\nu\frac{3Q^2}{(pq)^2 + Q^2M^2}\Big]W^{\mu\nu} \equiv F_2^{(a)} + F_2^{(b)}. \tag{7}$$

Two terms in the RHS of Eq. (7), $F_2^{(a)}$ and $F_2^{(b)}$, correspond to two tensor projectors, $g_{\mu\nu}$ and $p_\mu p_\nu$.

Note that the structure function $F_L = F_2 - 2xF_1$ is completely defined by the term $p_\mu p_\nu$ and, thus, proportional to $F_2^{(b)}$.

By definition, the gluon distribution $\Gamma^{\alpha\beta}$ can be rewritten in the form

$$\Gamma^{\alpha\beta} = \frac{1}{4\pi}\sum_n \delta(p + k - p_n)\langle p|I_\alpha^g(0)|n\rangle\langle n|I_\beta^g(0)|p\rangle, \tag{8}$$

where I_α^g is the conserved current. Both $|p\rangle$ and $|n\rangle$ are on shell states that result in

$$k^\alpha\Gamma_{\alpha\beta} = 0. \tag{9}$$

From an explicit form for $C^{\alpha\beta}_{\mu\nu}$ (see Ref. [4], Appendix I, for details) one can verify that it obeys the same condition:

$$k^\alpha C^{\mu\nu}_{\alpha\beta} = 0. \tag{10}$$

Equations (9) and (10) allow us to simplify expression (4) and get ($r = a, b$):

$$\frac{1}{x} F_2^{(r)} = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^4} C^{(r)}_{\alpha\beta}(q, k; m_q) \Gamma^{\alpha\beta}(k, p), \tag{11}$$

with the notations

$$\begin{aligned} C^{(a)}_{\alpha\beta} &= -g_{\mu\nu} C^{\mu\nu}_{\alpha\beta}, \\ C^{(b)}_{\alpha\beta} &= \frac{3Q^2}{(pq)^2 + Q^2M^2} p_\mu p_\nu C^{\mu\nu}_{\alpha\beta}. \end{aligned} \tag{12}$$

The tensor $\Gamma^{\alpha\beta}$ can be expanded in Lorentz structures

$$\begin{aligned} \Gamma^{\alpha\beta} &= \left(g_{\alpha\beta} - \frac{k_\alpha k_\beta}{k^2}\right)\Gamma_1 + \left(p_\alpha - k_\alpha \frac{pk}{k^2}\right)\left(p_\beta - k_\beta \frac{pk}{k^2}\right)\frac{1}{k^2}\Gamma_2 \\ &+ \left(k_\alpha - n_\alpha \frac{k^2}{kn}\right)\left(k_\beta - n_\beta \frac{k^2}{kn}\right)\frac{1}{k^2}\Gamma_3 + \left(p_\alpha - n_\alpha \frac{pk}{kn}\right)\left(p_\beta - n_\beta \frac{pk}{kn}\right)\frac{1}{k^2}\Gamma_4 \end{aligned} \tag{13}$$

with $\Gamma_i = \Gamma_i(k^2, M^2, pk)$.

Let us consider a contribution of the invariant function Γ_1 into the structure function F_2 (11). With the accounting for (9) and (10) we obtain

$$\frac{1}{x} F_2^{(r)} = e_q^2 \int\limits_x^1 \frac{dz}{z} \int\limits_{Q_0^2}^{Q^2(z/x)} \frac{dl^2}{l^2} \frac{1 - l^2x^2/Q^2z^2}{1 + M^2x^2/Q^2} C^{(r)}\left(\frac{Q^2}{l^2}, \frac{m_q^2}{l^2}, \frac{x}{z}\right) \frac{\partial}{\partial \ln l^2} g(l^2, z), \tag{14}$$

where

$$l^2 = -k^2 > 0, \tag{15}$$

$$z = \frac{kn}{pn} \tag{16}$$

and

$$Q_0^2 = \frac{M^2z^2}{1 - z}. \tag{17}$$

Here we used the notation:

$$C^{(r)} = -g^{\alpha\beta} C^{(r)}_{\alpha\beta}. \tag{18}$$

To be more correct, one has to write $z > x(1+4m^2/Q^2)$ and $l^2 < Q(z/x) - 4m^2 z/(z-x)$ in (14), but me neglect power corrections $\mathrm{O}(m^2/Q^2)$.

In Eq. (14) the gluon distribution, $g(l^2, z)$, is introduced:

$$g(l^2, z) = \frac{1}{2(2\pi)^4} \int\limits_{Q_0^2}^{l^2} \frac{dl'^2}{l'^4} \int d^2 k_\perp \Gamma_1(l'^2, k_\perp, z). \tag{19}$$

If we use the new variable

$$\xi = \frac{-k^2}{pk + \sqrt{(pk)^2 - k^2 M^2}} \tag{20}$$

instead of $k_\perp^2$, we will arrive at the expression

$$g(l^2, z) = \frac{z}{32\pi^3} \int\limits_{Q_0^2}^{l^2} \frac{dl'^2}{l'^4} \int\limits_{z}^{1} d\xi \left(M^2 + \frac{l'^2}{\xi^2} \right) \Gamma_1(l'^2, \xi). \tag{21}$$

A thorough analysis shows, however, that the main contribution to F_2 at small x comes from Γ_2 and Γ_4 in (13) and F_2 is given by the formula (see [4] for details):

$$\frac{1}{x} F_2 = e_q^2 \sum_{r=a,b} \int\limits_{z}^{1} \frac{dz}{z} \int\limits_{Q_0^2}^{Q^2(z/x)} \frac{dl^2}{l^2} \quad \left[\tilde{C}^{(r)} \left(\frac{Q^2}{l^2}, \frac{m_q^2}{l^2}, \frac{x}{z} \right) \frac{\partial}{\partial \ln l^2} G(l^2, z) \right.$$
$$\left. + \hat{C}^{(r)} \left(\frac{Q^2}{l^2}, \frac{m_q^2}{l^2}, \frac{x}{z} \right) \frac{\partial}{\partial \ln l^2} \hat{G}(l^2, z) \right]. \tag{22}$$

As we are interested in a calculation of the difference of the structure functions corresponding to the massive and massless cases, we preserve those terms in $C^{(r)}$ which give a leading contribution to ΔF_2. In [4] we have calculated the functions $C^{(a)}$ in lowest order in the strong coupling α_s:

$$\begin{aligned} \tilde{C}^{(a)}(u,v,y) &= \frac{\alpha_s}{4\pi} \{ [(1-y)^2 + y^2] L(u,v,y) - [(1-y)^2 + y^2 - 2v] M(v,y) - 1 \}, \\ \hat{C}^{(a)}(u,v,y) &= \frac{\alpha_s}{\pi} y(1-y) M(v,y), \end{aligned} \tag{23}$$

where

$$
\begin{aligned}
L(u,v,y) &= \ln \frac{u(1-y)}{y[v+y(1-y)]}, \\
M(v,y) &= \frac{y(1-y)}{v+y(1-y)}.
\end{aligned} \tag{24}
$$

As for the gluon distributions, they are given by the formulae:

$$
G = \frac{1}{32\pi^3 z} \int\limits_{Q_0^2}^{l^2} \frac{dl'^2}{l'^4} \int\limits_z^1 \frac{d\xi}{\xi}(\xi - z)\left(M^2 + \frac{l'^2}{\xi^2}\right)[\Gamma_2(l'^2,\xi) + \Gamma_4(l'^2,\xi)], \tag{25}
$$

$$
\hat{G} = \frac{1}{32\pi^3 z} \int\limits_{Q_0^2}^{l^2} \frac{dl'^2}{l'^4} \int\limits_z^1 d\xi \left(M^2 + \frac{l'^2}{\xi^2}\right)\left[\frac{(2\xi - z)^2}{4\xi^2}\Gamma_2(l'^2,\xi) + \Gamma_4(l'^2,\xi)\right] \tag{26}
$$

The analogous expressions for the functions $C^{(b)}$ are the following [4]:

$$
\begin{aligned}
\tilde{C}^{(b)}(u,v,y) &= \frac{3\alpha_s}{2\pi}\frac{1}{u} y\{2y[(1-2y)(1-y) - v]L(u,v,y) \\
&+ (1-y)[(1-y)^2 + y^2 - 2v]M(v,y)\} + \frac{3\alpha}{2\pi}y(1-y), \\
\hat{C}^{(b)}(u,v,y) &= -\frac{12\alpha_s}{\pi}\frac{1}{u}y^2(1-y)^2 M(v,y).
\end{aligned} \tag{27}
$$

It may be shown that the leading contribution to ΔF_2 comes from the region $l^2 \sim m_q^2$, $k^2 = -l^2$ being the gluon virtuality. Then one can easily see from (24) and (27) that the first two terms in $\tilde{C}^{(b)}$ are suppressed by the factor k^2/Q^2 with respect to $\tilde{C}^{(a)}$, while the third terms in $\tilde{C}^{(b)}$ do not contribute to the difference $C^{(b)}|_{m=0} - C^{(b)}|_{m\neq 0}$.

In the leading logarithmic approximation (LLA), only the function L remaines in Eqs. (23), which results in

$$
\frac{1}{x}\frac{\partial}{\partial \ln Q^2} F_2(Q^2,x) = \frac{\alpha_s}{2\pi}\int\limits_x^1 \frac{dz}{z} P_{qg}\left(\frac{x}{z}\right) G(Q^2,z), \tag{28}
$$

where $P_{qg}(z)$ is the Altarelli–Parisi splitting function and $G(Q^2,z)$ is the gluon distribution in LLA defined by Eq. (25).

It is clear from (22) that $\Delta F_2 = F_2|_{m=0} - F_2|_{m\neq 0}$ is defined by the quantities ($r = a, b$)

$$\Delta C^{(r)}(u, v, y) = C^{(r)}(u, 0, y) - C^{(r)}(u, v, y). \tag{29}$$

By using Eq. (23) we obtain the important result

$$\begin{aligned} \Delta\tilde{C}^{(a)} &= \Delta\tilde{C}^{(a)}(v, y) \\ \Delta\hat{C}^{(a)} &= \Delta\hat{C}^{(a)}(v, y), \end{aligned} \tag{30}$$

while from (27) we get

$$\begin{aligned} \Delta\tilde{C}^{(b)} &= \frac{1}{u}\Delta\tilde{C}^{(b)}(v, y), \\ \Delta\hat{C}^{(b)} &= \frac{1}{u}\Delta\hat{C}^{(b)}(v, y). \end{aligned} \tag{31}$$

In this, we have

$$\Delta\tilde{C}^{(a)}, \Delta\hat{C}^{(a)}|_{-k^2\to\infty} \sim \frac{m_q^2}{k^2}. \tag{32}$$

So, we get [4]

$$\begin{aligned} \frac{1}{x}\Delta F_2(Q^2, m_q^2, x)|_{Q^2\to\infty} &= e_q^2 \int\limits_x^1 \frac{dz}{z} \int\limits_{Q_0^2}^{\infty} \frac{dl^2}{l^2}\Big[\Delta\tilde{C}\left(\frac{m_q^2}{l^2}, \frac{x}{z}\right) \frac{\partial}{\partial \ln l^2} G(l^2, z) \\ &+ \Delta\hat{C}\left(\frac{m_q^2}{l^2}, \frac{x}{z}\right) \frac{\partial}{\partial \ln l^2}\hat{G}(l^2, z)\Big]. \end{aligned} \tag{33}$$

Here

$$\begin{aligned} \Delta\tilde{C}(v, y) &= \frac{\alpha_s}{4\pi}\Big\{[(1-y)^2 + y^2]\ln\Big[1 + \frac{v}{y(1-y)}\Big] - \frac{v}{v + y(1-y)}\Big\}, \\ \Delta\hat{C}(v, y) &= \frac{\alpha_s}{\pi} y(1-y)\frac{v}{v + y(1-y)} \end{aligned} \tag{34}$$

with $G(l^2, z)$ and $\hat{G}(l^2, z)$ being defined by Eqs. (25) and (26).

The integral in l^2 (33) converges because of condition (32). Contributions from $\Delta\tilde{C}^{(b)}$ and $\Delta\hat{C}^{(b)}$ are suppressed by the factors $(m^2/Q^2)\ln Q^2$ and can thus be omitted.

Let us consider the gluon distribution $\hat{G}$ (26). At small z the leading contribution to $\hat{G}(l^2,z)$ comes from the region $z \ll \xi$, and we have

$$\hat{G}(l^2,z) \simeq G(l^2,z). \tag{35}$$

Taking expression (35) into account, the structure function F_2 (22) has the following form at low x (with the term of the order of k^2/Q^2 and m^2/Q^2 subtracted)

$$\frac{1}{x}F_2 = e_q^2 \int\limits_x^1 \frac{dz}{z} \int\limits_{Q_0^2}^{Q^2(z/x)} \frac{dl^2}{l^2} C\left(\frac{Q^2}{l^2}, \frac{m_q^2}{l^2}, \frac{x}{z}\right) \frac{\partial}{\partial \ln l^2} G(l^2,z), \tag{36}$$

where

$$C(u,v,y) = \frac{\alpha_s}{4\pi}\{[(1-y)^2+y^2]L(u,v,y) - [(1-3y)^2 - 3y^2 - 2v]M(v,y) - 1\}. \tag{37}$$

As for the difference of the structure function, we obtain the following prediction

$$\begin{aligned} \frac{1}{x}\Delta F_2(Q^2, m_q^2, x)|_{Q^2\to\infty} &= \frac{1}{x}\Delta F_2(m_q^2, x) \\ &= e_q^2 \int\limits_x^1 \frac{dz}{z} \int\limits_{Q_0^2}^{\infty} \frac{dl^2}{l^2} \Delta C\left(\frac{m_q^2}{l^2}, \frac{x}{z}\right) \frac{\partial}{\partial \ln l^2} G(l^2,z), \end{aligned} \tag{38}$$

where

$$\Delta C(v,y) = \frac{\alpha_s}{4\pi}[(1-y)^2+y^2]\left\{\ln\left[1+\frac{v}{y(1-y)}\right] - (1-2y)^2\frac{v}{v+y(1-y)}\right\}. \tag{39}$$

3 Relation between measurable structure functions

Up to now, we considered those contributions to F_2 that came from the quark with electric charge e_q and mass m_q, $\tilde{F}_2|_{m\neq 0}$. Then we have taken the analogous contributions from the massless quark with the same e_q, $\tilde{F}_2|_{m=0}$, and have calculated the quantity $\tilde{F}_2|_{m=0} - \tilde{F}_2|_{m\neq 0}$.

The total structure function F_2 has the form

$$F_2(Q^2,x) = \sum_q e_q^2 \tilde{F}_2^q(Q^2,x), \tag{40}$$

where the functions $\tilde{F}_2^q$ are introduced ($q = u, d, s, c, b$).

The structure functions describing open charm and bottom production in DIS, F_2^c and F_2^b respectively, are related to $\tilde{F}_2^c$ and $\tilde{F}_2^b$ by the formulae

$$\begin{aligned} F_2^c &= \frac{4}{9}\tilde{F}_2^c, \\ F_2^b &= \frac{1}{9}\tilde{F}_2^b. \end{aligned} \tag{41}$$

At low x one can put ($m_u = m_d = m_s = 0$ is assumed)

$$\tilde{F}_2^u = \tilde{F}_2^d = \tilde{F}_2^s = \tilde{F}_2 \tag{42}$$

and define the difference between heavy and light flavour contributions to F_2:

$$\begin{aligned} \Delta\tilde{F}_2^c &= \tilde{F}_2 - \tilde{F}_2^c, \\ \Delta\tilde{F}_2^b &= \tilde{F}_2 - \tilde{F}_2^b. \end{aligned} \tag{43}$$

Notice that there are the functions $\tilde{F}_2$ and $\tilde{F}_2^q$ that have been calculated in the previous section (see Eqs (36) and (38)).

From Eqs. (38) and (43) one readily obtains that a linear combination

$$\Sigma_\alpha(Q^2,x) \equiv F_2(Q^2,x) + \alpha F_2^c(Q^2,x) - (4\alpha+11)F_2^b(Q^2,x) \tag{44}$$

scales at $Q^2 \to \infty$ and arbitrary parameter α. In terms of $\Delta\tilde{F}_2$ introduced in (38)

$$\lim_{Q^2\to\infty} \Sigma_\alpha(Q^2,x) = -\frac{4}{9}(1+\alpha)\Delta\tilde{F}_2(m_c^2,x) + \frac{1}{9}(4\alpha+10)\Delta\tilde{F}_2(m_b^2,x). \tag{45}$$

Let us now represent the function $\tilde{F}_2$ (36) in the following form

$$\frac{1}{x}\tilde{F}_2 = \int_x^1 \frac{dy}{y} \int_0^Y d\eta\, C(\eta,y) G'\left(Y-\eta, \frac{x}{y}\right), \tag{46}$$

where we denote

$$Y = \ln \frac{Q^2}{yQ_0^2} \tag{47}$$

and introduce the variable $\eta = \ln(k^2/Q_0^2)$. Here G' means the derivative of $G(Q^2, x)$ with respect to the variable $\ln Q^2$.

Analogously, we get from (38)

$$\frac{1}{x}\Delta\tilde{F}_2^q = \int_x^1 \frac{dy}{y} \int_{-\infty}^{Y_m} d\eta\, \Delta C(\eta, y) G'\left(Y_m - \eta, \frac{x}{y}\right), \tag{48}$$

with

$$Y_m = \ln \frac{m_q^2}{yQ_0^2}. \tag{49}$$

Here $\eta = \ln(m_q^2/k^2 y(1-y)) \simeq \ln(m_q^2/k^2 y)$ (remember that we consider small x).

The expression for ΔC is given by Eq. (39) and, in terms of the variables η and y, looks like

$$\Delta C = \frac{\alpha_s}{4\pi}[(1-y)^2 + y^2]\left[\ln(1+e^\eta) - (1-2y)^2 \frac{e^\eta}{1+e^\eta}\right]. \tag{50}$$

As for the expression for C, it has to be defined via relation (11) and exact formulae ([4]) taken at $m = 0$. The result is of the form

$$C(\eta, y) = \frac{\alpha_s}{2\pi}\left[\frac{1}{2U}\ln\frac{1+U}{1-U}\left(1 - \frac{3}{U^2}V + V\right) - \left(1 - \frac{3}{U^2}V\right)\right], \tag{51}$$

where

$$\begin{aligned} U &= \sqrt{1 - 4y(1-y)e^{-\eta}}, \\ V &= (1-y)\left[y + (1-y)e^{-\eta}\right]\left(1 - e^{-\eta}\right). \end{aligned} \tag{52}$$

It is clear from (50) that

$$\Delta C(\eta, y) > 0 \tag{53}$$

for $-\infty < \eta < \infty$, $0 \le y \le 1$ and $\Delta C(\eta, y)$ is negligible at $\eta < 0$ (see Figs. 1a-1d).

Moreover, the quantitative analysis shows that at most at $y \leq 0.2$, which is relevant for small x as under consideration, one has

$$C(\eta, y) > \Delta C(\eta, y), \qquad \eta > 0, \tag{54}$$

(see Figs. 2a-2d). Neglecting the small contribution to $\tilde{F}_2$ from the region $\eta < 0$ and taking into account that $\partial G(Q^2, x)/\partial \ln Q^2 > 0$ at small x (cf. [6]), we thus conclude

$$\Delta \tilde{F}_2^q(m_q^2, x) < \tilde{F}_2(Q^2, x)|_{Q^2=m_q^2}. \tag{55}$$

From Eqs. (45), (55) we obtain the following inequality which holds for $-2.5 \leq \alpha \leq -1$

$$\begin{aligned} 0 < \Sigma_\alpha(Q^2, x)|_{Q^2 \gg 1\mathrm{GeV}^2} \; < \; & -\frac{2}{3}(1+\alpha)(F_2 - F_2^c - F_2^b)(Q^2, x)|_{Q^2=m_c^2} \\ & + \frac{1}{3}(5+2\alpha)(F_2 - F_2^c - F_2^b)(Q^2, x)|_{Q^2=m_b^2}. \end{aligned} \tag{56}$$

At the endpoints $\alpha = -2.5$ and $\alpha = -1$ we get

$$\begin{aligned} \left(F_2 - 2.5F_2^c - F_2^b\right)(Q^2, x) \; & < \; \left(F_2 - F_2^c - F_2^b\right)(Q^2, x)|_{Q^2=m_c^2}, \\ \left(F_2 - F_2^c - 7F_2^b\right)(Q^2, x) \; & < \; \left(F_2 - F_2^c - F_2^b\right)(Q^2, x)|_{Q^2=m_b^2}. \end{aligned} \tag{57}$$

Data on the total structure function F_2 for Q^2 between 1.5 GeV^2 and 5000 GeV^2 and x between 3×10^{-5} and 0.32 are now available [7]. As for the charm structure function, there are recent data on F_2^c at $Q^2 = 12$ GeV^2, 25 GeV^2 and 45 GeV^2 with rather large errors [8].

Using the first of the inequalities (57) we get (assuming $F_2^c(m_c^2, x)$, $F_2^b(m_c^2, x) \simeq 0$ (cf. [9]))

$$F_2^c(Q^2, x) > 0.4\left[F_2(Q^2, x) - F_2^b(Q^2, x) - F_2(m_c^2, x)\right]. \tag{58}$$

Taking use of the available data on $F_2(Q^2, x)$ [7] and neglecting F_2^b (as F_2^b/F_2 reaches at most $2 \div 3\%$ at HERA) we estimate the lower bound according to (58). The result is exhibited in Figs. 3a-3c with $m_c^2 = 2.5$ GeV^2. Experimental data on F_2^c at several values of Q^2 and x are taken from Ref. [8].

Tables 1-3 present the result of our calculations of the lower bounds on F_2^c at the same Q^2 and x. Three different values of F_2^c for each Q^2, x correspond to $m_c = 1.3$ GeV, $m_c = 1.5$ GeV, $m_c = 1.7$ GeV, respectively.

$\langle x \rangle$	F_2^c (theor.)	F_2^c (exper.) [8]
.0008	0.173 0.161 0.145	$0.211 \pm 0.049 \ {}^{+0.045}_{-0.040}$
.0020	0.137 0.128 0.116	$0.263 \pm 0.036 \ {}^{+0.043}_{-0.041}$
.0032	0.120 0.112 0.101	$0.190 \pm 0.054 \ {}^{+0.052}_{-0.049}$

Table 1. The lower bounds on $F_2^c(Q^2, x)$ for $Q^2 = 12$ GeV^2.

$\langle x \rangle$	F_2^c (theor.)	F_2^c (exper.) [8]
.0008	0.258 0.247 0.231	$0.324 \pm 0.099 \ {}^{+0.065}_{-0.058}$
.0020	0.205 0.196 0.184	$0.253 \pm 0.069 \ {}^{+0.041}_{-0.040}$
.0032	0.179 0.172 0.161	$0.222 \pm 0.066 \ {}^{+0.044}_{-0.039}$

Table 2. The lower bounds on $F_2^c(Q^2, x)$ for $Q^2 = 25$ GeV^2.

$\langle x \rangle$	F_2^c (theor.)	F_2^c (exper.) [8]
.0020	0.258 0.249 0.237	$0.156 \pm 0.070 \ {}^{+0.031}_{-0.028}$
.0032	0.226 0.218 0.207	$0.275 \pm 0.074 \ {}^{+0.045}_{-0.042}$
.0080	0.165 0.160 0.152	$0.200 \pm 0.064 \ {}^{+0.040}_{-0.035}$

Table 3. The lower bounds on $F_2^c(Q^2, x)$ for $Q^2 = 45$ GeV^2.

These estimates of F_2^c agree with the recent data on the charm contribution to F_2 [8]. Our inequalities (56)-(58) are also in agreement with the results of Ref. [10] where the ratio F_2^c/F_2 was estimated. For a detailed comparison of our predictions with the data, an improved measurement of the charm component F_2^c is required.

Conclusions

In this paper we have demonstrated that the lowest–order quark loop contributions to the structure functions at small x contain mass–dependent terms which scale at high Q^2. This effect can be observed experimentally, and we predict theoretical bounds for the corresponding contributions from c-quarks (see Eqs. (56) and (57), Figs. 3a-3c, Tabs. 1-3).

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Figure Captions

Figs. 1a-1d: $\Delta C(\eta, y)$ as a function of the variable η at several fixed values of y.

Figs. 2a-2d: $C(\eta, y)$ (continuous curves) and $\Delta C(\eta, y)$ (dashed curves) as functions of the variable η ($\eta \geq 0$) at several fixed values of y.

Figs. 3a-3c: The lower bounds on $F_2^c(Q^2, x)$ (continuous curves) together with results from H1 [8] (open and closed circles).