Beam Dynamics Studies for the PITZ Undulator

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Introduction
THz SASE FEL at DESY PITZ

Motivation:
Development of a high power, tunable THz source for European XFEL
⇒ THz radiation at $\lambda \sim 100$ $\mu$m, $\sim 6$ ns pulse length, and $\sim 38$ MW peak power

THz-FEL Parameters:
- Bunch charge $Q_b \sim 4$ nC
- Beam energy $E_b \sim 16.7$ MeV
- Und. length $L_u \sim 3.4$ m
- Und. period $\lambda_U \sim 3$ cm

Challenge:
Transport of SC dominated beam
⇒ SC beam dynamics simulation

Further Information:
M. Krasilnikov et al., „Start-to-End Simulations of THz SASE FEL Proof-of-Principle Experiment at PITZ“, ICAP’18, Key West
Numerical Models
2D Undulator Model

Analytic 2D Undulator Field Model:

\[ \vec{B}(\vec{r}) = B_0(z) \cosh\left(\frac{2\pi}{\lambda_u} y\right) \sin\left(\frac{2\pi}{\lambda_u} z\right) \hat{e}_y + B_0(z) \sinh\left(\frac{2\pi}{\lambda_u} y\right) \cos\left(\frac{2\pi}{\lambda_u} z\right) \hat{e}_z \]

with period \( \lambda_u = 3 \text{ cm} \), total length \( L_u = 120 \lambda_u \), and tapered \( 0 \text{T} \leq B_0(z) \leq 1.28 \text{T} \)

2D Undulator Field on Axis:
Numerical Models
Idealized 2D Undulator

Periodic field
map w. tapering:

⇒ $B_y(x, y, z)$

Focusing in $y$:

⇒ $B_z(x, y, z)$

Linear tapering:

⇒ $\nabla \cdot \vec{B} \neq 0$
Numerical Models
2D Undulator Model

Beam Dynamics in the Undulator w/o SC:

![Graph showing beam dynamics](image)
Numerical Models
Space Charge Models and Codes

Inertial Frame Approach:

Electrostatic solver in bunch „rest“ frame
⇒ Approximation exact if $\beta = \text{const.}$ and $\Delta \beta = 0$
⇒ Missing effects: Non-inertial frame, velocity dispersion, radiation
⇒ Codes: Astra3D (PIC-FFT), Krack (PIC-FFT), TEMF-Code (PP), REPTIL (FMM)

\[ \gamma \approx 32 \]
Numerical Models
Space Charge Models and Codes

Local Inertial Frame Approach:

Electrostatic solver for local particle frame
⇒ Approximation exact if $\beta_{\text{particle}} = \text{const}$.
⇒ Missing effects: Nonlinear trajectory, radiation
⇒ Codes: TEMF-Code (PP), “REPTIL (w. energy binning)”
Numerical Models
Space Charge Models and Codes

Liénard-Wiechert Approach:

Full electromagnetic solver
⇒ Evaluation of time-retardation $|\vec{r_i} - \vec{r_j}| = c \left( t_i - t_j \right)$
⇒ Liénard-Wiechert fields include radiation
⇒ Codes: TEMF-Code (PP)

For $t_j < t_0$ rigid bunch initialization:

⇒ $z_j \left( t_j \right) = z_j \left( t_0 \right) - c \beta_z \left( t_0 \right) \left( t_0 - t_j \right)$

Radiation Off:

⇒ Setting $\frac{d\beta}{dt} = 0$ neglects radiation field
Simulation Results
Retardation and Radiation Effects

Transversal Bunch Size Growth:

- Bunch size $\Delta X_{rms}$ for IF model larger than for LIF model
  $\Rightarrow$ Artificial increase of static space charge effects
- Bunch size $\Delta X_{rms}$ for RO smallest
  $\Rightarrow$ Static space charge effects not dominating
Simultation Results
Retardation and Radiation Effects

$\Rightarrow$ Inertial frame approaches overestimate static space charge field
Simulation Results
Retardation and Radiation Effects

Transversal Bunch Size Growth:

- Good agreement of IF, Astra3D, Krack and Reptil simulations
- Small difference in $\Delta X_{rms}$ between IF and LW in bunch size
- Bunch size for RO significantly smaller than for LW
  $\Rightarrow$ Radiation leads to higher emittance
Simulation Results
Retardation and Radiation Effects

⇒ Radiation fields dominate space charge beam dynamics
Simulation Results
Retardation and Radiation Effects

Longitudinal Bunching:

THz-Wavelength $\lambda_{THz} = \frac{\lambda_U}{2\gamma^2} \left(1 + \frac{K^2}{2}\right) \approx 105 \, \mu m$

Micro-bunching consistent with $\lambda_{THz}$
Simulation Results
Retardation and Radiation Effects

THz-Wavelength Dependency:

Longitudinal Phase Space $B_0 = 1.00$ T

Longitudinal Phase Space $B_0 = 1.28$ T
Simulation Results
Retardation and Radiation Effects

Particle Bunch at Undulator Exit:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>LW-model</th>
<th>IF-model</th>
<th>Rel. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta x_{rms}$</td>
<td>3.1 mm</td>
<td>3.1 mm</td>
<td>+0.5%</td>
</tr>
<tr>
<td>$\Delta y_{rms}$</td>
<td>2.6 mm</td>
<td>2.6 mm</td>
<td>−0.9%</td>
</tr>
<tr>
<td>$\Delta z_{rms}$</td>
<td>2.2 mm</td>
<td>2.2 mm</td>
<td>−2.3%</td>
</tr>
<tr>
<td>$\delta E_{rms}$</td>
<td>347.4 keV</td>
<td>95.7 keV</td>
<td>−72.5%</td>
</tr>
<tr>
<td>$\epsilon_x$</td>
<td>17 $\pi$ mrad mm</td>
<td>13 $\pi$ mrad mm</td>
<td>−24.6%</td>
</tr>
<tr>
<td>$\epsilon_y$</td>
<td>10 $\pi$ mrad mm</td>
<td>7 $\pi$ mrad mm</td>
<td>−30.5%</td>
</tr>
<tr>
<td>$\epsilon_z$</td>
<td>1366 $\pi$ mrad mm</td>
<td>282 $\pi$ mrad mm</td>
<td>−79.4%</td>
</tr>
</tbody>
</table>

- IF-model provides reasonable estimate for bunch size
- IF-model underestimates transversal emittance
- Strong deviation for energy spread and long. emittance
Simulation Results
Retardation and Radiation Effects

Radiation Field:

Screen at $z = 30.8 \text{ m}$

Spatial res. $\Delta x = 100 \mu\text{m}$
($\lambda_{THz} \approx 105 \mu\text{m}$)

Temporal res. $\Delta t = 0.1 \text{ ps}$
($T_{THz} \approx 0.35 \text{ ps}$)

$\Rightarrow$ Memory limitation for $\Delta t$

... work in progress
Simulation Results
Retardation and Radiation Effects

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… work in progress
Summary & Outlook

Summary:
- Liénard-Wiechert simulations of THz-FEL undulator with $N \leq 100k$ particles
- Inertial frame models overestimate static space charge effects
- Radiation effects dominate space charge beam dynamics
- Astra3D, Krack, and Reptil provide reasonable estimates for beam size, but cannot reproduce momentum space (no general statement)

Outlook:
- Validation of Liénard-Wiechert simulations with CST EM-PIC
- Implementation of realistic undulator field map
- Study of bunch parameters: charge, size, etc…
- Approximation of particle world lines $\Rightarrow$ red. LW-code memory requirements
- Approximation of retardation & retardation effects in Reptil $\Rightarrow$ red. runtime

...thanks to M. Krasilnikov for the provided data.