Introduction to Elementary Particle Physics

Philip Bechtle

August 2011

Before we start

Please

- For the next two lectures: You may want to print the slides and take notes on them during the lecture
- Please ask questions anytime whenever you have one
- Interrupt if I'm too fast, or
- Speed me up if I'm telling you stuff which has been told several times before
- Sometimes, you'll hear about some crazy stuff which is not completely explained in this lecture. In this case: Ask questions and look forward to the more advanced lectures later on.
- Let's have as much interesting discussion as possible!

Some (typically more theory-oriented) literature

- Martin, Shaw: Particle Physics; Wiley 1997
- Halzen, Martin: Quarks and Leptons; Wiley 1984
- Griffiths: Introduction to Elementary Particle Physics; Wiley 2008
- Perkins: Introduction to High Energy Physics
- Particle data booklet, see `http://pdg.lbl.gov` or `http://pdg.web.cern.ch`

Motivation

- We live in truly exciting times
- The LHC is a huge success
- Recent results could mean that the Higgs boson might be discovered soon
- The end of the reign of the SM is eagerly avaited
- You have the chance to witness and actively contribute to a new era of revolution in particle physics

	Hauptgruppen-Elemente		Nebengruppen-Elemente (d-Elemente)										Hauptgruppen-Elemente					Edelgase
	IA	IIA	IIIB	IVB	VB	VIB	VIIB	VIIIB			IB	IIB	IIIA	IVA	VA	VIA	VIIA	VIIIA
Gruppe	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
	1 H 1.0079																	2 He 4.0026
	3 Li 6.941	4 Be 9.0122											5 B 10.81	6 C 12.01115	7 N 14.0067	8 O 15.9994	9 F 18.9984	10 Ne 20.179
	11 Na 22.9898	12 Mg 24.305											13 Al 26.9815	14 Si 28.086	15 P 30.9738	16 S 32.08	17 Cl 35.453	18 Ar 39.948
	19 K 39.09	20 Ca 40.08	21 Sc 44.956	22 Ti 47.90	23 V 50.941	24 Cr 51.996	25 Mn 54.9380	26 Fe 55.847	27 Co 58.9332	28 Ni 58.71	29 Cu 63.45	30 Zn 65.37	31 Ga 69.72	32 Ge 75.59	33 As 74.9216	34 Se 78.96	35 Br 79.909	36 Kr 83.80
	37 Rb 85.467	38 Sr 87.62	39 Y 88.906	40 Zr 91.22	41 Nb 92.9064	42 Mo 95.94	43 Tc 98.906	44 Ru 101.07	45 Rh 102.905	46 Pd 106.4	47 Ag 107.868	48 Cd 112.40	49 In 114.82	50 Sn 118.69	51 Sb 121.75	52 Te 127.60	53 I 126.904	54 Xe 131.30
	55 Cs 132.905	56 Ba 137.33	57 La 138.905	72 Hf 178.49	73 Ta 180.947	74 W 183.85	75 Re 186.2	76 Os 190.2	77 Ir 192.2	78 Pt 195.09	79 Au 196.967	80 Hg 200.59	81 Tl 204.37	82 Pb 207.2	83 Bi 208.980	84 Po (209)	85 At (210)	86 Rn (222)
	87 Fr (223)	88 Ra 226.025	89 Ac (227)	104 Rf 261.109	105 Db 262.114	106 Sg 263.118	107 Bh 262.123	108 Hs	109 Mt	110 Ds	111 Uuu	112 Uub	113	114 Uuq	115	116	117	118

f-Elemente (Seltene Erden)

Lanthaniden	58 Ce 140.12	59 Pr 140.907	60 Nd 144.24	61 Pm (145)	62 Sm 150.4	63 Eu 151.96	64 Gd 157.25	65 Tb 158.925	66 Dy 162.50	67 Ho 164.930	68 Er 167.26	69 Tm 168.934	70 Yb 173.04	71 Lu 174.97
Actiniden	90 Th 232.038	91 Pa 231.036	92 U 238.029	93 Np 237.05	94 Pu (244)	95 Am (243)	96 Cm (247)	97 Bk (247)	98 Cf (251)	99 Es (254)	100 Fm (257)	101 Md (258)	102 No (259)	103 Lr (260)

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No, I didn't choose the wrong subject . . .

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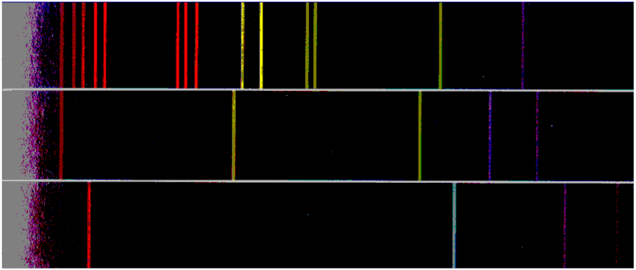
Even more order on the level of Atoms



Ba

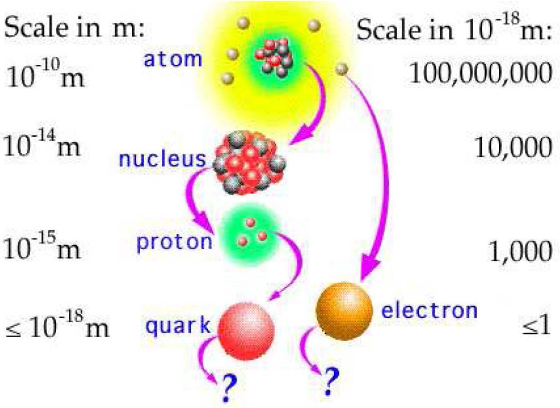
He

H

The Search for the Fundamental Order of nature

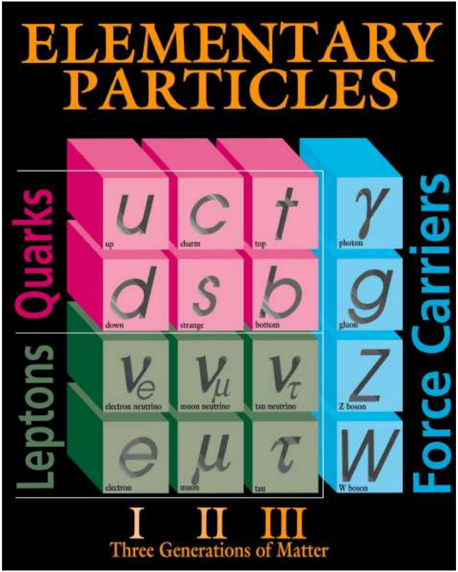
Rutherford
Geiger Marsden
1909

HERA
LEP
Tevatron
LHC
2009

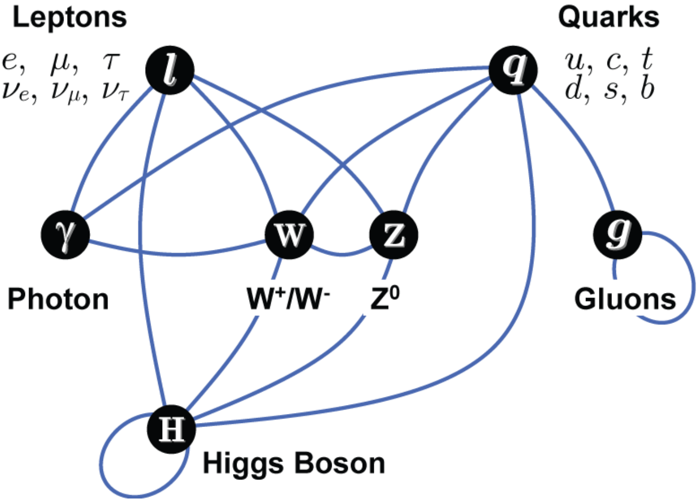


We achieved a lot in the last 100 years . . .

Our Current Picture of Elementary Particles



The Standard Model of Elementary Particles



„Dass ich erkenne, was die Welt im Innersten zusammenh"alt"

Why we know that we missed something

- Experimental Knowledge: The SM is incomplete!

- In the SM, there are no particles with the correct properties for Dark Matter


© supermagnete.de

Why is the electromagnetic force of the tiny magnet stronger than the gravity of all the earth combined?

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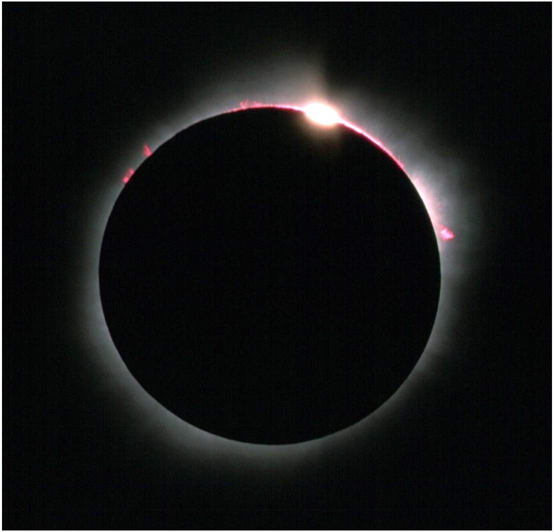
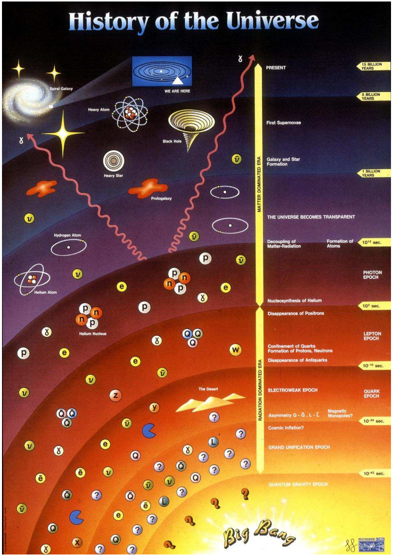
A warning: Order without fundamental reason



Particle Physics is also about History



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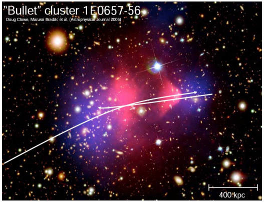
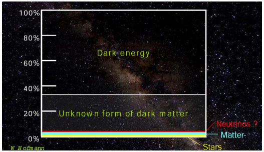
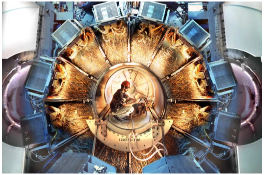
The “everlasting” goals of particle physics

- What are the fundamental building blocks of Nature?
- What are the interactions between them?
- Where does the mass of the particles originate?
- What is the structure of space and time?
- What is dark matter? Or even dark energy?
- Why is antimatter different from matter?







The "Common Knowledge" about particles

Three Generations of Matter (Fermions)

		I	II	III	Bosons (Forces)
Quarks	mass → charge → spin → name →	2.4 MeV, ⅔, ½, **u**, up	1.27 GeV, ⅔, ½, **c**, charm	171.2 GeV, ⅔, ½, **t**, top	0, 0, 1, **γ**, photon
Quarks		4.8 MeV, -⅓, ½, **d**, down	104 MeV, -⅓, ½, **s**, strange	4.2 GeV, -⅓, ½, **b**, bottom	0, 0, 1, **g**, gluon
Leptons		<2.2 eV, 0, ½, **ν_e**, electron neutrino	<0.17 MeV, 0, ½, **ν_μ**, muon neutrino	<15.5 MeV, 0, ½, **ν_τ**, tau neutrino	91.2 GeV, 0, 1, **Z^0**, weak force
Leptons		0.511 MeV, -1, ½, **e**, electron	105.7 MeV, -1, ½, **μ**, muon	1.777 GeV, -1, ½, **τ**, tau	80.4 GeV, ±1, 1, **$W^\pm$**, weak force

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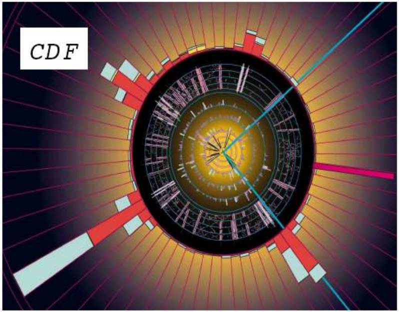
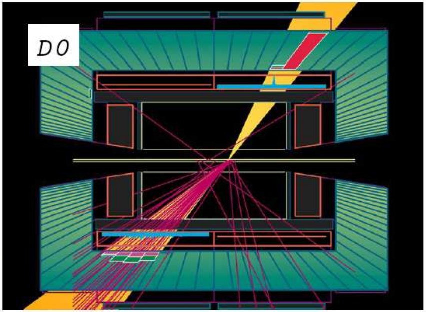
Discoveries at Accelerators

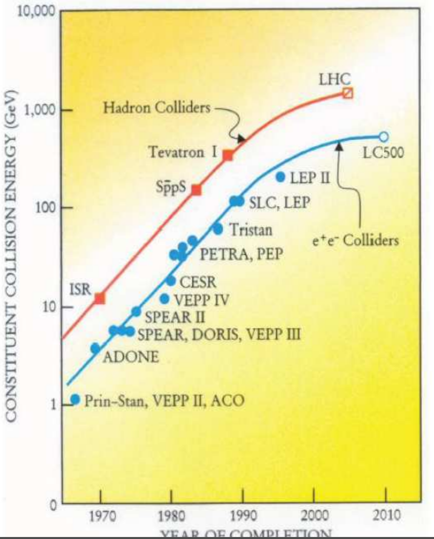
Predicted discovery of the top quark at the Tevatron 1995:





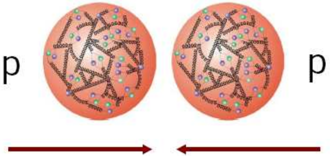
- The history of physics is full of predicted discoveries: $e^+, n, \pi, q, g, W, Z, c, b$
- Most recent example: top quark
- Future examples: Higgs, SUSY ???

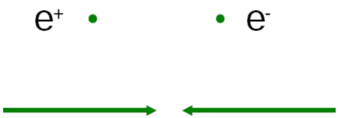
High Energy Physics: Shifting the The Energy Frontier



- The interplay between electron and hadron machines has a long and fruitful tradition
  - J/ψ at SPEAR (e^+e^-) and AGS (proton fixed target)
  - Υ discovery at E288 (p fixed target), precision B studies at the e^+e^- B factories
  - ...
  - top quark at LEP and Tevatron
- To be continued in the form of LHC and ILC

Complementarity of pp and e^+e^- machines

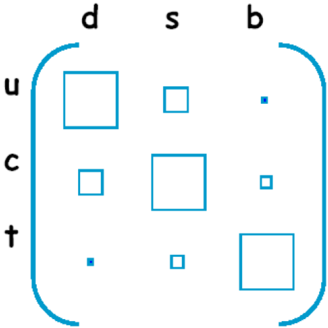




- Proton-(Anti-)Proton Colliders
  - Higher energy reach (limited by magnets)
  - Composite particles: unknown and different colliding constituents, energies in each collision
  - Confusing final states
- Discovery machines (W, Z, t)
- In some cases: precision measurements possible (W mass at the Tevatron)

- Electron-Positron-Colliders
  - Energy reach limited by RF
  - Point like particles, exactly definded initial system, quantum numbers, energy, spin polarisation possible
  - Hadronic final states with clear signatures
- Precision machines
- Discovery potential, but not at the energy frontier

High Energy Physics is not ONLY about discovering particles



- Quark mass eigenstates = eigenstates of the quark-Higgs-interaction
- Quark mass eigenstates $\neq$ eigenstates of the weak interaction
- Wqq' vertex: transition between different quarks: CKM matrix
- Kobayashi, Maskawa 1973: If at least 3 generations, matrix can be complex $\Rightarrow$ CP-violation
- Prediction of the b and t mesons
- Discovery of the b 1977
- Precision tests at e^+e^- B-factories

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Time to Breath, Think and Ask

Accelerators: Basics

- Want: As many colliding particles as possible at the highest possible energy
- Energy is connected to resultion: deBroglie wave length $\lambda = \frac{h}{p} = \frac{hc}{\beta E}$
- Energy is connected to the mass of particles that can be produced: $E = mc^2$
- Therefore, we probe the smallest things at the highest possible energies

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- Therefore, we probe the smallest things at the highest possible energies
- For highest energies: Want colliding beams, not fixed target:
  - Fixed target:

$$m_X^2 = p_x^2 = (p_B + p_T)^2 = ((E_B, 0, 0, p_B) + (m_T, 0, 0, 0))^2$$

$$m_X \leq \sqrt{s} \approx \sqrt{2 m_T p_B}$$

  - Colliding Beams:

$$m_X^2 = p_x^2 = (p_B + p_B)^2 = ((E_B, 0, 0, p_B) + (E_B, 0, 0, -p_B))^2$$

$$m_X \leq\leq \sqrt{s} = 2\sqrt{E_B E_B} = 2E_B$$

Accelerators: Basics

Requirements:

- Highest possible beam energy ($\sqrt{s}$, heavy m_X, small $\lambda \rightarrow$ resolution)
- Highest possible beam intensity: Luminosity

$$\mathcal{L} = \frac{\mathrm{d}N}{\mathrm{d}t} / \sigma \approx \frac{nfN_1 N_2}{\sigma_x \sigma_y}$$

- Best possible beam quality: Energy spread, focussing
- For more details see lecture on accelerators

Accelerators: The Synchrotron

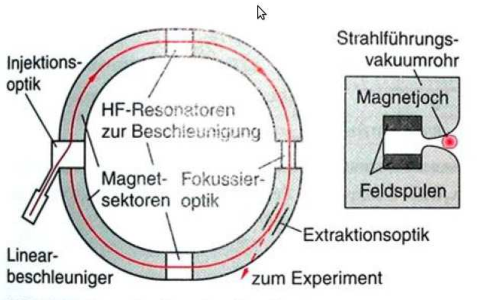
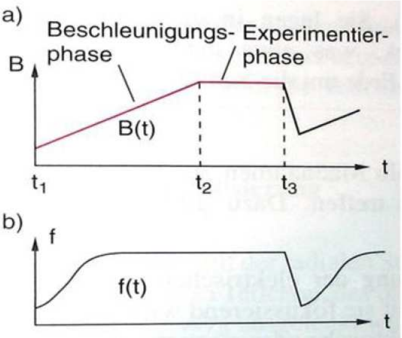


Abb. 4.17. Grundaufbau des Synchrotrons



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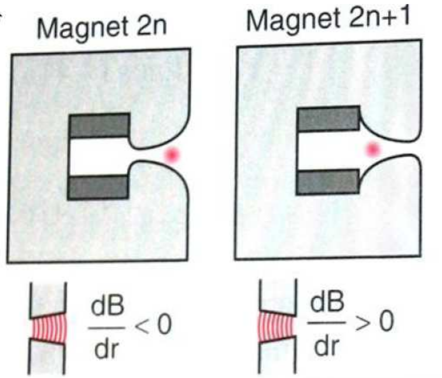
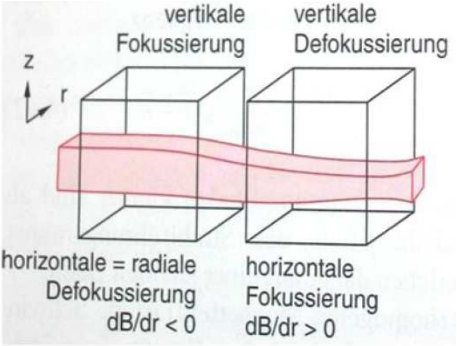
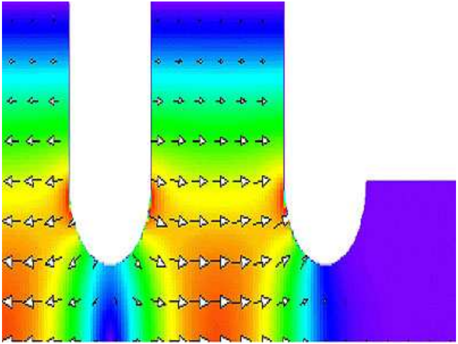




Accelerators: Acceleration

- Accelerate by using a Cavity: Radio Frequency Resonator with $f \approx$ GHz
- Use superconducting material like Nb at $T = 1.8\,K$
- Problem: $E(t) \rightarrow B(t)$, but B fiekd destroys Cooper pairs
- $E_{eff} \approx 35\,\mathrm{MV/m}$ in the currently best cavities
- Not problematic for hadron colliders



Accelerators: Circular vs. Linear

Forgive me, it's in German, but the formulas are enough . . .

Die abgestrahlte Leistung ist:

$$P = \frac{2}{3}\frac{e^2}{4\pi\varepsilon_0}\frac{e^2c^3}{(mc^2)^4}E_0^2B^2$$

Für den Krümmungsradius ρ gilt: $B = \frac{p_0}{e\rho} \approx \frac{E_0}{e\rho c}$

Damit ist der Energieverlust pro Umlauf

$$\Delta E_0 = \frac{e^2}{3\varepsilon_0}\left(\frac{E_0}{mc^2}\right)^4\frac{1}{\rho} \Rightarrow \Delta E_0[GeV] = 8.85x10^{-5}\frac{(E[GeV])^4}{\rho[m]} \text{ für Elektronen}$$

Beispiel LEP (CERN):

$$E_0 = 100\,\text{GeV und } \rho = 4.2km \rightarrow \Delta E_0 = 2.8GeV!$$

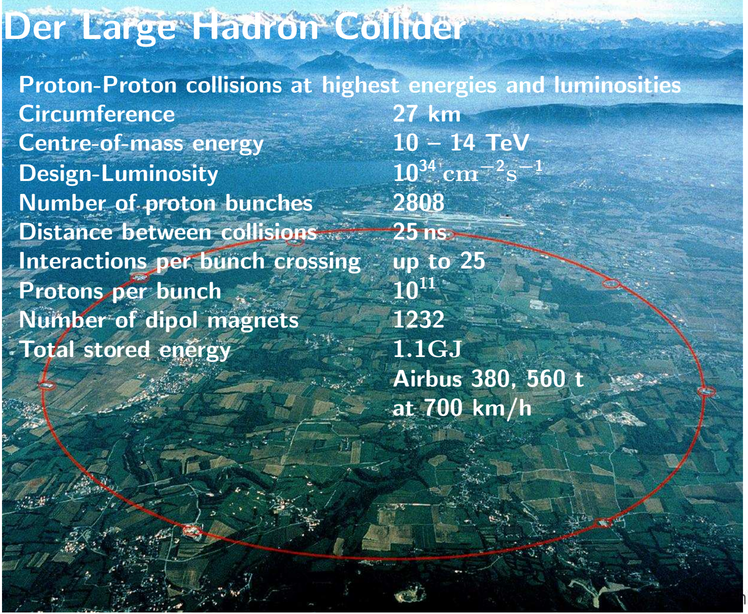
→ Hohe Elektronenenergien nur mit Linearbeschleunigern!

Time to Breath, Think and Ask

The large Hadron Collider LHC

The most powerfull collider ever
27km long, 100m below surface

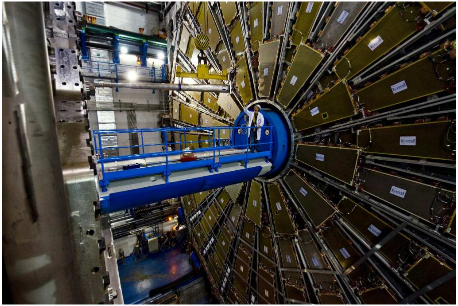
Der Large Hadron Collider

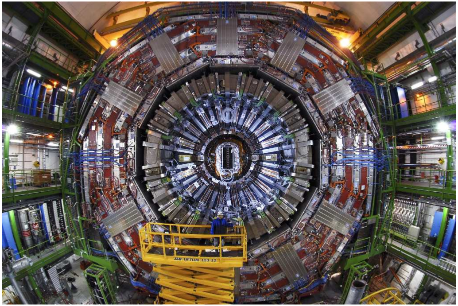


Proton-Proton collisions at highest energies and luminosities

Circumference	27 km
Centre-of-mass energy	10 – 14 TeV
Design-Luminosity	$10^{34}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$
Number of proton bunches	2808
Distance between collisions	25 ns
Interactions per bunch crossing	up to 25
Protons per bunch	10^{11}
Number of dipol magnets	1232
Total stored energy	$1.1\mathrm{GJ}$
	Airbus 380, 560 t at 700 km/h

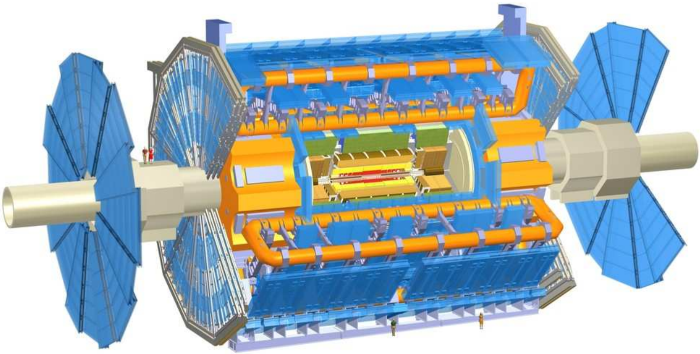
The LHC





Example: The ATLAS Experiment

Together with CMS: The fastest and biggest digital camera on earth:



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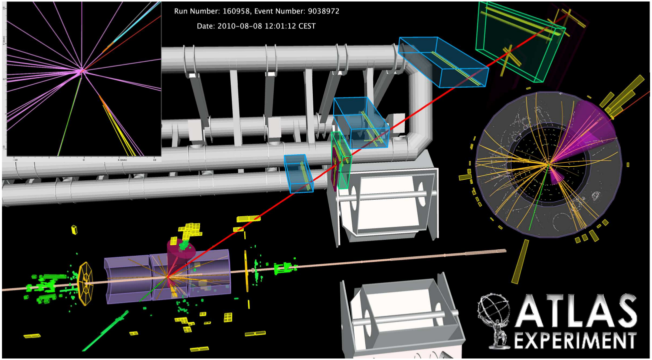

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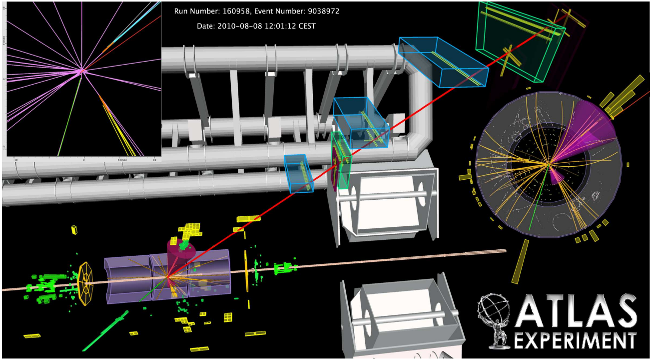
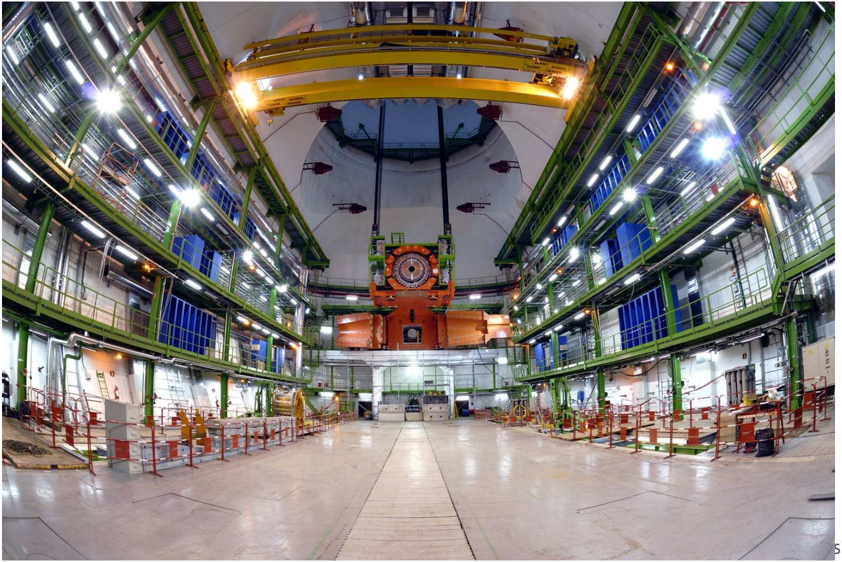


Design: 40 Millionen Pictures per second!

Currently: about 1/10th of the design, but 4 M Pictures per second is already pretty impressive

Data stream corresponds to 250 000 DVDs per second

Setting up the experiments



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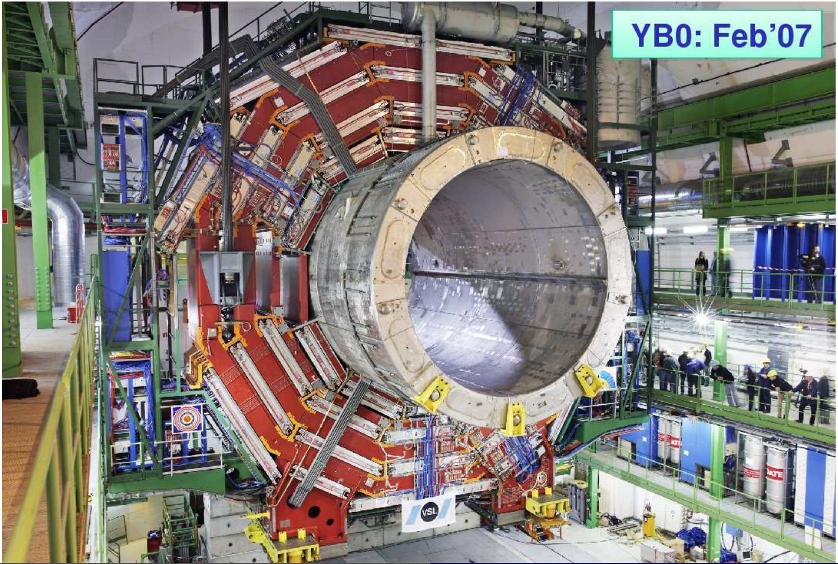

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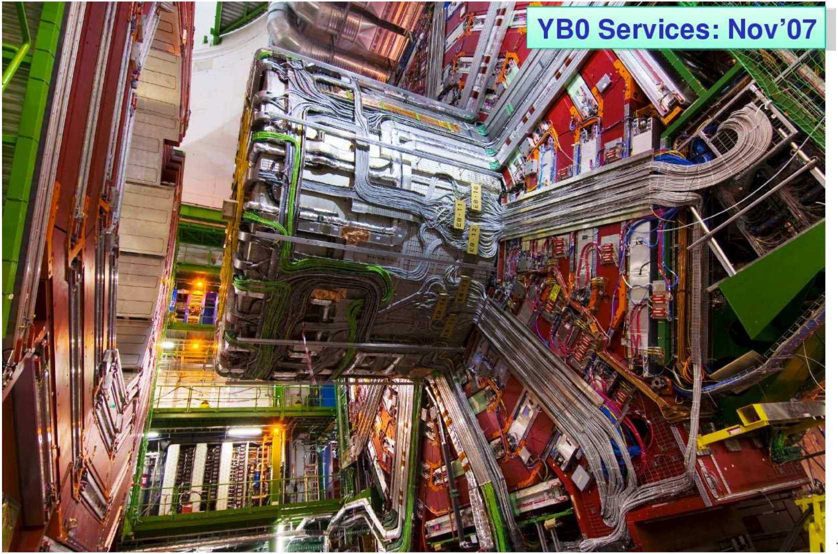

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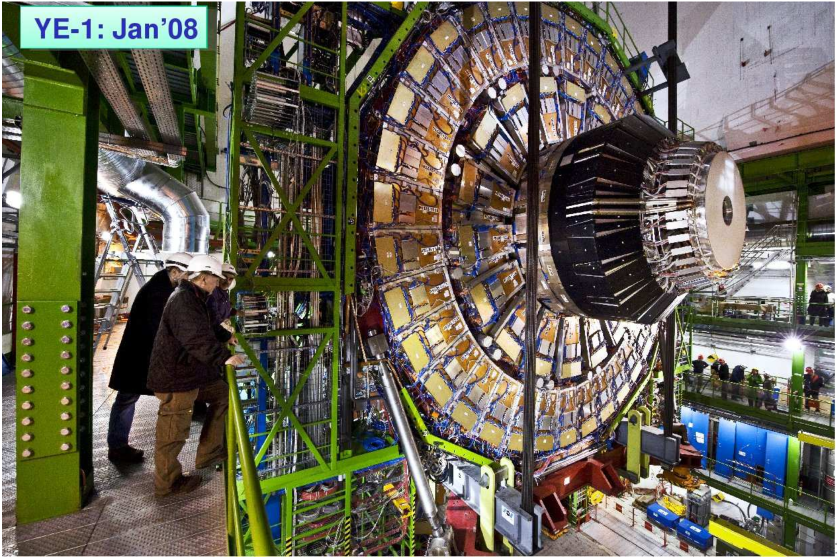
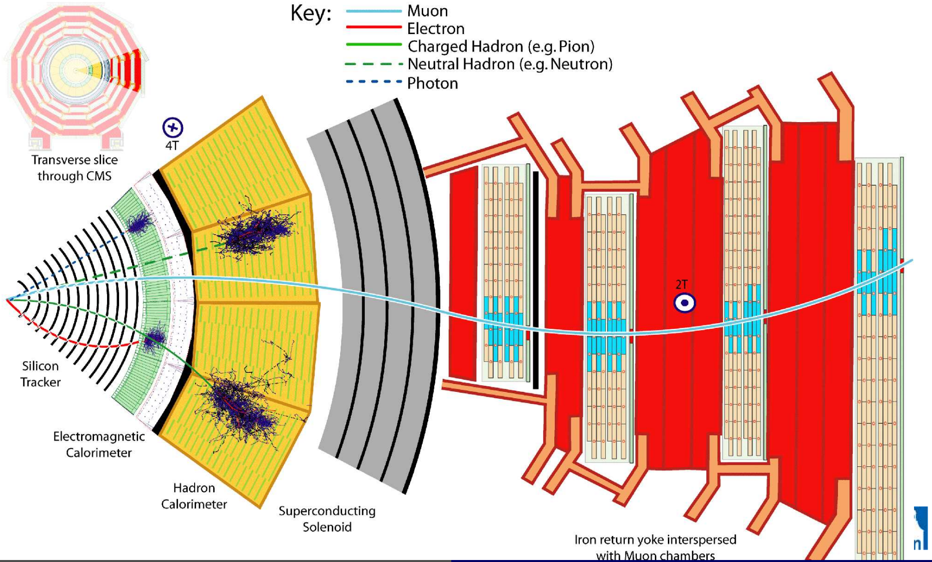


Setting up the experiments

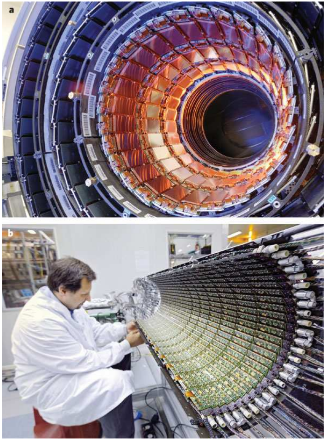
Let's have a detailed look

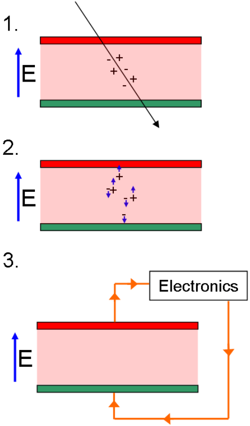


What do we need to measure?

- From where do all the particles come? $\rightarrow$ Vertex Detector
- Are there secondary decays (e.g. $B^o \rightarrow W^- c + X$)?
  $\rightarrow$ Vertex Detector
- Where do all the particles point to?
  $\rightarrow$ Vertex Detector, Tracking Detector
- What are all the momenta of the charged particles ($r = p/(eB)$)?
  $\rightarrow$ Tracking Detector, Magnetic Field
- What is the energy of all particles? $\rightarrow$ Calorimeters
- Identify the particles $\rightarrow$ all detectors!
  - $\pi^\pm, K^\pm, e$: e.g. $\mathrm{d}E/\mathrm{d}x$ in Tracking Detector
  - $\pi^\pm, e$: Fraction of energy in the gebinning and the end of the calorimeter
  - μ: Muon System outside of the calorimeters
  - $D, B, \ldots$: Vertex Detector
  - ...

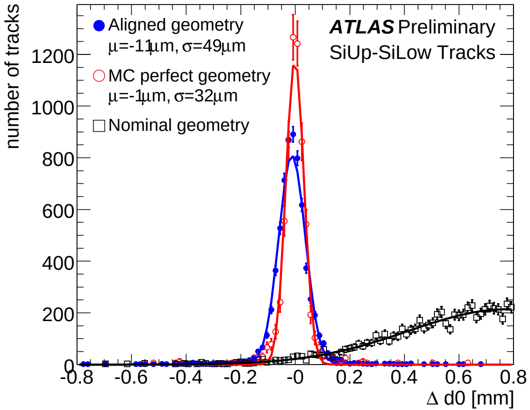
Vertexing and Tracking: Vertexing

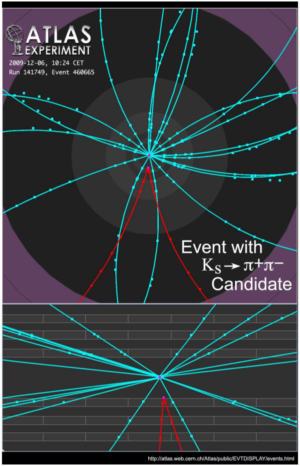




Extreme requirements: Radiation hard, extremely fast (timestamping within 25ns), readout of all channels at $> 100kHz$, high occupancies $> 10^{-4}$

Examples for Vertexing Performance





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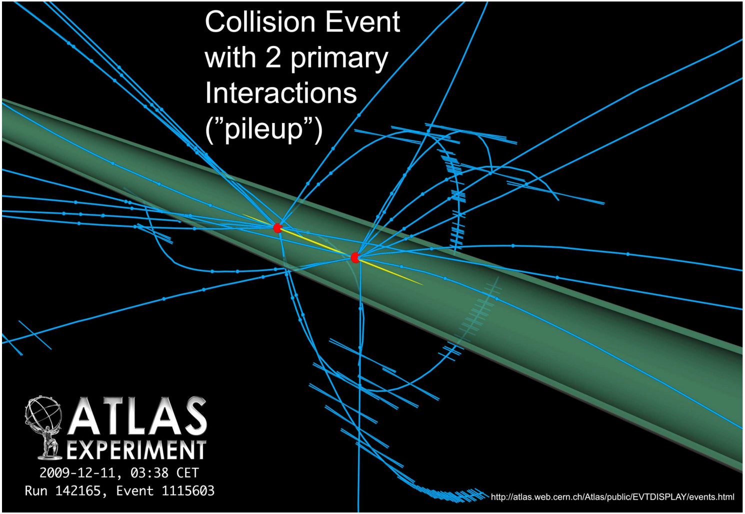
Examples for Vertexing Performance



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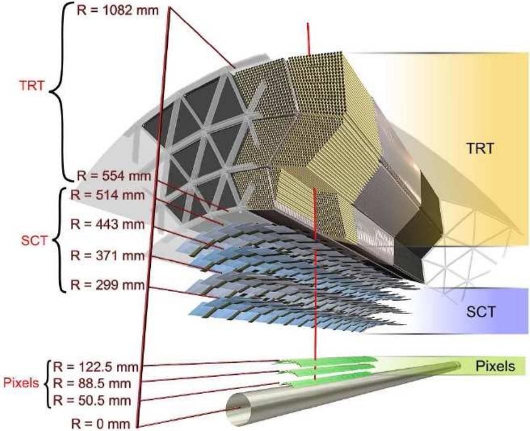
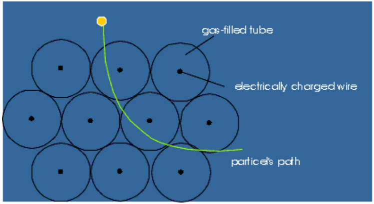
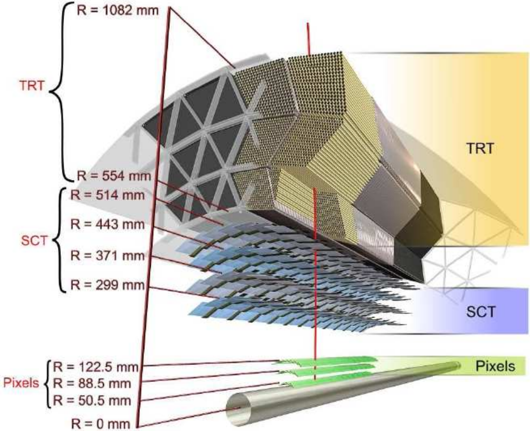
Vertexing and Tracking: The ATLAS tracking

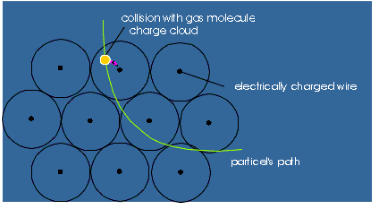




Tracking detectors should in principle be build out of nothing – they should not disturb the path of the particles and lead to no significant energy loss . . .

Vertexing and Tracking: The ATLAS tracking





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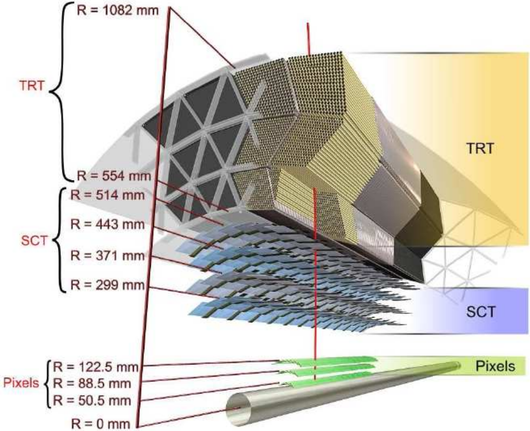

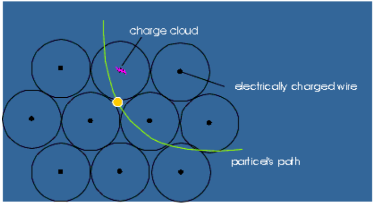

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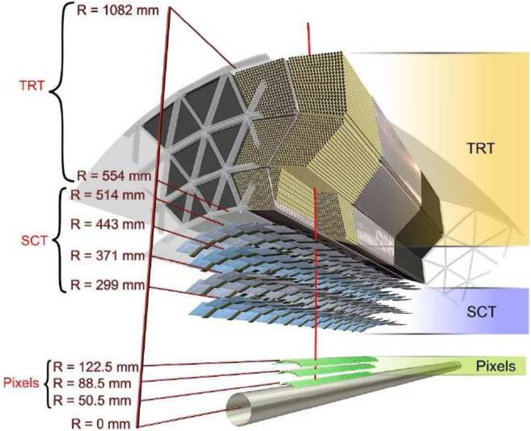

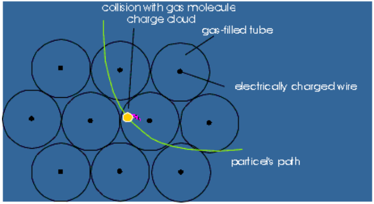

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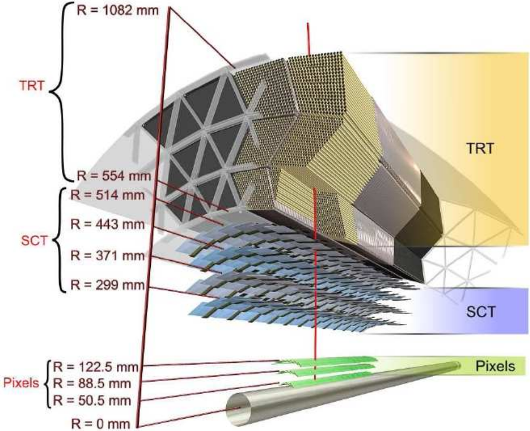

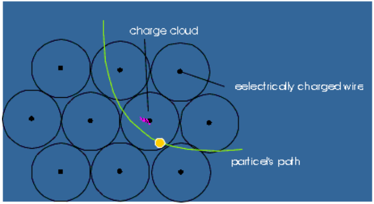

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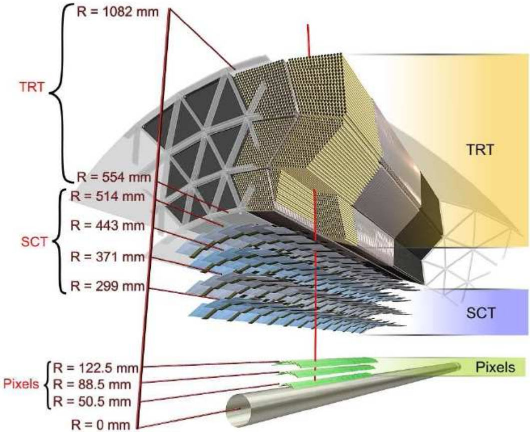

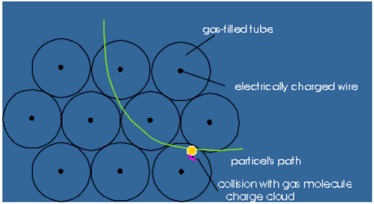

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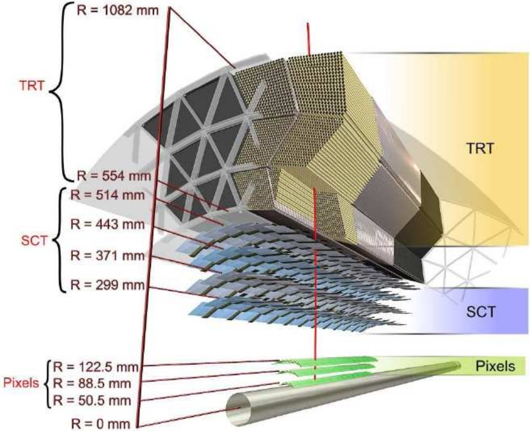
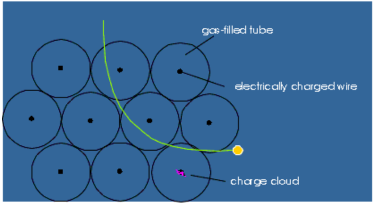
Vertexing and Tracking: The ATLAS tracking

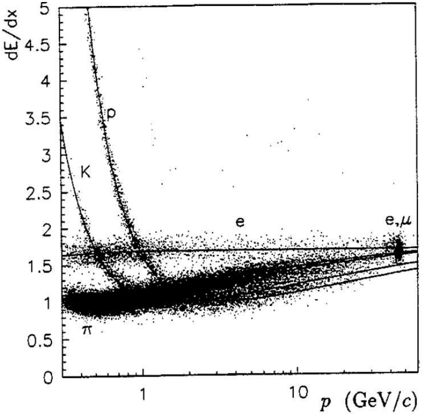




Tracking detectors should in principle be build out of nothing – they should not disturb the path of the particles and lead to no significant energy loss . . .

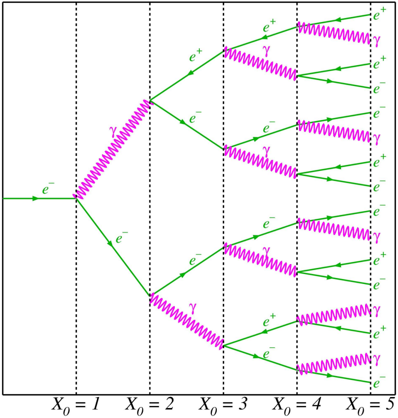
Particle Identification: Example

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = 4\pi\, N_\mathrm{A}\, r_\mathrm{e}^2\, m_\mathrm{e}\, c^2\, z^2\, \frac{Z}{A}\, \frac{1}{\beta^2} \left[\ln\left(\frac{2m_\mathrm{e}\, c^2\, \gamma^2\, \beta^2}{I} \right) - \beta^2 - \frac{\delta}{2} \right]$$



- Maesure $\mathrm{d}E/\mathrm{d}x$ from signal hight
- Measure p from $r = \frac{p}{eB}$
- Get β from $p = \beta\gamma m$
- Only one solution for m!

Electromagnetic Calorimeter

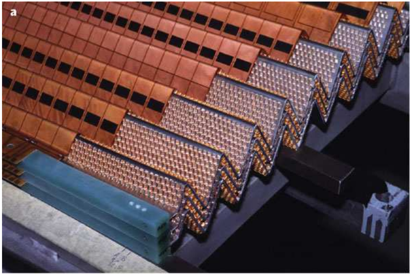


- For particles which interact only electromagnetically (γ, e, μ):
- Every radiation length X_0: approximately 1 oair production or Bremsstrahlung
- Hardly any energy transfer to the material
- All Energy visible!

Play around yourself:

`http://www2.slac.stanford.edu/vvc/egs/basicsimtool.html`

Sampling vs. Monolithic Calorimeters



- ATLAS: sampling ECAL made of LiAr and Pb
- Dense, short X_0
- Resulution reduced by energy captured in Pb
- CMS: Homogenious $PbWO_4$ crystals
- Expensive, difficult to make
- No energy lost in inactive detector part: Great resolution

Examples for ECAL resolutions

Photons at 100 GeV

ATLAS: 1-1.5%

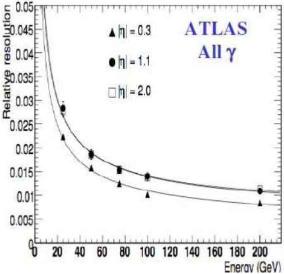
energy resol. (all γ)

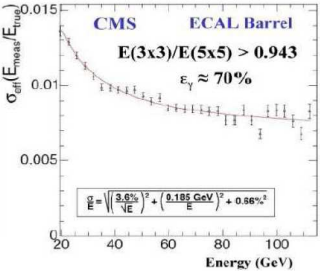
CMS: 0.8%

energy resol.

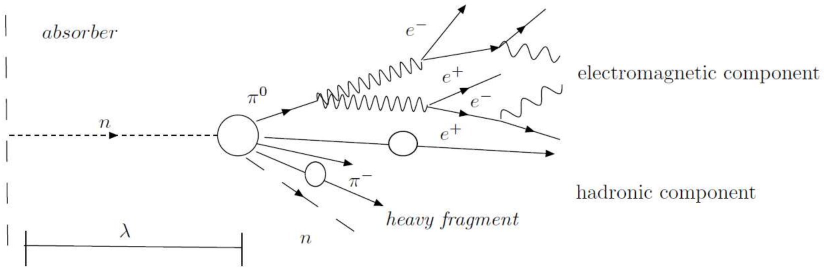
($\varepsilon_\gamma \sim 70\%$)

Electrons at 50 GeV



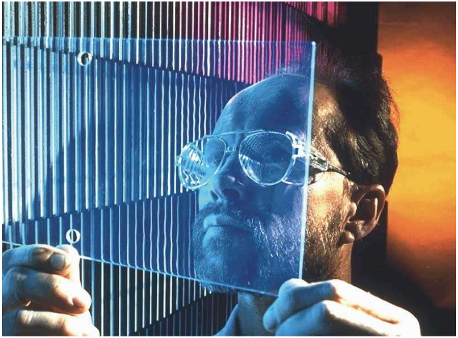


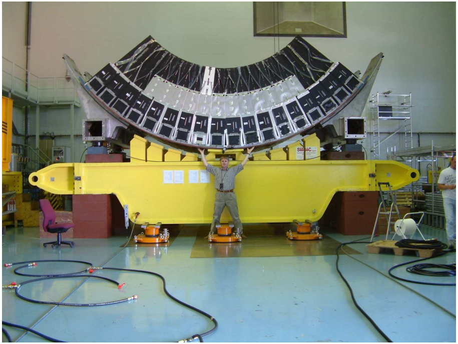
Hadronic Calorimeter



- complex composition
- hadronic interactioon lengtt $\lambda >> X_0$ (why?)
- Energy transfer to disrupt nuclei $\rightarrow$ not all energy visible!

Hadronic Calorimeters





- Usually want $10\,\lambda$: Always use sampling calorimeters, and they are huge!
- E.g. stainless steel and plastic scintillator

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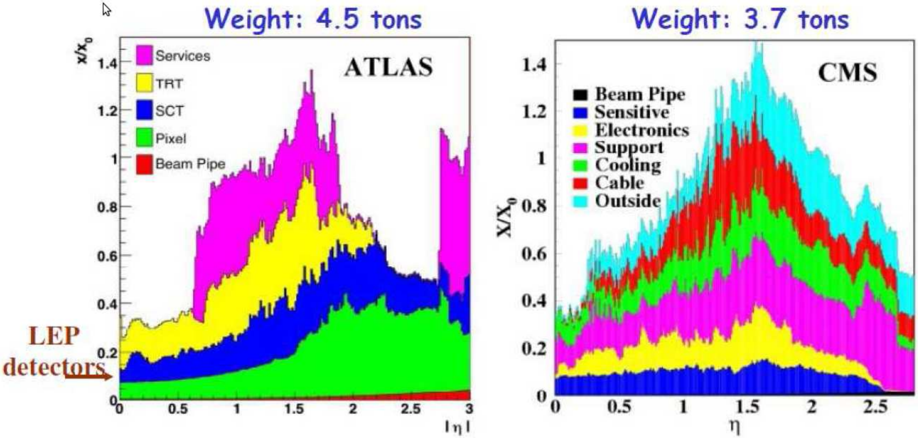
Hadronic Calorimeters: Typical Resolutions at Hadron Collider Detectors

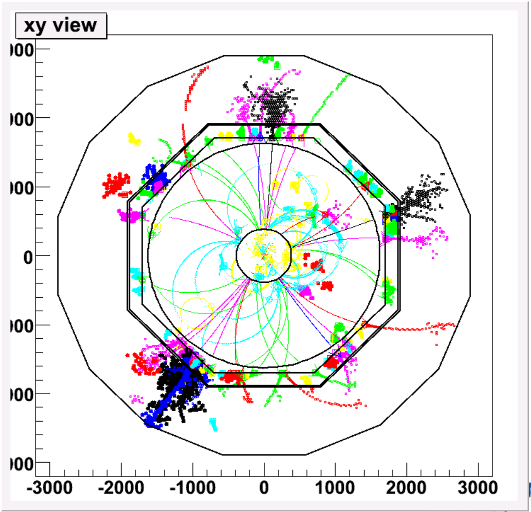
	ATLAS				CMS	
	Barrel LAr/Tile		End-cap LAr			
	Tile	Combined	HEC	Combined	Had. barrel	Combined
Electron/hadron ratio	1.36	1.37	1.49			
Stochastic term	$45\%/\sqrt{E}$	$55\%/\sqrt{E}$	$75\%/\sqrt{E}$	$85\%/\sqrt{E}$	$100\%/\sqrt{E}$	$70\%/\sqrt{E}$
Constant term	1.3%	2.3%	5.8%	$< 1\%$		8.0%
Noise	Small	3.2 GeV		1.2 GeV	Small	1 GeV

No energy loss in front of the calorimeters?



$$\eta = -\ln\left(\tan\left(\frac{\theta}{2}\right)\right)$$

There might be much more precise detectors than ATLAS and CMS in the Future...



e.g. at an e^+e^- linear collider:

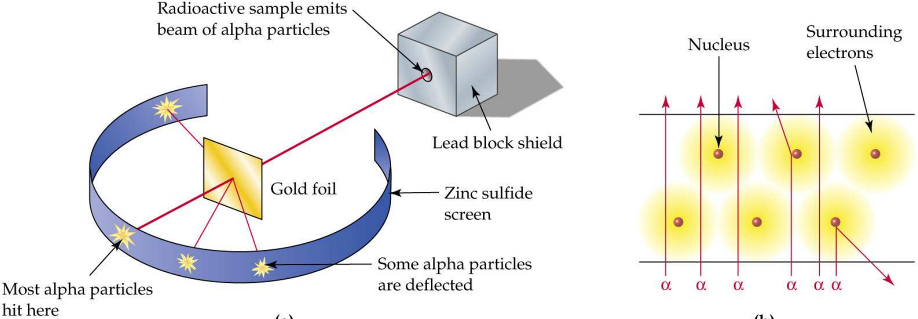
Time to Breath, Think and Ask

Just very few historical landmarks of particle physics

- The Rutherford Experiment performed by Geiger and Marsden
- The discovery of the positron
- The discovery of the electroweak Standardmodel: $W^{\pm}$, Z^0

The “Rutherford” Experiment

The discovery of the complex substructure of the atom
The fundamental principle of this experiment from 1909 is the same as what we do at the LHC

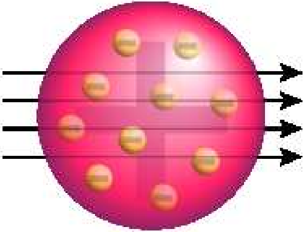


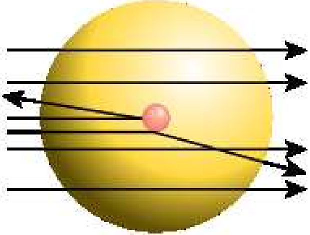
(a) (b)

The "Rutherford" Experiment

SCATTERING OF ALPHA PARTICLES BY GOLD
(Experimental test by Geiger and Marsden)

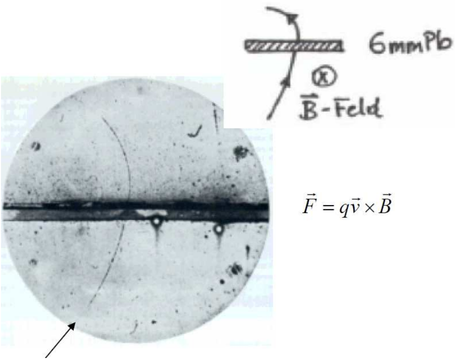
EXPERIMENTAL MEASUREMENTS		TEST OF THEORETICAL PREDICTION	
Angle of Deflection A*	*Experimental Count† N*	*Proportion predicted (on a special scale)*	*The test* $\frac{N}{\text{proportion predicted}}$
150°	33	1.15	29
135°	43	1.15	31
120°	52	1.79	29
105°	69.5	2.53	28
75°	211	7.25	29
60°	477	16.0	30
45°	1 435	46.6	31
30°	7 800	223	33
15°	120 570	3 445	35
10°	502 570	17 330	29
5°	8 289 000	276 300	30





The Discovery of the Positron

- Antiparticles were predicted by Dirac in 1927 (see later how he predicted them)



$$\vec{F} = q\vec{v} \times \vec{B}$$

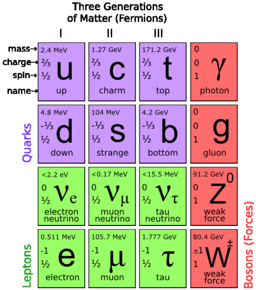
- Use very high B-field (1.5T)
- Get the direction from $\mathrm{d}E/\mathrm{d}x$
- Get momentum and charge from curvature:
  charge positive, $p = 23\,\mathrm{MeV}$
- Proton with same p would have get stuck in Pb
- highly relativistic particle doesn't get stuck
- Must be new, yet unknown positively charged light particle: positron

The Discovery of the $W^{\pm}$ and Z^0

By the time of the 1970's, a lot of particles were discovered, and everybody wondered about the ordering principle, the theory behind.

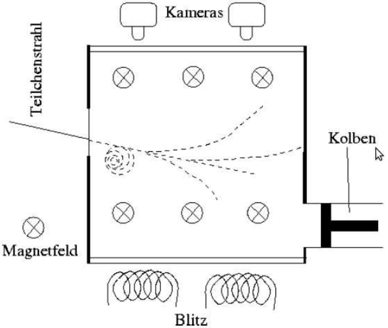
One crazy idea was the Standard Model, invetend mostly by Glashow, Salam and Weinberg. It predicted heavy gauge bosons $W^{\pm}$ and Z^0 (see later) with precisely predicted properties.

The Z^0 should be something like the photon γ, but with a heavy mass and with coupling to neutrinos.



Prelude to the Discovery of the Z^0

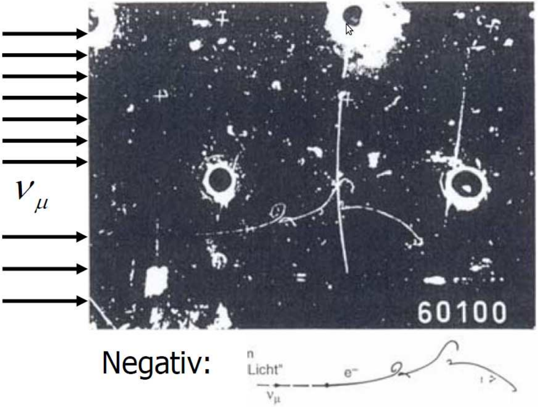
Look for neutrinos interacting with matter:



Prelude to the Discovery of the Z^0

Indeed, we find interactions without visible incoming or outgoing particle:

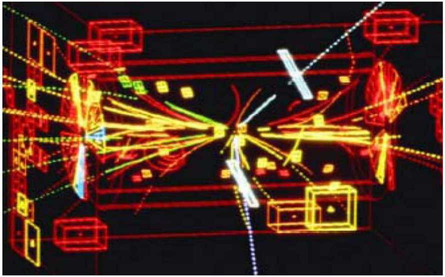
a new interaction

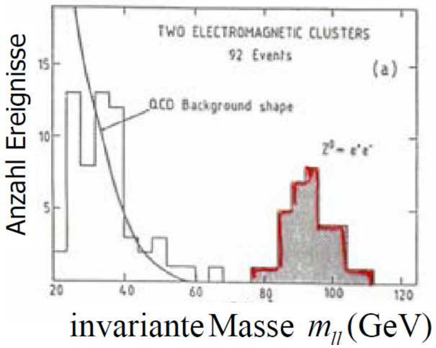


The Discovery of the Z^0

But can we see the new particle and measure it's properties? Yes, we can

Beispielereignis (UA1-Experiment):





An era of discoveries in the 70's and early 80's

History of Discoveries

- 1897 Electron discovered by J.J. Thompson
- 1899 Alpha particle discovered by Ernest Rutherford in uranium radiation
- 1900 Gamma ray (i.e. photon) discovered by Paul Villard in uranium decay.
- 1911 Atomic nucleus identified by Ernest Rutherford, based on scattering observed by Hans Geiger and Ernest Marsden.
- 1919 Proton discovered by Ernest Rutherford
- 1932 Neutron discovered by James Chadwick
- 1932 Positron discovered by Carl D. Anderson (proposed by Paul Dirac in 1927)
- 1937 Muon discovered by Seth Neddermeyer, Carl Anderson, J.C. Street, and E.C. Stevenson, using cloud chamber measurements of cosmic rays. (It was mistaken for the pion until 1946.)
- 1947 Pion discovered by Cecil Powell (predicted by Hideki Yukawa in 1934)
- 1947 Kaon, the first strange particle, discovered by G.D. Rochester and C.C. Butler
- 1955 Antiproton discovered by Owen Chamberlain, Emilio Segre, Clyde Wiegand, and Thomas Ypsilantis
- 1956 Neutrino detected by Frederick Reines and Clyde Cowan (proposed by Wolfgang Pauli in 1931 to explain the apparent violation of energy conservation in beta decay)
- 1962 Muon neutrino proved distinct from electron neutrino by group headed by Leon Lederman
- 1964 Higgs boson predicted as a result of a mechanism for electroweak symmetry breaking proposed by Peter Higgs (remains hypothetical as of 2005, but widely expected to be found at the Large Hadron Collider at CERN in the early 2010s)
- 1969 Partons (internal constituents of hadrons) observed in deep inelastic scattering experiments between protons and electrons at SLAC; this was eventually associated with the quark model (predicted by Murray Gell-Mann and George Zweig in 1963) and thus constitutes the discovery of the up quark, down quark, and strange quark.
- 1974 J/Ψ particle discovered by groups headed by Burton Richter and Samuel Ting, demonstrating the existence of the charm quark (proposed by Sheldon Glashow, John Iliopoulos, and Luciano Maiani in 1970)
- 1975 Tau lepton discovered by group headed by Martin Perl
- 1977 Upsilon particle discovered at Fermilab, demonstrating the existance of the bottom quark (proposed by Kobiyashi and Maskawa in 1973)
- 1979 Gluon observed in three jet events at DESY.
- 1983 W and Z bosons discovered by Carlo Rubbia, Simon van der Meer, and the CERN UA-1 collaboration (widely expected, predicted in detail by Sheldon Glashow, Abdus Salam, and Steven Weinberg in the 1960s)
- 1995 Top quark discovered at Fermilab
- 2000 Tau neutrino proved distinct from other neutrinos at Fermilab

Time to Breath, Think and Ask

Fundamental Properties of “Fundamental” Particles

From `http://pdg.lbl.gov`:

$\boxed{\boldsymbol{\mu}}$

$$J = \frac{1}{2}$$

Mass $m = 0.1134289256 \pm 0.0000000029$ u

Mass $m = 105.658367 \pm 0.000004$ MeV

Mean life $\tau = (2.197034 +\text{-} 0.000021) \times 10^{-6}$ s $\quad (S = 1.2)$

$\tau_{\mu^+}/\tau_{\mu^-} = 1.00002 \pm 0.00008$

$c\tau = 658.654$ m

Magnetic moment anomaly $(g-2)/2 = (11659209 \pm 6) \times 10^{-10}$

$(g_{\mu^+} - g_{\mu^-})\ /\ g_{\text{average}} = (-0.11 \pm 0.12) \times 10^{-8}$

Electric dipole moment $d = (-0.1 \pm 0.9) \times 10^{-19}$ e cm

Fundamental Properties of “Fundamental” Particles

From http://pdg.lbl.gov:

μ^- **DECAY MODES**			Fraction (Γ_i/Γ)		Confidence level	p (MeV/c)
$e^- \overline{\nu}_e \nu_\mu$			$\approx 100\%$			53
$e^- \overline{\nu}_e \nu_\mu \gamma$		[d]	(1.4 ± 0.4) %			53
$e^- \overline{\nu}_e \nu_\mu e^+ e^-$		[e]	$(3.4\pm0.4) \times 10^{-5}$			53
Lepton Family number (LF) violating modes						
$e^- \nu_e \overline{\nu}_\mu$	LF	[f]	< 1.2	%	90%	53
$e^- \gamma$	LF		< 1.2	$\times 10^{-11}$	90%	53
$e^- e^+ e^-$	LF		< 1.0	$\times 10^{-12}$	90%	53
$e^- 2\gamma$	LF		< 7.2	$\times 10^{-11}$	90%	53

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Properties of Composite Particles

LIGHT UNFLAVORED MESONS
$(S = C = B = 0)$

For $I = 1$ (π, b, ρ, a): $u\overline{d}$, $(u\overline{u} - d\overline{d})/\sqrt{2}$, $d\overline{u}$;
for $I = 0$ $(\eta, \eta', h, h', \omega, \phi, f, f')$: $c_1(u\overline{u} + d\overline{d}) + c_2(s\overline{s})$

$\pi^{\pm}$

$$I^G(J^P) = 1^-(0^-)$$

Mass $m = 139.57018 \pm 0.00035$ MeV (S = 1.2)
Mean life $\tau = (2.6033$+-$0.0005) \times 10^{-8}$ s (S = 1.2)
$c\tau = 7.8045$ m

$\pi^{\pm} \rightarrow \ell^{\pm} \nu \gamma$ form factors [a]

$F_V = 0.0254 \pm 0.0017$
$F_A = 0.0119 \pm 0.0001$
F_V slope parameter $a = 0.10 \pm 0.06$
$R = 0.059^{+0.009}_{-0.008}$

Properties of Composite Particles

π^+ DECAY MODES			Fraction (Γ_i/Γ)	Confidence level	p (MeV/c)
$\mu^+\nu_\mu$		[b]	(99.98770 ± 0.00004) %		30
$\mu^+\nu_\mu\gamma$		[c]	$(2.00 \pm 0.25) \times 10^{-4}$		30
$e^+\nu_e$		[b]	$(1.230 \pm 0.004) \times 10^{-4}$		70
$e^+\nu_e\gamma$		[c]	$(7.39 \pm 0.05) \times 10^{-7}$		70
$e^+\nu_e\pi^0$			$(1.036 \pm 0.006) \times 10^{-8}$		4
$e^+\nu_e e^+ e^-$			$(3.2 \pm 0.5) \times 10^{-9}$		70
$e^+\nu_e\nu\overline{\nu}$			$< 5 \times 10^{-6}$	90%	70
Lepton Family number (*LF*) or Lepton number (*L*) violating modes					
$\mu^+\overline{\nu}_e$	L	[d]	$< 1.5 \times 10^{-3}$	90%	30
$\mu^+\nu_e$	LF	[d]	$< 8.0 \times 10^{-3}$	90%	30
$\mu^- e^+ e^+ \nu$	LF		$< 1.6 \times 10^{-6}$	90%	30

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How are particles described theoretically

Very short example: QED is a local abelian $U(1)$ gauge symmetry

Fermions (particles with Spin $\frac{1}{2}$, which form the matter of the SM) are the quanta of fields ψ obeying the Dirac equation:

$$(i\partial_\mu \gamma^\mu - m)\psi = 0$$

This equation of motion is derived from a formula called Lagrangian:

$$\mathcal{L}_{\text{free}} = \bar{\psi}(i\partial\!\!\!/ - m)\psi$$

using $\partial\!\!\!/ = \partial_\mu \gamma^\mu$, which contains the fundamental input which we put into the theory, in terms of masses, couplings and relations between fields.

For the field ψ, two solutions of the Dirac equation exist:
One with Energy $+E$, and one with energy $-E$. The first one are the particles. The latter ones are the antiparticles.

How are particles described theoretically

So, that's the particles. How do we get the forces? Simple:
Make the theory gauge invariant under local gauge transformations:

$$\psi(x) \rightarrow e^{i\alpha(x)}\psi(x)$$

What is the transformation behaviour of the free Lagrangian?

How are particles described theoretically

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$$\psi(x) \to e^{i\alpha(x)}\psi(x)$$

What is the transformation behaviour of the free Lagrangian?

$$\mathcal{L}_{\mathrm{free}} \to \mathcal{L}_{\mathrm{free}} - \bar{\psi}\gamma_\mu\psi(\partial^\mu\alpha(x))$$

How are particles described theoretically

So, that's the particles. How do we get the forces? Simple:
Make the theory gauge invariant under local gauge transformations:

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That's not invariant!

How are particles described theoretically

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Make the theory gauge invariant under local gauge transformations:

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What is the transformation behaviour of the free Lagrangian?

$$\mathcal{L}_{\text{free}} \to \mathcal{L}_{\text{free}} - \bar{\psi}\gamma_\mu\psi(\partial^\mu\alpha(x))$$

That's not invariant!
But luckily it's also not QED . . .

How are forces described theoretically

In order to save QED under the transformation $U(x) = e^{-1\alpha(x)}$, add a gauge field A_μ (Spin 1) obeying:

$$A_\mu(x) \to U^{-1} A_\mu U + \frac{1}{q} U^{-1} \partial_\mu U = A_\mu(x) - \frac{1}{q} \partial_\mu \alpha(x)$$

A miracle has occured: we introduced not only a gauge field, but also a charge q. Also, we would have needed the photon A_μ anyway . . .

Now modify the derivative:

$$\partial_\mu \to \partial_\mu + iqA_\mu(x) = D_\mu$$

How is everything described theoretically

Let's write $\mathcal{L}$ again with all possible Lorentz and gauge invariant terms:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\partial\!\!\!/ - m)\psi - q\bar{\psi}\gamma^{\mu}\psi A_{\mu}$$

The last term describes the interaction between a current

$$\bar{\psi}\gamma^{\mu}\psi$$

and the gauge field of the photon

$$A_\mu$$

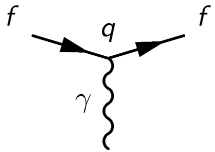
with coupling (here: em charge)

$$q.$$

The mass of the particle in the term

$$m\bar{\psi}\psi$$

will lead to big problems later on, but we'll not discuss that in this lecture – wait for later!

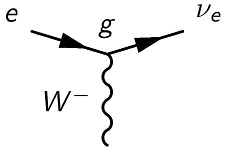


Feynman-Diagram

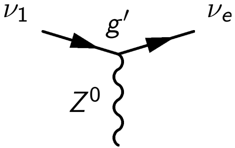
Time to Breath, Think and Ask

Phenomenons of the Weak Force

The weak force works as QED. Just, it's a more complex gauge group: A non-abelian gauge group $SU(2)_L$, acting only on the lefthanded particles. Here, let's look at the phenomenons only arising from the 3 gauge particles W^+, W^-, Z^0, a more detailed look will come later.



Example Feynman-Diagram of a W exchange



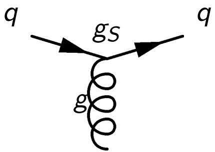
Example Feynman-Diagram of a Z exchange

However, there are many complications here which I won't mention directly. Wait a bit for the Higgs mechanism and later lectures.

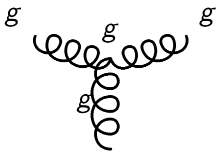
Phenomenons of the Strong Force

The strong force also works as QED, Just, it is based on an even bigger non-abelian gauge group: $SU(3)_C$

It has 8 gauge particles, the massless gluons. They interact only on quarks, not on leptons. In principle it's easy, but the coupling constant g_S is strong and the gluons interact with themselves, which leads to interesting phenomenons.

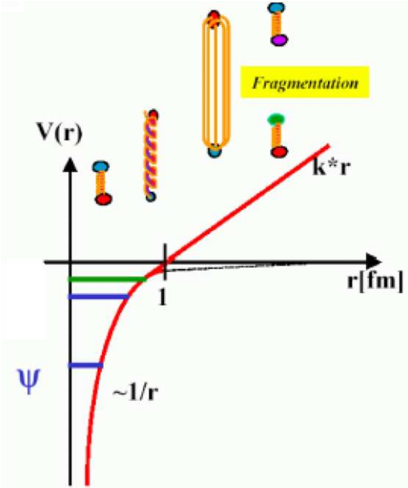


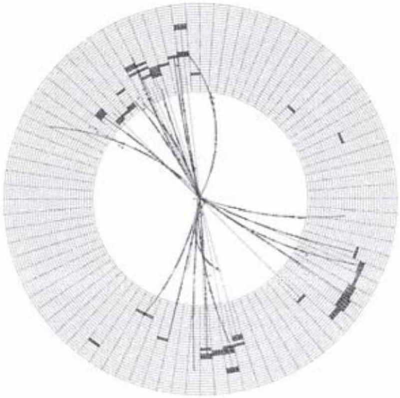
Example Feynman-Diagram of a g exchange



Example Feynman-Diagram of a g interaction

Phenomenons of the Strong Force

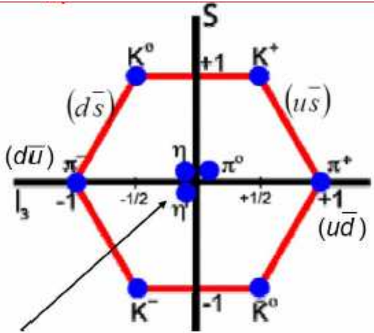




Confinement and Jets

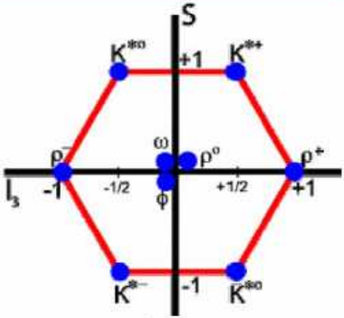
Phenomenons of the Strong Force

The strong force is also the one which holds all the complex hadrons together: Protons, Neutrons, π, K, . . .



$$\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$$
$$\eta = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$
Oktett

$$\eta' = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$
Singulett



$$\rho^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$$

Für Vektormesonen sind ϕ und ω Mischung aus η^* und $\eta^{*\prime}$:

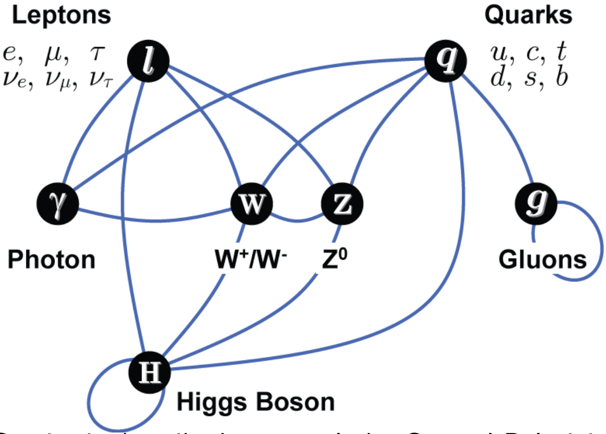
$$\phi = s\bar{s} \quad \omega = \frac{1}{2}(u\bar{u} + d\bar{d})$$

The Standard Model

The Standrad Model is the combination of the Gauge groups

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

including the Higgs Mechansism



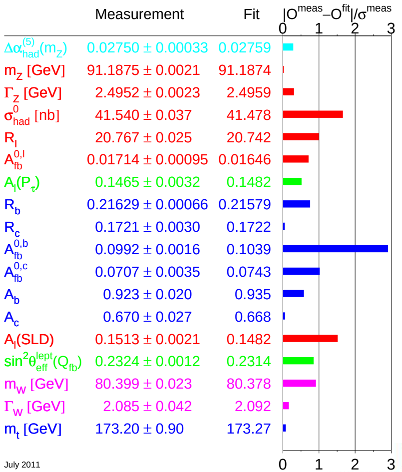
Gravity is described separately by General Relativity

Time to Breath, Think and Ask

Describes all precision experiments performed yet

Within expected statistical fluctuations... Measurements include

- Particle content complete up to Higgs boson
- All masses, couplings, asymmetries are described
- Measured CP violation (mostly) described
- ...



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QFD: $SU(2)_L \times U(1)_Y$ EWSB

We have seen before, that the SM has the interactions $SU(2)_L \times U(1)_Y$. The gauge bosons of the SM have the following mass terms:

$$\frac{1}{4} g^2 v^2 W_\mu^+ W_\mu^- + \frac{1}{8} v^2 (B^\mu, W_\mu^3) \begin{pmatrix} g'^2 & -gg' \\ -gg' & g^2 \end{pmatrix} \begin{pmatrix} B^\mu \\ W^{3\mu} \end{pmatrix}$$

We have the mass term on the $W^\pm$ already. Let's diagonalize the mass matrix of the hypercharge field B_μ and the third component of the $SU(2)_L$ gauge field W_μ^3:

$$\begin{pmatrix} A_\mu \\ Z_\mu^0 \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B^\mu \\ W^{3\mu} \end{pmatrix}$$

Now another miracle has occured: The photon field A_μ drops out of EWSB!

QFD: $SU(2)_L \times U(1)_Y$ EWSB

we have now introduced the Weinberg angle

$$\sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}$$

From the diagonalization of the mass matrix for W^3_μ and B_μ

$$A_\mu = \frac{1}{\sqrt{g^2 + g'^2}}(g' W^3_\mu + g B_\mu), \quad m_A^2 = 0$$

$$Z^0_\mu = \frac{1}{\sqrt{g^2 + g'^2}}(g W^3_\mu - g' B_\mu), \quad m^2_{Z^0} = \frac{(g^2 + g'^2) v^2}{4}$$

QFD: $SU(2)_L \times U(1)_Y$ EWSB

We also obtain the charged current and its coupling to the W_μ^+ as

$$\frac{g}{2\sqrt{2}}(\bar{\nu}_L \gamma^\mu e_L W_\mu^+ + h.c.)$$

In addition, as the first tested firm prediction of this theory, the neutral currents have been introduced ('74 November revolution: Gargamelle):

$$\frac{\sqrt{g^2+g'^2}}{4}(\overline{L}\gamma^\mu \tau_3 L - 2\frac{g'^2}{g^2+g'^2}\bar{e}\gamma^\mu e)Z_\mu^0, \qquad \frac{gg'}{\sqrt{g^2+g'^2}}\bar{e}\gamma^\mu e\, A_\mu$$

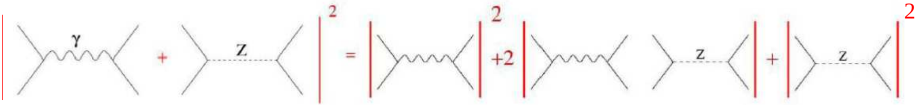
where

$$L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L = \frac{1}{2}(1-\gamma^5)\begin{pmatrix} \nu \\ e \end{pmatrix}, \quad e_R = \frac{1}{2}(1+\gamma^5)e,$$

$$q_e = \frac{gg'}{\sqrt{g^2+g'^2}}, \quad e = e_L + e_R$$

This formalism can now be used to predict the detailed behaviour of the Z^0

3 Parameters for the Shape of the Differential $e^+e^- \to Z^0$ Cross-Section



$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = N_C \frac{\alpha_{em}^2}{4s} \left\{ (1+\cos^2\theta) \left[Q_f^2 - 2\chi_1 v_e v_f Q_f - \chi_2 (a_e^2 + v_e^2)(a_f^2 + v_f^2) \right] \right.$$
$$\left. +2\cos\theta \left[-2\chi_1 a_e a_f Q_f + 4\chi_2 a_e a_f v_e v_f \right] \right\}$$

$$\chi_1 = \frac{s(s-M_Z^2)}{16\sin^2\theta_W \cos^2\theta_W \left((s-M_Z^2)^2 + M_Z^2\Gamma_Z^2 \right)}$$

$$\chi_2 = \frac{s^2}{256\sin^4\theta_W \cos^4\theta_W \left((s-M_Z^2)^2 + M_Z^2\Gamma_Z^2 \right)}$$

$$a_e = -1; \quad v_e = -1 + 4\sin^2\theta_W; \quad a_f = 2I_f; \quad v_f = 2I_f - 4Q_f\sin^2\theta_W$$

Jump to Angular Distribution

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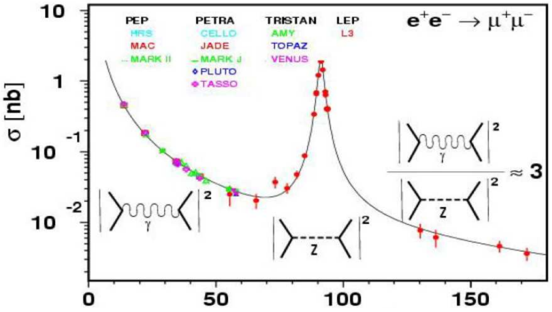
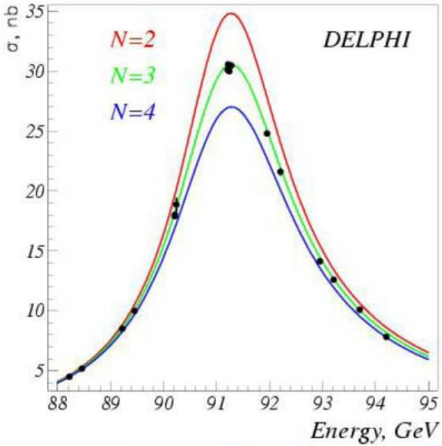
The Total $e^+e^- \to Z^0$ Cross-Section



- Perfectly described by the 3 non-digital parameters from before!
- Theory curve is not the one from before but it includes radiative corrections
- Z^0 is a dramatic resonance!

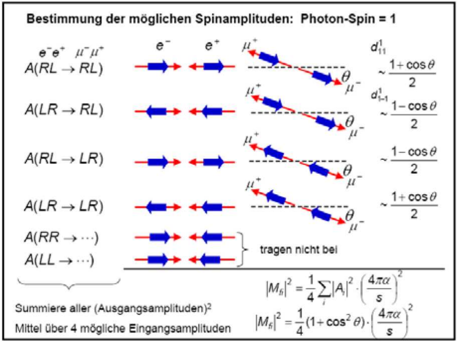
Counting Invisible particles: Neutrinos



$$\Gamma_{\mathrm{tot}} = \Gamma_{\ell\ell} + \Gamma_{qq} + N_{\mathrm{fam}}\Gamma_{\nu\nu}$$

- Total width depends on the number of neutrino families!
- Result:
  $N_{\mathrm{fam}} = 2.9841 \pm 0.0083$
- Result before LEP: $N_{\mathrm{fam}} < 5.9$

Even more Detail: Angular Distributions

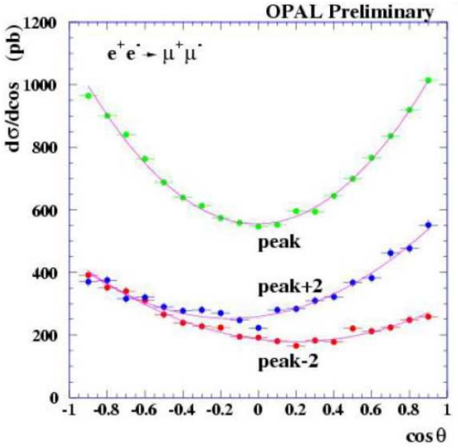


- Linear Term in $\cos\theta_W$ on page Jump to Differential Cross-Section causes a forward-backward Asymmetry A_{FB}:

$$A_{FB} = \frac{\sigma(\cos\theta > 0) - \sigma(\cos\theta < 0)}{\sigma(\cos\theta > 0) + \sigma(\cos\theta < 0}$$

- Pure A_{FB} is better than a fit to the whole distribution, since detector systematics cancels (as long as the detector is symmetrical)

Even more Detail: Angular Distributions



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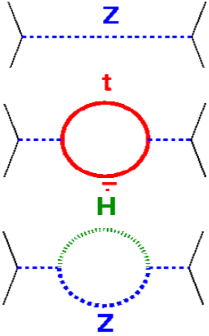
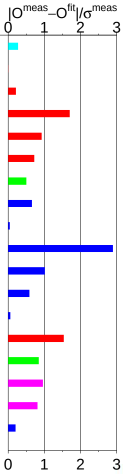
Precision Tests of Loop Corrections

e^+e^- machines can see effects of virtual particles



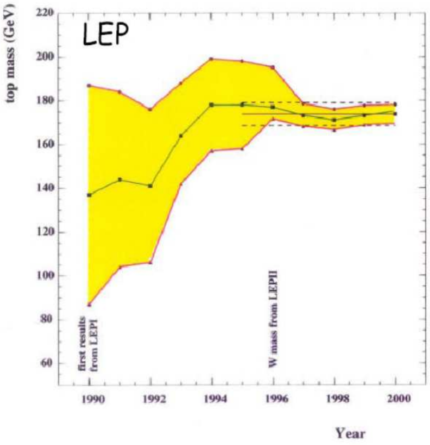
	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02768
m_Z [GeV]	91.1875 ± 0.0021	91.1875
Γ_Z [GeV]	2.4952 ± 0.0023	2.4957
σ_{had}^0 [nb]	41.540 ± 0.037	41.477
R_l	20.767 ± 0.025	20.744
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21586
R_c	0.1721 ± 0.0030	0.1722
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1038
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.398 ± 0.025	80.374
Γ_W [GeV]	2.140 ± 0.060	2.091
m_t [GeV]	170.9 ± 1.8	171.3

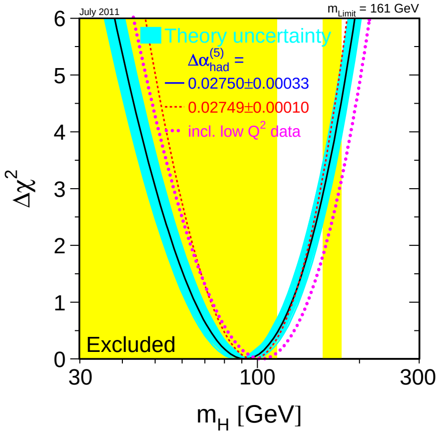


$$M_Z^2 = M_Z^{2\,\textcolor{red}{0th\ order}}\left(1+\mathcal{O}(m_t^2)+\mathcal{O}(\ln m_h^2)+\cdots\right)$$

Precision Tests of Loop Corrections

e^+e^- machines can see effects of virtual particles

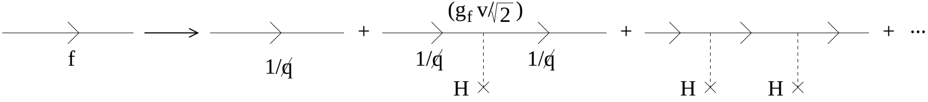




Graphical Representation of how Mass is Created

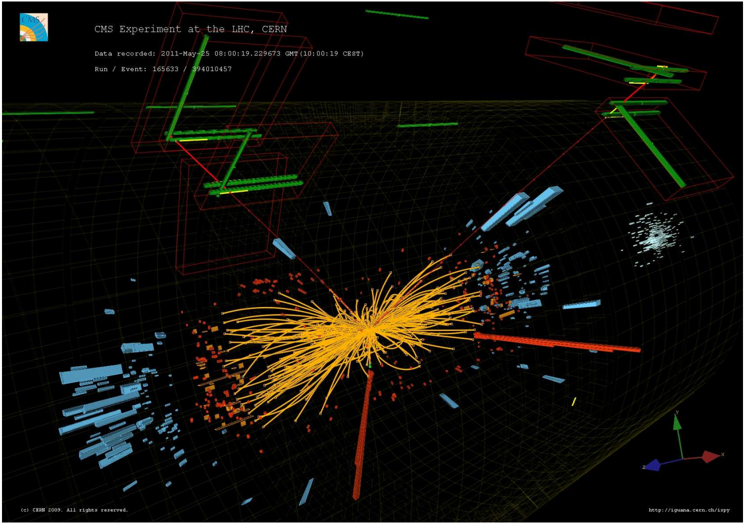
The Higgs mechanism is like a boring cocktail party:

„famousness" g_f of a particle determines its mass:



$$\frac{1}{\not{q}} + \frac{1}{\not{q}}\left(\frac{g_f\, v}{\sqrt{2}}\right)\frac{1}{\not{q}} + \cdots = \frac{1}{\not{q}}\sum_{n=0}^{\infty}\left[\left(\frac{g_f\, v}{\sqrt{2}}\right)\frac{1}{\not{q}}\right]^n = \frac{1}{\not{q} - \left(\frac{g_f\, v}{\sqrt{2}}\right)}$$

The first glimpse of the Higgs?



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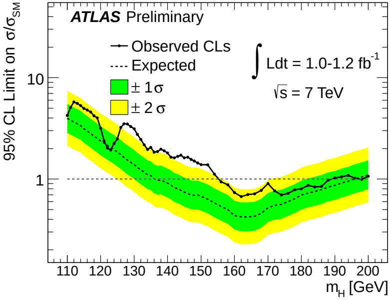
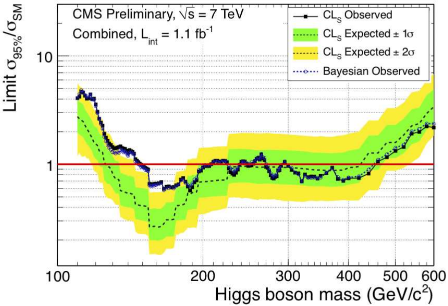


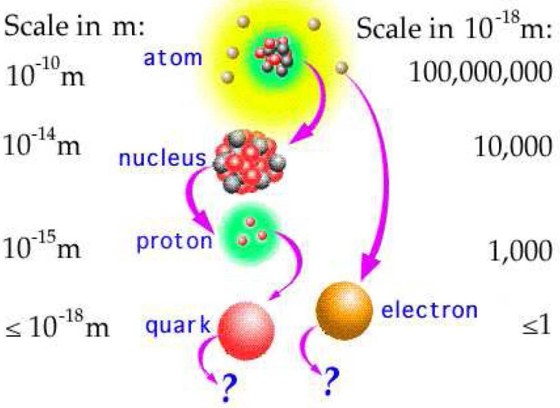


If this turns out to be the SM Higgs, it will be an unprecedented success: a prediction more than 40 years old would come true!

Time to Breath, Think and Ask

Let's revisit the progress of Particle Physics

Rutherford
Geiger Marsden
1909



HERA
LEP
Tevatron
LHC
2009

Will we go on like that, finding more and more fundamental scales?

I think: NO, we already found a very fundamental scale, we need to understand it!

Why we assume we have found something incredibly fundamental

- Quantum Mechanics seems to work on the most fundamental scale we know. So using QM, we can show the follwing:
- The electron cannot be a composit particle How do we show that incredible claim (within the principles of QM)?

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- The electron cannot be a composit particle How do we show that incredible claim (within the principles of QM)?
- Heisenbergs uncertainty principle tells us:

$$\Delta x \Delta p \geq \hbar/2 = 3.29 \times 10^{-16}\,\mathrm{eV\,s}$$

- Let's apply that on the electron. From scattering experiments, we know its size is tiny: $r_e < 10^{-18}$ m

$$10^{-18}\,\mathrm{m}\,\Delta p \geq \hbar/2 \rightarrow \Delta p \geq 98\,\mathrm{GeV/c}$$

- But the electron has a mass which is much smaller: $m_e = 511\,\mathrm{keV/c^2}\ldots$

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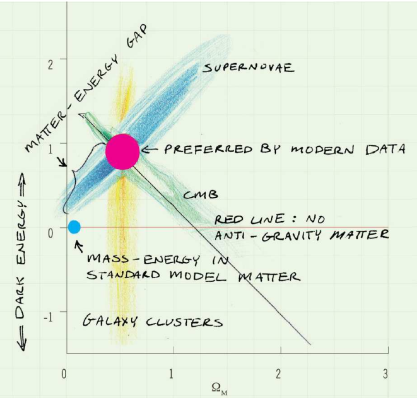
- But the electron has a mass which is much smaller: $m_e = 511\,\mathrm{keV/c^2}\ldots$
- The electron must be elemental, it cannot be composed of more fundamental constituents

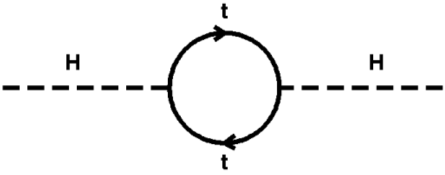
How do we Know About Dark Matter



- In many models, the dark matter is a thermal relic WIMP: Weakly Interacting Massive (stable) Particle
- Once in thermal equlibrium, they've 'frozen out' due to the expansion of the universe (Can't decay on their own – need a partner to annihilate with)
- Calculable density
- Naturally appear in SUSY with R-parity:
  - $m_{DM} \approx 100\,\mathrm{GeV}$
  - SM QFD couplings

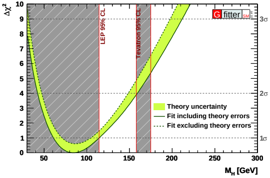
Supersymmetry

- Even if we find the Higgs, we still have a problem . . .



$$\Delta m_h \sim \Lambda^2$$
natural $m_h = M^2_{Planck}$
Finetuning:

- From indirect measurements: $m_h < 140$ GeV



$$m_{h,obs} = \underbrace{10^{2\cdot 19}\,\mathrm{GeV}}_{\text{nat. mass}} - \underbrace{(1-\epsilon)10^{2\cdot 19}\,\mathrm{GeV}}_{\textit{Renormalisation}} \approx 100\,\mathrm{GeV}$$

Supersymmetry

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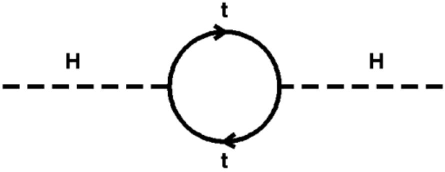


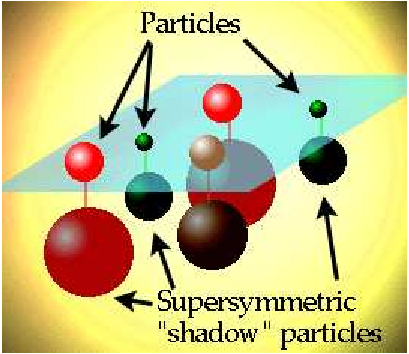
$$\Delta m_h \sim \Lambda^2$$

$$\Delta m_h \sim \ln \Lambda$$

- From indirect measurements: $m_h < 140$ GeV
- To prevent quadratic divergencies: Introduce shadow world: One SUSY partner for each SM d.o.f.
- Nice addition for free: If R-parity conserved, automatically the Lightest SUSY Particle (LSP) is a stable DM candidate
- But: Where are all those states?

Supersymmetry

- Even if we find the Higgs, we still have a problem . . .



In any case: $m_{Hlike} < 1\,\mathrm{TeV}$
$m_{SUSY} \leq \mathcal{O}(\mathrm{TeV})$
$\Rightarrow$ Terascala

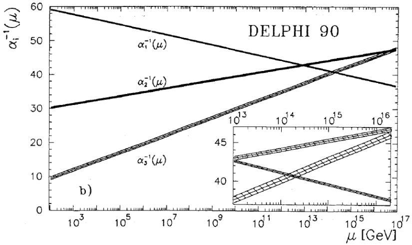
- From indirect measurements: $m_h < 140$ GeV
- To prevent quadratic divergencies: Introduce shadow world: One SUSY partner for each SM d.o.f.
- Nice addition for free: If R-parity conserved, automatically the Lightest SUSY Particle (LSP) is a stable DM candidate
- But: Where are all those states?
- SUSY breaking introduces a lot of additional parameters
  Understand model: Measure parameters!

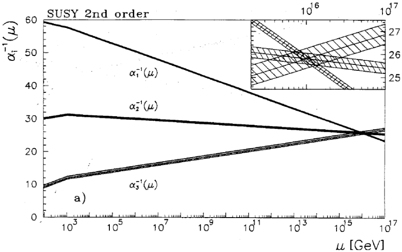
Why try (trust?) SUSY?

Wim de Boer *et al.* (1991):

It was shown that the evolution of the coupling constants within the minimal Standard Model with one Higgs doublet does not lead to Grand Unification, but if one adds five additional Higgs doublets, unification can be obtained at a scale below $2 \cdot 10^{14}$ GeV. However, such a low scale is excluded by the limits on the proton lifetime.

On the contrary, the minimal supersymmetric extension of the Standard Model leads to unification at a scale of $10^{16.0 \pm 0.3}$ GeV. Such a large unification scale is compatible with the present limits on the proton lifetime of about 10^{32} years. Note that the Planck mass (10^{19} GeV) is well above the unification scale of 10^{16} GeV, so presumably quantum gravity does not influence our results.

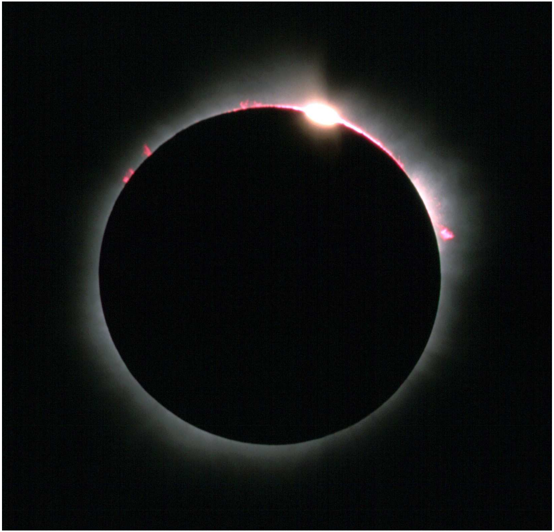




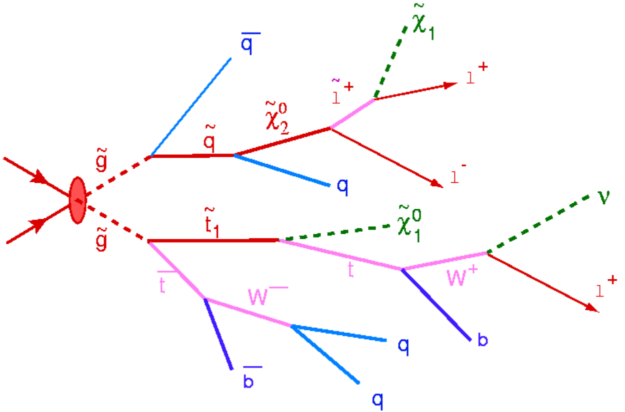
„Prediction“ of $\sin^2\theta_W$:

$$\sin^2\theta_W^{SUSY} = 0.2335(17), \qquad \sin^2\theta_W^{exp} = 0.2315(02)$$

A Warning: Apparent Finetuning



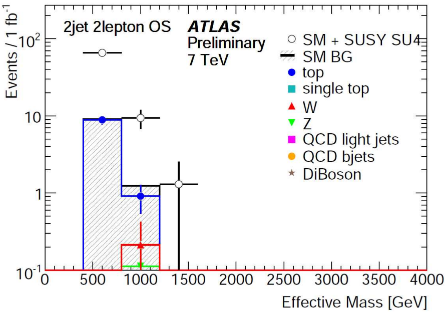
What do we hope to find?



Need everything: MET, Jets, B-Jets, elektrons, myons, taus

The possible discovery of Physics at the Terascale

- inclusive spectra: probably fastest way to discover SUSY-like physics
- Challenging because very good detector understanding with relatively little data needed (ca. $\mathcal{L} \approx 1\,\mathrm{fb}^{-1}$)



$$M_{eff} = \sum_i p_{T,i} + E_{Tmiss}$$

ATLAS MC $1\,\mathrm{fb}^{-1}$ @ $7\,\mathrm{TeV}$

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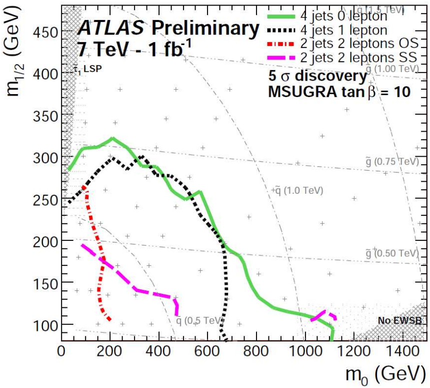

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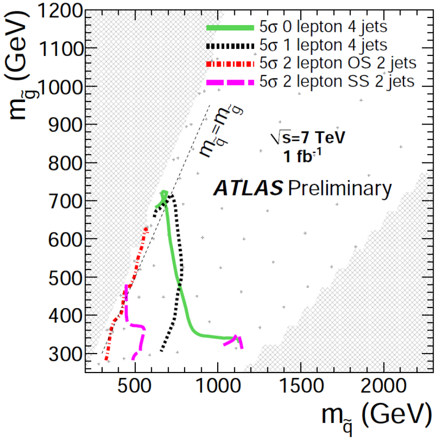

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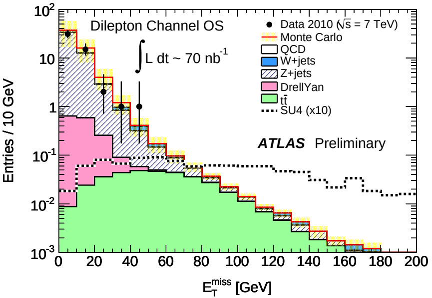

ATLAS data @ $7\,\mathrm{TeV}$ only 70 nb!

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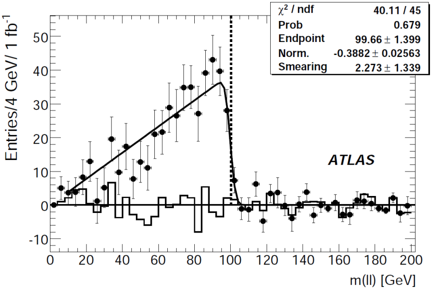

ATLAS MC $1\,\mathrm{fb}^{-1}$ @ $14\,\mathrm{TeV}$

kinematic edges

$\Rightarrow$ mass information

Still Searching for the Unexpected!

Miracles and open questions – incomplete

- Dark Matter
- Explanation for EWSB and Hierarchy problem
- Gauge Coupling Unification
- Matter Asymmetry of the Universe
- Smallness of the neutrino masses and absence of their righthanded couplings
- Mass hierarchy of the SM particles
- Dark Energy
- How does gravity fit into the picture?

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- How does gravity fit into the picture?
- My favourite reason why the SM is wrong (i.e. incomplete):

$$q_\ell = -n_C(q_u - q_d)$$

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Particle Physics is Philosophy

Not from the beginning the gods disclosed everything to us,
but in the course of time we find, searching, a better knowledge.
These things have seemed to me to resemble the truth.
There never was nor will be a person who has certain knowledge
about the gods and about all the things I speak of.
Even if he should chance to say the complete truth,
yet he himself can not know that it is so.

XENOPHANES OF KOLOPHON, ca. 500 b.c.

Backup Slides

Prerequisites: γ_μ, ∂^μ and the $\dagger$

The notation is a little bit confusing sometimes, so let's try to sort things a little bit:
Fermions are represented by 4-dimensional spinors:

$$\psi(p) = \sqrt{p_0 + m} \begin{pmatrix} \chi_s \\ \frac{\vec{\sigma}\vec{p}}{p_0+m}\chi_s \end{pmatrix}, \quad \chi_{1/2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \chi_{-1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The 4×4 γ matrices are acting on the 4 dimensions of ythe spinors.

An index (γ_μ, A_μ or $F_{\mu\nu}$) always denotes a 4-dimensional Lorentz vector. This 4-dimensional space is independent of the 4-dimensional spinor space.

∂^μ denotes a partial derivative for x^0, x^1, x^2, x^3 respecively.

Einstein convention:
4-vector: x^μ
scalar: $x^\mu y_\mu$
matrix: $x^\mu y^\nu$

Prerequisites: γ_μ, ∂^μ and the $\dagger$

Dirac matrices (each matrix acting on a 4-dim spinor):

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}, \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\gamma^5 := i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Hermitean adjoint: $\psi^\dagger$: $a_{ij} = a^*_{ji}$, Dirac adjoint: $\bar{\psi} = \psi^\dagger\gamma^0$

The Lagrangian

Require that the action S remains invariant under small changes of the fiends ϕ:

$$\frac{\delta \mathcal{S}}{\delta \varphi_i} = 0$$

S is determined by the Lagrangian (classically: $\mathcal{L} = T - V$)

$$\mathcal{S}[\varphi_i] = \int \mathcal{L}[\varphi_i(s)] \, \mathrm{d}^n s,$$

where s_α denotes the parameters of the system.

The equations of motion of the system can then be derived from the Euler-Lagrange equation:

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \right) - \frac{\partial \mathcal{L}}{\partial \varphi} = 0$$

The Lagrangian

Classical Example in three-dimensional space:

$$L(\vec{x}, \dot{\vec{x}}) = \frac{1}{2}\, m\, \dot{\vec{x}}^2 - V(\vec{x}).$$

Then, the Euler-Lagrange equation is:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_i}\right) - \frac{\partial L}{\partial x_i} = 0$$

with $i = 1, 2, 3$. The derivation yields:

$$\frac{\partial L}{\partial x_i} = -\frac{\partial V}{\partial x_i}$$

$$\frac{\partial L}{\partial \dot{x}_i} = \frac{\partial}{\partial \dot{x}_i}\left(\frac{1}{2}\, m\, \dot{\vec{x}}^2\right) = \frac{1}{2}\, m\, \frac{\partial}{\partial \dot{x}_i}\left(\dot{x}_i\, \dot{x}_i\right) = m\, \dot{x}_i$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_i}\right) = m\, \ddot{x}_i$$

From the Euler-Lagrange-equation we get the equation of motion:

Gauge Transformations

- Global Gauge Invariance:
  Require that $\mathcal{L}$ (i.e. the equation of motion) is invariant under the transformation:

  $$\psi(x) \rightarrow e^{i\alpha}\psi(x)$$

  with α being the same everywhere.

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This principle is the foundation of the SM

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Group Theory in a Tiny Nutshell

A group is a set G (the "underlying set") under a binary operation satisfying three axioms:

- The operation is associative.
- The operation has an identity element.
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Very simple example: 2 is the generator of all numbers 2^n, $n = [0, \inf[$

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Construct the SM particles as elements of a group invariant under operations within the group.

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Some Mathematics: $SU(2)$

For the special unitary group $SU(2)$, the generators are proportional to the Pauli matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The generators of the group are $\tau_i = \frac{1}{2}\sigma_i$. The Pauli matrices obey

$$\begin{aligned} [\sigma_i, \sigma_j] &= 2i\,\varepsilon_{ijk}\,\sigma_k \\ \{\sigma_i, \sigma_j\} &= 2\delta_{ij} \cdot I \end{aligned}$$

Example for an $SU(2)$ transformation:

$$\psi(x) \to e^{i\tau_i \alpha^i(x)} \psi(x)$$

$SU(2)$ and $SU(3)$ are not abelian, i.e. the generators of the group do not commute.

Some Mathematics: $SU(3)$

The analog of the Pauli matrices for $SU(3)$ are the Gell-Mann matrices:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

The generators of $SU(3)$ are defined as T by the relation

$$T_a = \frac{\lambda_a}{2}.$$

Some Mathematics: $SU(3)$

The generators T obey the relations

$$[T_a, T_b] = i \sum_{c=1}^{8} f_{abc} T_c$$

where f is called structure constant and has a value given by

$$f^{123} = 1$$

$$f^{147} = f^{165} = f^{246} = f^{257} = f^{345} = f^{376} = \frac{1}{2}$$

$$f^{458} = f^{678} = \frac{\sqrt{3}}{2}$$

$$\mathrm{tr}(T_a) = 0$$

Introduction: QED

QED is a local abelian $U(1)$ gauge symmetry

Using our knowledge about the Lagrangian, we construct the Lagrangian which gives us the equation of motion of the Dirac equation $((i\partial_\mu\gamma^\mu - m)\psi = 0)$:

$$\mathcal{L}_{\text{free}} = \bar{\psi}(i\partial\!\!\!/ - m)\psi$$

using $\partial\!\!\!/ = \partial_\mu\gamma^\mu$.
Make the theory gauge invariant under local $U(1)$ transformations:

$$\psi(x) \to e^{i\alpha(x)}\psi(x)$$

What is the transformation behaviour of the free Lagrangian?

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That's not invariant!
But luckily it's also not QED . . .

Introduction: QED

In order to save QED under the transformation $U(x) = e^{-1\alpha(x)}$, add a gauge field obeying:

$$A_\mu(x) \to U^{-1} A_\mu U + \frac{1}{q} U^{-1} \partial_\mu U = A_\mu(x) - \frac{1}{q} \partial_\mu \alpha(x)$$

A miracle has occured: we introduced not only a gauge field, but also a charge q. Also, we would have needed the photon A_μ anyway...

Now modify the derivative:

$$\partial_\mu \to \partial_\mu + iqA_\mu(x) = D_\mu$$

Let's write $\mathcal{L}$ again with all possible Lorentz and gauge invariant terms:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\partial\!\!\!/ - m)\psi - q\bar{\psi} A\!\!\!/ \psi$$

using

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Introduction: QED

Let's check the transformational behaviour under local U(1) again:

$$\mathcal{L} \to \mathcal{L}' = -\frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} + \bar{\psi}'(i\partial\!\!\!/ - m)\psi' - q\bar{\psi}' A\!\!\!/' \psi'$$

$$= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\partial\!\!\!/ - m)\psi - \bar{\psi}\gamma_\mu\psi(\partial^\mu \alpha(x)) - q\bar{\psi}\gamma_\mu\psi A^\mu + \bar{\psi}\gamma_\mu\psi(\partial^\mu \alpha(x))$$

$$= \mathcal{L}$$

with

$$F'_{\mu\nu} = \partial_\mu(A_\nu - \frac{1}{q}\partial_\nu \alpha(x)) - \partial_\nu(A_\mu - \frac{1}{q}\partial_\nu \alpha(x))$$

$$= F_{\mu\nu} - \partial_\mu \frac{1}{q}\partial_\nu \alpha(x) + \partial_\nu \frac{1}{q}\partial_\mu \alpha(x) = F_{\mu\nu}$$

QED including a gauge field is invariant under local U(1)!

Use this principle to construct the SM

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QCD: $SU(3)_C$

The fundamental states of QCD are the three color states of the quarks:

$$q = \begin{pmatrix} q_R \\ q_G \\ q_B \end{pmatrix},$$

which are transforming under the fundamental representation of $SU(3)_C$:

$$q_i \to {q_i}' = \left(e^{i\alpha^a(x)\frac{\lambda_a}{2}}\right)_{ij} q_j,$$

where λ_a with $a = 1, \ldots, 8$ are the eight 3×3 Gell-Mann-Matrices and $i, j = R, G, B$ run over the color indices.

The transformation works in principle just as in case of the QED, it's just slightly more complex due to the eight dimensions of the $SU(3)$ generators. As in QED before, the transformation renders the free Lagrangian not invariant under $SU(3)$. We need to introduce a gauge field A_μ^a transforming according to the adjoint representation:

QCD: $SU(3)_C$

Using the quarks q and the gluons A^a_μ we can now write the Lagrangian

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a + i\,\bar{q}_i \left(\not{\partial} \delta_{ij} + i g_C \left(\frac{\lambda_a}{2} \right)_{ij} \not{A}^a \right) q_j$$

with

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_C f^a_{\ bc} A^b_\mu A^c_\nu$$

which is different than in $U(1)$ due to the non-abelian character of $SU(3)$.
A little bit more detail: The full form of the field operators can be written as:

$$q_i(x) = \sum_{\text{spins}\,\lambda} \int \frac{\mathrm{d}^3 p}{\sqrt{(2\pi)^3 2 p_0}} \left(a_{i\lambda}(p) u_{i\lambda}(p) e^{-ipx} + b^+_{i\lambda}(p) v_{i\lambda}(p) e^{ipx} \right),$$

analogously without the spinors u, v for the gluon field.

QCD: $SU(3)_C$: Just for completeness

What's all that stuff in the previous equation? Important are the creation and annihilation operators $a_{i\lambda}$ and $b_{i\lambda}$, obeying

$$[b_i(p), b_j^+(p')]_{\substack{+\,\mathrm{Quarks} \\ -\,\mathrm{Gluonen}}} = \delta_{ij}\delta^3(\vec{p} - \vec{p}\,'),$$

$$[a_\lambda(k), a_{\lambda'}^+(k')]_{\substack{+\,\mathrm{Quarks} \\ -\,\mathrm{Gluonen}}} = \delta_{\lambda\lambda'}\delta^3(\vec{k} - \vec{k}\,')$$

All of the above has to be done separately for $q = u, d, c, s, b, t$.

The only input parameter is $\alpha_s = \frac{g_C^2}{4\pi} \approx 0.3$ for a scale of $Q^2 \approx 1$ GeV^2

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That's it ... a beautifully simple theory with awfully complex consequences ...

QFD: $SU(2)_L \times U(1)_Y$ Leptonic Sector

We choose the $\mathrm{SU}(2)_L$ doublett

$$L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L = \frac{1}{2}(1-\gamma^5)\begin{pmatrix} \nu \\ e \end{pmatrix}, \quad \begin{array}{l} I_3 = +\frac{1}{2},\ Q = 0,\ Y = -1 \\ I_3 = -\frac{1}{2},\ Q = -1,\ Y = -1 \end{array}$$

and the singlett

$$R = e_R = \frac{1}{2}(1+\gamma^5)e,\ \ I_3 = 0,\ Q = -1,\ Y = -2$$

which transform $\mathrm{SU}(2)_L$ according to

$$L \to L' = e^{i\alpha^a \frac{\tau_a}{2}} L, \quad R \to R' = R$$

and under $\mathrm{U}(1)_Y$ according to

$$L \to L' = e^{i\beta^a \frac{Y}{2}} L, \quad R \to R' = e^{i\beta^a \frac{Y}{2}} R$$

QFD: $SU(2)_L \times U(1)_Y$ Leptonic Sector

Now we construct the gauge fields W_μ^a for $\mathrm{SU}(2)_L$ analogously to $SU(3)_C$ before and B_μ of $\mathrm{U}(1)_Y$ analously to the QED before. We get the covariant derivative

$$D_\mu = \partial_\mu + ig\frac{\tau_a}{2}W_\mu^a + ig'\frac{Y}{2}B_\mu.$$

Using this, we can construct the first part of the QFD Lagrangian

$$\mathcal{L}^1_{\mathrm{QFD}} = -\frac{1}{4}W^a_{\mu\nu}W_a^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + i\bar{L}\not{D}L + i\bar{R}\not{D}R,$$

with

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g\epsilon^a{}_{bc}W^b_\mu W^c_\nu$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu.$$

QFD: $SU(2)_L \times U(1)_Y$ Masses

- Mass of the gauge bosons

  Now we would like to add gauge boson masses:

$$\frac{1}{2}M^2 B^\mu B_\mu$$

  However, this is not invariant under $SU(2)$:

$$\rightarrow \frac{1}{2}M^2(B^\mu - \frac{1}{g'}\partial^\mu \alpha(x))(B_\mu - \frac{1}{g'}\partial_\mu \alpha(x))$$

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  However, this is not invariant under $SU(2)$:
$$\rightarrow \frac{1}{2}M^2(B^\mu - \frac{1}{g'}\partial^\mu\alpha(x))(B_\mu - \frac{1}{g'}\partial_\mu\alpha(x))$$
- Mass of the fermions
$$-m\bar{e}e = -m\bar{e}\left(\frac{1}{2}(1-\gamma^5) + \frac{1}{2}(1+\gamma^5)\right)e$$
$$= -m(\bar{e}_R e_L + \bar{e}_L e_R)$$
  But only e_L and not e_R is transforming under $SU(2)$!

QFD: $SU(2)_L \times U(1)_Y$ Masses

- Mass of the gauge bosons
  Now we would like to add gauge boson masses:
  $$\frac{1}{2}M^2 B^\mu B_\mu$$
  However, this is not invariant under $SU(2)$:
  $$\rightarrow \frac{1}{2}M^2(B^\mu - \frac{1}{g'}\partial^\mu\alpha(x))(B_\mu - \frac{1}{g'}\partial_\mu\alpha(x))$$
- Mass of the fermions
  $$-m\bar{e}e = -m\bar{e}\left(\frac{1}{2}(1-\gamma^5) + \frac{1}{2}(1+\gamma^5)\right)e$$
  $$= -m(\bar{e}_R e_L + \bar{e}_L e_R)$$
  But only e_L and not e_R is transforming under $SU(2)$!

We have a beautiful theory of massless particles!

QFD: $SU(2)_L \times U(1)_Y$ EWSB

In order to allow masses for the gauge bosons, we introduce the Higgs doublett into the theory:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix},\ Y = +1 \quad \text{which is gauged like} \quad \Phi = e^{i\frac{\sigma_a \alpha^a}{2v}} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+\eta \end{pmatrix}$$

We obtain $v = \sqrt{-\mu^2/\lambda}$ as vacuum expectation value of the field in the potential

$$V(\Phi) = \frac{\mu^2}{2}\Phi^+\Phi + \frac{\lambda}{4}(\Phi^+\Phi)^2$$

with $\lambda > 0$ and $\mu^2 < 0$, such that there is spontaneous symmetry breaking (the ground state does not obey the symmetries of the theory). ϕ^+ has to be gauged to 0 in order to render the charge operator $Q = I_3 + \frac{Y}{2}$ unbroken. Otherwise the photon acquires mass.

QFD: $SU(2)_L \times U(1)_Y$ EWSB

Using the global $SU(2)_L$ gauge transformation from before

$$L \to L' = e^{-i\frac{\sigma^a \alpha_a}{2v}} L \Rightarrow \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \eta \end{pmatrix}$$

we obtain the following expression for the mass sector of the QFD:

$$\mathcal{L}^2_{\mathrm{QFD}} = -\sqrt{2} f (\overline{L}\Phi R + \overline{R}\Phi^+ L) + |D_\mu \Phi|^2 - V(\Phi)$$

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$$\mathcal{L}^2_{\mathrm{QFD}} = -\sqrt{2} f (\overline{L} \Phi R + \overline{R} \Phi^+ L) + |D_\mu \Phi|^2 - V(\Phi)$$

From where do we get the fermion masses?

$$-\sqrt{2} f (\overline{L} \Phi R + \overline{R} \Phi^+ L)$$

acts as a mass term with the Yukawa coupling parameter f determining the mass of the fermion.

QFD: $SU(2)_L \times U(1)_Y$ EWSB

The gauge boson masses are coming from

$$|D_\mu \Phi|^2 = \frac{1}{8} g^2 v^2 (W^a_{\mu\nu})^2 + \frac{1}{8} g'^2 v^2 B_\mu B^\mu - \frac{1}{4} g g' v^2 B^\mu W^3_\mu$$

using

$$(W^1_\mu)^2 + (W^2_\mu)^2 = (W^1_\mu + iW^2_\mu)(W^1_\mu - iW^2_\mu) = 2 W^+_\mu W^-_\mu$$

introducing the charged currents. That yields

$$\frac{1}{4} g^2 v^2 W^+_\mu W^-_\mu + \frac{1}{8} v^2 (B^\mu, W^3_\mu) \begin{pmatrix} g'^2 & -gg' \\ -gg' & g^2 \end{pmatrix} \begin{pmatrix} B^\mu \\ W^{3\mu} \end{pmatrix}$$

We have the mass term on the $W^\pm$ already. Let's diagonalize the mass matrix of the hypercharge field B_μ and the third component of the $SU(2)_L$ gauge field W^3_μ:

$$\begin{pmatrix} A_\mu \\ Z^0_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B^\mu \\ W^{3\mu} \end{pmatrix}$$

QFD: $SU(2)_L \times U(1)_Y$ EWSB

we have now introduced the Weinberg angle

$$\sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}$$

From the diagonalization of the mass matrix for W_μ^3 and B_μ

$$A_\mu = \frac{1}{\sqrt{g^2 + g'^2}}(g' W_\mu^3 + g B_\mu), \quad m_A^2 = 0$$

$$Z_\mu^0 = \frac{1}{\sqrt{g^2 + g'^2}}(g W_\mu^3 - g' B_\mu), \quad m_{Z^0}^2 = \frac{(g^2 + g'^2)v^2}{4}$$

QFD: $SU(2)_L \times U(1)_Y$ EWSB

We also obtain the charged current and its coupling to the W^+_μ as

$$\frac{g}{2\sqrt{2}}(\bar{\nu}_L \gamma^\mu e_L W^+_\mu + h.c.)$$

In addition, as the first tested firm prediction of this theory, the neutral currents have been introduced ('74 November revolution: Gargamelle):

$$\frac{\sqrt{g^2+g'^2}}{4}(\overline{L}\gamma^\mu \tau_3 L - 2\frac{g'^2}{g^2+g'^2}\bar{e}\gamma^\mu e)Z^0_\mu, \qquad \frac{gg'}{\sqrt{g^2+g'^2}}\bar{e}\gamma^\mu e\, A_\mu$$

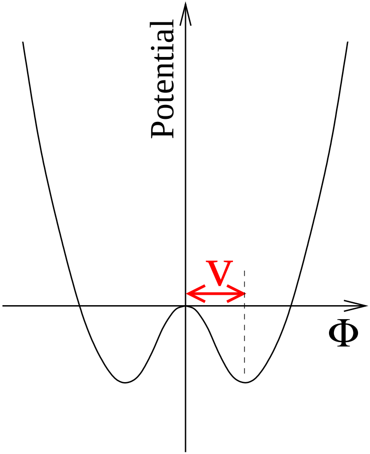
where

$$q_e = \frac{gg'}{\sqrt{g^2+g'^2}}$$

is the electromagnetic charge and $e = e_L + e_R$

This formalism has to be written for all three lepton families $\ell = e, \mu, \tau$.

QFD: $SU(2)_L \times U(1)_Y$ Properties of the Higgs



- The heavier the particle, the stronger the Higgs coupling to it (or the other way around!)
- The position of the minimum of the potential

$$V(\Phi) = \frac{\mu^2}{2}\Phi^+\Phi + \frac{\lambda}{4}(\Phi^+\Phi)^2$$

is known: Compare

$$\frac{g}{2\sqrt{2}}\bar{\nu}_L\gamma^\mu e_L W^+_\mu$$

with $V-A$ theory: $\mathcal{L}_{eff}^{V-A} \sim -\frac{G_F}{2}\ldots$

$$\left(\frac{g}{2\sqrt{2}}\right)^2 \frac{1}{M^2} = \frac{G_F}{2} \Rightarrow v = 246\,\mathrm{GeV}$$

QFD: $SU(2)_L \times U(1)_Y$ Remarks

There are a few non-trivial observations about EWSB in the SM:

- It is not trivial that the photon field A_μ fullfills

$$m_A = 0$$

$$q_e \bar{e} \gamma^\mu e A_\mu$$

  (i.e. no coupling to the neutrino and the same coupling to the left and right fields) at the same time!
- All three elements of

$$\frac{M_W}{M_Z} = \cos\theta_W$$

  can be measured independently ⇒ precision tests
- The Higgs has been introduced to give mass to the gauge bosons, but it offers an elegant way to introduce masses of the fermions, too.
- There is a self-interaction among the gauge bosons in the $-\frac{1}{4} W^a_{\mu\nu} W^{\mu\nu}_a$ term. This just pops out of the theory, it was not constructed as the gauge boson fermion interactions. Does Nature obey the SM also in

Quarks

For the quarks, we choose the fundamental states differently for the mass and the interaction operators:

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L, \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L, \qquad u_R, \quad d_R, \quad c_R, \quad s_R, \quad t_R, \quad b_R$$

being the weak interaction eigenstates. We get the mass eigenstates using the CKM matrix:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{\mathrm{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \approx \begin{pmatrix} 1 & \lambda & A\rho\lambda^3 e^{i\delta} \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1-\rho e^{i\delta}) & -A\lambda^2 & 1 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$VV^+ = 1$$

Quarks

Then the QFD of the quarks can be written in exact analogy to the leptons. We ge tadditional terms for the right-handed up-type quarks, for which we have no corresponding leptons in the SM wit hmassles sneutrinos. We use a $SU(2)$ transform of the Higgs field for the right-handed up-type quark mass terms.

$$-\sqrt{2}f_d(\bar{u},\bar{d}')\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}d_R-\sqrt{2}f_u(\bar{u},\bar{d}')\begin{pmatrix}-\phi^0\\ \phi^+\end{pmatrix}u_R.$$

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Input parameters to the QFD:

m_e	≈ 511 keV	m_μ	≈ 105 MeV	m_τ	$\approx 1,7$ GeV
m_u	≈ 5 MeV	m_d	≈ 5 MeV	m_s	≈ 150 MeV
m_c	$\approx 1,5$ GeV	m_b	≈ 4.7 GeV	m_t	≈ 174 GeV
m_H	$\approx$?	m_W	≈ 81 GeV	$\alpha(Q^2\approx 0)$	$\approx 1/137$
$\sin\theta_W$	$\approx 0,23$	λ	$\approx 0,22$	ρ	≈ 0.8
A	$\approx 0,5$	δ	$\approx 0,004$		

This has to be slightly extended if neutrino masses and mixing are added.

Reading the Feynman Rules

1. Draw your Feynman diagram
2. Follow the fermion lines in opposite direction of the arrows. For each outgoing (anti)particle, write $\bar{u}(v)$, for each incoming (anti)particle $u(\bar{v})$.
3. For each incoming(outgoing) photon, write $\epsilon_\mu(\epsilon^*_\mu)$
4. For each internal line, write a propagator:
   - Fermion: $1/(\not{p} - m)$
   - Photon: $-ig_{\mu\nu}/p^2$
   - Boson: $-i(g_{\mu\nu} - p_\mu p_\nu/M^2)/(p^2 - M^2)$
5. Read the couplings from the Lagrangian:
   QED example: $\mathcal{L}_{int} = -q_e\bar{\psi}\gamma_\mu\psi A^\mu$
   denotes the coupling of an incoming fermion ψ and an outgoing fermion $\bar{\psi}$ to the photon A^μ with coupling q_e.
   In this case, we get

$$iq_e\gamma_\mu$$

   for each photon-electron vertex.