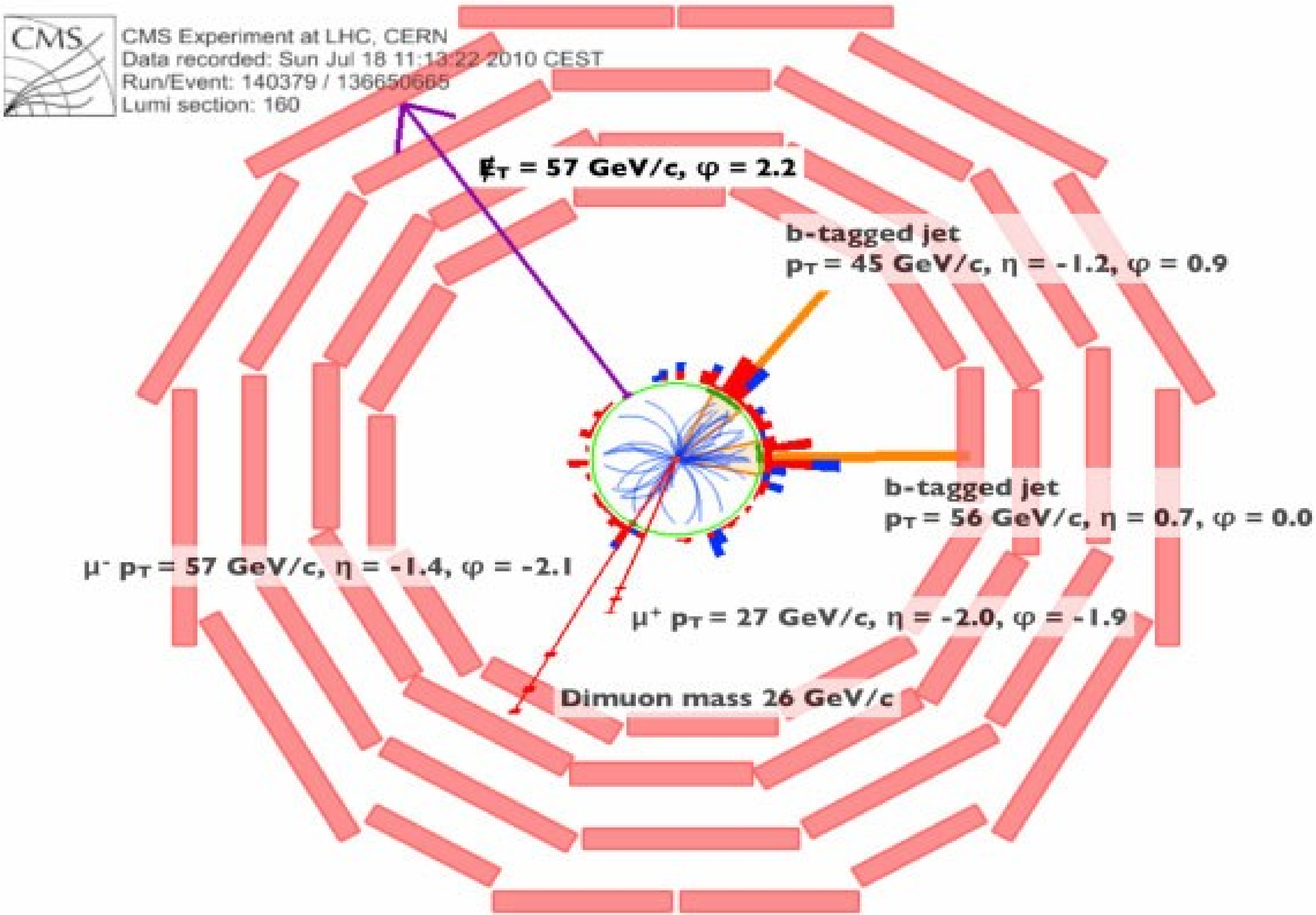
Monte Carlo Simulations

H. Jung (DESY, University Antwerp)

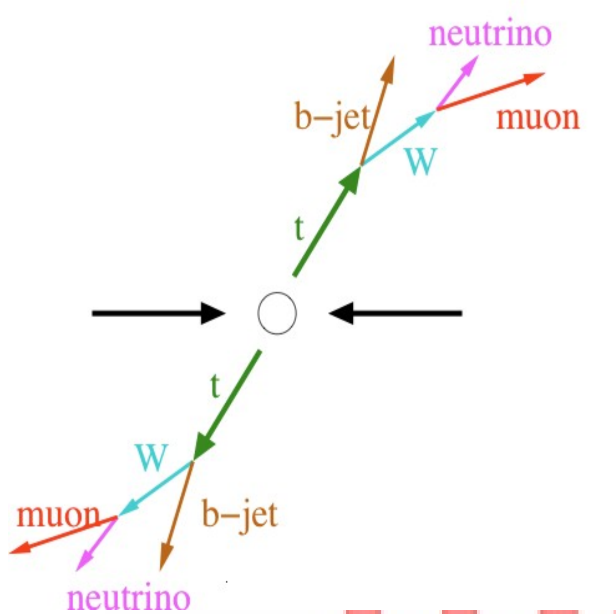
LHC rap

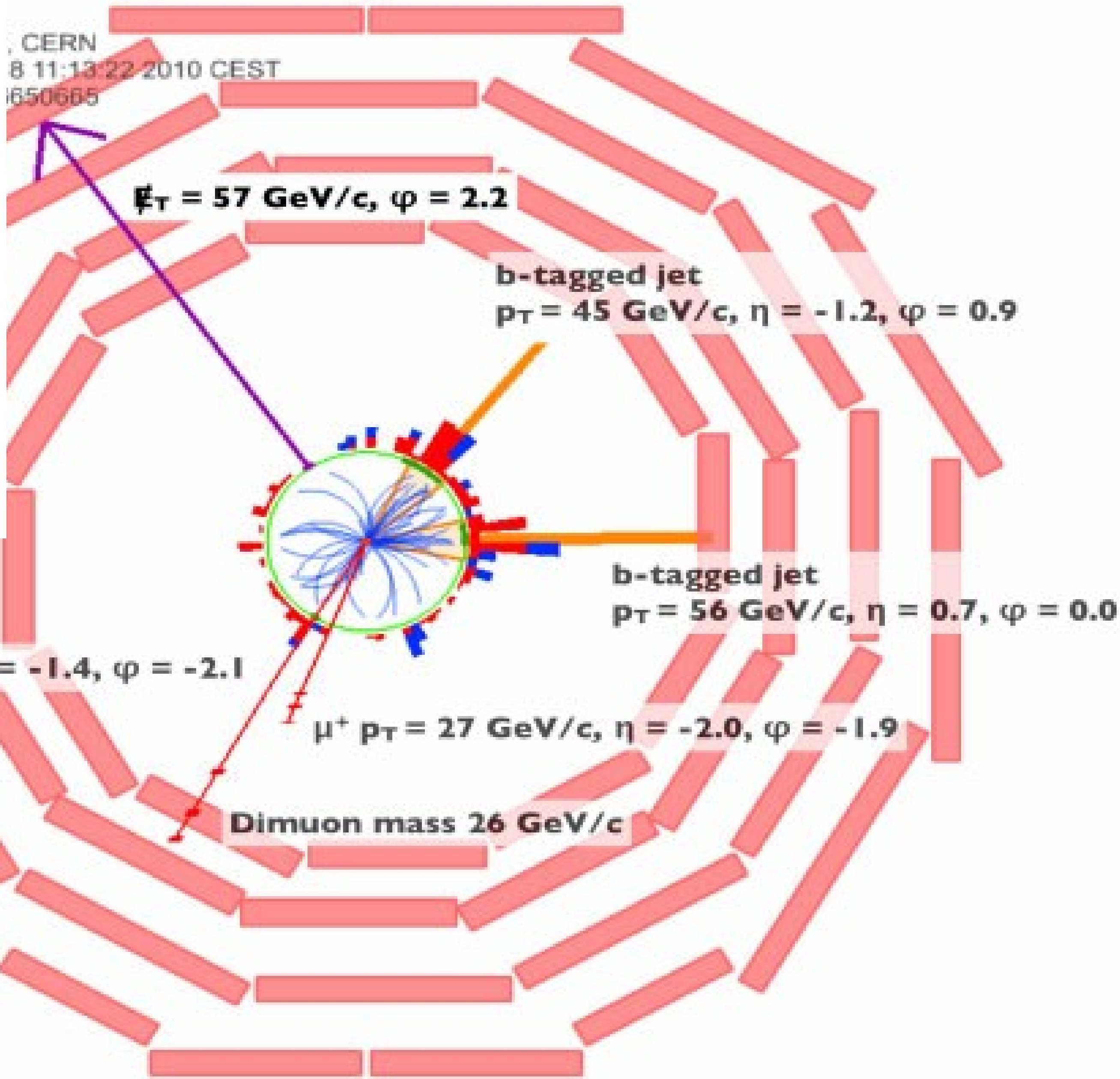
A real event - from July 18, 2010



What is this ?

A real event - from July 18, 2010





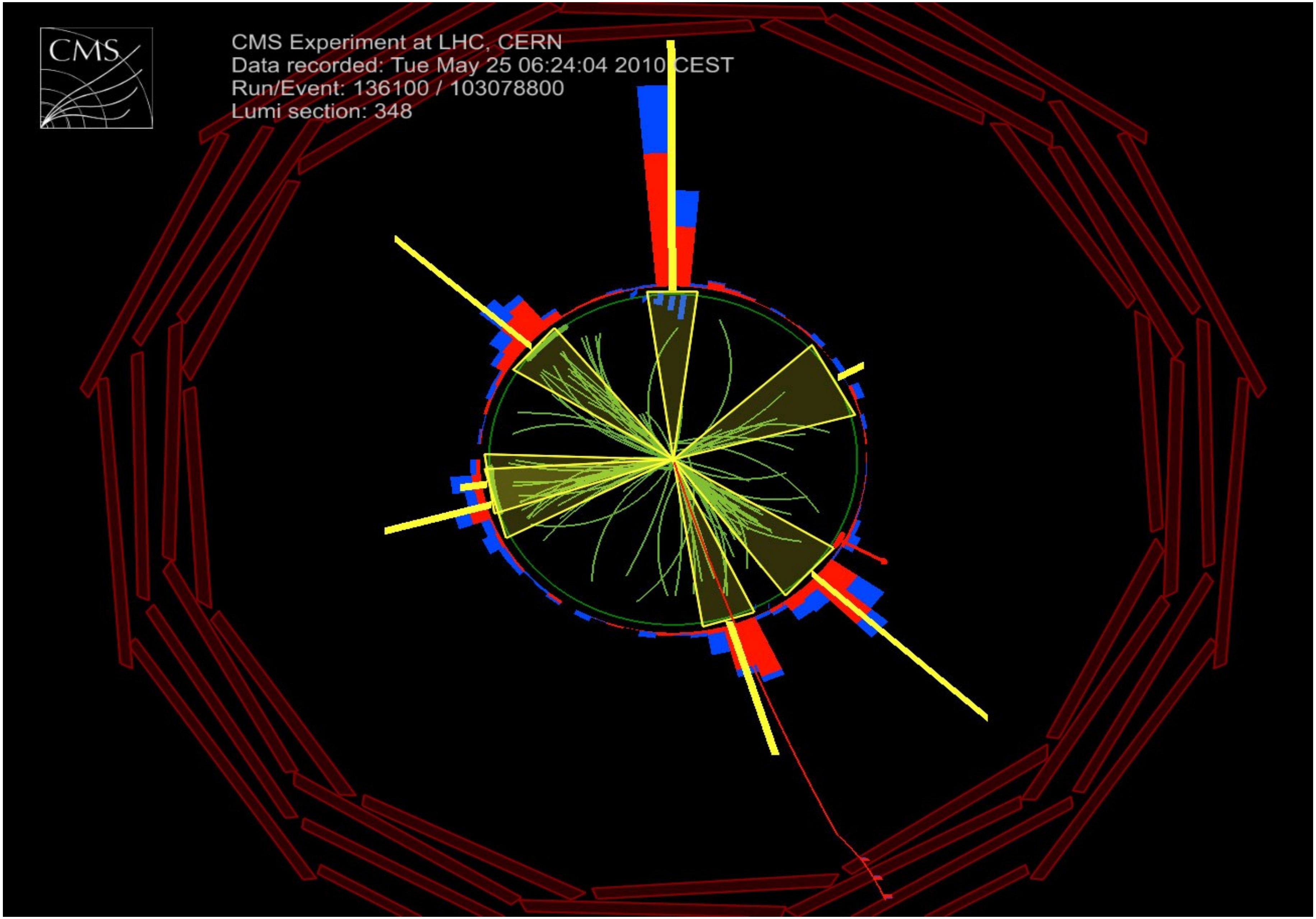
$$pp \rightarrow t\bar{t} + X$$

$$t(\bar{t}) \rightarrow W + b$$

$$W \rightarrow \mu + \nu$$

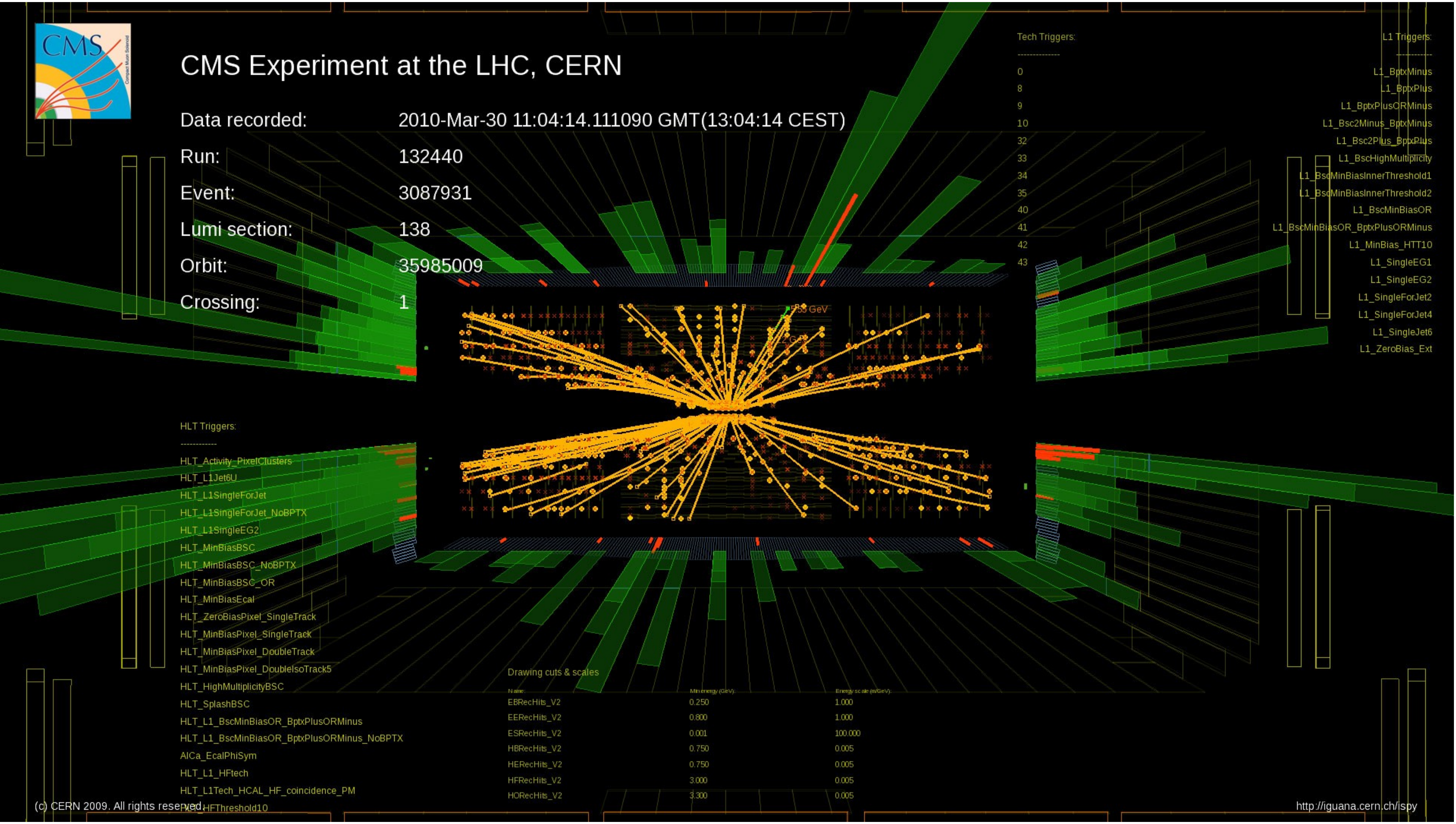
Is this a typical LHC event ?

Another real event ...

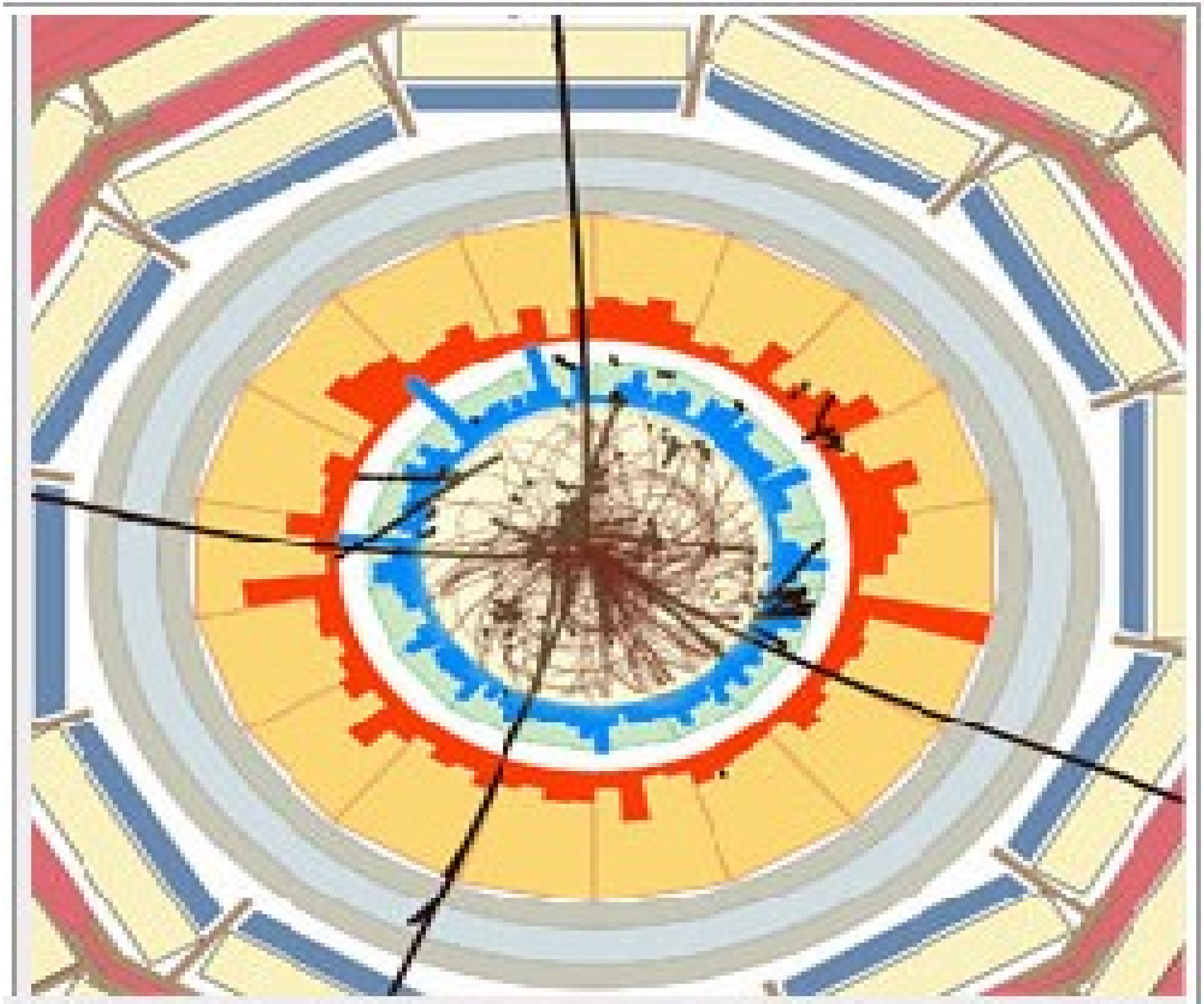


Is this then a typical LHC event ?

Another real event ...



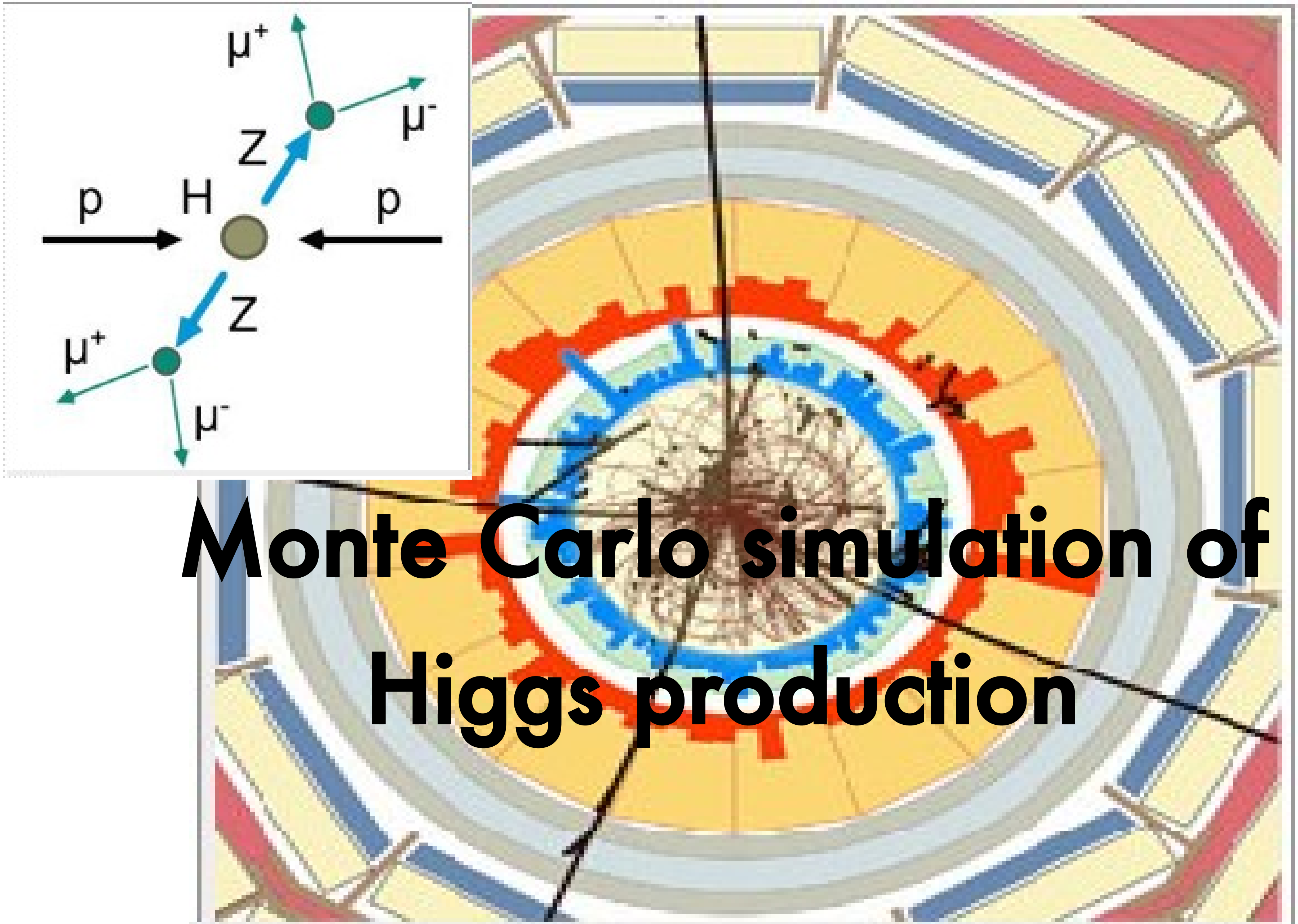
and what is this ?



and what is this ?

Monte Carlo simulation of Higgs production

and what is this ?



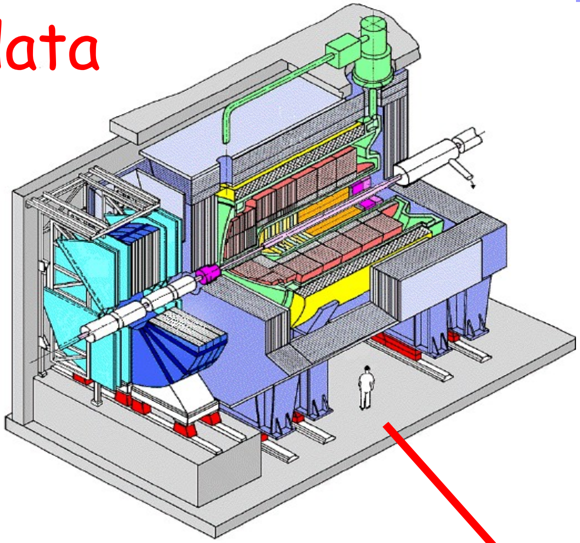
Monte Carlo simulation of Higgs production

How to extract anything useful from these events ?????

We need a simulation of the physics and the detector !

From experiment to measurement

take data

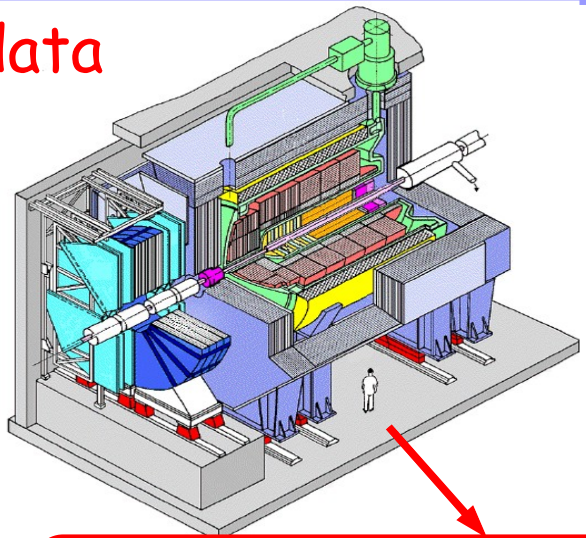


- run MC generator
- detector simulation

Upppps ...... all measurements rely on proper MC generators and MC simulation !!!!

From experiment to measurement

take data



- run MC generator
- detector simulation

define visible x - section in kinematic variables

calculate factor C_{corr} to correct from detector to hadron level

$$\frac{d\sigma^{data}_{had}}{dx} = \frac{d\sigma^{data}_{det}}{dx} C_{corr} \text{ with } C_{corr} = \frac{\frac{d\sigma^{MC}_{had}}{dx}}{\frac{d\sigma^{MC}_{det}}{dx}}$$

visible x-section on hadron level

Upppps ...... all measurements rely on proper MC generators and MC simulation !!!!

Monte Carlo – different applications

- MC simulation of detector response
  - input: hadron level events - output: detector level events
  - Calorimeter ADC hits
  - Tracker hits
  - need knowledge of particle passage through matter, x-section ...
  - need knowledge of actual detector

Monte Carlo – different applications

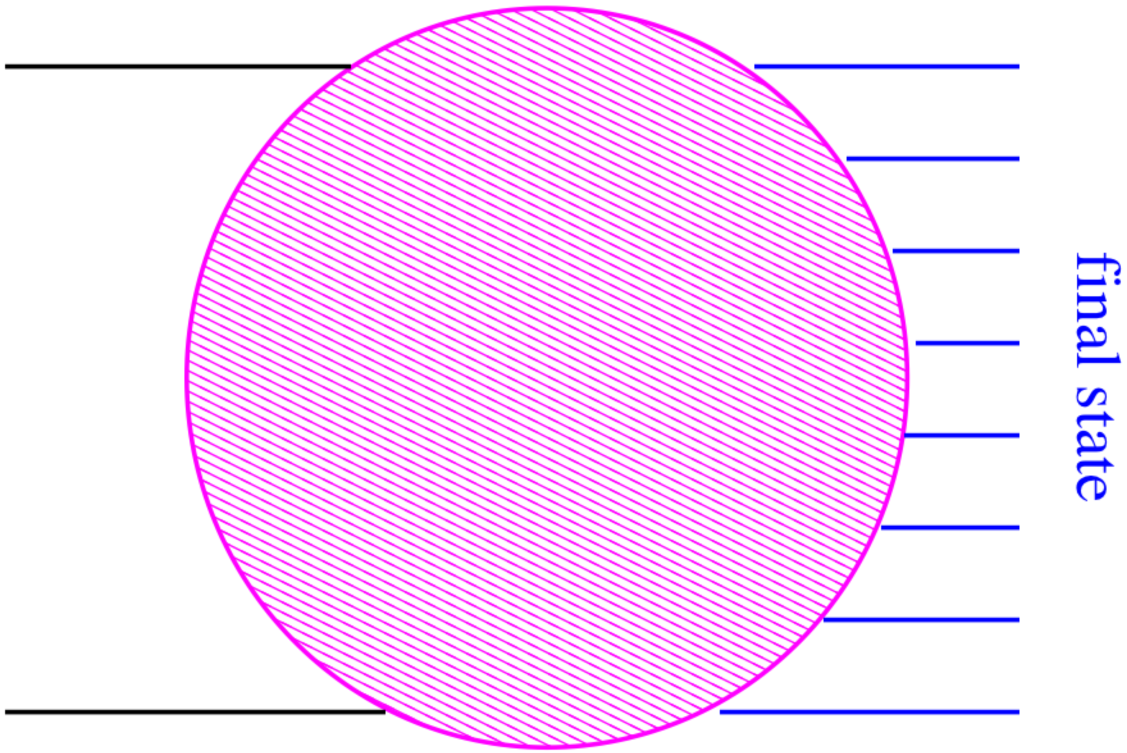
- MC simulation of detector response
  - input: hadron level events - output: detector level events
  - Calorimeter ADC hits
  - Tracker hits
  - need knowledge of particle passage through matter, x-section ...
  - need knowledge of actual detector

Not covered here

- multipurpose MC event generators:
  - x-section on parton level
  - including multi-parton (initial & final state) radiation
  - remnant treatment (proton remnant, electron remnant)
  - hadronization/fragmentation (more than simple fragmentation functions...)
- fixed order parton level ...... theorists like it !!!!!!!!!!!!!!!!!!!!!!
  - integration of multidimensional phase space

The general case

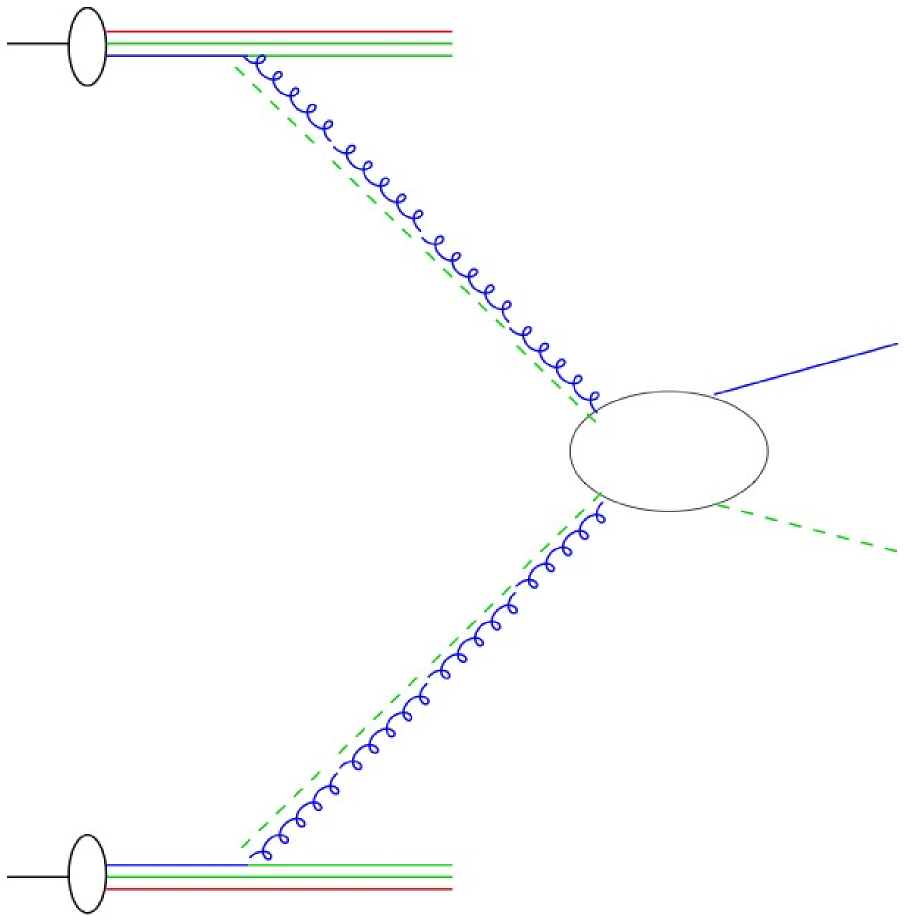
- Calculation of cross section of $A + B \rightarrow \text{anything}$



➔ Start with jet production ....

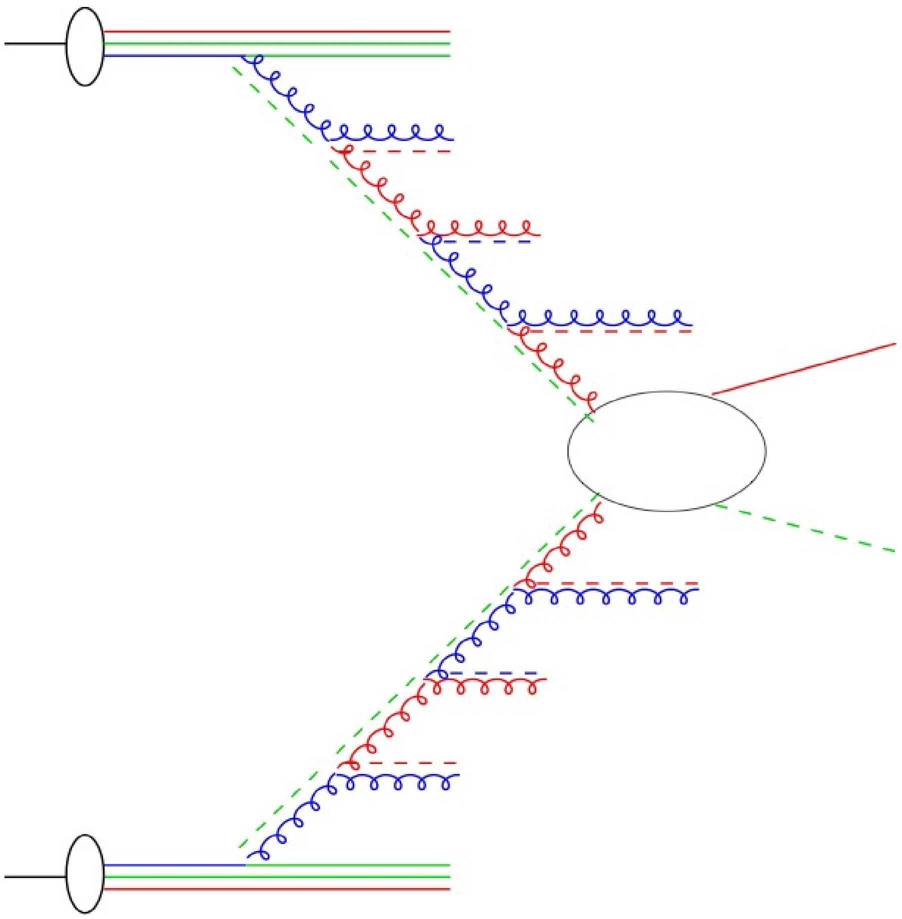
Picture of jet production

- General approach to hard scattering processes



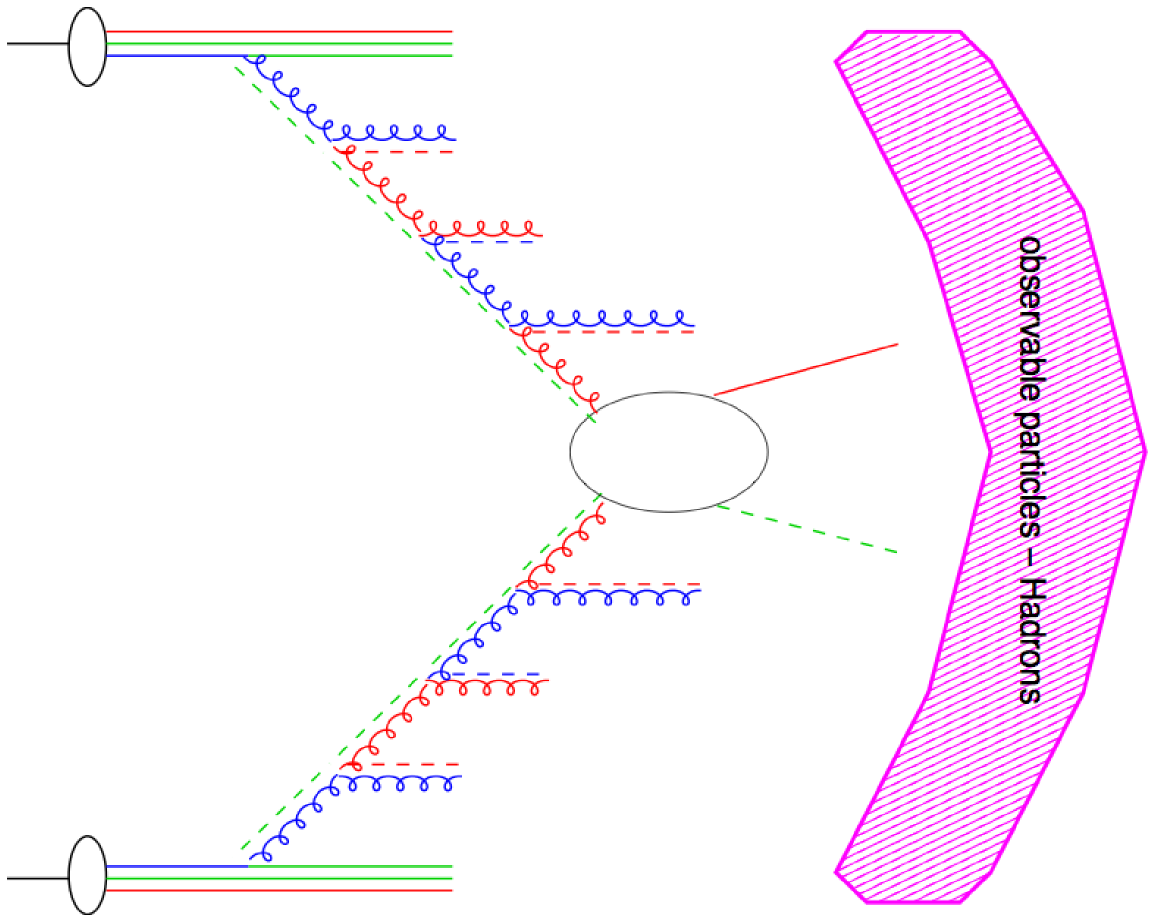
Picture of jet production

- General approach to hard scattering processes
  - including higher order parton radiation



Picture of jet production

- General approach to hard scattering processes
  - including higher order parton radiation
    - adding hadronization and fragmentation

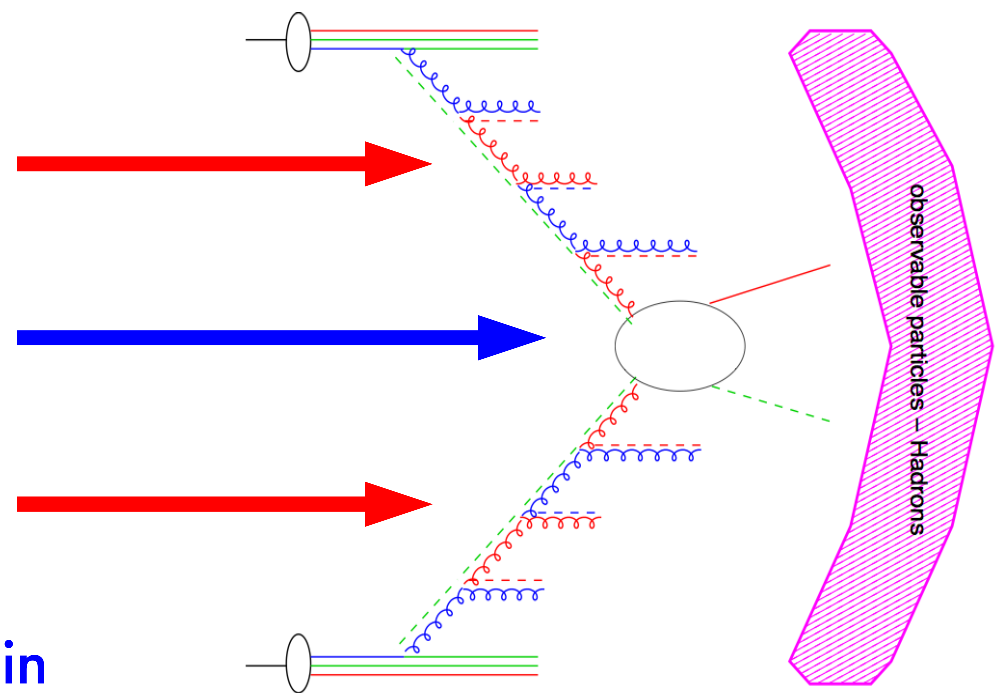


Picture of jet production

- General approach to hard scattering processes
  - including higher order parton radiation
    - adding hadronization and fragmentation

➔ leads to the concept of factorization:

$$\sigma = f(x_1,\mu^2) \otimes \hat{\sigma}(x_1,x_2,\mu^2) \otimes f(x_2,\mu^2)$$

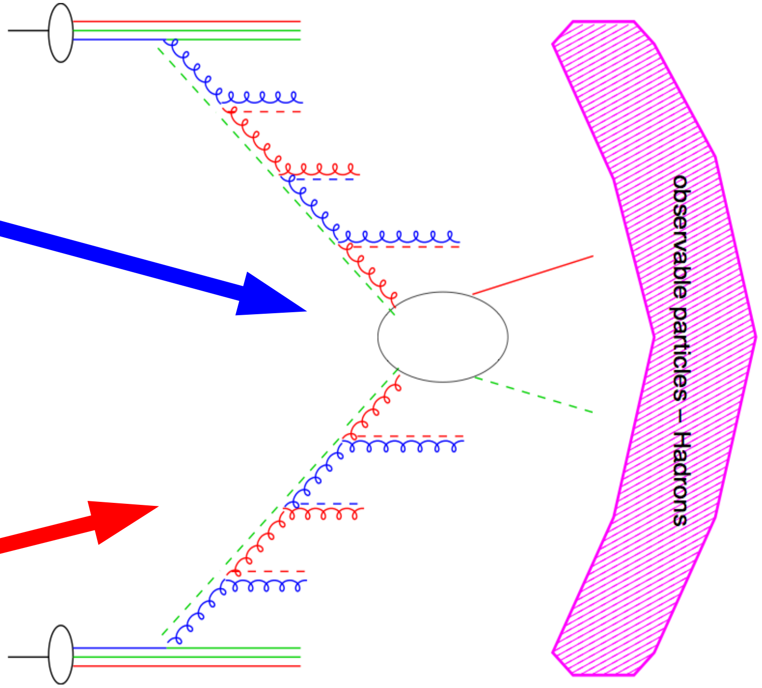


➔ factorization ansatz is used in

any calculation (LO,NLO, MC event generators ...)

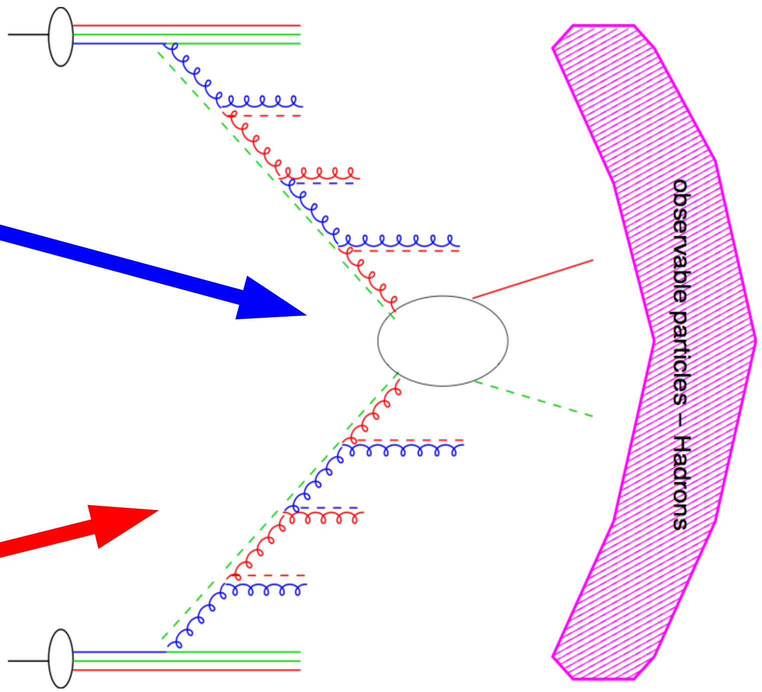
How to simulate these processes ?

- calculate hard scattering
  - example of $e^+e^- \to \mu^+\mu^-$
- hadronization
  - example of $e^+e^- \to q\bar{q}$
- multi parton radiation
  - example of $ep \to e'X$



How to simulate these processes ?

- calculate hard scattering
  - example of $e^+e^- \to \mu^+\mu^-$
- hadronization
  - example of $e^+e^- \to q\bar{q}$
- multi parton radiation
  - example of $ep \to e'X$

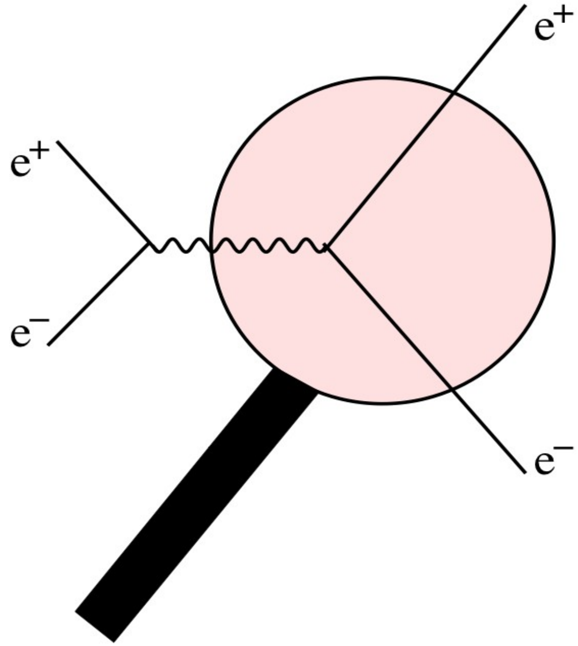


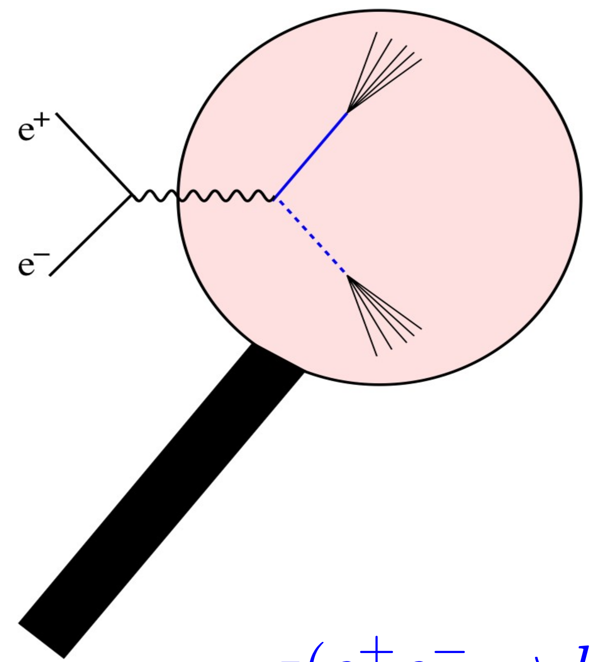
- and, is this then all ?

  ➔left for the discussion ....

The easy case: $e^+e^- \longrightarrow X$

- use $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to q\bar{q} \to \mathrm{hadrons}$



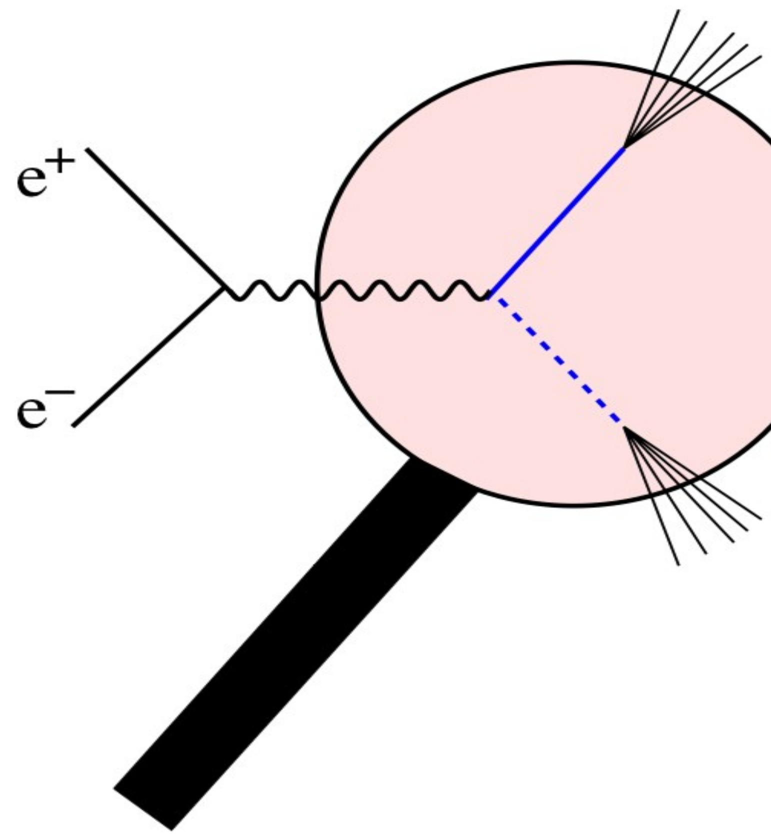


- cross sections can be calculated in QED: $\sigma(e^+e^- \to l^+l^-) = \dfrac{4\pi\alpha^2}{3s}$
- and for quarks $\sigma(e^+e^- \to q\bar{q}) = 3\dfrac{4\pi\alpha^2}{3s}e_q^2$

➔ but quarks carry color and fractional charge !!!!!

color charge

The early steps: $e^+e^- \to$ hadrons



➔How to compare a detailed measurement with a theoretical prediction ?

Simulate these processes with Monte Carlo method !!!

Basics for Simulation

- Need the cross section for a process:
  - ➔ probability that a process happens with given kinematics
  - ➔ simulate kinematics of event according to cross section
- Need a mechanism to select a given configuration according to a probability density function
  - ➔ Make use of Random Numbers

➔ Monte Carlo Method

Monte Carlo method

- Monte Carlo method
  - refers to any procedure that makes use of random numbers
  - uses probability statistics to solve the problem
- Random number:

Monte Carlo method

- Monte Carlo method
  - refers to any procedure that makes use of random numbers
  - uses probability statistics to solve the problem
- Random number:

  one of them is 3

Monte Carlo method

- Monte Carlo method
  - refers to any procedure that makes use of random numbers
  - uses probability statistics to solve the problem
- Random number:

one of them is 3
No such thing as a single random number
A sequence of random numbers is a set of numbers that have nothing to do with the other numbers in a sequence

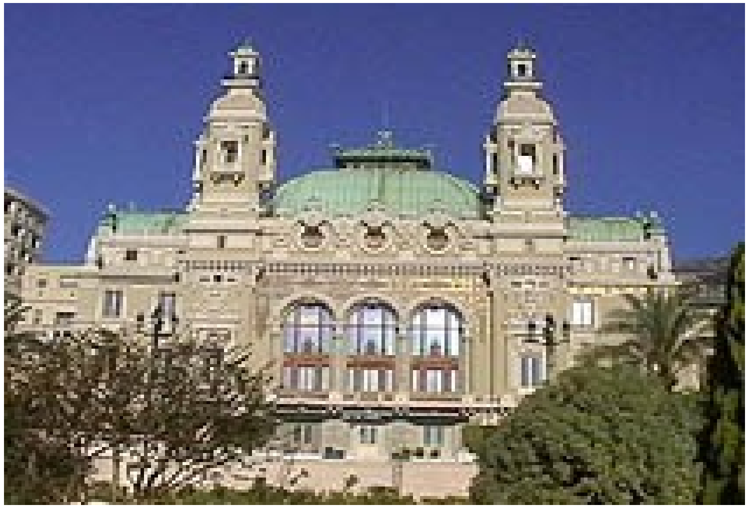
Monte Carlo method

- Monte Carlo method
  - refers to any procedure that makes use of random numbers
  - uses probability statistics to solve the problem
- Random number:

Monte Carlo method

- Monte Carlo method
  - refers to any procedure that makes use of random numbers
  - uses probability statistics to solve the problem
- Monte Carlo methods are used in:
  - Simulation of natural phenomena
  - Simulation of experimental apparatus
  - Numerical analysis

Why Monte Carlo ?



- Obtain true Random Numbers from Casino in Monte Carlo
- Puhhh... Going out every night ...

Random Numbers

- In a uniform distribution of random numbers in [0,1] every number has the same chance of showing up
- Note that 0.000000001 is just as likely as 0.5

To obtain random numbers:

- Use some chaotic system like roulette, lotto, 6-49, ...
- Use a process, inherently random, like radioactive decay
- Tables of a few million truly random numbers exist .....

(.....until ~ 30 years ago.....)

BUT not enough for most applications

→ .... we have true random number generators ...

True Random Numbers

- Random numbers from classical physics: coin tossing

  evolution of such a system can be predicted, once the initial condition is known... however it is a chaotic process ... extremely sensitive to initial conditions.

- Truly Random numbers used for
  - Cryptography

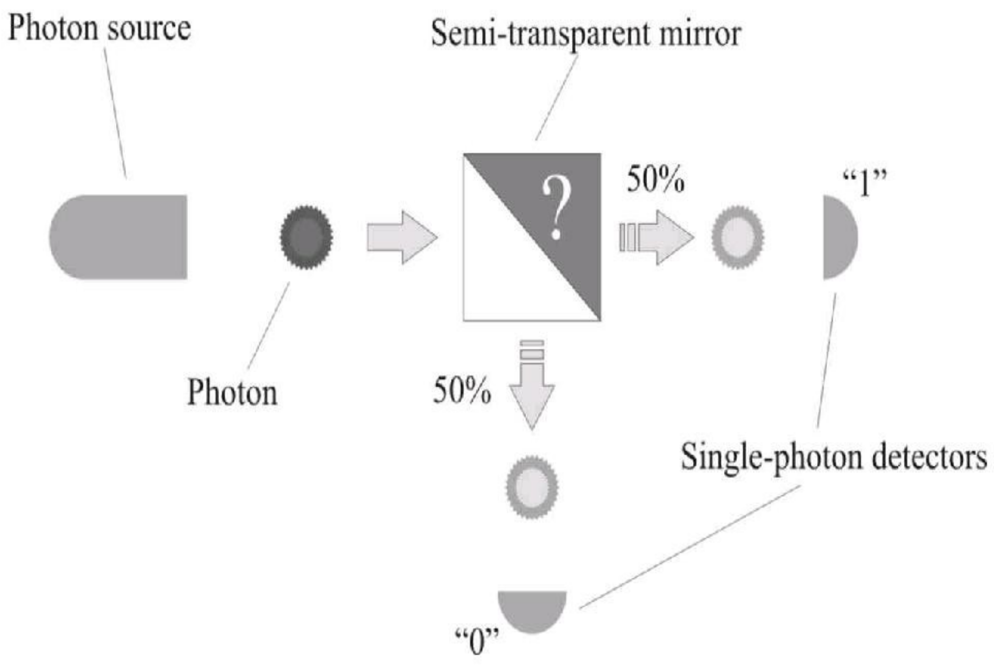
    Confidentiality

    Authentication
  - Scientific Calculation
  - Lotteries and gambling

    do not allow to increase chance of winning by having a bias .... too bad

- Random numbers from quantum physics: intrinsic random

  photons on a semi-transparent mirror



- Available and tested in MC generator by a summer student
- Generator is however very slow...

True Random Numbers

- atmospheric noise, which is quite easy to pick up with a normal radio: used by RANDOM.ORG
- much more can be found on the web ....

Home Introduction Statistics Numbers Drawings Testimonials Quota FAQ Contact Login What's New!

RANDOM.ORG

Search RANDOM.ORG

Google™ Custom Search Search

True Random Number Service

What's this fuss about *true* randomness?

Perhaps you have wondered how predictable machines like computers can generate randomness. In reality, most random numbers used in computer programs are *pseudo-random*, which means they are a generated in a predictable fashion using a mathematical formula. This is fine for many purposes, but it may not be random in the way you expect if you're used to dice rolls and lottery drawings.

RANDOM.ORG offers *true* random numbers to anyone on the Internet. The randomness comes from atmospheric noise, which for many purposes is better than the pseudo-random number algorithms typically used in computer programs. People use RANDOM.ORG for holding drawings, lotteries and sweepstakes, to drive games and gambling sites, for scientific applications and for art and music. The service has existed since 1998 and was built and is being operated by Mads Haahr of the School of Computer Science and Statistics at Trinity College, Dublin in Ireland.

True Random Number Generator

Min: 1

Max: 100

Generate

Result:

Powered by RANDOM.ORG

Like the widget?

Get Your Own

Pseudo Random Numbers

Pseudo Random Numbers

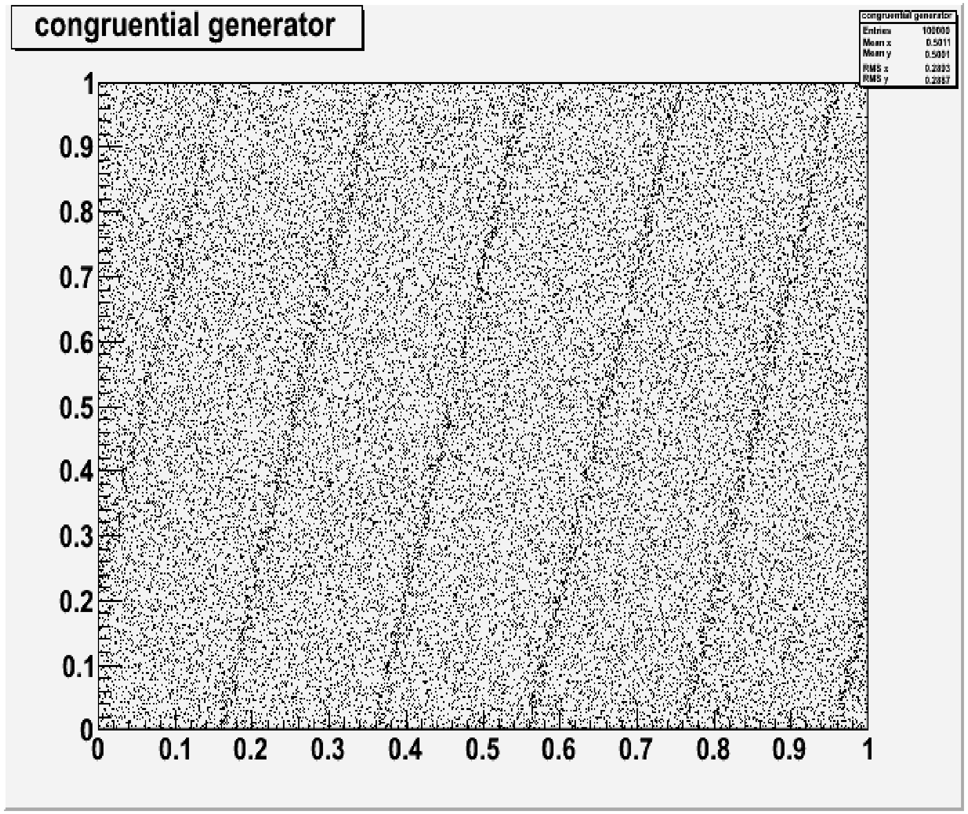
- are a sequence of numbers generated by a computer algorithm, usually uniform in the range [0,1]
- more precisely: algo's generate integers between 0 and M, and then $r_n = I_n/M$
- A very early example: Middles Square (John van Neumann, 1946) generate a sequence, start with a number of 10 digits, square it, then take the middle 10 digits from the answer, as the next number etc.:

  $5772156649^2 = 33317792380594909291$

  Hmmmm, sequence is not random, since each number is determined from the previous, but it appears to be random
- this algorithm has problems .....
  BUT a more complex algo does not necessarily lead to better random sequences ....
  Better use an algo that is well understood

Randomness tests

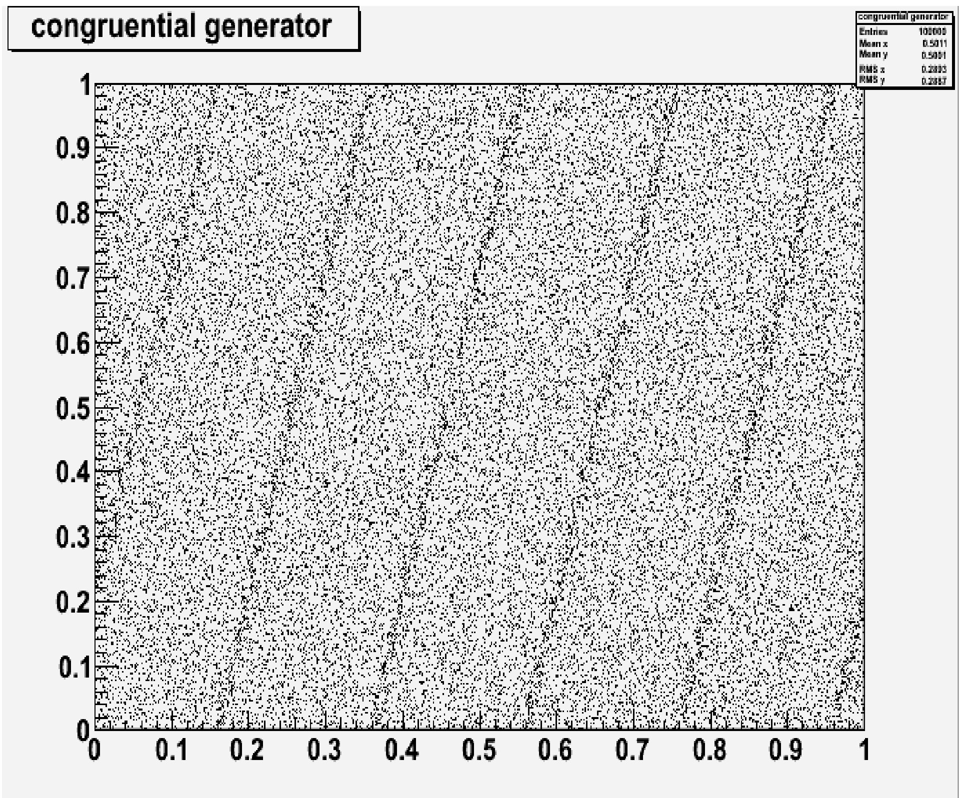
- Simple generator



➔ Simple generator is not bad …
but it could be better ….

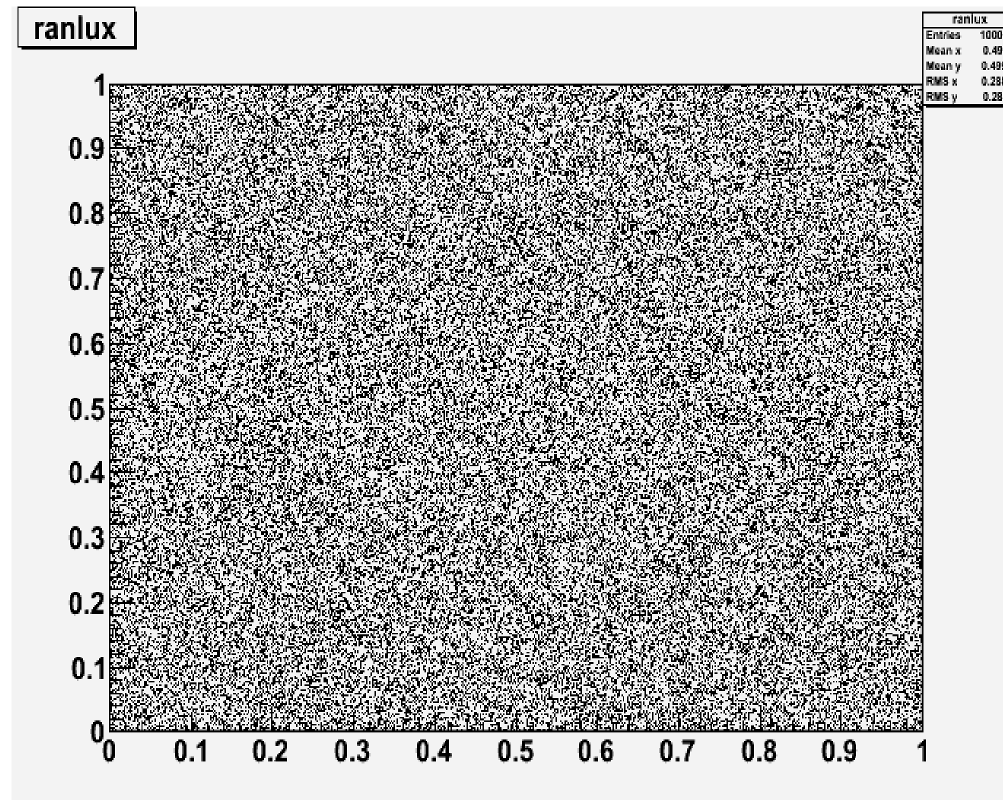
Randomness tests

- Simple generator



- RANLUX

M. Lüscher, A portable high-quality random number generator for lattice field theory simulations, Computer Physics Communications 79 (1994) 100
http://luscher.web.cern.ch/luscher/ranlux/index.html



➔ RANLUX much more sophisticated
Developed and used for QCD lattice calcs

Expectation values and variance

- Expectation value (defined as the average or mean value of function *f*):

$$E[f] = \int f(u)dG(u) = \left(\frac{1}{b-a} \int_a^b f(u)du \right) = \frac{1}{N}\sum_{i=1}^{N} f(u_i)$$

for uniformly distributed u in [a,b] *then* $dG(u) = du/(b-a)$

- rules for expectation values:

$$E[cx+y] = cE[x] + E[y]$$

- Variance

$$V[f] = \int (f - E[f])^2 \, dG = \left(\frac{1}{b-a} \int_a^b (f(u) - E[f])^2 \, du \right)$$

- rules for variance:

if x,y uncorrelated: $V[cx+y] = c^2 V[x] + V[y]$

if x,y correlated $V[cx+y] = c^2 V[x] + V[y] + 2cE\left[(y - E[y])(x - E[x])\right]$

From uniform to other distributions

- cumulative distribution

$$F(x_{max}) = \int_{\infty}^{x_{max}} f(x)dx$$

probability that event happens between $\infty < x < x_{max}$

- Given a random number in [0,1], find a transformation such, that the resulting sequence of x_i is distributed according to f(x)
- inverse transform method:

let x be distributed according to f(x), with $F(x)$ in [0,1], then

$$x = F^{-1}(u)$$

with random number u=[0,1]

Generating distributions

- General form: $$\int_{x_{min}}^{x_j} f(x')dx' = u_j \int_{x_{min}}^{x_{max}} f(x')dx'$$

- linear p.d.f:

$$\begin{aligned} f(x) &= 2x \\ u(x) &= \int_0^x 2t dt = x^2 \\ x_j &= \sqrt{u_j} \end{aligned}$$

- 1/x distribution

$$\begin{aligned} f(x) &= \frac{1}{x} \\ u(x) &= \frac{\int_{x_{min}}^{x} \frac{1}{t} dt}{F_{max} - F_{min}} \\ x_j &= x_{min} \left(\frac{x_{max}}{x_{min}} \right)_j^u \end{aligned}$$

Generating distributions

- Brute Force or Hit & Miss method
  - use this if there is no easy way to find a analytic integrable function
  - find $c \leq \max f(x)$
  - reject if $f(x_i) < u_j \cdot c$
  - accept if $f(x_i) > u_j \cdot c$

- Improvements for Hit & Miss method by variable transformation
  - find $c \cdot g(x) > f(x)$
  - reject if $f(x) < u_j \cdot c \cdot g(x)$
  - accept if $f(x) > u_j \cdot c \cdot g(x)$

Monte Carlo technique: basics

- **Law of large numbers**

  chose N numbers u_i randomly, with probability density uniform in [a,b],

  evaluate $f(u_i)$ for each u_i :

$$\frac{1}{N}\sum_{i=1}^{N} f(u_i) \to \frac{1}{b-a}\int_a^b f(u)du$$

  for large enough N Monte Carlo estimate of integral converges to correct answer.

- Convergence

  is given with a certain probability ...

THIS is a mathematically serious and precise statement !!!!

Central Limit Theorem

- Central Limit Theorem

for large N the sum of independent random variables is always normally (Gaussian) distributed:

$$f(x) = \frac{1}{s\sqrt{2\pi}} \exp\left[-\frac{(x-a)^2}{2s^2}\right]$$

$$\frac{\sum_i x_i - \sum_i \mu_i}{\sqrt{\sum_i \sigma_i^2}} \to N(0,1)$$

→ independent on the original sub-distributions

Central Limit Theorem

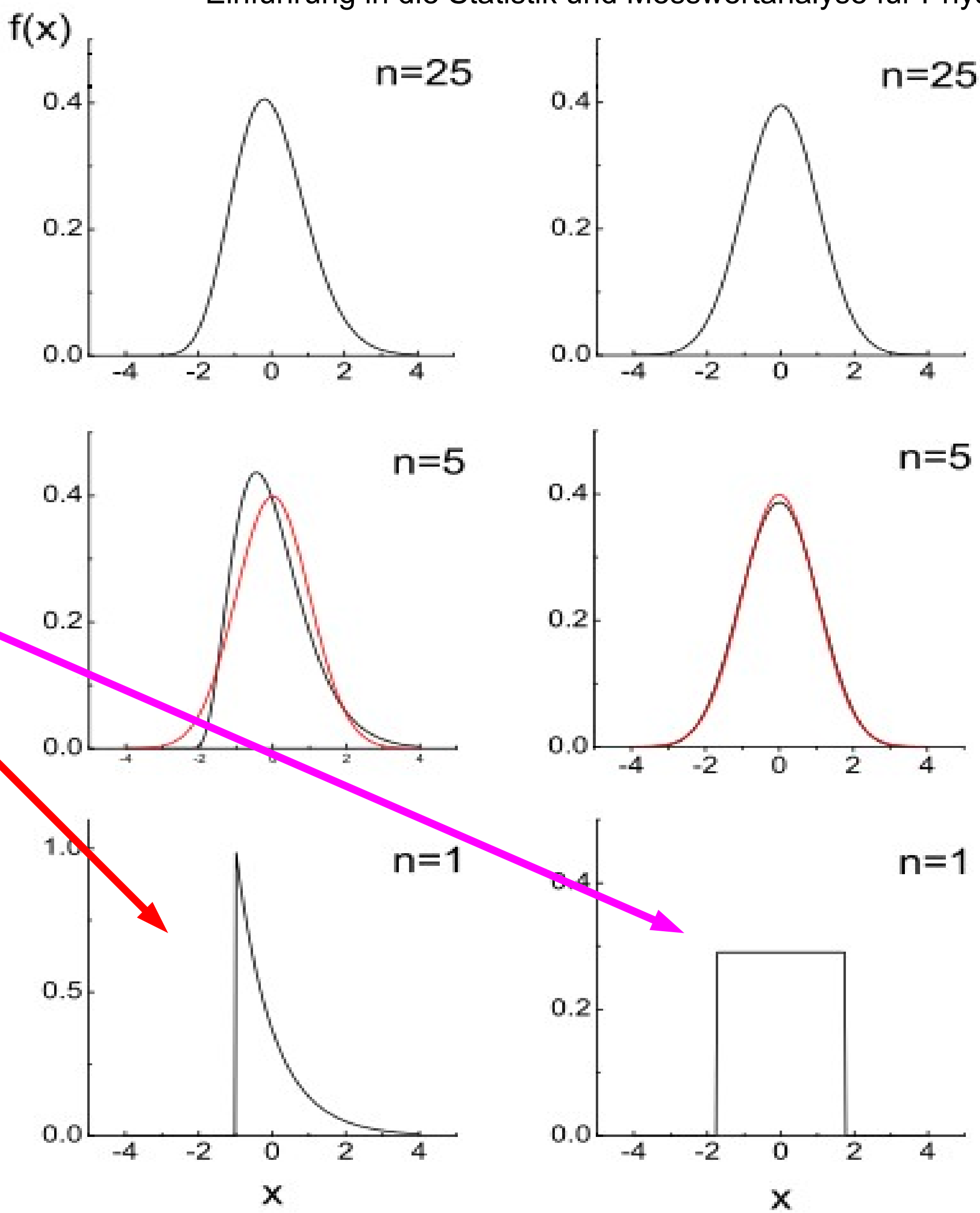
- Central Limit Theorem
  for large N the sum of independent random variables is always normally (Gaussian) distributed

➔ for any starting distribution

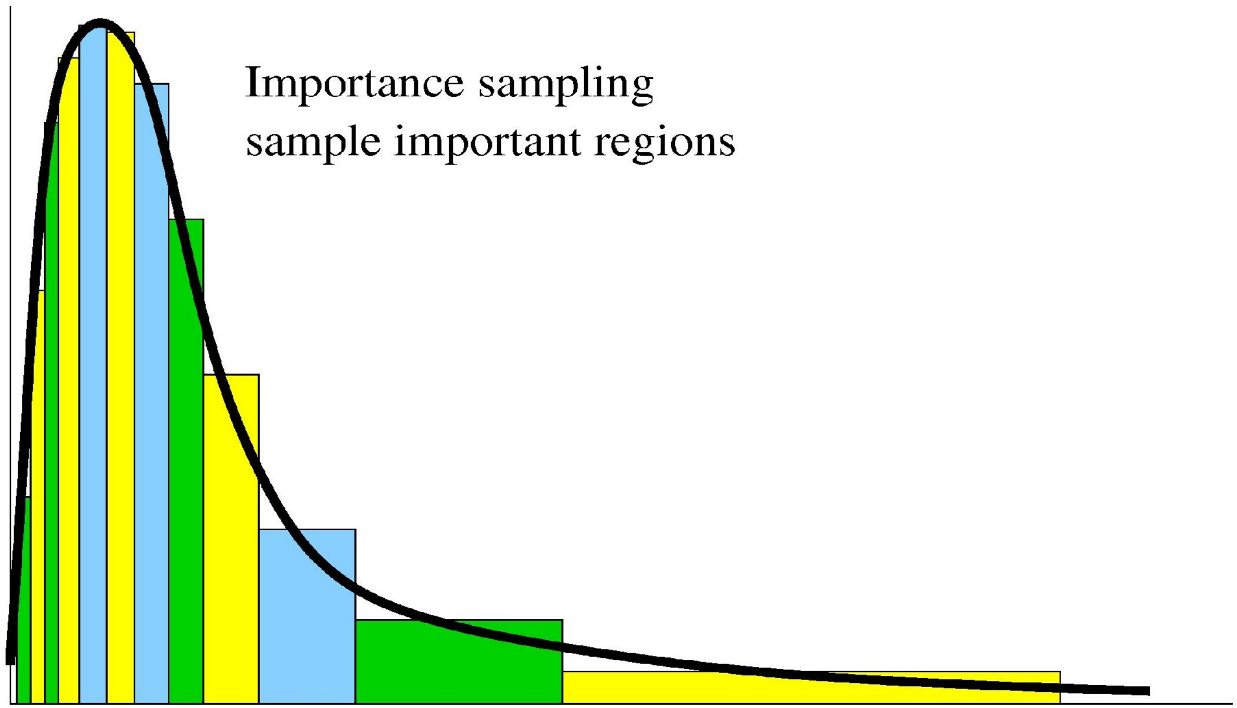
➔ for uniform distribution

➔ for exponential distribution

G. Bohm, G. Zech
Einfuhrung in die Statistik und Messwertanalyse fur Physiker



Importance Sampling



- MC calculations most efficient for small weight fluctuations:

  f(x)dx → f(x) dG(x)/g(x)

- chose point according to g(x) instead of uniformly

$$R \int_a^b g(x')dx' = \int_a^x g(x')dx'$$

We have the method,....

BUT

HOWTO

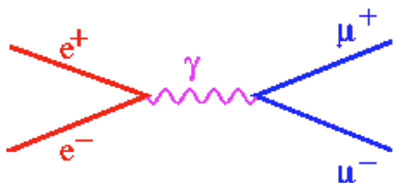
simulate the physics

????

The hard process:

the simple case in e^+e^-

Constructing a MC for e^+e^-

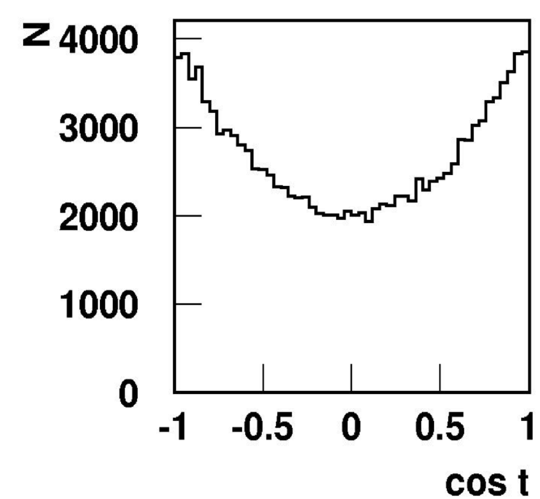


- process: $e^+e^- \to \mu^+ \mu^-$

- $$\frac{d\sigma}{d\cos\theta d\phi} = \frac{\alpha_{em}^2}{4s}\left(1+\cos^2\theta\right)$$

- goal: generate 4-momenta of μ's, need cm energy s, cos θ, ϕ

after 100000 events

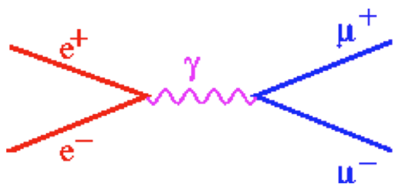


random number R1(0,1): $\phi = 2\pi R1$
random number R2(0,1): $\cos\theta = -1 + 2\,R2$

for every R1, R2 use weight with $\frac{d\sigma}{d\cos\theta\, d\phi}$
repeat many times

Constructing a MC for e^+e^-

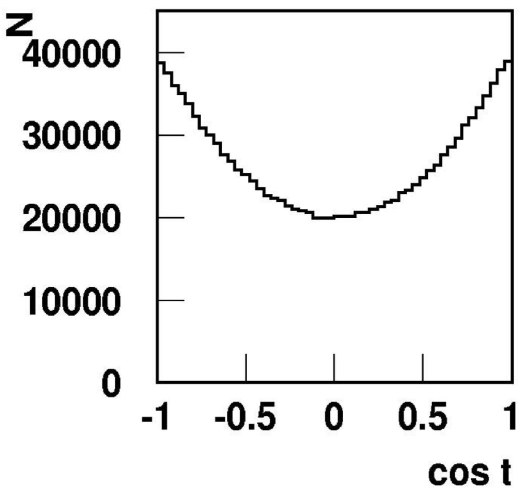
- process: $e^+e^- \rightarrow \mu^+ \mu^-$



- $$\frac{d\sigma}{d\cos\theta d\phi} = \frac{\alpha_{em}^2}{4s}(1+\cos^2\theta)$$

- goal: generate 4-momenta of μ's, need cm energy s, cos θ, ϕ

after 10^6 events



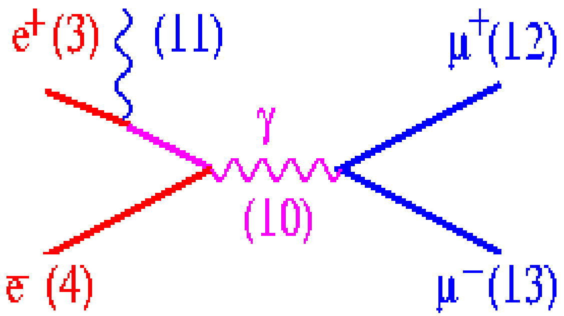
random number R1(0,1): $\phi = 2\pi R1$
random number R2(0,1): $\cos\theta = -1 + 2 R2$

for every R1, R2 use weight with $\frac{d\sigma}{d\cos\theta\, d\phi}$
repeat many times

Example event: $e^+e^- \rightarrow \mu^+ \mu^-$

- example from PYTHIA: Event listing

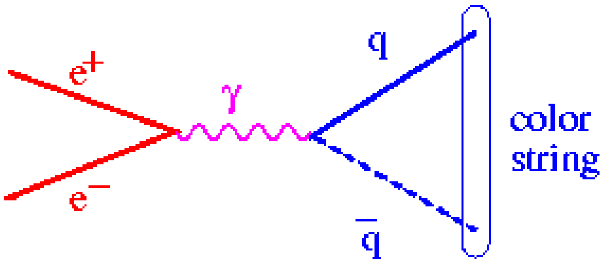
I	particle/jet	KS	KF	orig	p_x	p_y	p_z	E	m
1	!e+!	21	-11	0	0.000	0.000	30.000	30.000	0.001
2	!e-!	21	11	0	0.000	0.000	-30.000	30.000	0.001
3	!e+!	21	-11	1	0.000	0.000	30.000	30.000	0.000
4	!e-!	21	11	2	0.000	0.000	-30.000	30.000	0.000
5	!e+!	21	-11	3	0.143	0.040	26.460	26.460	0.000
6	!e-!	21	11	4	0.000	0.000	-29.998	29.998	0.000
7	!Z0!	21	23	0	0.143	0.040	-3.539	56.458	56.347
8	!mu-!	21	13	7	-9.510	1.741	24.722	26.546	0.106
9	!mu+!	21	-13	7	9.653	-1.700	-28.261	29.913	0.106
10	(Z0)	11	23	7	0.143	0.040	-3.539	56.458	56.347
11	gamma	1	22	3	-0.143	-0.040	3.539	3.542	0.000
12	mu-	1	13	8	-9.510	1.741	24.722	26.546	0.106
13	mu+	1	-13	9	9.653	-1.700	-28.261	29.913	0.106
		sum:	0.00		0.000	0.000	0.000	60.000	60.000



- technicalities/advantages
  - ➔ can work in any frame
  - ➔ Lorentz-boost 4-vectors back and forth
  - ➔ can calculate any kinematic variable
  - ➔ history of event process

Constructing a MC for $e^+e^- \to q\bar{q}$

- process $e^+e^- \to q\bar{q}$



- $$\frac{d\sigma}{d\cos\theta d\phi} = \frac{\alpha_{em}^2}{4s}\left(1+\cos^2\theta\right)$$

- generate scattering as for $e^+e^- \to \mu^+\mu^-$
- BUT what about fragmentation/hadronization ???
- use concept of local parton-hadron duality

Different approaches to fragmentation/hadronization:

→ independent fragmentation

→ cluster fragmentation (HERWIG model)

→ string fragmentation (Lund Model) NEXT PAGE

Hadronization - the simple case in e^+e^-

Transition from Quarks to Hadrons

- **Independent Fragmentation** (Feynman & Field: Phys. Rev D15 (1977)2590, NPB 138 (1978) 1)
  - quarks fragment independently

Transition from Quarks to Hadrons

- Independent Fragmentation
  - quarks fragment independently
  - not Lorentz invariant

PHYSICAL REVIEW D VOLUME 27, NUMBER 1 1 JANUARY 1983

Scaling violations in inclusive e^+e^- annihilation spectra

C. Peterson,* D. Schlatter, I. Schmitt,[†] and P. M. Zerwas[‡]
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 29 July 1982)

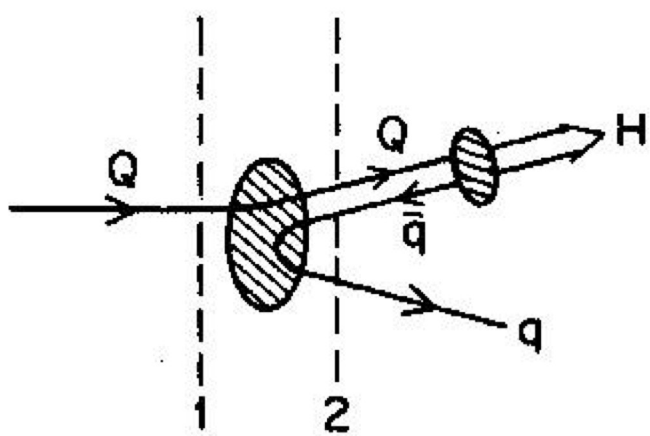


FIG. 3. The fragmentation of a heavy quark Q into a meson $H(Q\bar{q})$. Dashed lines are time slices used in the derivation of Eq. (3).

cussed in Ref. 18. The *gross features* of the amplitude for a fast moving heavy quark Q fragmentation into a hadron $H=(Q\bar{q})$ and light quark q (Fig. 3) are determined by the value of the energy transfer $\Delta E = E_H + E_q - E_Q$ in the breakup process,

$$\text{amplitude } (Q \to H + q) \propto \Delta E^{-1} . \tag{2}$$

Expanding the energies about the (transverse) particle masses ($m_H \simeq m_Q$ for simplicity),

$$\Delta E = (m_Q{}^2 + z^2P^2)^{1/2} + (m_q{}^2 + (1-z)^2P^2)^{1/2} - (m_Q{}^2 + P^2)^{1/2}$$
$$\propto 1 - (1/z) - (\epsilon_Q/1-z) \tag{3}$$

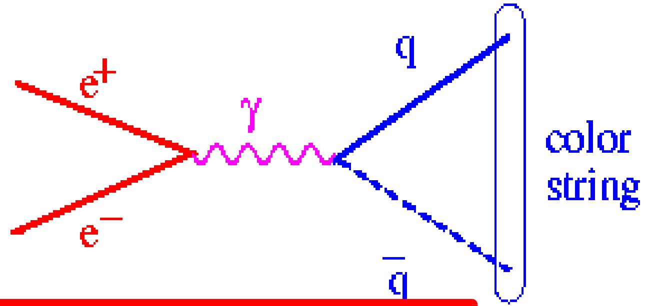
and taking a factor z^{-1} for longitudinal phase space, we suggest the following ansatz for the fragmentation function of heavy quarks Q

$$D_Q^H(z) = \frac{N}{z[1-(1/z)-\epsilon_Q/(1-z)]^2} . \tag{4}$$

Transition from Quarks to Hadrons

- Independent Fragmentation
  - quarks fragment independently
  - gluon are split: $g \to q\bar{q}$
  - not Lorentz invariant
- Lund String Fragmentation (Andersson, Gustafson, Peterson ZPC 1, 105 (1979), Andersson, Gustafson, Ingelman, Sjostrand Phys. Rep 97 (1983) 33)
  - use concept of local parton-hadron duality

$$\frac{d\sigma}{d\cos\theta d\phi} = \frac{\alpha_{em}^2}{4s}\left(1+\cos^2\theta\right)$$



linear confinement potential: *V(r) ~ -1/r + κr*
with *κ ~ 1* GeV/fm
qq connected via color flux tube of transverse size of hadrons (~1 fm)
color tube: uniform along its length → linearly rising potential

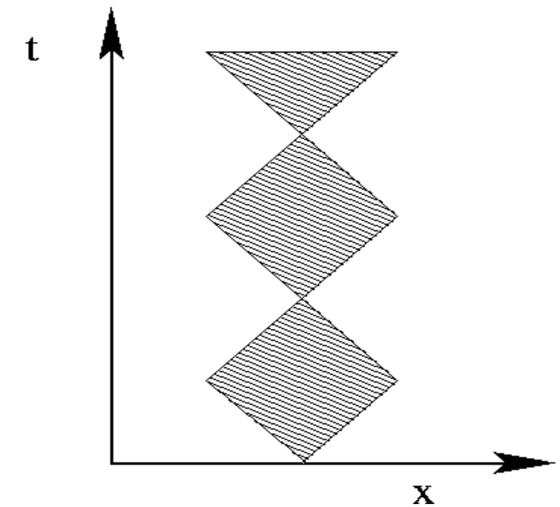
→ Lund string fragmentation

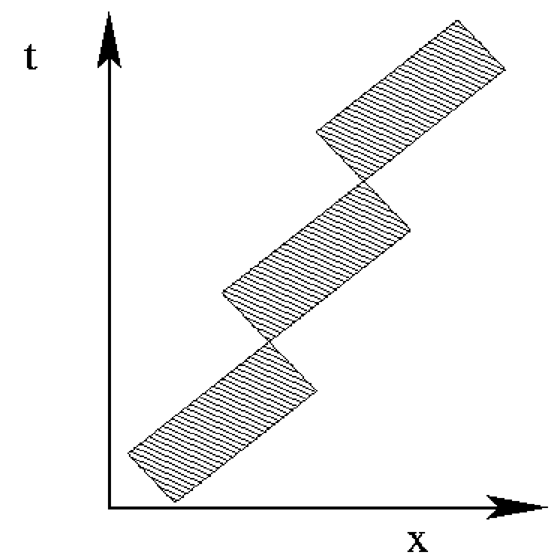
Lund string fragmentation

- in a color neutral qq-pair, a color force is created in between
- color lines of the force are concentrated in a narrow tube connecting q and q, with a string tension of:

  $\kappa \sim 1 GeV/fm \sim 0.2\ GeV^2$

- as q and q are moving apart in qq rest frame, they are de-accelerated by string tension, accelerated back etc ... (periodic oscillation)
- viewed in a moving system, the string is boosted





The first MC steps ...

- Monte Carlo source code of JETSET, fits on 1 page

T.Sjostrand, B. Soderberg LU-TP 78-18

Appendix

Listings of the program components.

```
      SUBROUTINE JETGEN(N)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /PAR/ PUD, PS1, SIGMA, CX2, EBEG, WFIN, IFLBEG
      COMMON /DATA1/ MESO(9,2), CMIX(6,2), PMAS(19)
      IFLSGN=(10-IFLBEG)/5
      W=2.*EBEG
      I=0
      IPD=0
C 1 FLAVOUR AND PT FOR FIRST QUARK
      IFL1=IABS(IFLBEG)
      PT1=SIGMA*SQRT(-ALOG(RANF(0)))
      PHI1=6.2832*RANF(0)
      PX1=PT1*COS(PHI1)
      PY1=PT1*SIN(PHI1)
  100 I=I+1
C 2 FLAVOUR AND PT FOR NEXT ANTIQUARK
      IFL2=1+INT(RANF(0)/PUD)
      PT2=SIGMA*SQRT(-ALOG(RANF(0)))
      PHI2=6.2832*RANF(0)
      PX2=PT2*COS(PHI2)
      PY2=PT2*SIN(PHI2)
C 3 MESON FORMED, SPIN ADDED AND FLAVOUR MIXED
      K(I,1)=MESO(3*(IFL1-1)+IFL2,IFLSGN)
      ISPIN=INT(PS1+RANF(0))
      K(I,2)=1+9*ISPIN+K(I,1)
      IF(K(I,1).LE.6) GOTO 110
      TMIX=RANF(0)
      KM=K(I,1)-6+3*ISPIN
      K(I,2)=8+9*ISPIN+INT(TMIX+CMIX(KM,1))+INT(TMIX+CMIX(KM,2))
C 4 MESON MASS FROM TABLE, PT FROM CONSTITUENTS
  110 P(I,5)=PMAS(K(I,2))
      P(I,1)=PX1+PX2
      P(I,2)=PY1+PY2
      PMTS=P(I,1)**2+P(I,2)**2+P(I,5)**2
C 5 RANDOM CHOICE OF X=(E+PZ)MESON/(E+PZ)AVAILABLE GIVES E AND PZ
      X=RANF(0)
      IF(RANF(0).LT.CX2) X=1.-X**(1./3.)
      P(I,3)=(X*W-PMTS/(X*W))/2.
      P(I,4)=(X*W+PMTS/(X*W))/2.
C 6 IF UNSTABLE, DECAY CHAIN INTO STABLE PARTICLES
  120 IPD=IPD+1
      IF(K(IPD,2).GE.8) CALL DECAY(IPD,I)
      IF(IPD.LT.I.AND.I.LE.96) GOTO 120
C 7 FLAVOUR AND PT OF QUARK FORMED IN PAIR WITH ANTIQUARK ABOVE
      IFL1=IFL2
      PX1=-PX2
      PY1=-PY2
C 8 IF ENOUGH E+PZ LEFT, GO TO 2
      W=(1.-X)*W
      IF(W.GT.WFIN.AND.I.LE.95) GOTO 100
      N=I
      RETURN
      END
```



```
      SUBROUTINE DECAY(IPD,I)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /DATA1/ MESO(9,2), CMIX(6,2), PMAS(19)
      COMMON /DATA2/ IDCO(12), CBR(29), KDP(29,3)
      DIMENSION U(3), BE(3)
C 1 DECAY CHANNEL CHOICE, GIVES DECAY PRODUCTS
      TBR=RANF(0)
      IDC=IDCO(K(IPD,2)-7)
  100 IDC=IDC+1
      IF(TBR.GT.CBR(IDC)) GOTO 100
      ND=(59+KDP(IDC,3))/20
      DO 110 I1=I+1,I+ND
      K(I1,1)=-IPD
      K(I1,2)=KDP(IDC,I1-I)
  110 P(I1,5)=PMAS(K(I1,2))
C 2 IN THREE-PARTICLE DECAY CHOICE OF INVARIANT MASS OF PRODUCTS 2+3
      IF(ND.EQ.2) GOTO 130
      SA=(P(IPD,5)+P(I+1,5))**2
      SB=(P(IPD,5)-P(I+1,5))**2
      SC=(P(I+2,5)+P(I+3,5))**2
      SD=(P(I+2,5)-P(I+3,5))**2
      TDU=(SA-SD)*(SB-SC)/(4.*SQRT(SB*SC))
      IF(K(IPD,2).GE.11) TDU=SQRT(SB*SC)*TDU**3
  120 SX=SC+(SB-SC)*RANF(0)
      TDF=SQRT((SX-SA)*(SX-SB)*(SX-SC)*(SX-SD))/SX
      IF(K(IPD,2).GE.11) TDF=SX*TDF**3
      IF(RANF(0)*TDU.GT.TDF) GOTO 120
      P(100,5)=SQRT(SX)
C 3 TWO-PARTICLE DECAY IN CM, TWICE TO SIMULATE THREE-PARTICLE DECAY
  130 DO 160 IL=1,ND-1
      IO=(IL-1)*100-(IL-2)*IPD
      I1=I+IL
      I2=(ND-IL-1)*100-(ND-IL-2)*(I+IL+1)
      PA=SQRT((P(IO,5)**2-(P(I1,5)+P(I2,5))**2)*
     &(P(IO,5)**2-(P(I1,5)-P(I2,5))**2))/(2.*P(IO,5))
  140 U(3)=2.*RANF(0)-1.
      PHI=6.2832*RANF(0)
      U(1)=SQRT(1.-U(3)**2)*COS(PHI)
      U(2)=SQRT(1.-U(3)**2)*SIN(PHI)
      TDA=1.-(U(1)*P(IO,1)+U(2)*P(IO,2)+U(3)*P(IO,3))**2/
     &(P(IO,1)**2+P(IO,2)**2+P(IO,3)**2)
      IF(K(IPD,2).GE.11.AND.IL.EQ.2.AND.RANF(0).GT.TDA) GOTO 140
      DO 150 J=1,3
      P(I1,J)=PA*U(J)
  150 P(I2,J)=-PA*U(J)
      P(I1,4)=SQRT(PA**2+P(I1,5)**2)
  160 P(I2,4)=SQRT(PA**2+P(I2,5)**2)
C 4 DECAY PRODUCTS LORENTZ TRANSFORMED TO LAB SYSTEM
      DO 190 IL=ND-1,1,-1
      IO=(IL-1)*100-(IL-2)*IPD
      DO 170 J=1,3
  170 BE(J)=P(IO,J)/P(IO,4)
      GA=P(IO,4)/P(IO,5)
      DO 190 I1=I+IL,I+ND
      BEP=BE(1)*P(I1,1)+BE(2)*P(I1,2)+BE(3)*P(I1,3)
      DO 180 J=1,3
  180 P(I1,J)=P(I1,J)+GA*(GA/(1.+GA)*BEP+P(I1,4))*BE(J)
  190 P(I1,4)=GA*(P(I1,4)+BEP)
      I=I+ND
      RETURN
      END
```



```
      SUBROUTINE EDIT(N)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /EDPAR/ ITHROW, PZMIN, PMIN, THETA, PHI, BETA(3)
      REAL ROT(3,3), PR(3)
C 1 THROW AWAY NEUTRALS OR UNSTABLE OR WITH TOO LOW PZ OR P
      I1=0
      DO 110 I=1,N
      IF(ITHROW.GE.1.AND.K(I,2).GE.8) GOTO 110
      IF(ITHROW.GE.2.AND.K(I,2).GE.6) GOTO 110
      IF(ITHROW.GE.3.AND.K(I,2).EQ.1) GOTO 110
      IF(P(I,3).LT.PZMIN.OR.P(I,4)**2-P(I,5)**2.LT.PMIN**2) GOTO 110
      I1=I1+1
      K(I1,1)=IDIM(K(I,1),0)
      K(I1,2)=K(I,2)
      DO 100 J=1,5
  100 P(I1,J)=P(I,J)
  110 CONTINUE
      N=I1
C 2 ROTATE TO GIVE JET PRODUCED IN DIRECTION THETA, PHI
      IF(THETA.LT.1E-4) GOTO 140
      ROT(1,1)=COS(THETA)*COS(PHI)
      ROT(1,2)=-SIN(PHI)
      ROT(1,3)=SIN(THETA)*COS(PHI)
      ROT(2,1)=COS(THETA)*SIN(PHI)
      ROT(2,2)=COS(PHI)
      ROT(2,3)=SIN(THETA)*SIN(PHI)
      ROT(3,1)=-SIN(THETA)
      ROT(3,2)=0.
      ROT(3,3)=COS(THETA)
      DO 130 I=1,N
      DO 120 J=1,3
  120 PR(J)=P(I,J)
      DO 130 J=1,3
  130 P(I,J)=ROT(J,1)*PR(1)+ROT(J,2)*PR(2)+ROT(J,3)*PR(3)
C 3 OVERALL LORENTZ BOOST GIVEN BY BETA VECTOR
  140 IF(BETA(1)**2+BETA(2)**2+BETA(3)**2.LT.1E-8) RETURN
      GA=1./SQRT(1.-BETA(1)**2-BETA(2)**2-BETA(3)**2)
      DO 160 I=1,N
      BEP=BETA(1)*P(I,1)+BETA(2)*P(I,2)+BETA(3)*P(I,3)
      DO 150 J=1,3
  150 P(I,J)=P(I,J)+GA*(GA/(1.+GA)*BEP+P(I,4))*BETA(J)
  160 P(I,4)=GA*(P(I,4)+BEP)
      RETURN
      END
```



```
      SUBROUTINE LIST(N)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /DATA3/ CHA1(9), CHA2(19), CHA3(2)
      WRITE(6,110)
      DO 100 I=1,N
      IF(K(I,1).GT.0) C1=CHA1(K(I,1))
      IF(K(I,1).LE.0) IC1=-K(I,1)
      C2=CHA2(K(I,2))
      C3=CHA3((47-K(I,2))/20)
      IF(K(I,1).GT.0) WRITE(6,120) I, C1, C2, C3, (P(I,J), J=1,5)
  100 IF(K(I,1).LE.0) WRITE(6,130) I, IC1, C2, C3, (P(I,J), J=1,5)
      RETURN
  110 FORMAT(////T11,'I',T17,'ORI',T24,'PART',T32,'STAB',
     &T44,'PX',T56,'PY',T68,'PZ',T80,'E',T92,'M'/)
  120 FORMAT(10X,I2,4X,A2,1X,2(4X,A4),5(4X,F8.1))
  130 FORMAT(10X,I2,4X,1X,I2,2(4X,A4),5(4X,F8.1))
      END
```

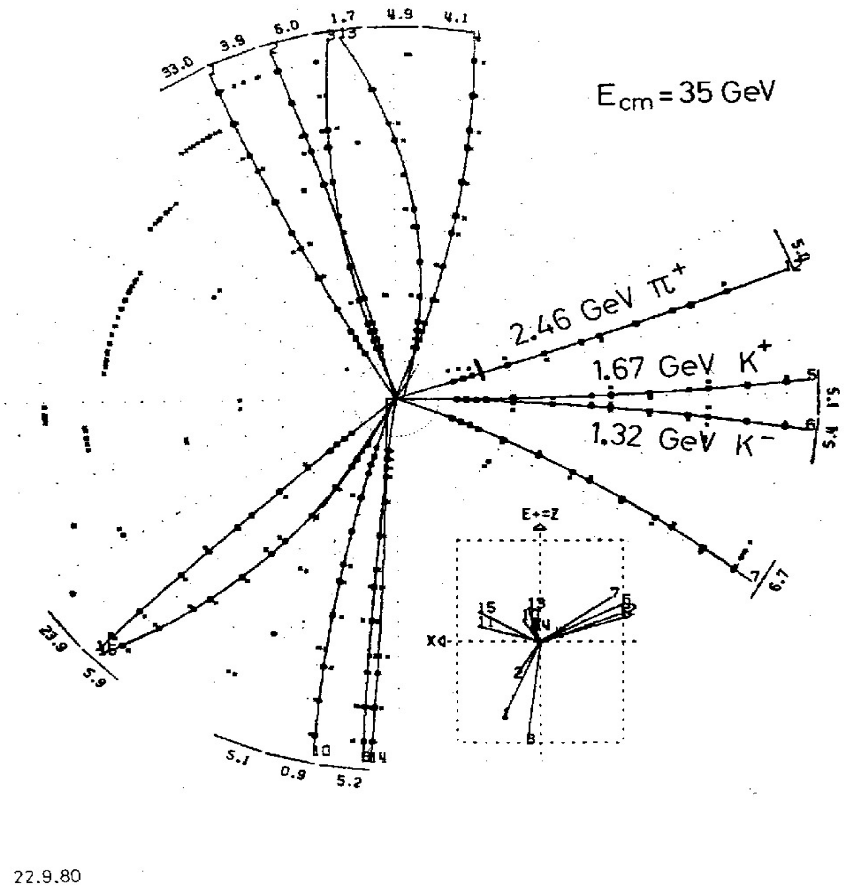
```
      BLOCK DATA
      COMMON /PAR/ PUD, PS1, SIGMA, CX2, EBEG, WFIN, IFLBEG
      COMMON /EDPAR/ ITHROW, PZMIN, PMIN, THETA, PHI, BETA(3)
      COMMON /DATA1/ MESO(9,2), CMIX(6,2), PMAS(19)
      COMMON /DATA2/ IDCO(12), CBR(29), KDP(29,3)
      COMMON /DATA3/ CHA1(9), CHA2(19), CHA3(2)
      DATA PUD/0.4/, PS1/0.5/, SIGMA/350./, CX2/0.77/,
     &EBEG/10000./, WFIN/100./, IFLBEG/1/
      DATA ITHROW/1/, PZMIN/0./, PMIN/0./, THETA,PHI,BETA/5*0./
      DATA MESO/7,1,3,2,8,5,4,6,9,7,2,4,1,8,6,3,5,9/
      DATA CMIX/2*0.5,1.,2*0.5,1.,2*0.25,0.5,2*0.,1./
      DATA PMAS/0.,2*139.6,2*493.7,2*497.7,135.,548.8,957.6,
     &2*765.9,2*892.2,2*896.3,770.2,782.6,1019.6/
      DATA IDCO/0,1,6,11,12,13,15,17,19,21,22,25/
      DATA CBR/1.,0.381,0.681,0.918,0.969,1.,0.426,0.662,0.959,
     &0.980,1.,1.,1.,0.667,1.,0.667,1.,0.667,1.,0.667,1.,1.,
     &0.899,0.987,1.,0.486,0.837,0.984,1./
      DATA KDP/1,1,8,2,1,1,2,8,1,1,1,2,3,6,4,7,5,4,6,5,7,2,2,
     &1,2,4,6,2,1,1,1,8,3,2,1,3,8,17,18,1,8,8,2,8,3,8,3,8,2,8,
     &3,3,8,3,5,7,3,9,0,0,8,8,3,8,9,9,14*0,8,4*0,8,0/
      DATA CHA1/'UD','DU','US','SU','DS','SD','UU','DD','SS'/
      DATA CHA2/'GAMM','PI+','PI-','K+','K-','K0','KB0','PI0','ETA',
     &'ETAP','RHO+','RHO-','K*+','K*-','K*0','KB*0','RHO0','OMEG','PHI'/
      DATA CHA3/'    ','STAB'/
      END
```

Hadronization and additional partons - the simple case in e^+e^-

Where is QCD - discovery of the Gluon

Gluon discovery in 1979

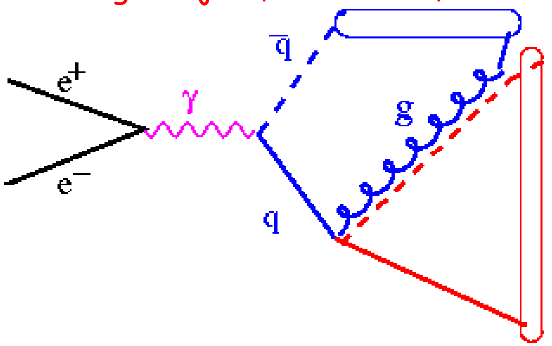
At the PETRA storage ring, the "gluon" was directly observed for the first time. For their discovery of the gluon in 1979, four DESY scientists received the Particle Physics Prize of the European Physical Society (EPS), considered the "European Nobel Prize in Physics", in 1995.

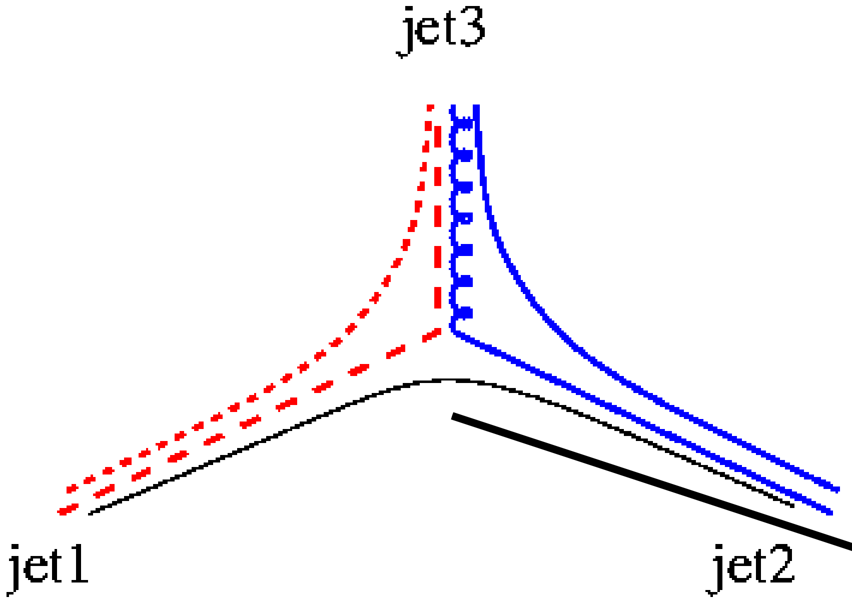


Gluons in Lund String Fragmentation

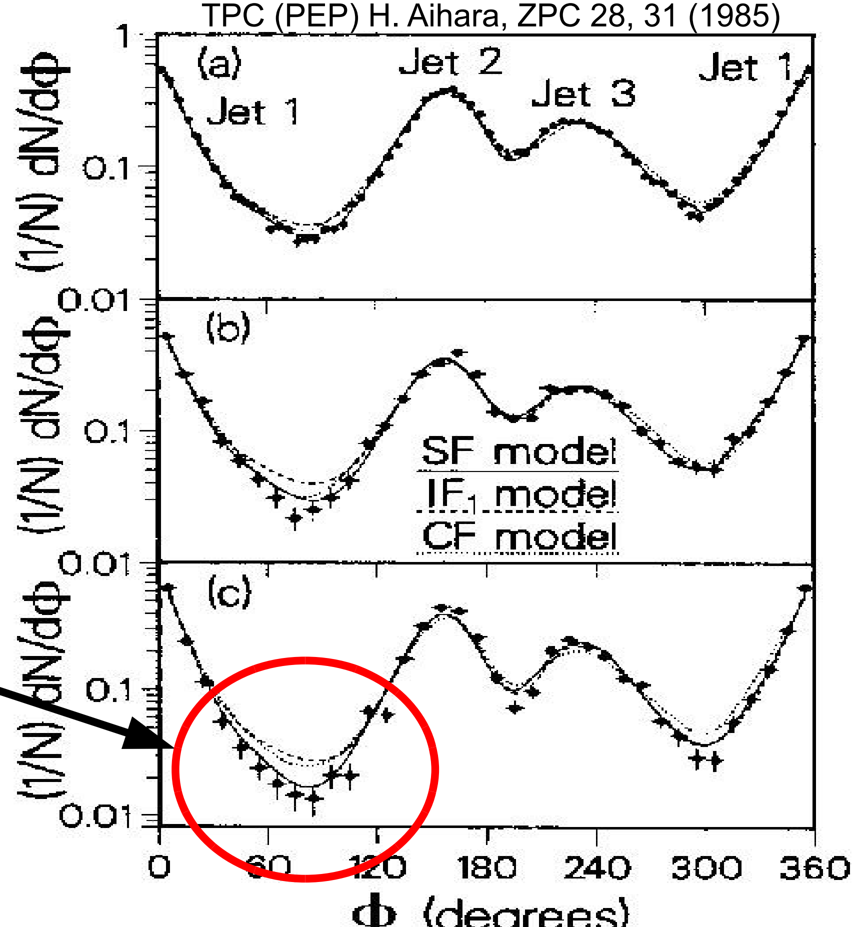
How to find the gluon jets, Andersson, Gustafson, Sjostrand, PLB 94,211 (1980)

- **process** $e^+e^- \to q\bar{q}g$
- watch out color flow !!!
- gluons act as kinks on strings
- string effect seen in experiment





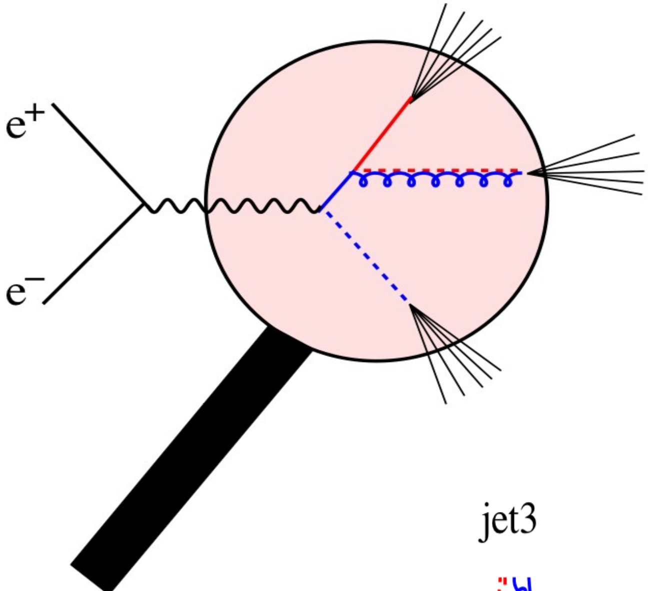
TPC (PEP) H. Aihara, ZPC 28, 31 (1985)

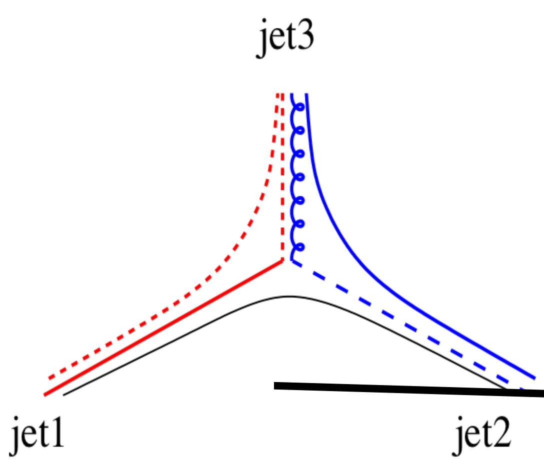


The Lund Model does describe it !!!

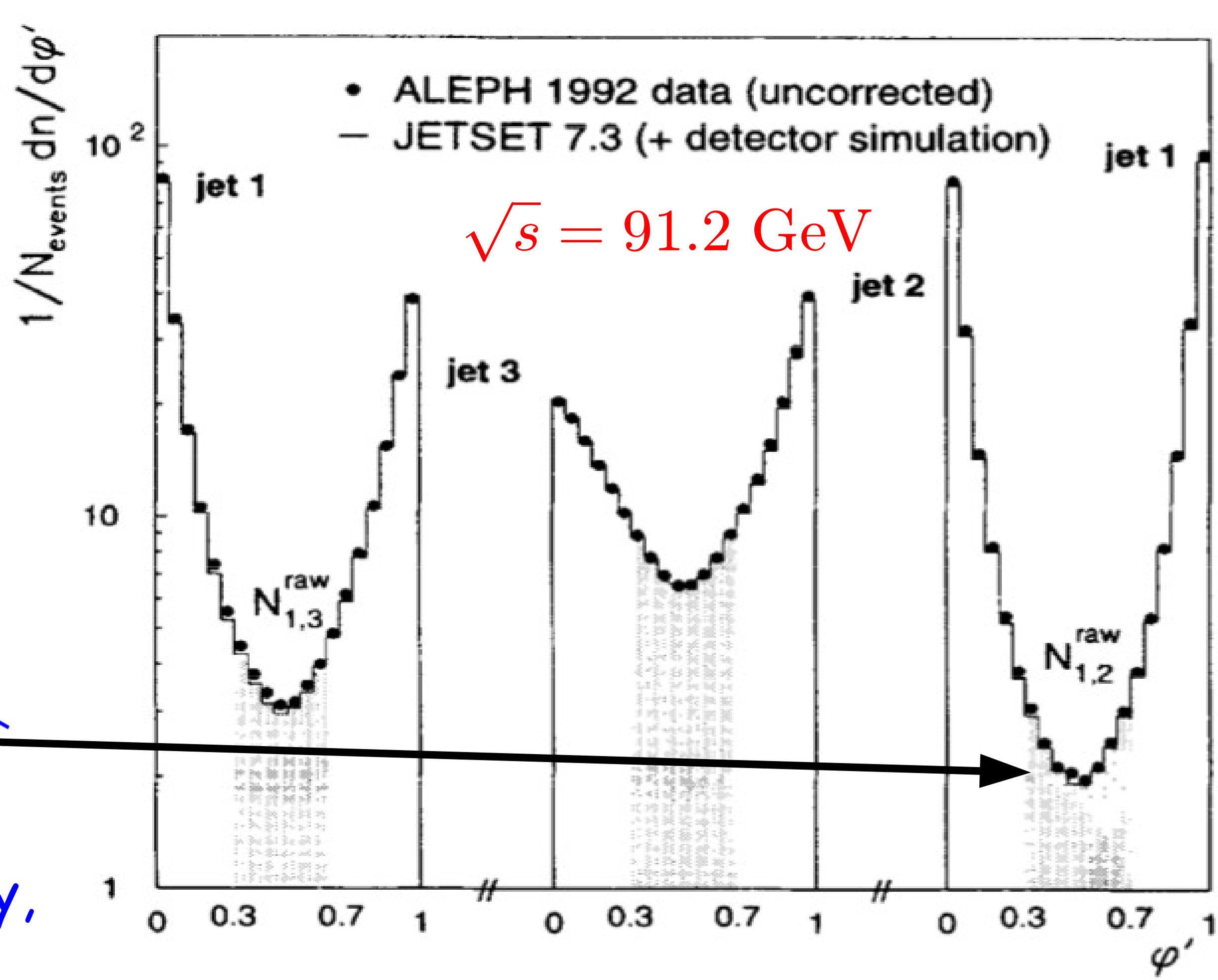
... and with more precise data ...

How to find the gluon jets, Andersson, Gustafson, Sjostrand, PLB 94,211 (1980)





ALEPH Collaboration / Physics Reports 294 (1998) 1–165



- jets ordered by energy,

  highest is quark (~94 %) ,

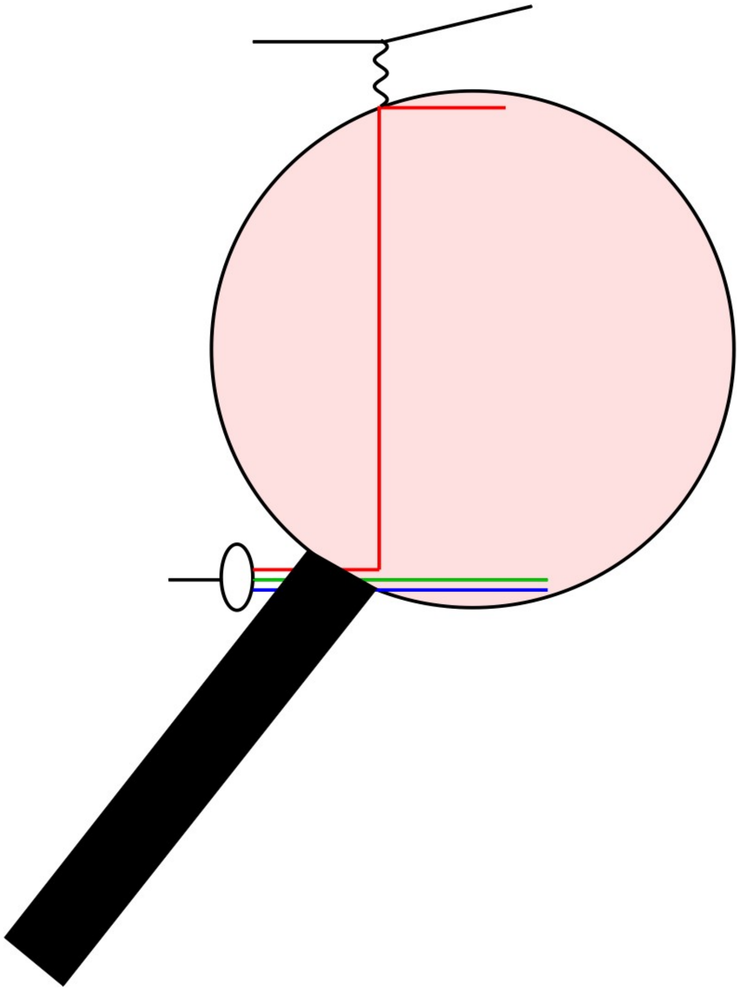
  lowest is gluon (~70%)

a more complicated case ...

... add a hadron in initial state ...

→ ep scattering

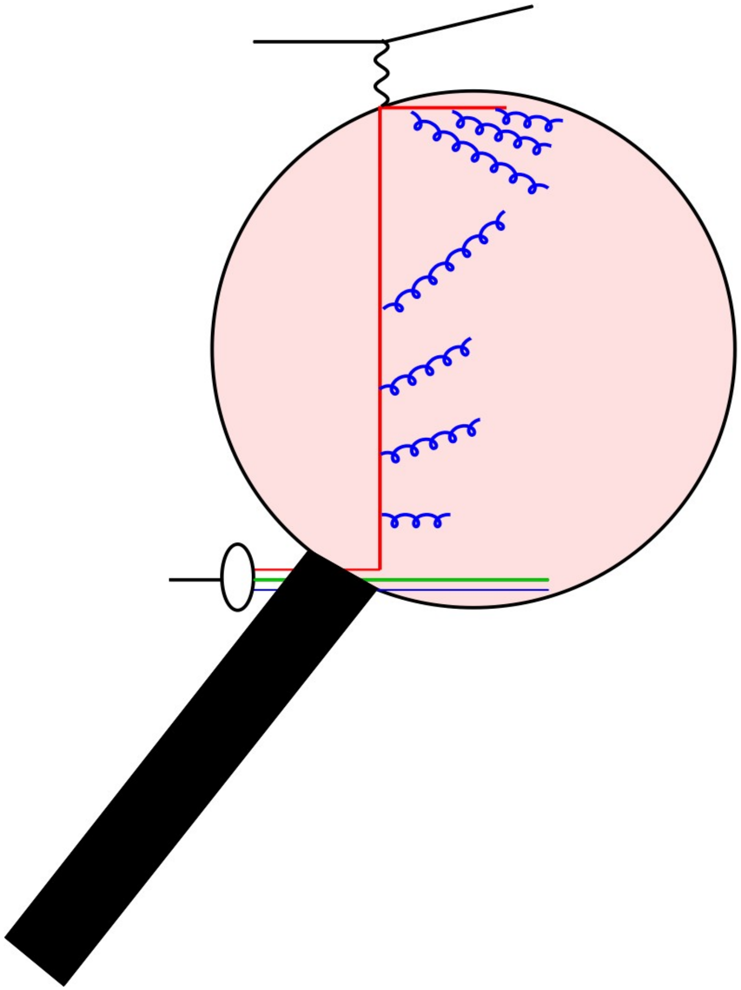
A proton in the initial state



- Deep Inelastic Scattering is a incoherent sum of $e^+q \to e + q$
- only 50 % of p momentum carried by quarks
- need a large gluon component
- partonic part convoluted with parton density function $f_i(x)$

$$\sigma(e^+p \to e^+X) = \sum_i f_i(x, \quad)\sigma(e^+q_i \to e^+q_i)$$

A proton in the initial state

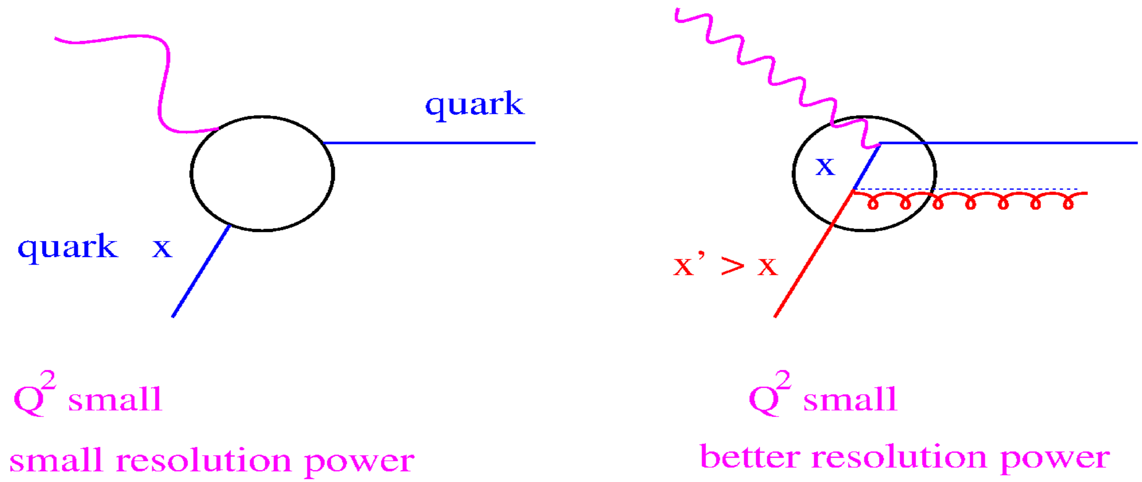


- Deep Inelastic Scattering is a incoherent sum of $e^+q \to e + q$
- only 50 % of p momentum carried by quarks
- need a large gluon component
- partonic part convoluted with parton density function $f_i(x)$
- BUT we know, PDF depends on resolution scale Q^2

$$\sigma(e^+p \to e^+X) = \sum_i f_i(x, Q^2)\sigma(e^+q_i \to e^+q_i)$$

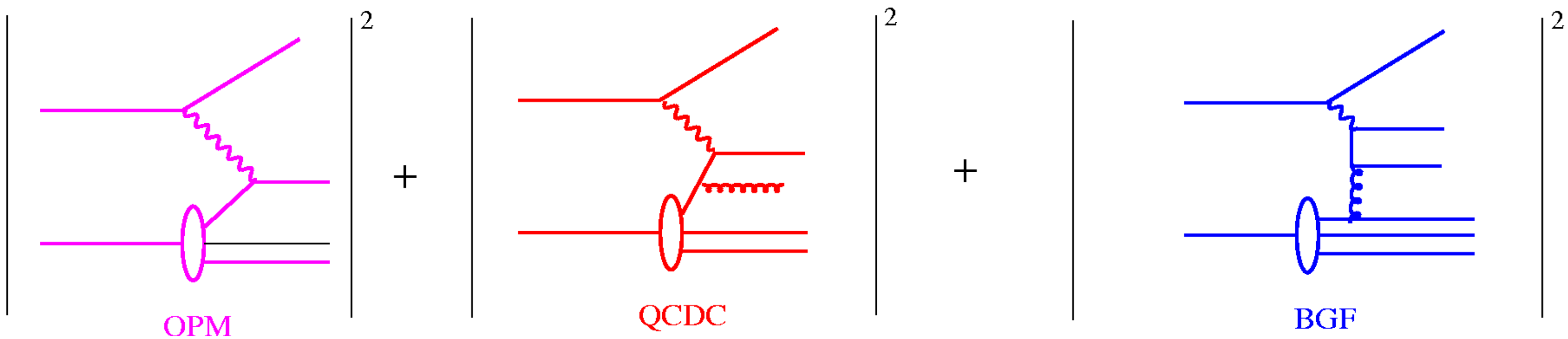
$F_2(x,Q^2)$: DGLAP evolution equation

- QPM: F_2 is independent of Q^2
- Q^2 dependence of structure function: **D**okshitzer **G**ribov **L**ipatov **A**ltarelli **P**arisi



➔ Probability to find parton at small x increases with Q^2

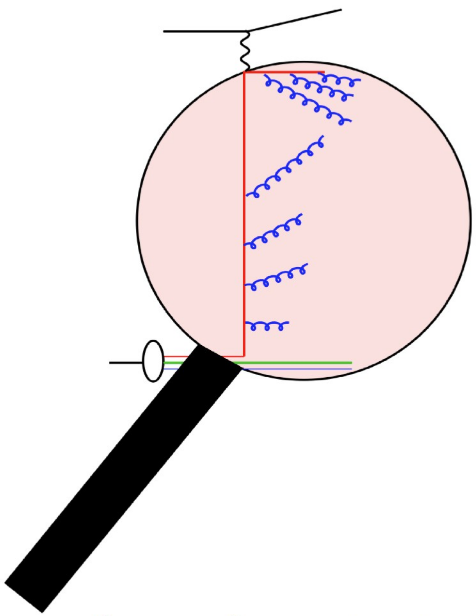
$F_2 =$

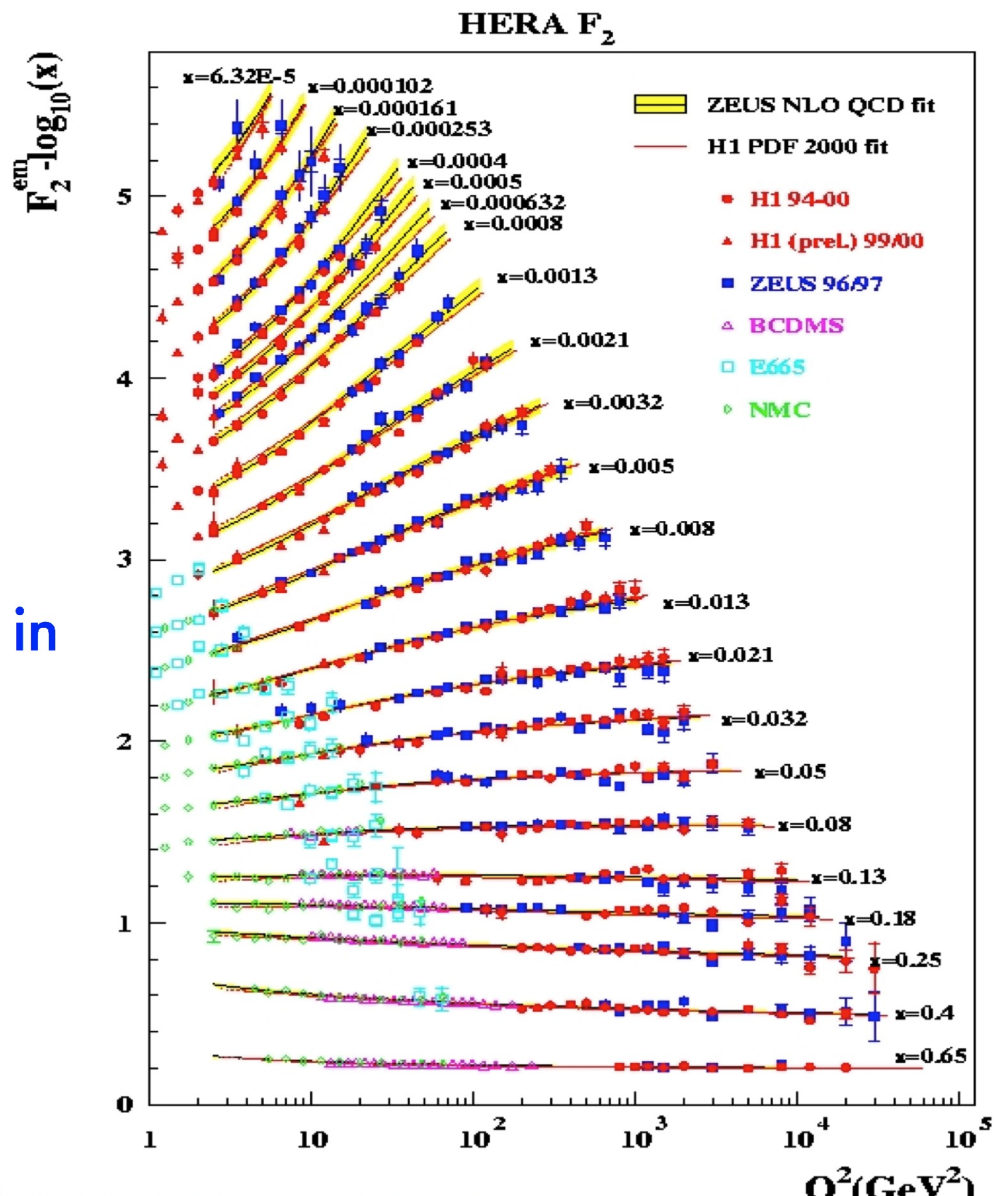


➔ **Test of theory: Q^2 evolution of $F_2(x,Q^2)$!!!!!**

A proton in the initial state

$$\sigma(e^+p \to e^+X) = \sum f_i(x,Q^2)\sigma(e^+q_i \to e^+q_i)$$





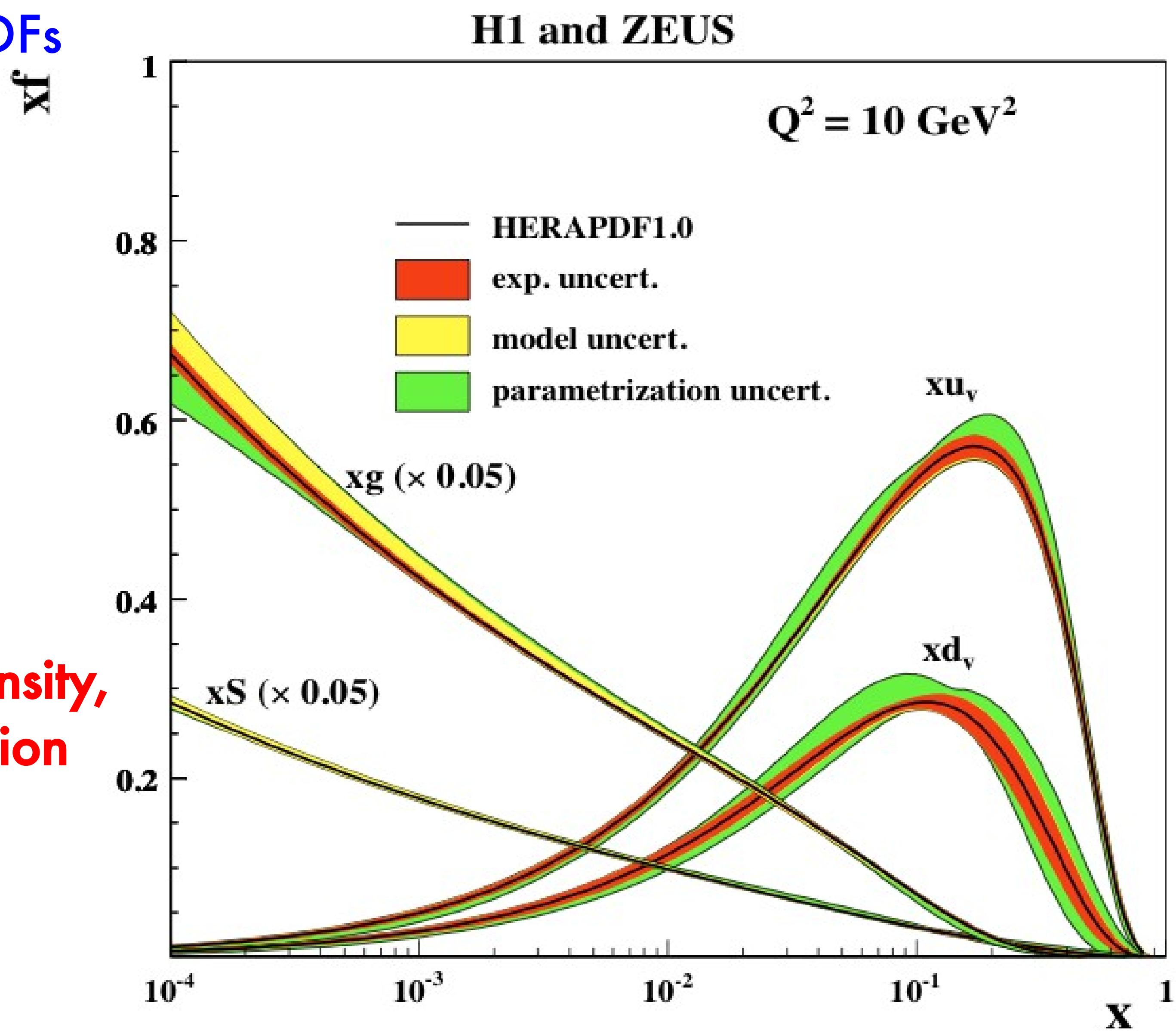
- perfect description of precise measurements of HUGE range in x and Q^2
- Theory works well.....

➔ extract parton densities, which are universal

➔ to be used at LHC.....

The proton PDFs ...

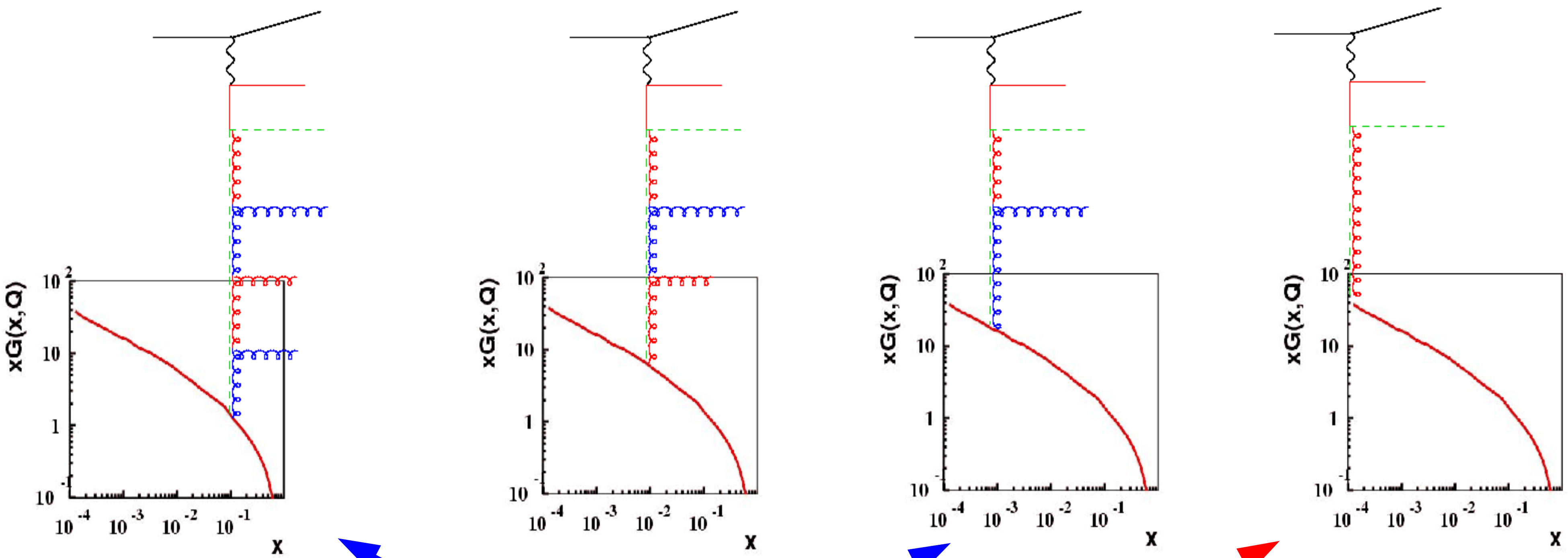
- quark and gluon PDFs



➔ Very large gluon density, even at small resolution scales Q^2

DGLAP evolution equation... again...

- for fixed x and Q^2 chains with different branchings contribute
- iterative procedure, spacelike parton showering

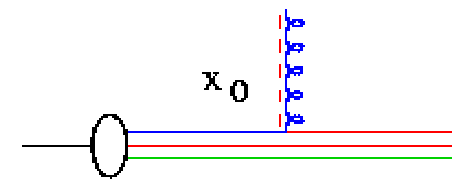


$$f(x,t) = \sum_{k=1}^{\infty} f_k(x_k, t_k) + f_0(x, t_0)\Delta_s(t)$$

Parton showers for the initial state

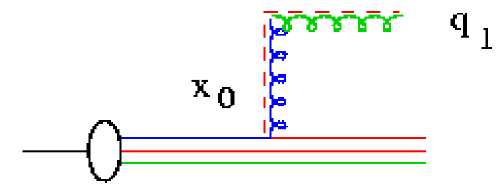
spacelike (Q<0) parton shower evolution

- starting from hadron (fwd evolution)

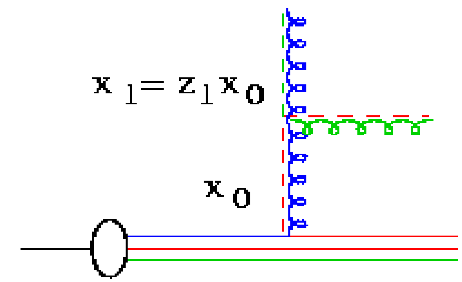


  or from hard scattering (bwd evolution)

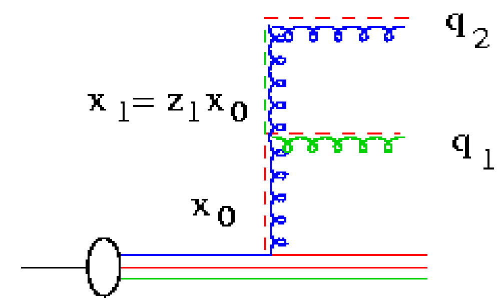
- select q_1



- select z_1 from splitting function

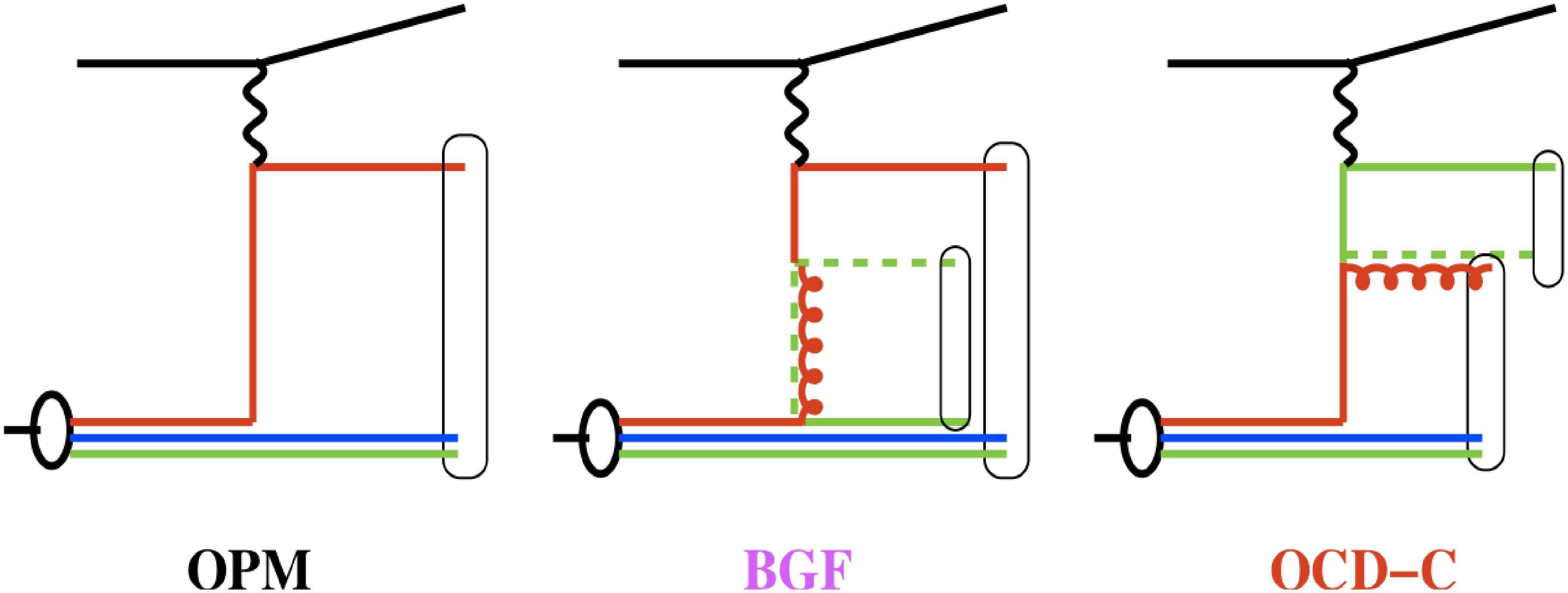


- select q_2



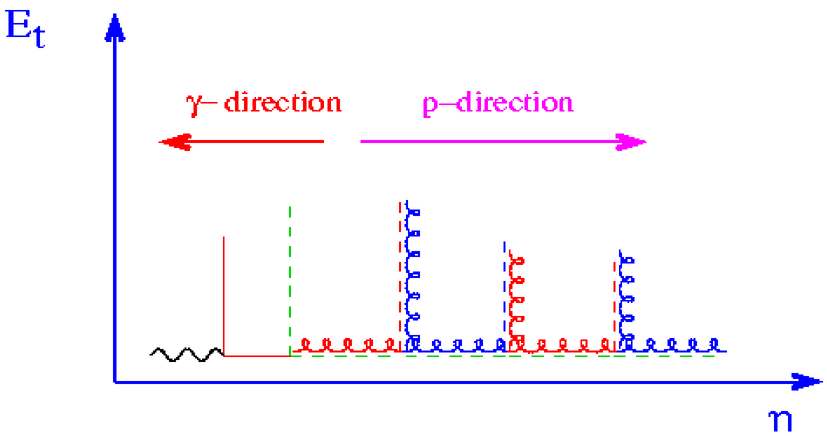
- select z_2 from splitting function
- stop evolution if $q_2 > Q_{hard}$

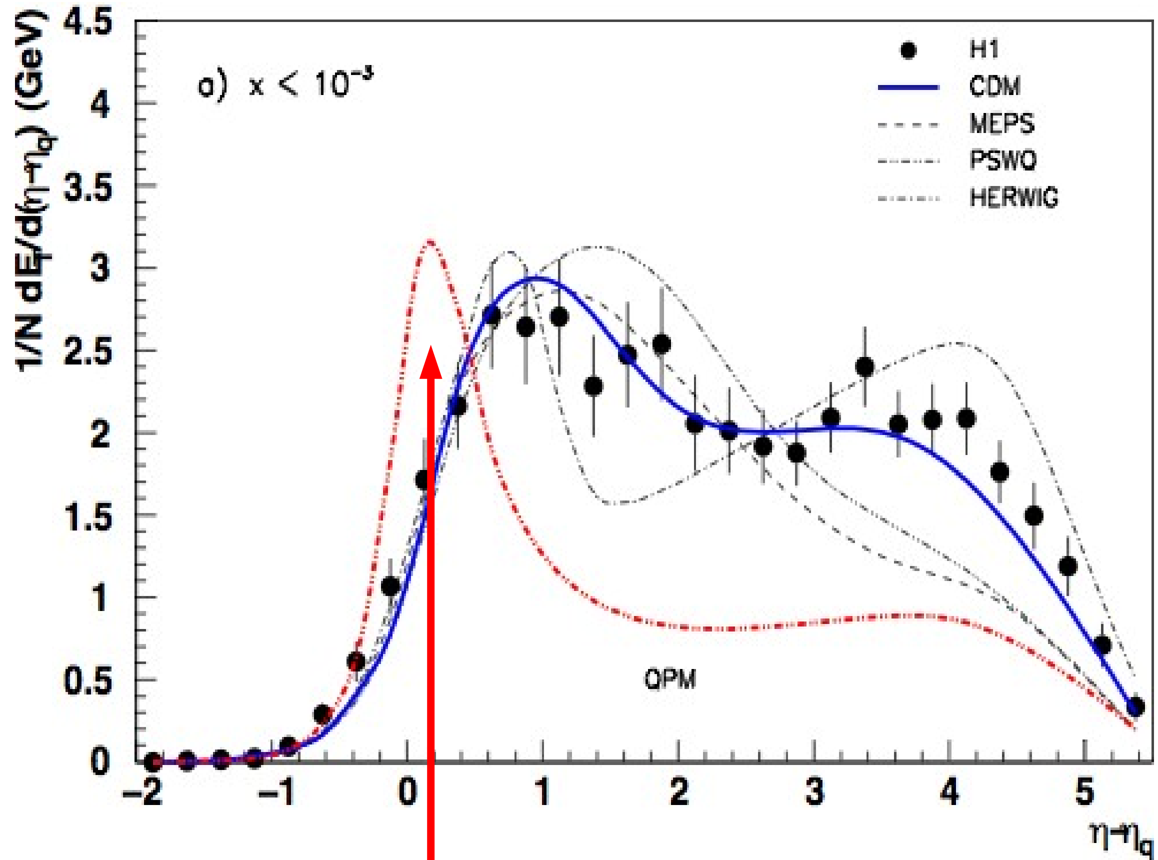
Color structure in ep



- Combine colored partons into color-singlet strings
- color singlet strings hadronize
    - the color structure determines the hadronic energy flow

Hadronic final state: Energy flow





- E_t flow in DIS at small x and forward angle (p-direction):

➔ QPM is not enough

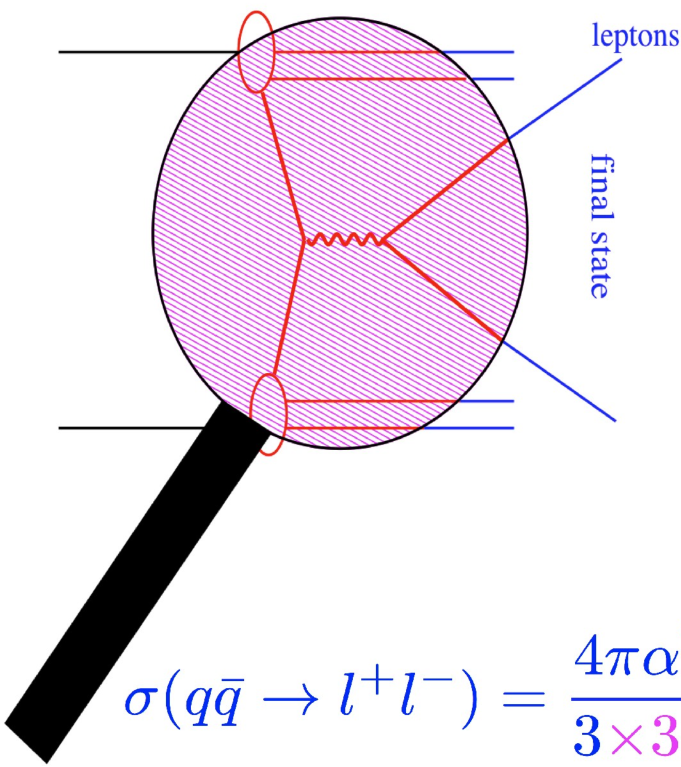
➔ clearly parton showers or higher order contributions needed

leading jet direction

A even more complicated case …

two hadrons in initial state: pp-scattering

Rotating the diagrams

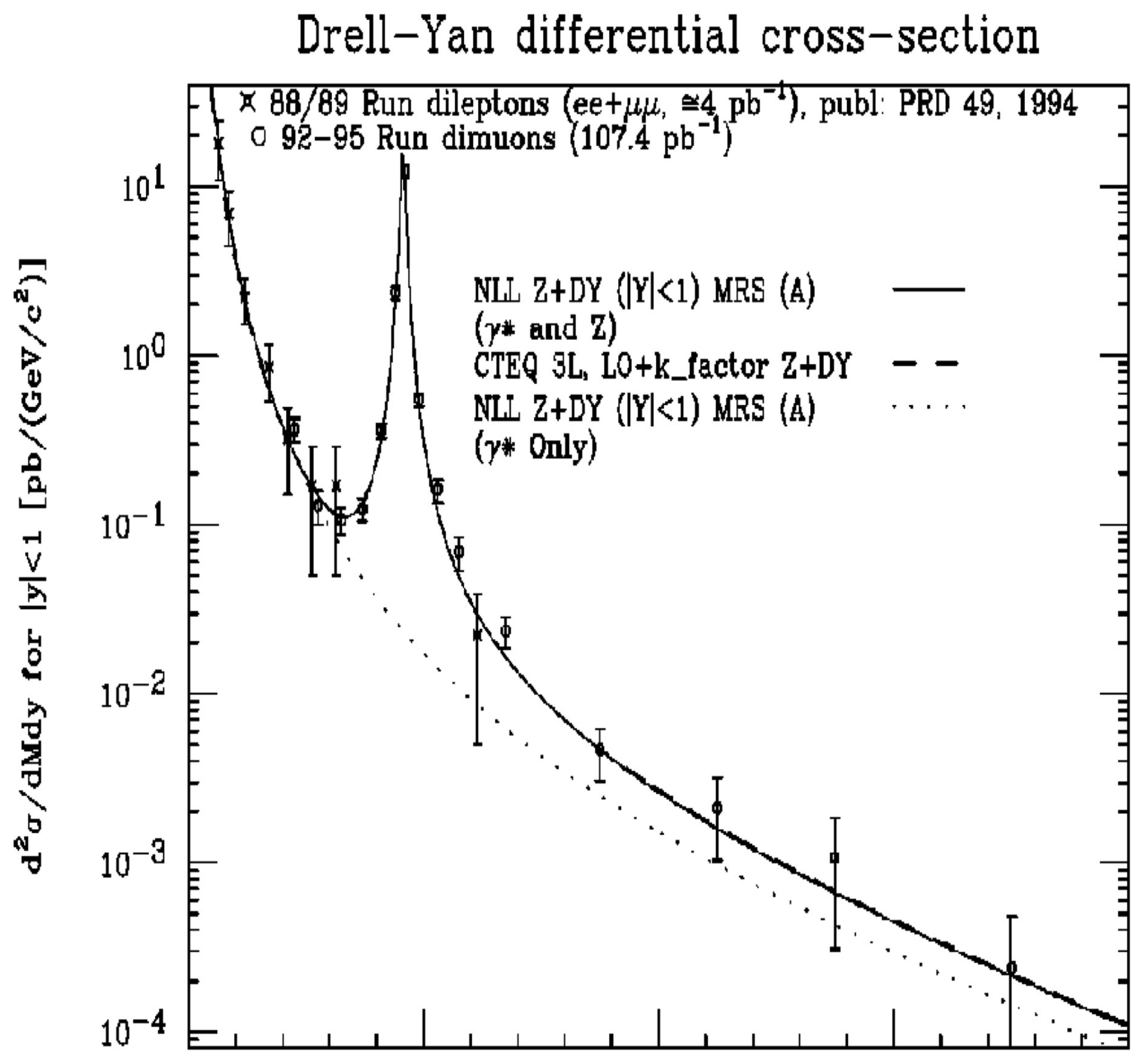


$$\sigma(q\bar{q} \to l^+ l^-) = \frac{4\pi\alpha^2}{3\times 3s} e_q^2$$

Factorization:

- hard process: $q\bar{q} \to l^+ l^-$
- parton densities

Measurement of Z0 and Drell-Yan production cross-section using dimuons in anti-p p collisions at S**(1/2) = 1.8-TeV.
CDF Collaboration F. Abe et al. Phys.Rev.D59:052002,1999.

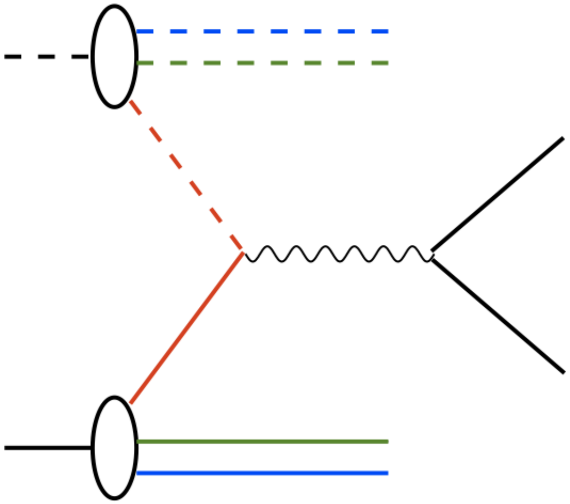


W & Z cross sections

- Basic process: Drell – Yan

  $p + p \to l^+ + l^- + X$

- Factorize process:
  - $q + \bar{q} \to \gamma^* \to l^+ + l^-$

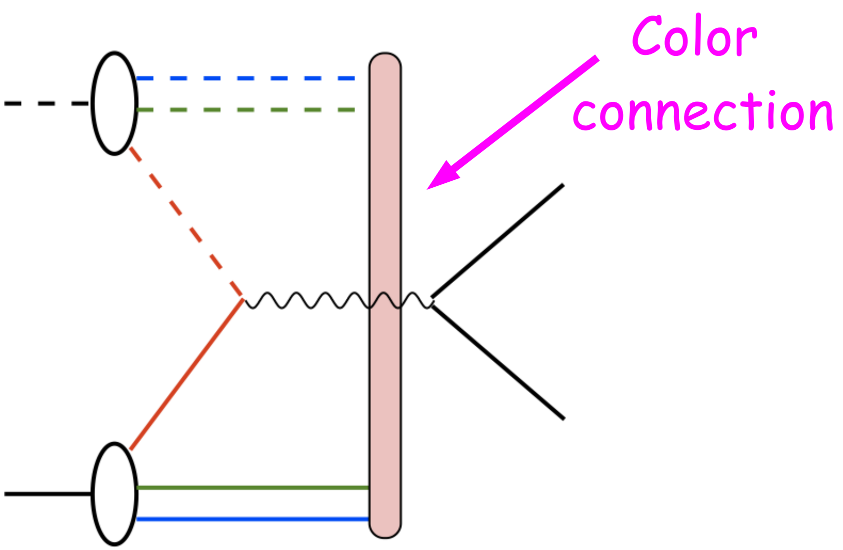


W & Z cross sections

- Basic process: Drell – Yan

  $p + p \to l^+ + l^- + X$

- Factorize process:
  - $q + \bar{q} \to \gamma^* \to l^+ + l^-$
- What about the colored remnants ?

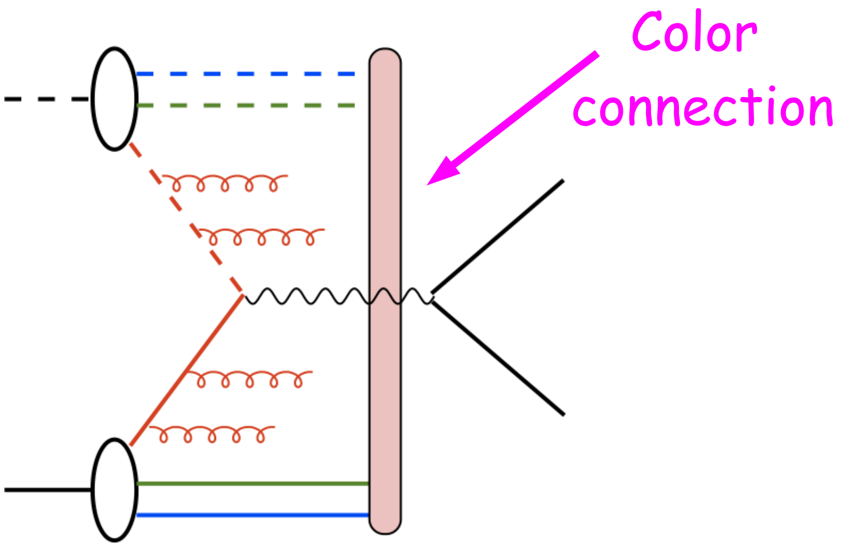


W & Z cross sections

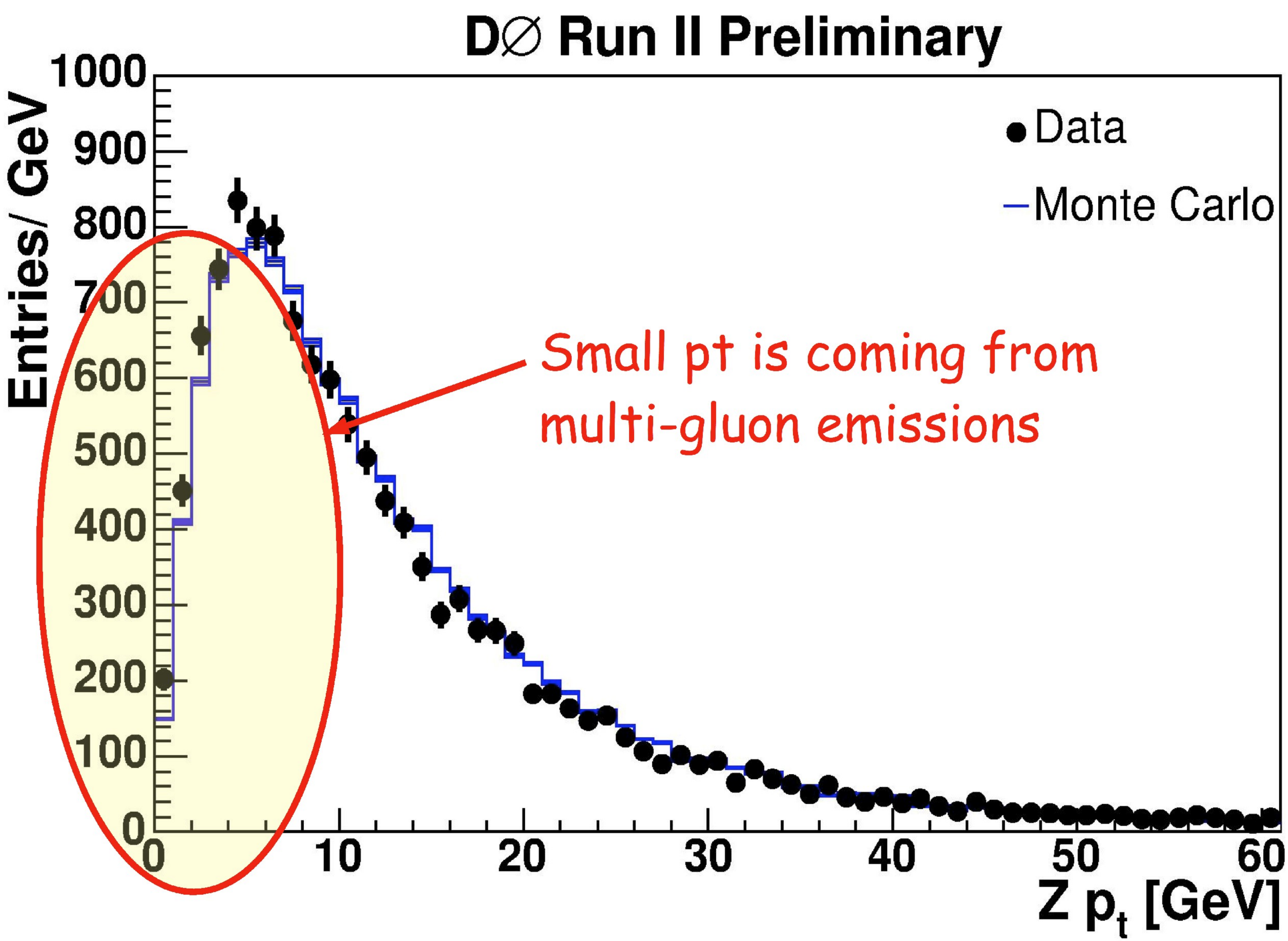
- Basic process: Drell – Yan

  $p + p \to l^+ + l^- + X$

- Factorize process:
  - $q + \bar{q} \to \gamma^* \to l^+ + l^-$
- What about the colored remnants ?
- What about parton radiation ?



The effect of parton radiation ...



Adding all together the complete hadronic final state in pp

Jet production in pp

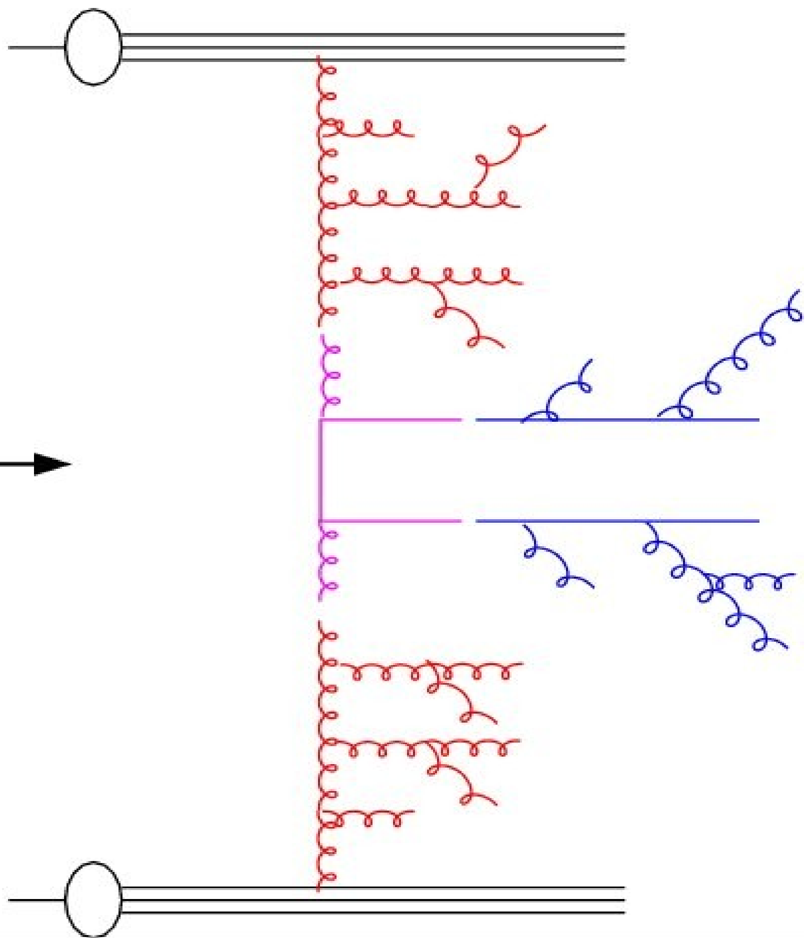
- x-section (i.e. for light and heavy quarks ($t\bar{t}$) production)

$$\sigma(\mathrm{pp} \to \mathrm{q\bar{q}X}) \quad = \quad \int \frac{dx_1}{x_1}\frac{dx_2}{x_2} x_1 G(x_1,\bar{q}) x_2 G(x_2,\bar{q}) \times \hat{\sigma}(\hat{s},\bar{q})$$

- with gluon densities $xG(x,\bar{q})$

- hard x-section:

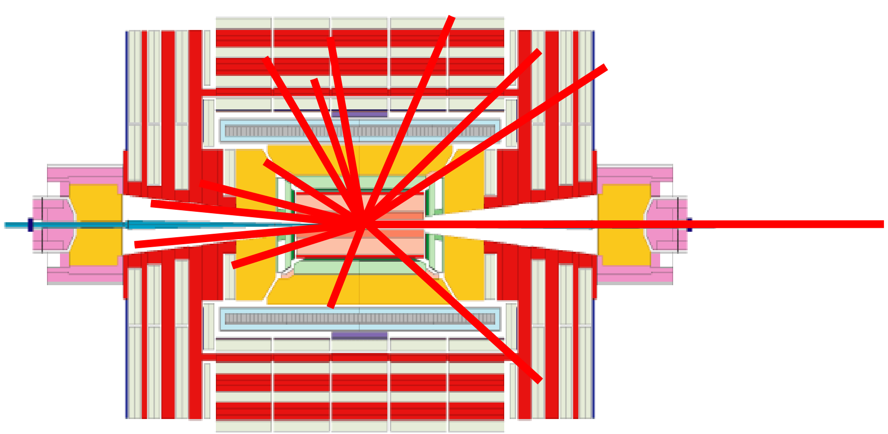
$$\frac{d\sigma}{dt} = \frac{1}{64\hat{s}^2}|M_{ij}|^2 \longrightarrow$$

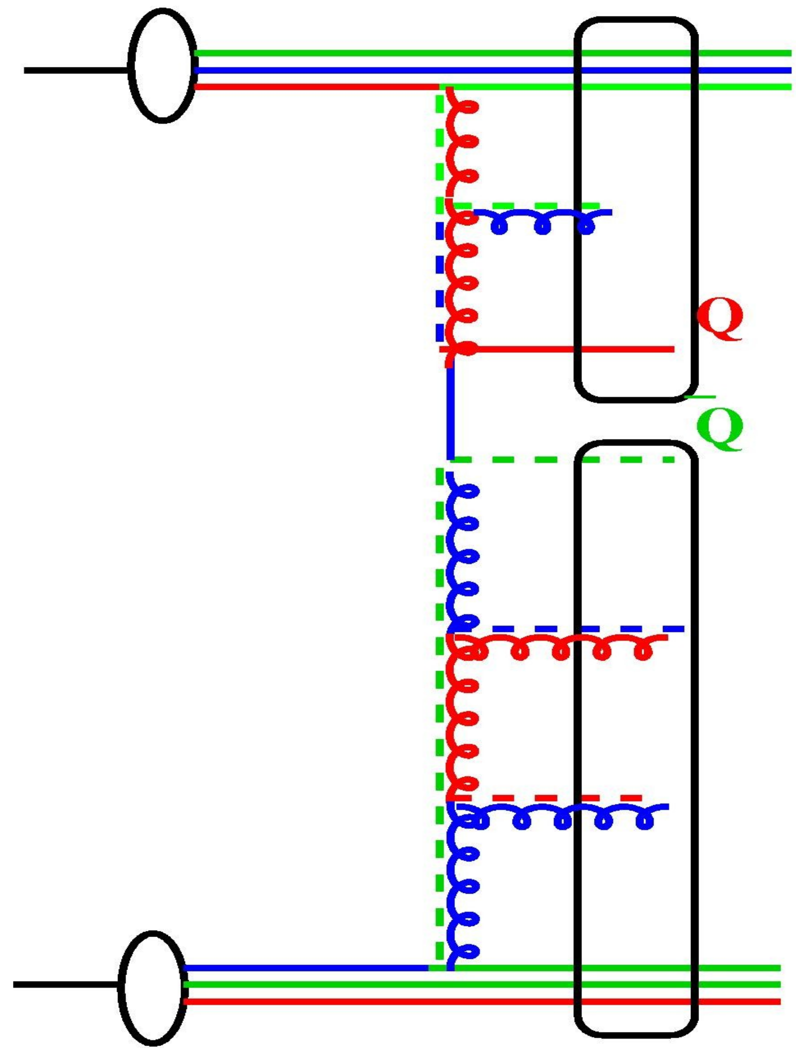


Color Flow in pp

- quarks carry color
- anti-quarks carry anticolor
- gluons carry color – anticolor
  - connect to color singlet systems
  - watch out pp or $p\bar{p}$

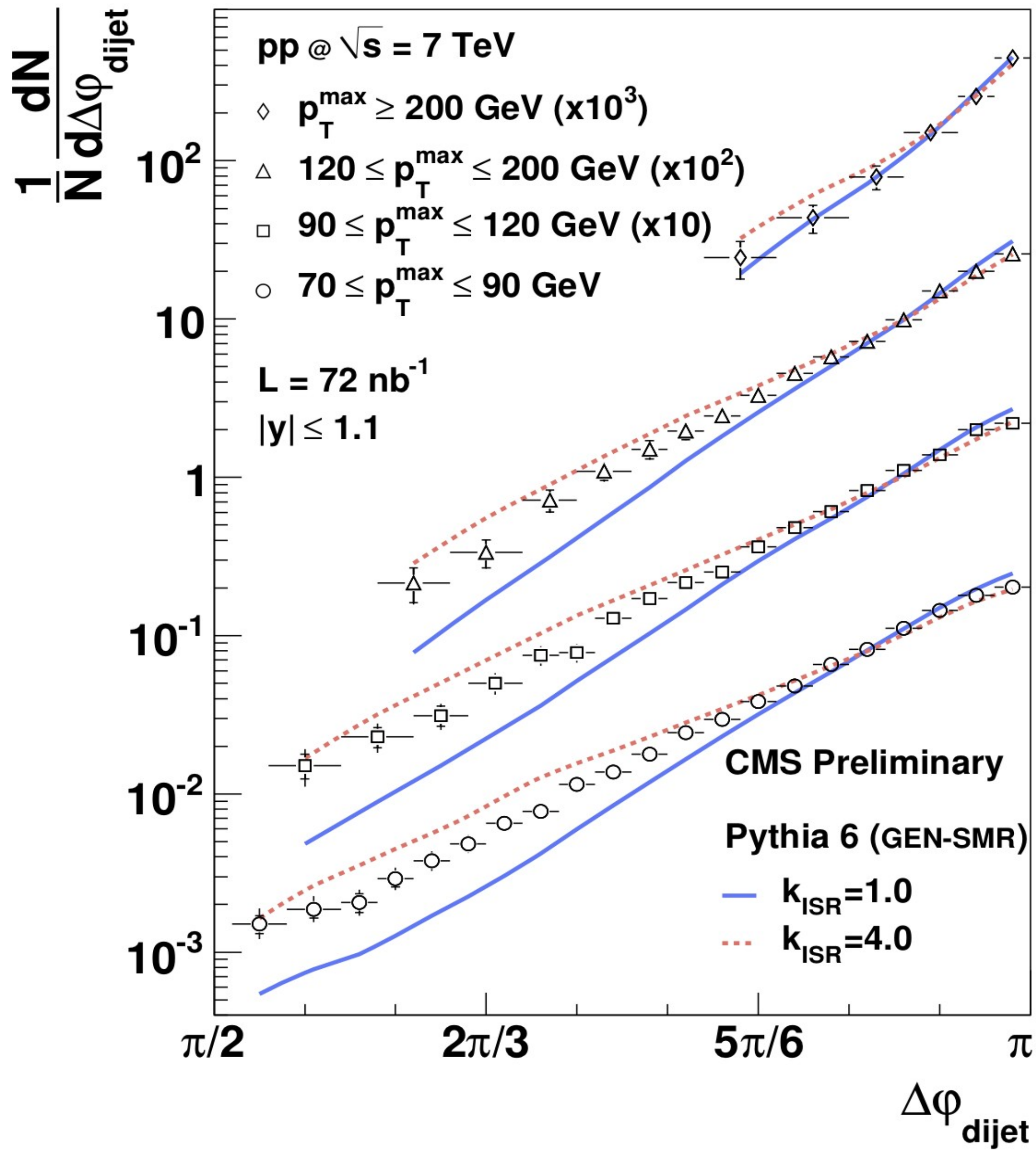
$$pp \to q\bar{q} + X$$





Jet production at LHC

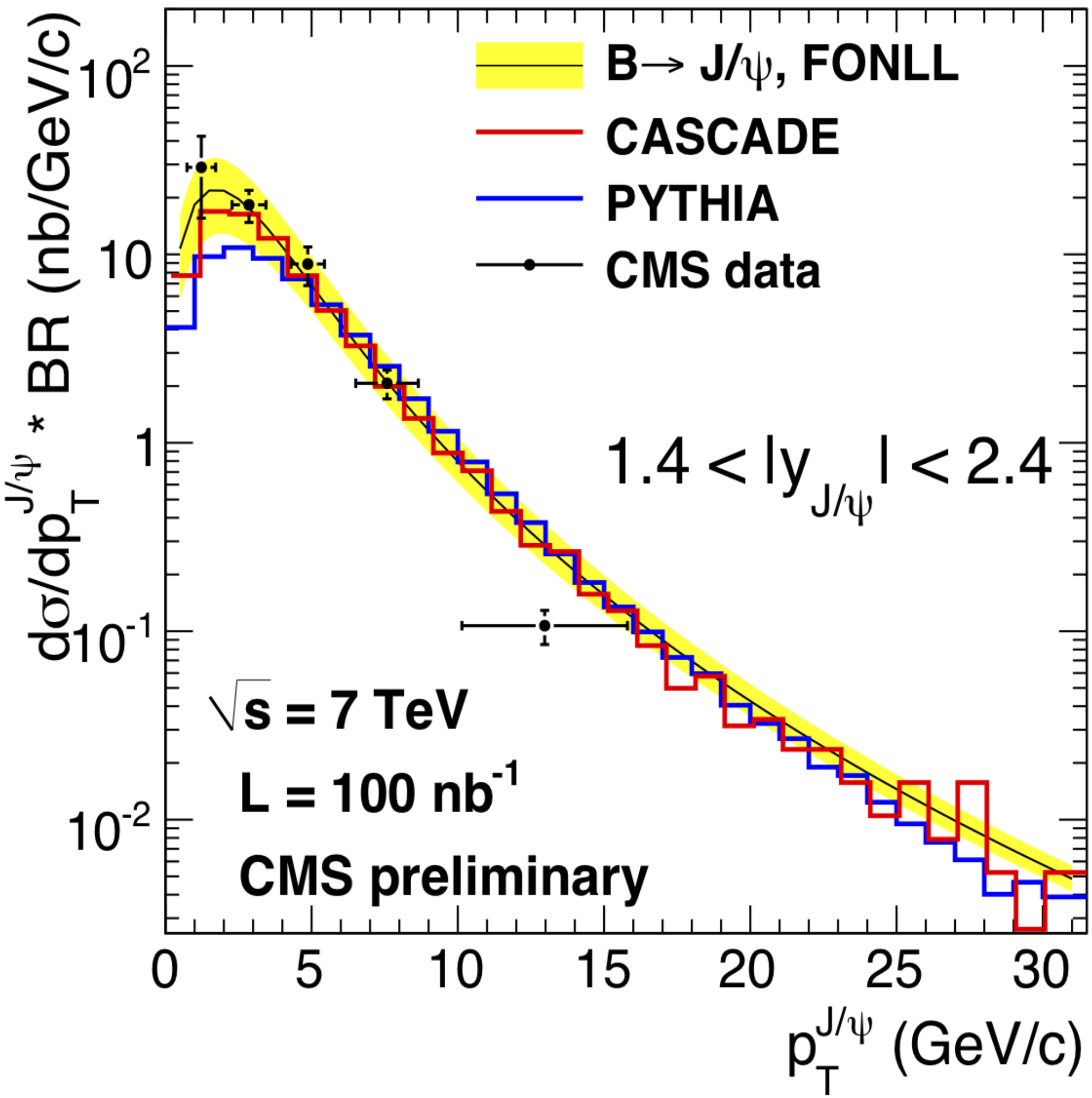
- reasonable description
  - need proper description of hard part of cross section
  - need multiparton radiation for delta phi
- sensitive to multi-gluon radiation



Heavy Flavor production at LHC

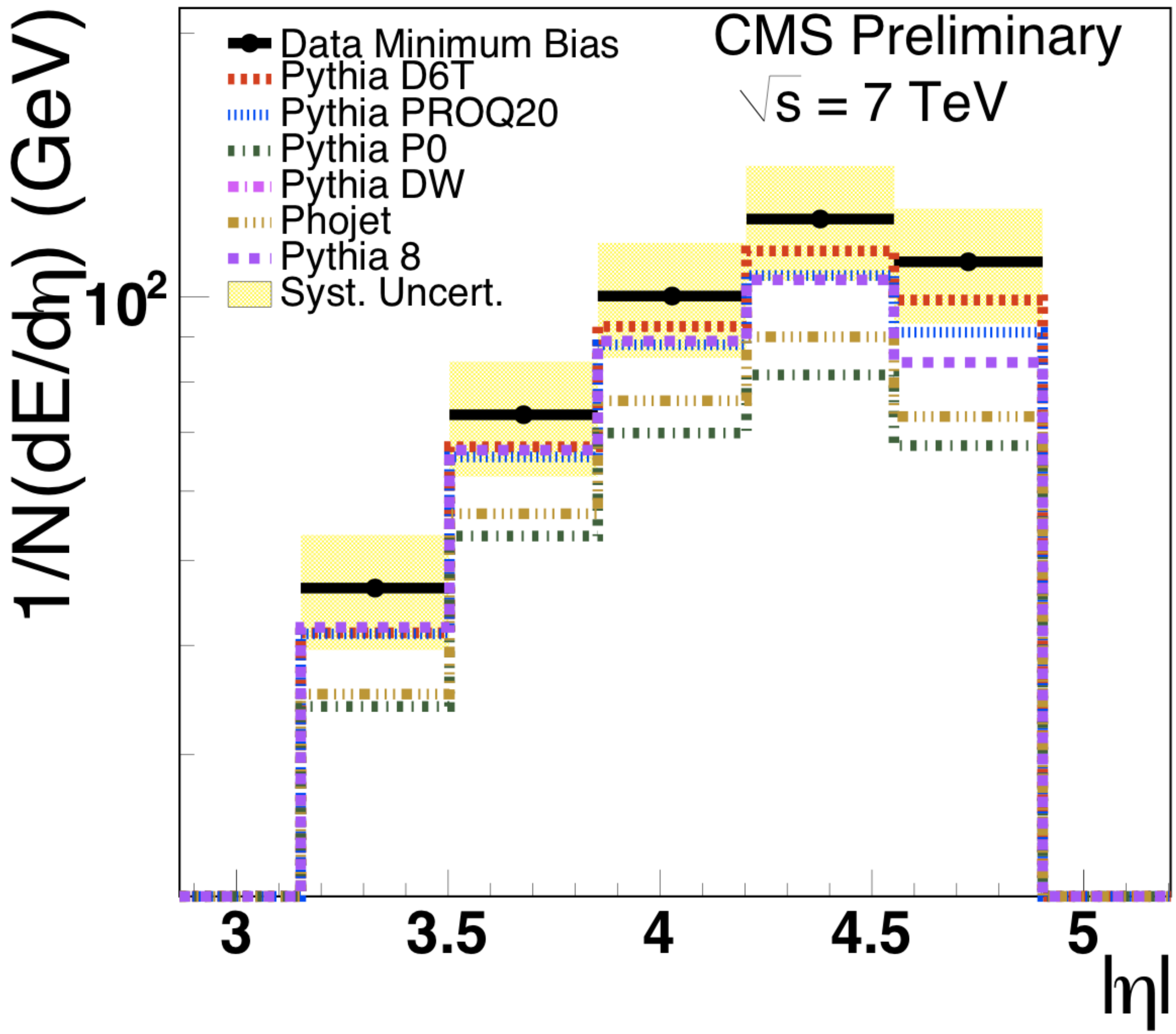
- good description of cross section
  - need hard part of x-section
  - need multi-parton emissions
  - need fragmentation

This process was for a long time a big problem at Tevatron !!!



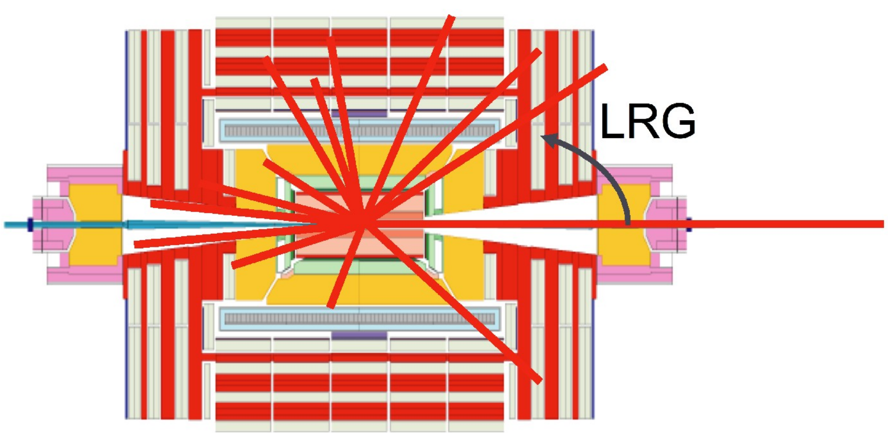
Measurement of Energy flow

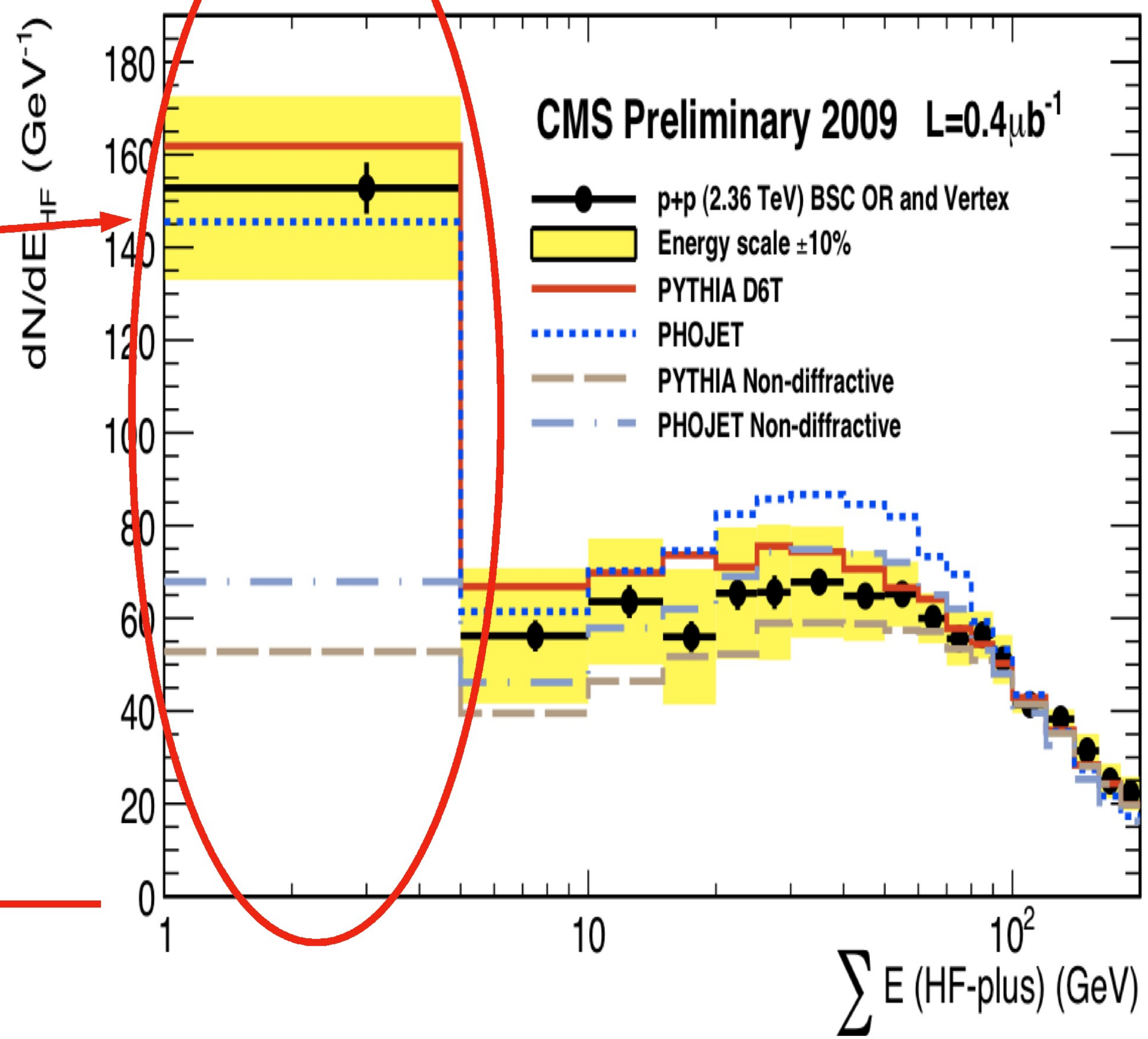
- Energy flow measured in forward region
- sensitive to multi-parton emissions
- Observe higher energy flow than in any MC prediction ...
  - WHY ?



Measurement of Energy flow

- Energy flow measured in forward region
- observe region without energy deposition
- WHAT IS THIS ?





Summary

- Monte Carlo event generators are needed to calculate multi-parton cross sections
- Monte Carlo method is a well defined procedure
  - parton shower are essential
  - hadronization is needed to compare with measurements
- MC approach extended from simple e+e- processes to

  ep processes

  pp processes and heavy Ion processes
- proper Monte Carlos are essential for any measurement

Monte Carlo event generators contain all our physics knowledge !!!!!

The Analysis Centre in the

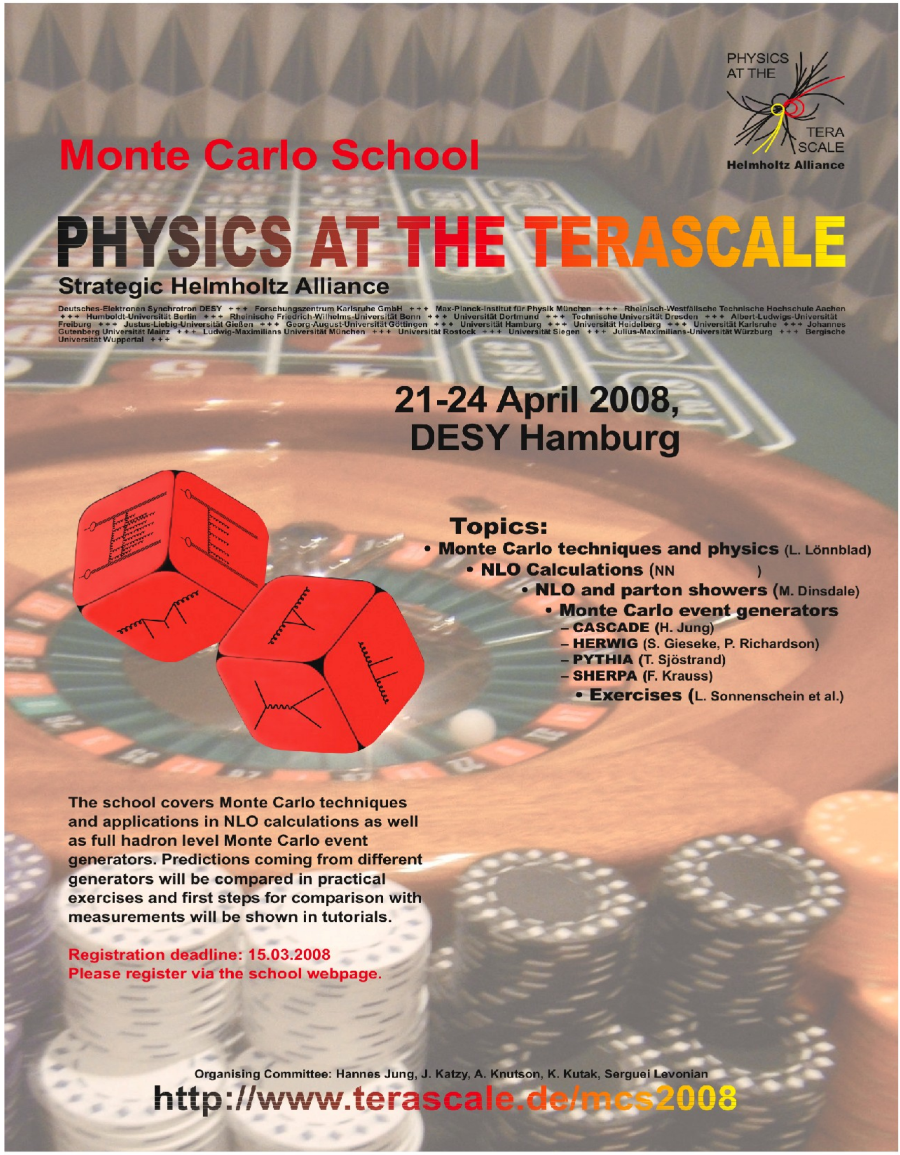
PHYSICS AT THE TERA SCALE

Helmholtz Alliance

- Areas
  - Monte Carlo (user support, tuning, development ...)
  - Parton Distribution Functions
  - Statistics Tools
  - Collaborative tools (web based infos etc)

Monte Carlo group activities

- Development of Monte Carlo generators
  - Tuning of MC generators
  - PDF4MC
  - User support
- Training (schools, seminars)
  - MC schools in spring 2008, 2009
- link to MC group page



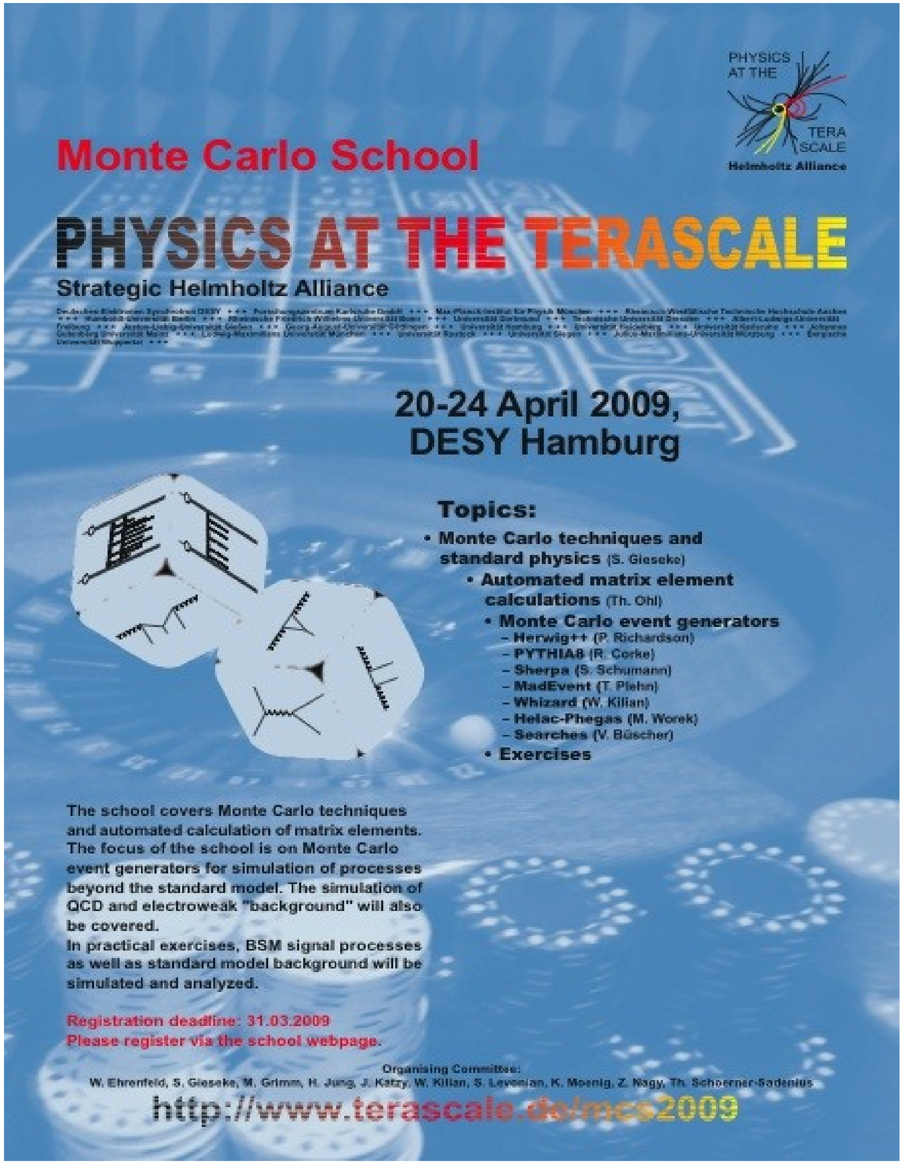
Monte Carlo group activities

If you are interested to do your

- diploma/masters thesis
- PhD
- postdoc

please get in contact with us...

There are plenty of possibilities and positions to do interesting physics with MC simulations ..... and help to find extra dimensions or SUSY or new phenomena in **QCD**



List of available MC programs

- HERA Monte Carlo workshop: www.desy.de/~heramc
- ARIADNE

  A program for simulation of QCD cascades implementing the color dipole model
- CASCADE

  is a full hadron level Monte Carlo generator for ep and pp scattering at small x build according to the CCFM evolution equation, including the basic QCD processes as well as Higgs and associated W/Z production
- HERWIG

  General purpose generator for Hadron Emission Reactions With Interfering Gluons; based on matrix elements, parton showers including color coherence within and between jets, and a cluster model for hadronization.
- JETSET

  The Lund string model for hadronization of parton systems.

List of available MC programs

- LDCMC

  A program which implements the Linked Dipole Chain (LDC) model for deeply inelastic scattering within the framework of ARIADNE. The LDC model is a reformulation of the CCFM model.

- PHOJET

  Multi-particle production in high energy hadron-hadron, photon-hadron, and photon-photon interactions (hadron = proton, antiproton, neutron, or pion).

- PYTHIA

  General purpose generator for e^+e^- pp and ep-interactions, based on LO matrix elements, parton showers and Lund hadronization.

- RAPGAP

  A full Monte Carlo suited to describe Deep Inelastic Scattering, including diffractive DIS and LO direct and resolved processes. Also applicable for photo-production and partially for pp scattering.

Literature & References

- F. James Rep. Prog. Phys., Vol 43, 1145 (1980)
- Glen Cowan STATISTICAL DATA ANALYSIS. Clarendon, 1998.
- Particle Data Book S. Eidelman et al., Physics Letters B592, 1 (2004)

  section on: Mathematical Tools (http://pdg.lbl.gov/)
- Michael J. Hurben Buffons Needle

  (http://www.angelfire.com/wa/hurben/buff.html)
- J. Woller (Univ. of Nebraska-Lincoln) *Basics of Monte Carlo Simulations*

  (http://www.chem.unl.edu/zeng/joy/mclab/mcintro.html)
- Hardware Random Number Generators:

  A Fast and Compact Quantum Random Number Generator (http://arxiv.org/abs/quant-ph/9912118)

  Quantum Random Number Generator (http://www.idquantique.com/products/quantis.htm)

  Hardware random number generator (http://en.wikipedia.org/wiki/)
- Monte Carlo Tutorals

  (http://www.cooper.edu/engineering/chemechem/MMC/tutor.html)
- History of Monte Carlo Method

  (http://www.geocities.com/CollegePark/Quad/2435/history.html)

Literature & References (cont'd)

- T. Sjostrand et al

  PYTHIA/JETSET manual - The Lund Monte Carlos

  http://www.thep.lu.se/tf2/staff/torbjorn/Pythia.html
- H. Jung

  RAPGAP manual

  http://www.desy.de/~jung/rapgap.html

  CASCADE manual

  http://www.desy.de/~jung/cascade.html
- V. Barger and R. J.N. Phillips

  Collider Physics

  Addison-Wesley Publishing Comp. (1987)
- R.K. Ellis, W.J. Stirling and B.R. Webber

  QCD and collider physics

  Cambridge University Press (1996)

General literature

- Many new books are available in DESY library NEW ... ask at the desk there ...
- Statistische und numerische Methoden der Datenanalyse

  V. Blobel & E. Lohrmann
- STATISTICAL DATA ANALYSIS. Glen Cowan.
- Particle Data Book S. Eidelman et al., Physics Letters B592, 1 (2004) (http://pdg.lbl.gov/)
- Applications of pQCD R.D. Field Addison-Wesley 1989
- Collider Physics V.D. Barger & R.J.N. Phillips Addison-Wesley 1987
- Deep Inelastic Scattering. R. Devenish & A. Cooper-Sarkar, Oxford 2
- Handbook of pQCD G. Sterman et al
- Quarks and Leptons, F. Halzen & A.D. Martin, J.Wiley 1984
- QCD and collider physics R.K. Ellis & W.J. Stirling & B.R. Webber Cambridge 1996
- QCD: High energy experiments and theory G. Dissertori,I. Knowles, M. Schmelling Oxford 2003