modeling of space charge effects and CSR in bunch compression systems

SC and CSR effects are crucial for the simulation of BC systems
CSR and related effects are challenging for EM field calculation
non-CSR effects are indispensable for design and simulation of BC systems

part 1: CSR codes
  effects: SC, CSR, shape variation ...
  approaches
    Vlasov-Maxwell
    paraxial approximation
    1d
    sub-bunch
  Zeuthen benchmark example

part 2: simulation of BC systems
  space charge
codes & tools, particle distributions
  μ bunching
  non linear effects in longitudinal phase space
  example “rollover compression” (FLASH)
  example “controlled compression” (European XFEL)
  compensation in 2-bc systems
  conclusion
what is different in magnetic BC systems (compared to usual LINACS)?

- $E_{56} = 127$ MeV, $R_{56} = 180$ mm
- $E_{56} = 380$ MeV, $R_{56} = 100$ mm
- $E = 450$ MeV for $\lambda = 30$nm

$r_{56}$: there are dispersive sections with non-linear trajectories
chirp: there is a strong linear correlation between energy and longitudinal position
there is a variation of bunch shape
the ratio $I_{\text{peak}}$/Energy after compression is quite high

uniform motion(*): forces scale as $1/\gamma^2$
circular motion: some coherent effects as CSR are not suppressed with increasing $\gamma$

→ new types of tracking codes with more general electromagnetic field solvers

*) uniform motion is an approximation for the motion of a particle distribution in a drift
... effects

radiation effects

overtaking

long transients:

\[ \frac{\vec{E} + \vec{v} \times \vec{B}}{E_c} \]

field in center of bunch

centrifugal

longitudinal
shape variation

without self-interaction

top view (horizontal plane), color = energy

with self-interaction

1 m

compression

emittance growth
### approaches

<table>
<thead>
<tr>
<th>Method</th>
<th>Reference</th>
<th>Equation Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1d (or projected)</td>
<td>(1) Schneidmiller, Stupakov, Emma, Borland, Dohlus, …; ELEGANT, CSRtrack, …</td>
<td>solution of Maxwell problem based on retarded sources</td>
</tr>
<tr>
<td>sub-bunch approach</td>
<td>(2) R.Li, Kabel, Dohlus, Limberg, Giannessi, Quattromini; ???, TrafiC⁴, CSRtrack, TREDI</td>
<td>solves integral equation</td>
</tr>
<tr>
<td>Maxwell-Vlasov</td>
<td>(3) Warnock, Bassi, Ellison</td>
<td>→ “natural” boundary condition: open shielding: mirror charges → two plane model</td>
</tr>
<tr>
<td>paraxial approximation</td>
<td>(4) Agoh, Yokoya</td>
<td>Maxwell equations on grid curvilinear coordinates solves PDE</td>
</tr>
<tr>
<td></td>
<td></td>
<td>→ “natural” boundary condition: closed efficient for strong shielding generalization: resistivity, geometric wakes</td>
</tr>
</tbody>
</table>
4d Vlasov equation in beam frame: \[
\frac{\partial f}{\partial s} + z \frac{\partial f}{\partial z} + p_z' \frac{\partial f}{\partial p_z} + x \frac{\partial f}{\partial x} + p_x' \frac{\partial f}{\partial p_x} = 0 \quad \text{with} \quad f = f(r, p, s)
\]
(horizontal)

3d charge and current density distributions: \[
\rho_L(R, Y, ct) = Q \cdot H(Y) \cdot \int f(r, p, \beta ct) dp \\
J_L(R, Y, ct) = \ldots
\]
(with \(Q\) = bunch charge \(H(Y)\) = fixed vertical profile (including mirror charges) \(Y\) = vertical coordinate)

2d \((Y\)-averaged) electromagnetic fields: \[
E(R, ct) = \langle E(R, Y, ct)H(Y) \rangle_{Y \in \text{gap}} \\
B(R, ct) = \ldots
\]

EoM in beam frame: \[
z' = -\kappa(s) x \\
x' = p_x \\
p_z' = F_z \\
p_x' = \kappa(s) p_z + F_x
\]

normalized Lorentz force: \[
F_x \propto E \cdot V(s, p_x) \\
F_z \propto \ldots
\]
the problems (direct time domain calculation):
calculation window is much bigger than bunch

e.g. 2cm×8cm×6cm, σ≈100µm → V≈10^8σ^3 → large mesh
number of time steps ∝ chicane length / σ ∝ 10^6
numerical dispersion

no way with explicit schemes (my personal opinion)

but: strong shielding; calculation window can be reduced
neglect backward waves; Field(x,y,s,t) is a slowly function of s-ct

the slowly variation should allow an algorithm with large steps in s-ct
not in time domain!
paraxial approximation


\[
\left( \nabla^2 - \frac{\partial^2}{\partial t^2} \right) \tilde{E} = \mu_0 \left( \nabla \tilde{J}_0 + \frac{\partial \tilde{J}}{\partial t} \right)
\]

wave equation in time domain

“accelerator coordinates”
and Fourier transformation

\[
E_\xi(x, y, k, s) = \int \tilde{E}_\xi(x, y, s, t = s + \tau) e^{-ik\tau} d\tau
\]

with \(\xi = x, y, s\)

weak s-dependence (forward propagation)

\[
\frac{\partial^2 E}{\partial s^2} \ll 2ik \frac{\partial E}{\partial s}
\]

pipe size small compared to bend radius
\(a \ll \rho\)

relativistic particles \(\gamma >> 1\)

paraxial approximation for transverse em-fields

\[
\frac{\partial E_\perp}{\partial s} = \frac{i}{2k} \left[ \left( \nabla_\perp^2 + \frac{2k^2 x}{\rho} \right) E_\perp - \mu_0 \nabla_\perp J_0 \right]
\]

\[
\rightarrow \quad E_s = \frac{i}{k} \left[ \frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} - \mu_0 J_0 \right]
\]
advantages:
(curved) rectangular beam-pipes defined by coordinate planes
bending radius needs not to be constant
mesh based computation (explicit, frequency by frequency)
resistive wall effects
generalization to arbitrary transverse
cross-sections and smooth variation of
longitudinal profile

special care:
singularity of 1d beams
transverse beam dimensions & SC effects
variation of bunch shape

problems:
free space or large chamber
non smooth variations → stimulation of backward waves
distributions with fine structure
\[ \dot{p}_v = q(e_{v||}E^{(\lambda)}(s_v, t) + v_v \times B^{(ext)}) \]

\[ \lambda^{(\delta)}(s-t_0c) = \sum q_v \delta((s-t_0c) - (s_v - s)) \]

no SC effect, 1d E-field without $\gamma^{-2}$ singularity:

\[ E^{(\lambda)}(s, t_0) = \int \lambda'(u + s - ct_0)K(s, u)du \]

some physics is missing

no transverse self-forces

no transverse dimensions, rigid 1d charge distribution:

very low numerical effort
differences of implementations (ELEGANT vs. CSRtrack)

a) trajectory: arc after line or line after arc (neglects longer interactions, uses artificial damping);
   general sequences of arcs and lines (→ interaction with waves from objects far beyond; that requires sometimes small track steps although the net effect is weak)

b) shielding: PEC planes

c) smoothing: crucial for suppression of artificial $\mu$-bunch effects
   binning & smoothing of histograms,
   sub-bunches & density dependent adaptive filters
“CSR” codes: sub-bunch approach

- \( N \) source distributions (sub-bunches)
- \( M \) test particles
- "macro" distribution

Individual trajectories

**Self-consistent tracking:**
- Pairs of source- and test-particles \( \bigcirc \)
- + additional test particles
- \( M \geq N \)

**Perturbative tracking:**
- Source- and test-particles independent

Effort "all to all" interaction = \( N^2 E_{s\text{to}\_s} \)

\( (M = N) \)
"CSR" codes: sub-bunch approach

calculation of sub-bunches

point particles & 1d sub-bunches → singular fields

3d sub-bunch → 3d integration

\[ E(r,t) = \int \frac{Q(r',t')}{\|r-r'\|} dV' \]

calculation of 3d sub-bunches by 1d integration:

a) convolution technique

\[ E^{(3d)}(r) = \eta \otimes E^{(\lambda)}(r) \]

(approximation based on near field expansion)

b) spherical Gaussian sub-bunches

\[ \rho(r,t) = \rho_s(r - r_t(t)) \]

\[ \Phi(r,t) = \frac{q}{\varepsilon(2\pi)^{3/2}\sigma^2} \int_0^\infty f\left( \frac{r'}{\sigma}, \frac{\|r - r_t(t - r'/c)\|}{\sigma} \right) dr' \]

with \( f(a,b) = \exp\left( -\frac{a^2 + b^2}{2} \right) \frac{\sinh(ab)}{b} \)
... “CSR” codes: sub-bunch approach

reduction of effort: Green’s function on mesh

\[ E^{(v)}(r, t_0) = R \cdot E^{(0)}(r_0(t_0)) + R^{-1}(r - r_0(t_0)), t_0 \]

“Green’s function”

\[ E^{(0)}(r, t_0) = \text{field of sub-bunch on ideal trajectory calculated on mesh with } M_g \text{ points} \]

effort “all to all” interaction = \[ M_g E_{s\text{ to } s} + N^2 E_{\text{interpolation}} \]
... “CSR” codes: sub-bunch approach

reduction of effort: em-field on mesh

calculate em-field (E&B) on a mesh with $M_{\text{em}}$ points

→ Lorentz force $E + v \times B$ for each point in that volume by interpolation to the grid

effort “all to all” interaction = $N \cdot M_{\text{em}} \cdot E_{s\_to\_s}$

+ $N \cdot E_{\text{em\_interpolation}}$
em-field on mesh

sub-bunch-field on mesh
("Green's function")
... “CSR” codes: sub-bunch approach

scaling of effort
(simplified)

\[ \text{effort} = a.u \]

- p to p
- Green’s
- EM-mesh
- EM-mesh + Green’s

\[ N = M \]
Zeuthen benchmark chicane


computed by many CSR codes still a reference for new developments e.g. Maxwell-Vlasov solver

4 magnet chicane
length = 15 m
r56 = 100 mm

energy = 500 MeV / 5 GeV
charge = 0.5 nC or 1 nC
compression factor = 10
(600 A → 6 kA)
shape = Gaussian / rectangular
Zeuthen benchmark chicane
longitudinal phase space

5GeV, 1nC, Gaussian

1d codes

agreement between 1d codes
e.g. relative loss @ 14m ≈ 0.04%

relative loss @ 14m ≈ 0.06%
(differences due to transverse beam dim.?!)

P.Emma

M. Borland

M. Dohlus

TRAFIC4

Bassi, Ellison, Warnock
PAC 2005
Zeuthen benchmark chicane
horizontal phase space

5GeV, 1nC, Gaussian

available results are comparable:
weak growth of slice emittance about 1% of $1 \times 10^{-6}$ m
projected emittance $\approx 1.5 \times 10^{-6}$ m

but:

500 MeV, 1nC, trapezoid

significant differences between 1d and sub-bunch methods for lower energy;
3d & space charge effects
part 2: simulation of BC systems  
  some problems

- particle description (macro particles,  
ensembles, sub-bunch distributions  
phase space density)

- tracking with different methods (different particle descriptions)

- μ-bunching → laser heater  
  → decoupled investigation → amplification  
  → noise suppression

- longitudinal sensitivity → a) controlled compression  
  → b) “over” compression

- transverse: space charge Q shift
SC contribution to longitudinal phase space

longitudinal SC effects per length $\propto I/\gamma^2$
longitudinal SC effect in accelerator ($\gamma_1 \rightarrow \gamma_2 >> \gamma_1$) $\propto \frac{I}{\gamma_1} \cdot \frac{1 + \ln(\gamma_1)}{\gamma'}$

e.g. European XFEL:

negative chirp compensated by LINAC wakes
positive chirp induced by space charge!
utility programs:
format and/or phase space conversion
some simple manipulations of phase space:
add cavity wakes
add space charge wakes (semi analytic model)
transverse matching, …

linear trajectory codes (=LT codes):
1st principles method: particle in cell codes as MAFIA T3
working horse: Runge-Kutta tracker + Poisson solver
PARMELA, ASTRA, GPT, …
or ELEGANT + external SC calculation
Simulation of BC systems

Particle distribution

Try to simulate the complete BC system (or even s2e) with one set of particles.

Injector simulation and linear trajectory codes:
- Typical number of particles: $\sim 10^5 \ldots 10^6$
- Equal charged
- Random or semi-random distribution in 6d phase space

Noise of particle distribution is a problem in general $\rightarrow$ amplification of $\mu$-bunching
- 1d: binning, filtering (e.g. sub-bunches), adaptive to density
- Mesh: too few particles per cell (e.g. of Poison solver) are a problem $\rightarrow$
  - Increase number of particles or
  - Decrease resolution of mesh or
  - Reduce dimension of mesh (e.g. rz in ASTRA or xy in CSR codes)

Use smooth source distribution:
- Track original (s2e) particles in the field that is created by the smooth source ($\rightarrow$ and track smooth source with self interaction)

Number of particles is a problem for some CSR methods
- 1d method: similar as LT codes
- Sub-bunch methods with point to point interaction*: $N \sim 10^3 \ldots 10^4$
- Sub-bunch methods + mesh techniques*: $N \sim 10^5 \ldots 10^6$

(*) 10 .. 20 CPUs
μ-bunching - amplification

impedances (steady state):

\[ Z'_{sc}(k, \sigma_r, R_{pipe}, \gamma) \approx \frac{iZ_0 k}{2\pi\gamma^2} \ln \left( \frac{\gamma}{k\sigma_r} \right) \]  \hspace{1cm} \text{(free space, } k\sigma_r/\gamma < < 1 \text{)} \hspace{1cm} \text{“SC-instability”}  \\

\[ Z'_{CSR}(k, R_{curv}) \approx Z_0 \frac{\Gamma(2/3)}{2\pi} \left( \frac{k}{3iR_{curv}^2} \right)^{1/3} \]  \hspace{1cm} \text{“CSR-instability”}
... µ-bunching - amplification

proposed by E. Schneidmiller 2002

laser heater

picture based on: Z. Huang FLS2006
1) it is difficult to simulate macroscopic & microscopic effects together (very high resolution, very many particles required)

2) → separate investigation of µ-bunching
   CSR: integral equation method (limited applicability)
   projected method: modulated beam, 1- and 2-stage compression
   SC: impedance + r56

example: European XFEL

3) s2e simulations without µ structure:
   avoid artificial instability
   e.g. due to shot noise of few macro-particles → noise reduction
non linear effects in long. phase space
controlled compression vs. rollover compression

before BC

controlled compression:
\[ \Delta E = \text{non lin. function}(z) \]
\[ z_2 - z_1 = \text{non lin. function}(\Delta E) \]
compensation of both effects with higher harmonics rf

rollover compression:
use rollover

after BC

controlled compression:
~ uniform compression of complete distribution
very sensitive to parameter fluctuations

rollover compression:
sharp spike with less charge
insensitive to parameter fluctuations, few knobs

lost control:
magnet strength changed by 0.5%

\[ I_{\text{peak}} \approx 15 \text{ kA} \]
\[ I_{\text{peak}} \approx 3.5 \text{ kA} \]
rollover compression
example: FLASH (=VUF-FEL=TTF2)
... rollover compression
example: FLASH s2e simulation

$E = 127 \text{ MeV}$
$R_{16} = 180 \text{ mm}$

$E = 380 \text{ MeV}$
$R_{16} = 100 \text{ mm}$

$E = 450 \text{ MeV}$
for $\lambda = 30 \text{nm}$

$W = \text{wake of one TTF module}$

$W_L = \text{wake of LOLA structure}$

$\text{TM} = \text{transverse matching to design optic}$
example: FLASH s2e simulation

$q = 0.5 \text{ nC}$

Current

Energy

RMS energy spread

Horizontal emittance

Vertical emittance
example: FLASH s2e simulation
example: FLASH s2e simulation
example: FLASH s2e simulation
example: FLASH s2e simulation

\[ \Phi_{rf} \]

- RF-GUN
- ACC1
- BC2
- ACC2
- BC1
- ACC3
- BC3
- ACC4
- ACC5
- LOLA
- BYPASS
- UNDULATOR
- DUMP

**rms**

- horizontal rms 0.2 MeV
- vertical rms

**emittance**

- horizontal emittance
- vertical emittance

**energy spread**

- \( \delta E_{\text{rms}} = 0.2 \text{ MeV} \)

- current
- energy

- horizontal current
- energy with horizontal
… rollover compression
example: FLASH, extreme case

strong ‘over compression’
FLASH, 1nC, more chirp
controlled compression
eexample: European XFEL

- **gun** → $1nC \sim 50A \sim 7$MeV
- **1 x module** → $130$MeV
- **dogleg**
  - $4 x$ module + $2 x$ module-3$^{rd}$ → $500$MeV
  - bc1 → $\sim 1kA$
  - $12 x$ module → $2$GeV
  - bc2 → $\sim 5kA$
  - **main linac** → $17.5$GeV
  - collimator
  - beam distribution … undulators …

- **CSR & SC, guiding fields**
- **dispersive**
- **linear**

3$^{rd}$ harm. rf
10 keV by laser heater

1.3GHz: 442.85 MV  1.42 deg
3.9GHz:   90.63 MV 143.35 deg

≈ 50A
≈ 1kA
≈ 5kA

equp: European XFEL
example: European XFEL

after BC2

- Current: ≈5kA
- Longitudinal phase space
- RMS energy spread

after collimator

- Current
- Azimuthal phase space
- Horizontal and vertical emittance

norm. emittance

hor
vert
the CSR related growth of projected emittance in one bc can be partially compensated in a second stage if some conditions are fulfilled:
right phase advance,
right compression ratio (chirp as well as r56),
no interference with other effects as shielding or resistive walls
... compensation in 2-bc systems
shielding & resistive walls

element: compression from 100 μm → 20 μm with gap \( h \)

free space condition for CSR in circular motion:
\[
\frac{h^3}{\sigma^2} \gg R_0 \quad \text{curvature radius} \quad \frac{h^3}{\sigma^2} \propto 10^3
\]

free space condition for wave propagation after bend:
\[
\frac{2}{\pi^2} \frac{h^2}{\sigma} \gg L_d \quad \text{length of drift} \quad \frac{2}{\pi^2} \frac{h^2}{\sigma} \propto 1
\]

inside of a chicane:
if is difficult to avoid shielding
shielding in a long drift: resistive wall effects
part II
- effects in BC **systems** are challenging (many physical effects are involved)
- μ-bunching effects beyond the resolution of non-1d-codes
- several types of codes needed (LT- and CSR-codes)

part I
- 1d- and sub-bunch codes are available
  Vlasov-Maxwell approach and paraxial approximation under development
- resolution of sub-bunch method increased
- ‘CSR’ methods cover all important physical effects
  (SC, CSR, shape variation, shielding, resistive walls)

**in reach: code that covers all effects**