

# Simulations

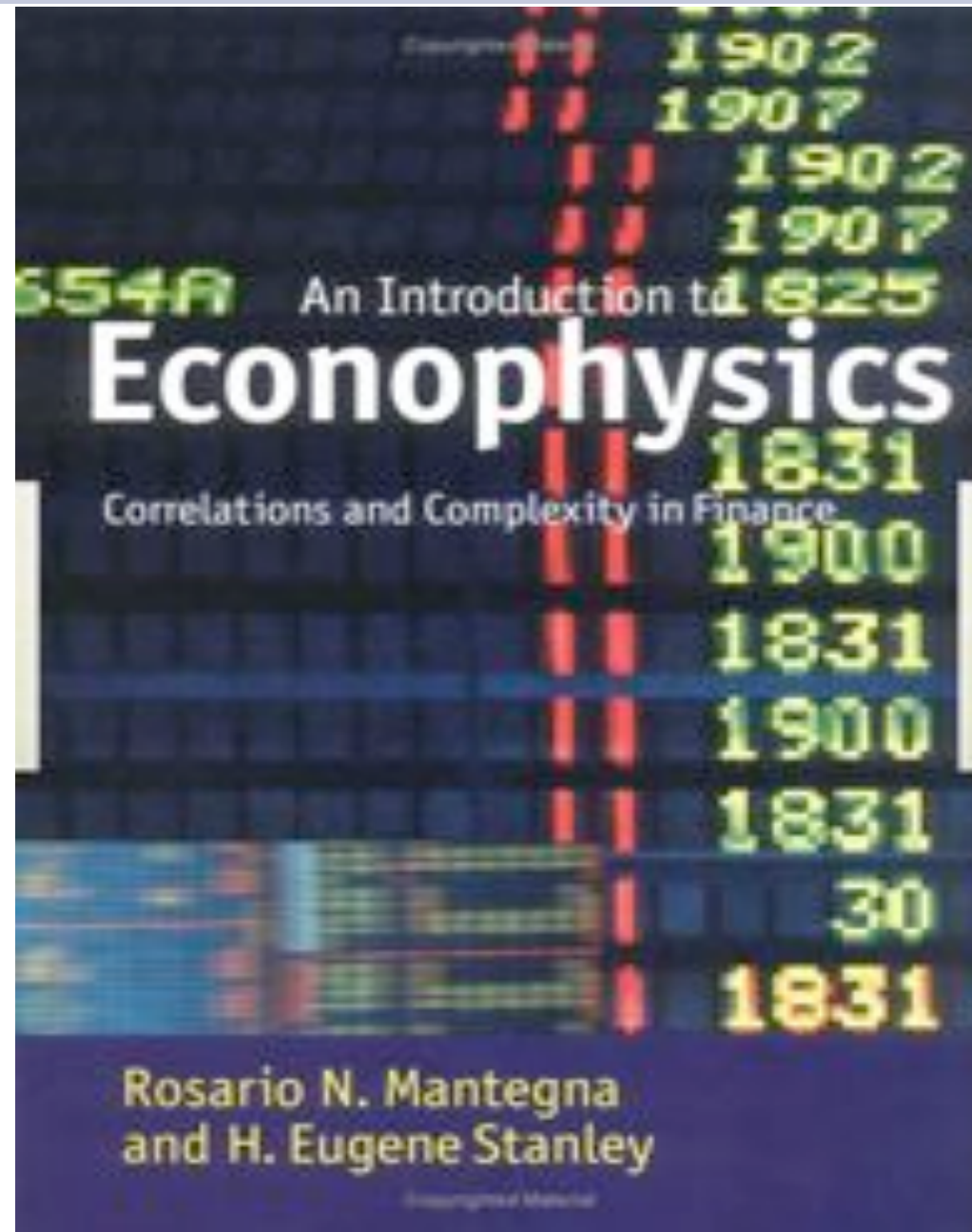
H. Jung (DESY, University Antwerp)

LHC rap

# From gambling ...



.... to



# ... via applications in risk management

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#### News

- Pension, Insurance Researchers Develop @RISK Model to Project Key Indices**  
Three leading actuarial authorities teamed up to develop an @RISK model for pension and insurance risk planning. The resulting model provides a free, and publicly accessible integrated framework for sampling future financial scenarios.
- 2006 Palisade User Conference: Americas, November 13-14 Miami**  
Join us in Miami for America's most innovative risk and decision analysis forum. From hands-on software training to real-world case studies and best practices, the 2006 Palisade User Conference has something for everyone.
- @RISK Models EU Blood Screening**  
The only significant disease for which blood transfusions continue to pose a health risk is hepatitis B. The Hospital Clinic in Barcelona, Spain used @RISK to model the best ways to manage this risk.
- Palisade Featured in Quality Digest**  
Highlighting the growing popularity of Palisade tools in the quality control community, trade leader Quality Digest recently published the article "Neural Networks Software Crunches the Big Numbers," highlighting NeuralTools.
- Introducing NeuralTools**  
NeuralTools, Palisade's new Neural Networks add-in for Excel, is now available. Over 2000 people participated in the NeuralTools Beta. Featuring Live Prediction that works with Evolver and Solver, NeuralTools

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- London: 21 - 22 September**  
Risk Assessment

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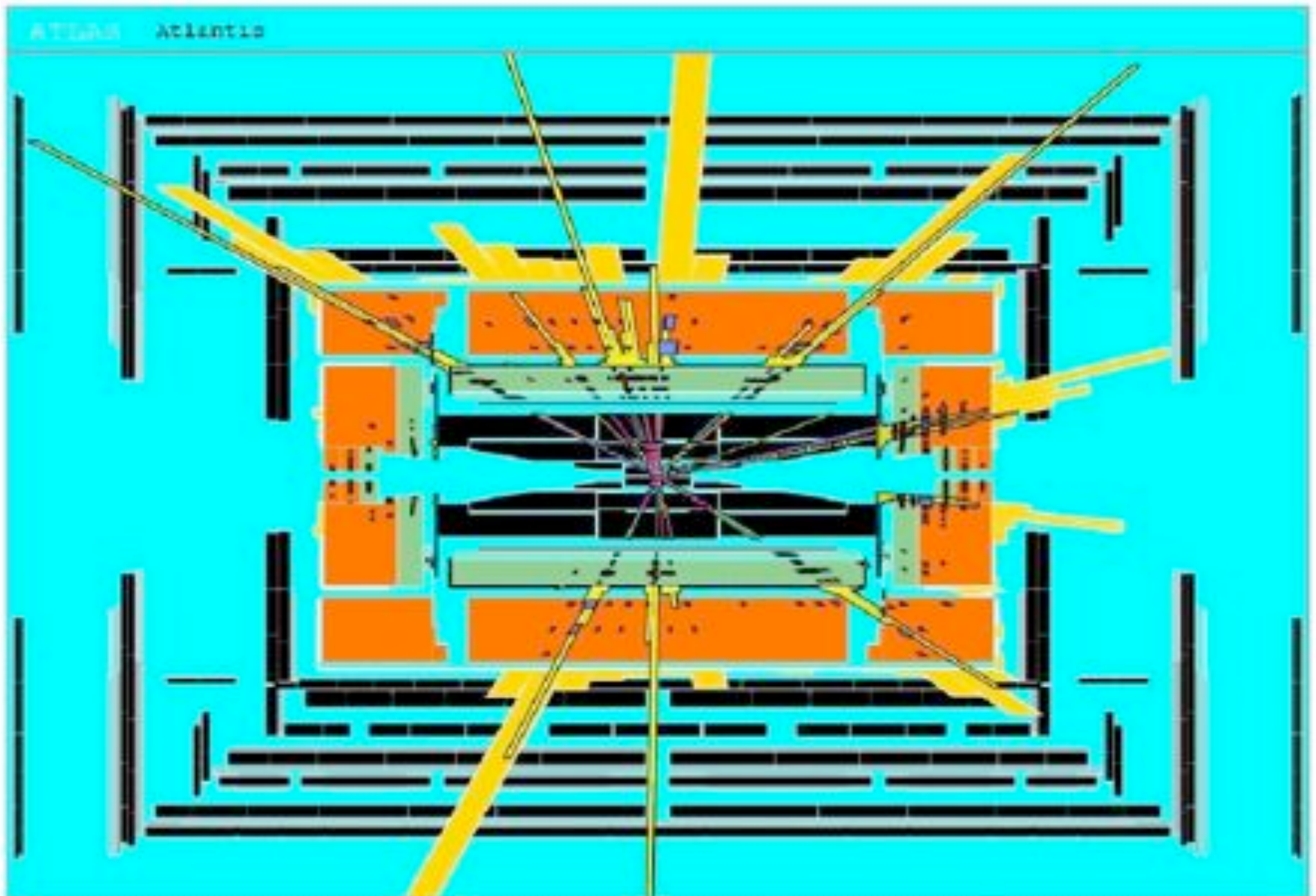
# Why do we need all that ?

- because physics and life is more complicated than a simple equation, which can be solved analytically ....
- **Monte Carlo techniques are**
  - widely used
  - are of enormous advantages
  - can be used to simulate any complicated process
  - are now **EVEN** used in **particle physics theory** ..... !!!!! ....

... and in particle physics ?

Do we also need simulations ?

# What is this ?



# What is this ?



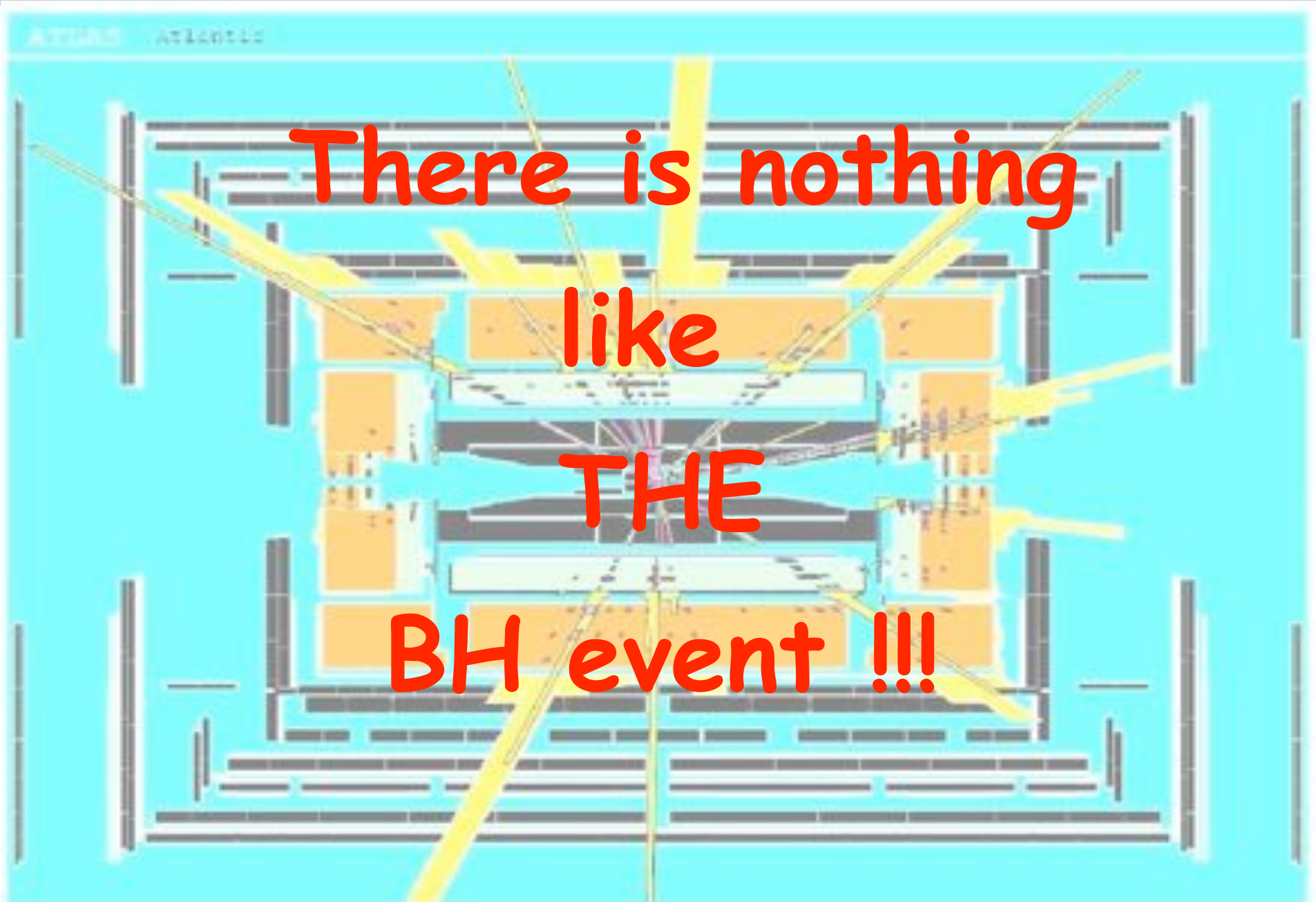
**Tell the difference  
between the production  
of a mini black hole  
and a multijet event ?**



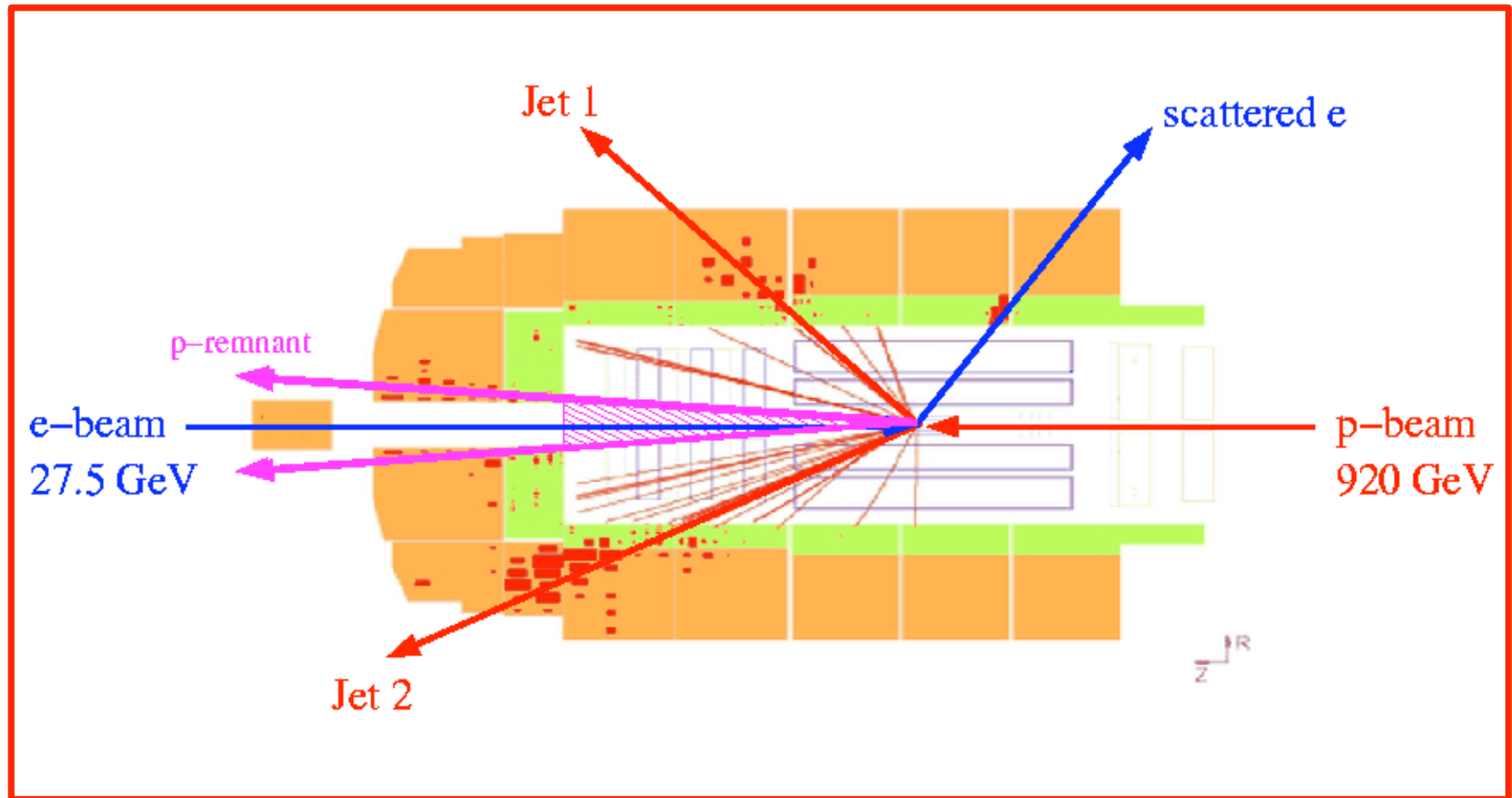
# What is this ?



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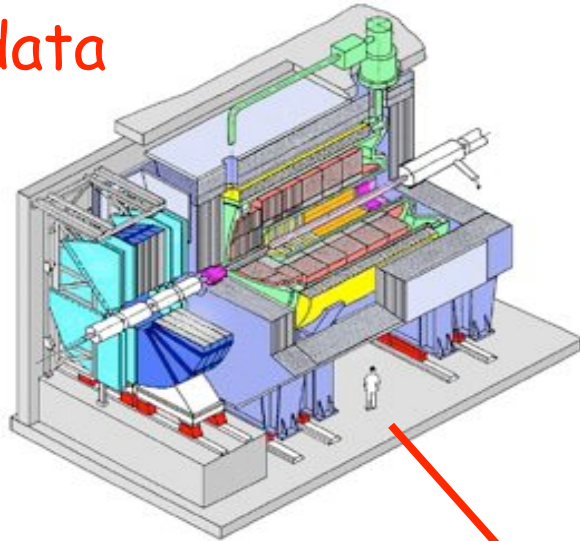
# Events at HERA ...



$$\sqrt{s} \sim 318 \text{ GeV}$$
$$x \sim 7 \cdot 10^{-5} \text{ at } Q^2 = 4 \text{ GeV}^2$$

# From experiment to measurement

take data



- run MC generator
- detector simulation

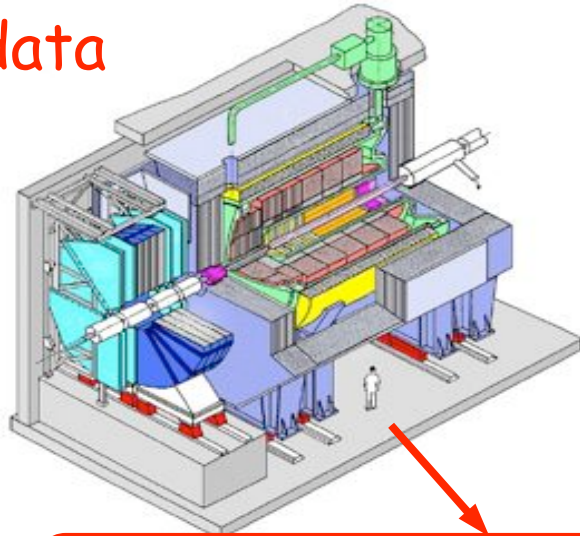


Compare measurement  
with  
simulation

**Upppps ..... all measurements rely on proper  
MC generators and MC simulation !!!!**

# From experiment to measurement

take data



- run MC generator
- detector simulation

define visible  $x$  - section in kinematic variables  
calculate factor  $C_{corr}$  to correct from detector to hadron level

$$\frac{d\sigma_{had}^{data}}{dx} = \frac{d\sigma_{det}^{data}}{dx} C_{corr} \quad \text{with} \quad C_{corr} = \frac{\frac{d\sigma_{had}^{MC}}{dx}}{\frac{d\sigma_{det}^{MC}}{dx}}$$

visible  $x$ -section on hadron level

**Upppps ..... all measurements rely on proper  
MC generators and MC simulation !!!!**

# Monte Carlo - different applications

- MC simulation of detector response
  - input: hadron level events - output: detector level events
  - Calorimeter ADC hits
  - Tracker hits
  - need knowledge of particle passage through matter, x-section ...
  - need knowledge of actual detector

# Monte Carlo - different applications

- MC simulation of detector response
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  - Calorimeter ADC hits
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  - need knowledge of actual detector
- multipurpose MC event generators:
  - x-section on parton level
  - including multi-parton (initial & final state) radiation
  - remnant treatment (proton remnant, electron remnant)
  - hadronization/fragmentation (more than simple fragmentation functions...)
- fixed order parton level ..... theorists like it !!!!!!!!!!!!!!!!!!!!!!!!!!!!!
  - integration of multidimensional phase space

Not covered here

# What this is about ...

From

$$e^+ e^- \rightarrow e^+ e^-$$

via

$$ep \rightarrow e' X$$

to

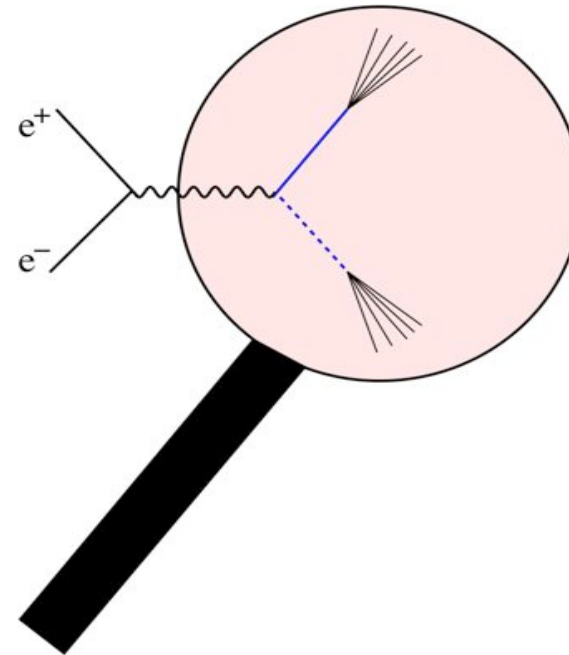
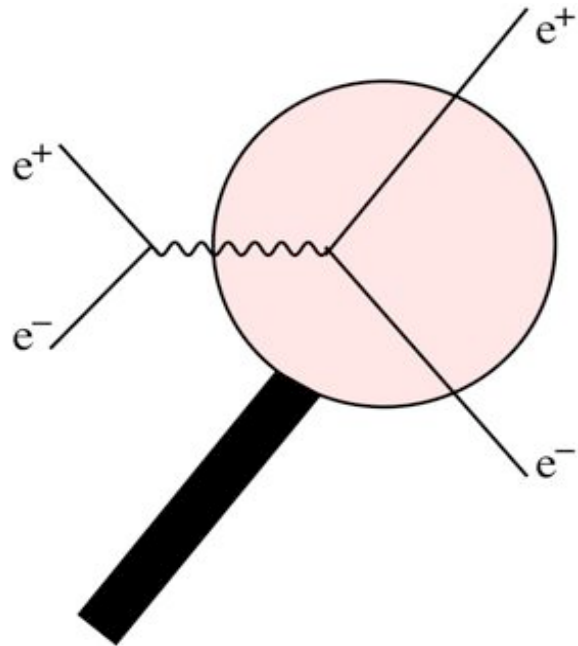
$$pp \rightarrow h + X$$



# The easy case: $e^+e^- \rightarrow X$

• use  $e^+e^- \rightarrow \mu^+\mu^-$  and

$e^+e^- \rightarrow q\bar{q} \rightarrow \text{hadrons}$



• cross sections can be calculated in QED:  $\sigma(e^+e^- \rightarrow l^+l^-) = \frac{4\pi\alpha^2}{3s}$

• and for quarks

$$\sigma(e^+e^- \rightarrow q\bar{q}) = 3 \frac{4\pi\alpha^2}{3s} e_q^2$$

→ but quarks carry color and fractional charge !!!!!

color

charge

# The easy case: $e^+e^- \rightarrow X$

PDG 2008

- measure ratio of hadronic / leptonic cross section

$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

$$= N_c \sum_i e_q^2 = 3 \sum_i e_q^2$$

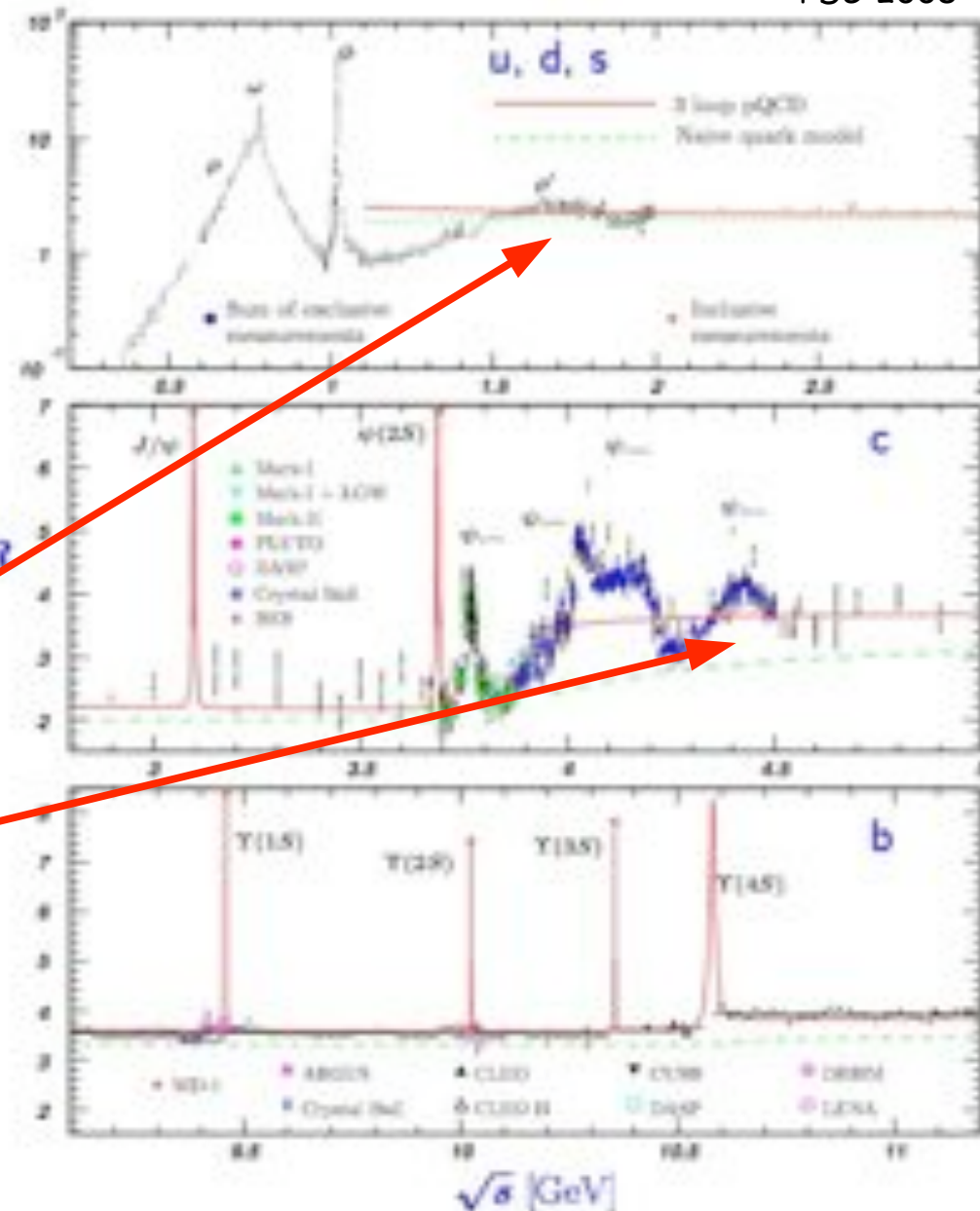
- for 3 quarks:

$$R = 3 \left[ \left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 \right] = 2$$

- including charm

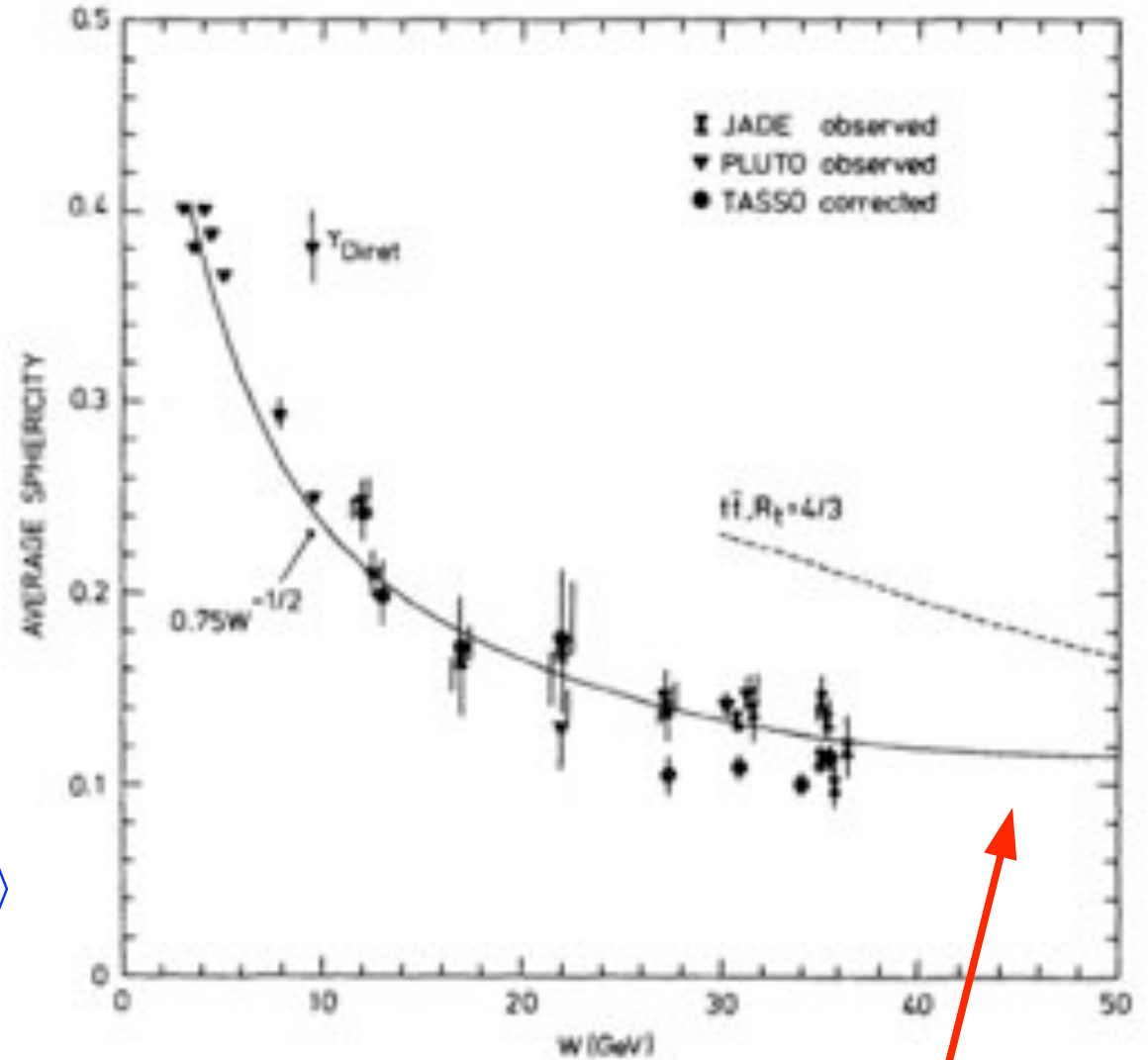
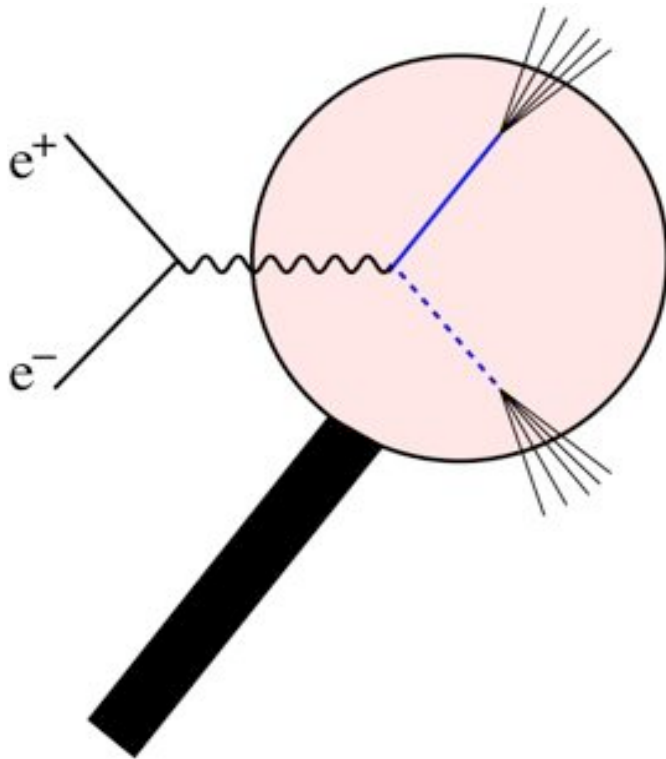
$$R = 3 \left[ \frac{2}{3} + \left(\frac{2}{3}\right)^2 \right] = 3.333$$

- "direct" observation of fractional charge of quarks and
- 3 different colors ....



# The early steps: $e^+e^- \rightarrow$ hadrons

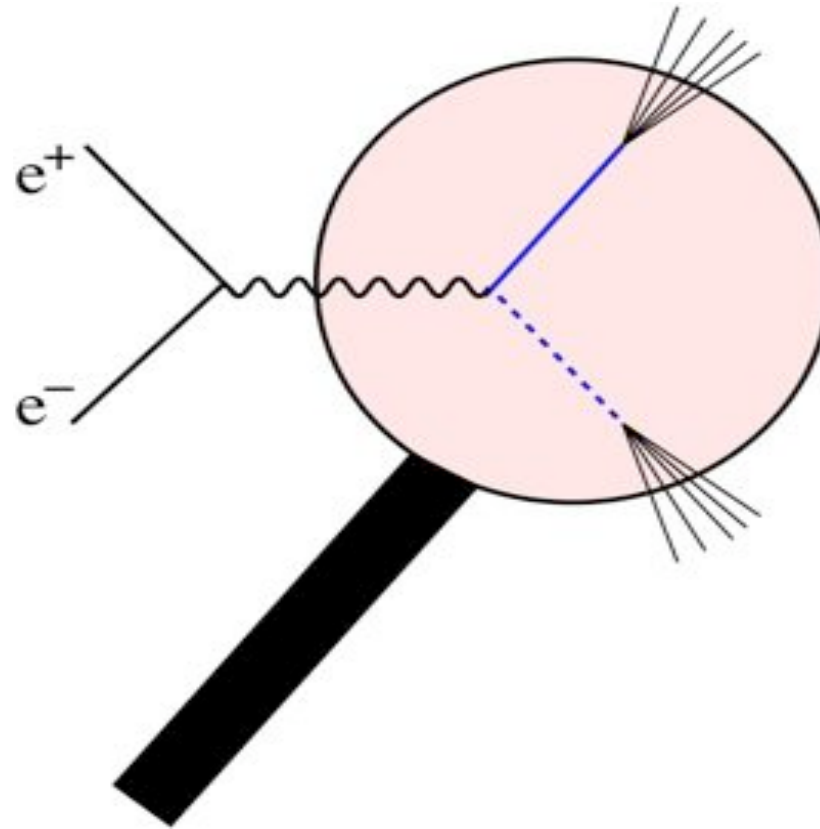
Soding, Wolf Experimental Evidence on QCD  
Ann Rev Nucl Part Sci 1981, 31, 231



- sphericity:  $S \sim 3/2 \langle \delta^2 \rangle$   
jet opening angle  $\langle \delta \rangle = \langle P_t / P_{||} \rangle$
- $S \sim 0$  for extreme jets  
 $S \rightarrow 1$  for spherical events

→ evidence for 2-jet structure

# The early steps: $e^+e^- \rightarrow \text{hadrons}$



→ How to compare a detailed measurement with a theoretical prediction ?

Simulate these  
processes with  
Monte Carlo  
method !!!

# Monte Carlo method

- Monte Carlo method
  - **refers** to any procedure that makes use of random numbers
  - **uses** probability statistics to solve the problem
- Monte Carlo methods are used in:
  - Simulation of natural phenomena
  - Simulation of experimental apparatus
  - Numerical analysis
- Random number:

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# Monte Carlo method

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  - Numerical analysis
- Random number:
  - one of them is **3**
  - No such thing as a single random number**
  - A sequence of random numbers is a set of numbers that have nothing to do with the other numbers in a sequence

# Going out to Monte Carlo



- Obtain true Random Numbers from Casino in Monte Carlo
- Puhhh... Going out every night ...



# Random Numbers

- In a uniform distribution of random numbers in  $[0,1]$  every number has the same chance of showing up
- Note that 0.000000001 is just as likely as 0.5

To obtain random numbers:

- Use some chaotic system like roulette, lotto, 6-49, ...
- Use a process, inherently random, like radioactive decay
- Tables of a few million truly random numbers exist .....

(.....until a few years ago.....)

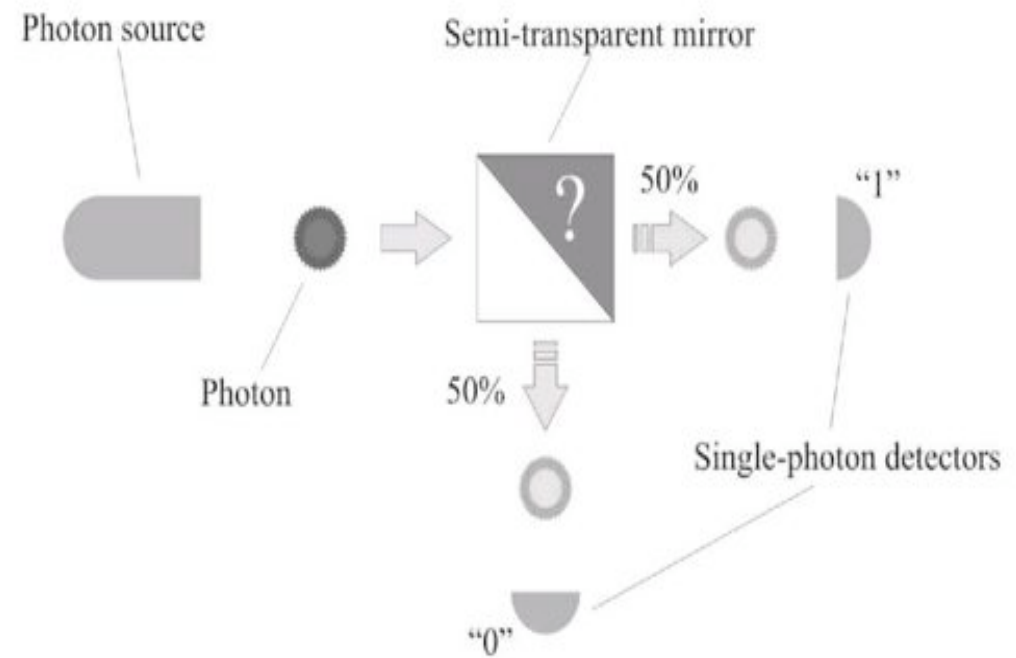
**BUT** not enough for most applications

→ ..... we have true random number generators ...

# True Random Numbers

- Random numbers from **classical physics: coin tossing**  
evolution of such a system can be predicted, once the initial condition is known... however it is a chaotic process ... extremely sensitive to initial conditions.
- Truly Random numbers used for
  - Cryptography
    - Confidentiality
    - Authentication
  - Scientific Calculation
  - Lotteries and gambling
    - do not allow to increase chance of winning by having a bias .... too bad

- Random numbers from **quantum physics: intrinsic random**  
**photons on a semi-transparent mirror**



- Available and tested in MC generator by a summer student
- Generator is **however** very slow...

# True Random Numbers

- atmospheric noise, which is quite easy to pick up with a normal radio: used by RANDOM.ORG (thanks to Albert)
- much more can be found on the web ....

The screenshot shows the homepage of RANDOM.ORG. At the top, there is a navigation menu with links: Home, Introduction, Statistics, Numbers, Drawings, Quota, Testimonials, FAQ, Contact, Login, and What's New. Below the menu is the site's logo "RANDOM.ORG" and a search bar with the text "Search RANDOM.ORG" and "Google™ Custom Search". To the right of the search bar is a "Search" button. Below the logo is the text "True Random Number Service".

The main content area features the heading "What's this fuss about *true* randomness?" followed by a paragraph: "Perhaps you have wondered how predictable machines like computers can generate randomness. In reality, most random numbers used in computer programs are *pseudo-random*, which means they are generated in a predictable fashion using a mathematical formula. This is fine for many purposes, but it may not be random in the way you expect if you're used to dice rolls and lottery drawings."

Below this is another paragraph: "RANDOM.ORG offers true random numbers to anyone on the Internet. The randomness comes from atmospheric noise, which for many purposes is better than the pseudo-random number algorithms typically used in computer programs. People use RANDOM.ORG for holding drawings, lotteries and sweepstakes, to drive games and gambling sites, for scientific applications and for art and music. The service has existed since 1998 and was built and is being operated by Mads Haahr of the School of Computer Science and Statistics at Trinity College, Dublin in Ireland."

On the right side of the page, there is a "True Random Number Generator" widget. It includes input fields for "Min:" (set to 1) and "Max:" (set to 100), a "Generate" button, and a "Result:" field. Below the widget is a link for "[more options]" and a note "[put this widget on your site]".

## Fun & Free

- Win an iPod! -- **new!**
- Coin Flipper
- Die Roller
- Playing Card Shuffler
- Lottery Quick Pick
- Keno Quick Pick
- Jazz Scale Generator
- Bitmap Generator
- Sound Generator

## Background & Stats

- [About Randomness](#)
- [History of RANDOM.ORG](#)
- [Randomness Quotations](#)
- [General FAQ](#)
- [Guide to Random Drawings](#)
- [Video Guide to Giveaways](#) -- **new!**
- [Real-Time Statistics](#)
- [Statistical Analysis](#)
- [Your Quota](#)

## Premium & Advanced

- [Login/Register](#)
- [Premium Generator](#)
- [Randomness Trails](#)
- [Integer Set Generator](#)
- [Third-Party Draw](#)
- [Gaussian Generator](#)
- [Fraction Generator](#)
- [Clock Time Generator](#)
- [Calendar Date Generator](#)

# Pseudo Random Numbers

## Pseudo Random Numbers

- are a sequence of numbers generated by a computer algorithm, usually uniform in the range  $[0,1]$
- **more precisely**: algo's generate integers between  $0$  and  $M$ , and then  $r_n = I_n / M$
- A very early example: **Middle Square (John van Neumann, 1946)**:  
generate a sequence, start with a number of 10 digits, square it, then take the middle 10 digits from the answer, as the next number etc.:

$$5772156649^2 = 33317792380594909291$$

**Hmmmm**, sequence is not random, since each number is determined from the previous, but it **appears** to be random

- this algorithm has problems .....

**BUT** a more complex algo does not necessarily lead to better random sequences ....

**Better** us an algo that is well understood

# Congruential linear generator

- develop our own simple generator

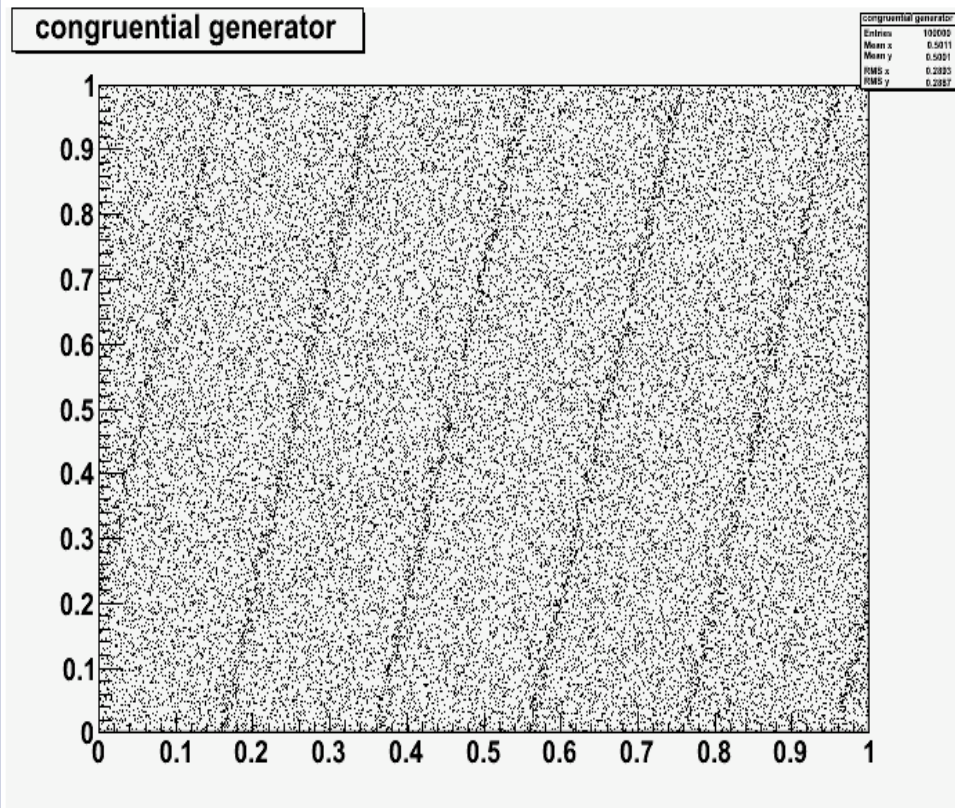
$$I_j = \text{mod}(aI_{j-1} + c, m)$$

$$R_j = \frac{I_j}{m}$$

- with seed  $I_0$
- and multiplicative constant  $a$  and additive constant  $c$
- modulus  $m$
- maximal repetition period:  $\mathcal{O}(m)$

# Randomness tests

- Congruential generator

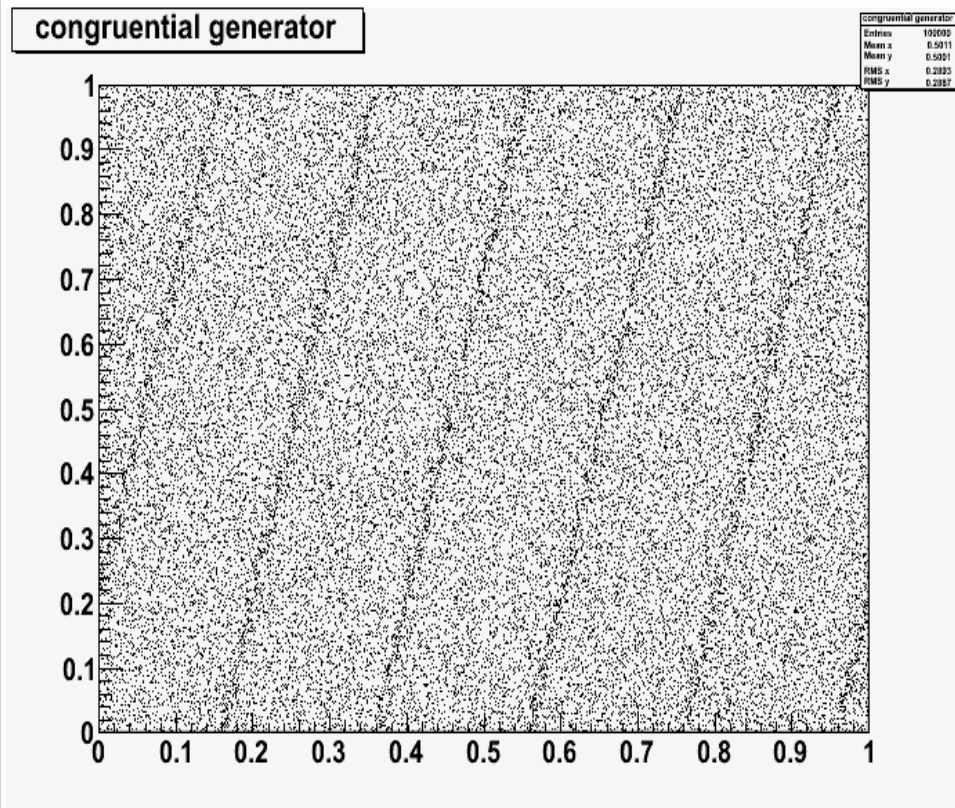


→ Congruential generator is not bad ... but it could be better ....



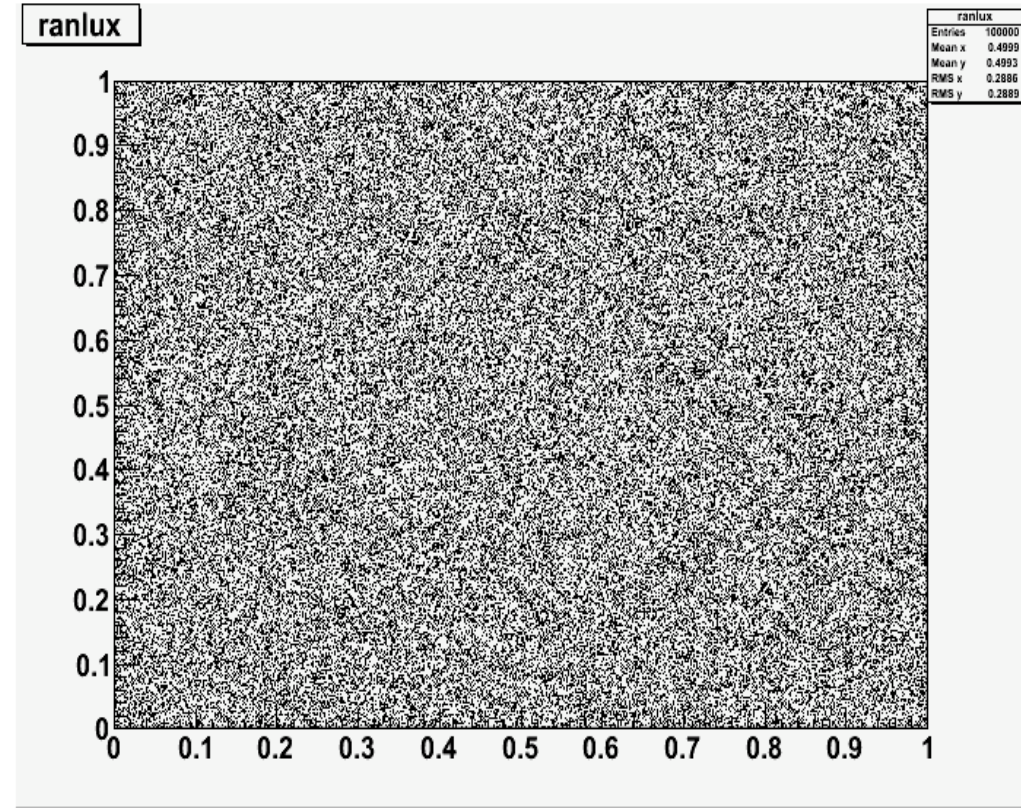
# Randomness tests

- Congruential generator



- RANLUX

M. Lüscher, A portable high-quality random number generator for lattice field theory simulations, Computer Physics Communications 79 (1994) 100  
<http://luscher.web.cern.ch/luscher/ranlux/index.html>



→ RANLUX much more sophisticated  
Developed and used for QCD  
lattice calcs

# Simulating Radioactive Decay

- radioactive decay is a truly random process
- $dN = -N \alpha dt$  i.e.  $N = N_0 e^{-\alpha t}$
- probability of decay is **constant** ... independent of the age of the nuclei:  
probability that nucleus undergoes radioactive decay in time  $\Delta t$  is  $p$ :  
 $p = \alpha \Delta t$  (for  $\alpha \Delta t \ll 1$ )
- **Problem:**  
consider a system initially having  $N_0$  unstable nuclei.  
**How does the number of parent nuclei,  $N$ , change with time ?**
- **Algorithm:**

```
LOOP from t=0 to t, step  $\Delta t$ 
  LOOP over each remaining parent nucleus
    Decide if nucleus decays:
      IF ( random # <  $\alpha \Delta t$  ) then
        reduce number of parents by 1
      ENDIF
    END LOOP over nuclei
  Plot or record  $N$  vrs  $t$ 
END LOOP over time
END
```

# The first simulation: radioactive decay

- implement algo into a small program
- show results after 3000 sec for:

$$N_0 = 100, \alpha = 0.01 \text{ s}^{-1}$$

$$\Delta t = 1\text{s}$$

$$N_0 = 5000, \alpha = 0.03 \text{ s}^{-1}$$

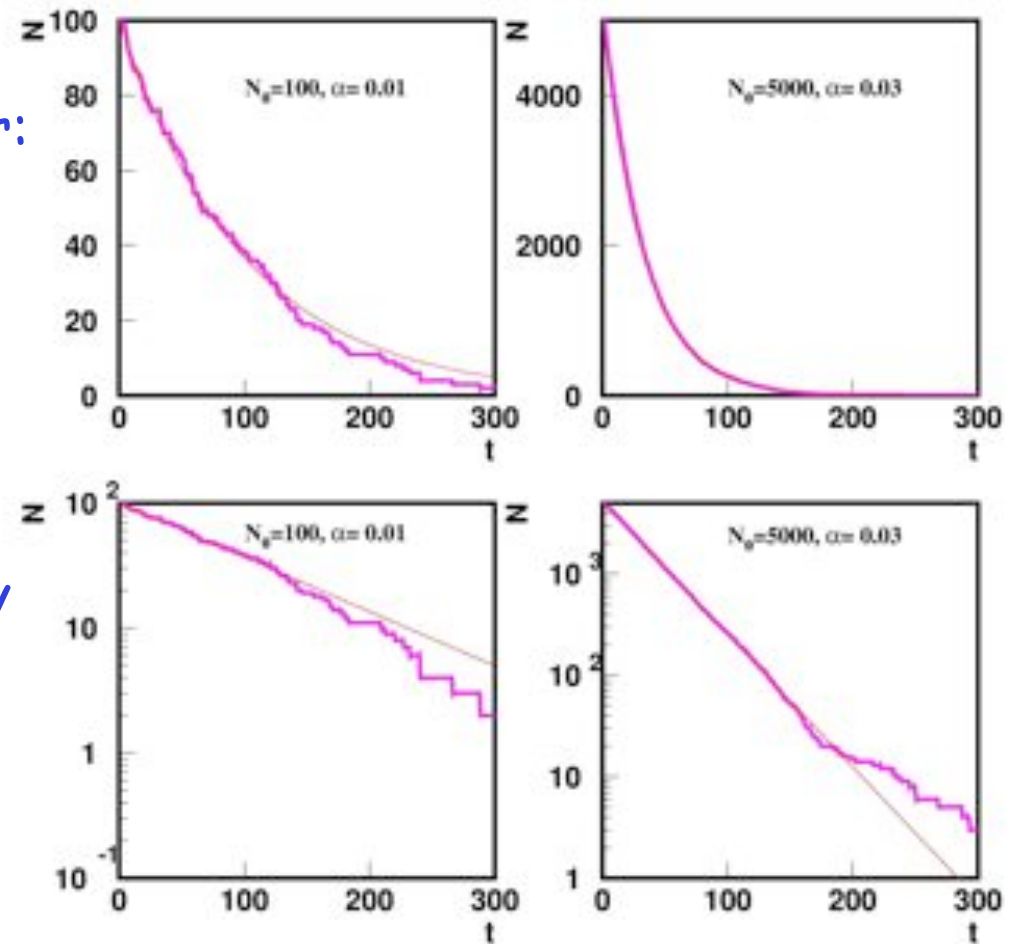
$$\Delta t = 1\text{s}$$

- algo:

```
alpha1 = 0.01
N01 = 100
deltat = 1
do I=1,300
  it = it + 1
  do j = 1, N01
    x = RN1
    fr = deltat*alpha1
    if(x.lt.fr) then
c   reduce number of parents N01
      N01 = N01 - 1
    endif
c   fill for each time it number N01
      call hfill(400,real(it+0.3),0,1.) !
    enddo
```

# The first simulation: radioactive decay

- implement algo into a small program
- show results after 3000 sec for:
  - $N_0 = 100, \alpha = 0.01 \text{ s}^{-1}$
  - $\Delta t = 1\text{s}$
  - $N_0 = 5000, \alpha = 0.03 \text{ s}^{-1}$
  - $\Delta t = 1\text{s}$
- MC experiment does not exactly reproduce theory ....
- results from MC experiment show statistical fluctuations ...
- .....as expected .....



# Expectation values and variance

- Expectation value (defined as the average or mean value of function  $f$ ):

$$E[f] = \int f(u) dG(u) = \left( \frac{1}{b-a} \int_a^b f(u) du \right) = \frac{1}{N} \sum_{i=1}^N f(u_i)$$

for uniformly distributed  $u$  in  $[a,b]$  **then**  $dG(u) = du/(b-a)$

- rules for expectation values:

$$E[cx + y] = cE[x] + E[y]$$

- Variance

$$V[f] = \int (f - E[f])^2 dG = \left( \frac{1}{b-a} \int_a^b (f(u) - E[f])^2 du \right)$$

- rules for variance:

if  $x,y$  uncorrelated:  $V[cx + y] = c^2V[x] + V[y]$

if  $x,y$  correlated  $V[cx + y] = c^2V[x] + V[y] + 2cE[(y - E[y])(x - E[x])]$

# Generating distributions

- From uniform distributions to distributions for any probability density function
  - use variable transformation

- linear p.d.f:

$$f(x) = 2x$$

$$u(x) = \int_0^x 2t dt = x^2$$

$$x_j = \sqrt{u_j}$$

- 1/x distribution

$$f(x) = \frac{1}{x}$$

$$u(x) = \frac{\int_{x_{min}}^x \frac{1}{t} dt}{F_{max} - F_{min}}$$

$$x_j = x_{min} \left( \frac{x_{max}}{x_{min}} \right)_j^u$$

# Generating distributions

- Brute Force or Hit & Miss method
  - use this if there is no easy way to find a analytic integrable function
  - find  $c \leq \max f(x)$
  - reject if  $f(x_i) < u_j \cdot c$
  - accept if  $f(x_i) > u_j \cdot c$
- Improvements for Hit & Miss method by variable transformation
  - find  $c \cdot g(x) > f(x)$
  - reject if  $f(x) < u_j \cdot c \cdot g(x)$
  - accept if  $f(x) > u_j \cdot c \cdot g(x)$

# Monte Carlo technique: basics

- **Law of large numbers**

chose  $N$  numbers  $u_i$  randomly, with probability density uniform in  $[a,b]$ ,  
evaluate  $f(u_i)$  for each  $u_i$  :

$$\frac{1}{N} \sum_{i=1}^N f(u_i) \rightarrow \frac{1}{b-a} \int_a^b f(u) du$$

for large enough  $N$  Monte Carlo estimate of integral converges to **correct answer**.

- **Convergence**

is given with a certain probability ...

**THIS is a mathematically serious and  
precise statement !!!!**



# Central Limit Theorem

- Central Limit Theorem

for large N the sum of independent random variables is **always** normally (Gaussian) distributed:

$$f(x) = \frac{1}{s\sqrt{2\pi}} \exp \left[ -\frac{(x-a)^2}{2s^2} \right]$$

$$\frac{\sum_i x_i - \sum_i \mu_i}{\sqrt{\sum_i \sigma_i^2}} \rightarrow N(0, 1)$$

→ independent on the original sub-distributions

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$$f(x) = \frac{1}{s\sqrt{2\pi}} \exp \left[ -\frac{(x-a)^2}{2s^2} \right]$$

- example: take sum of uniformly distributed random numbers:

$$R_n = \sum_{i=1}^n R_i$$

$$E[R_1] = \int u du = 1/2,$$

$$V[R_1] = \int (u - 1/2)^2 du = 1/12$$

$$E[R_n] = n/2$$

$$V[R_n] = n/12$$

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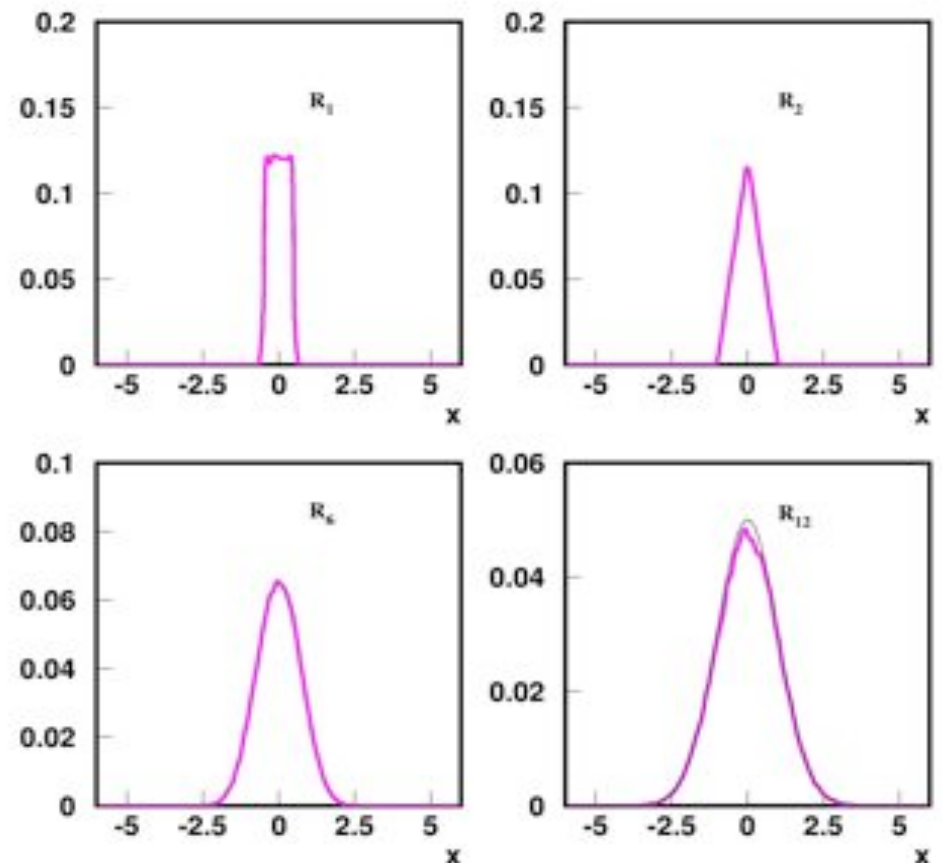
$$V[R_1] = \int (u - 1/2)^2 du = 1/12$$

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$$V[R_n] = n/12$$

- for Gaussian with mean=0 and variance=1, take for n=12:

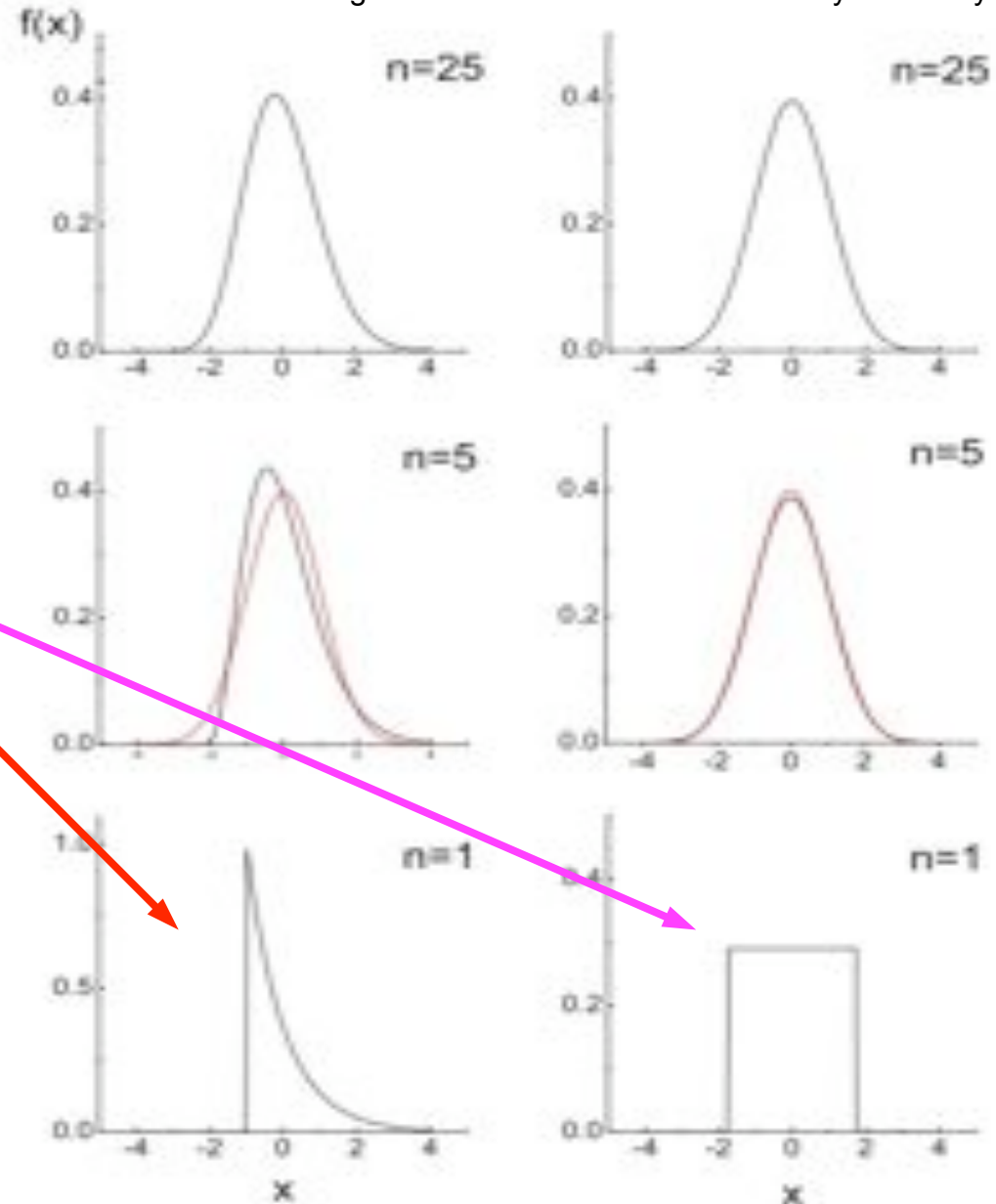
$$N(0, 1) \rightarrow \frac{R_n - n/2}{\sqrt{n/12}}$$



# Central Limit Theorem

- Central Limit Theorem
  - for large  $N$  the sum of independent random variables is **always** normally (Gaussian) distributed
  - for any starting distribution
  - for uniform distribution
  - for exponential distribution

G. Bohm, G. Zech  
Einführung in die Statistik und Messwertanalyse für Physiker



# MC method: advantage of hit & miss

- integration → weighting events
  - large fluctuations from large weights
  - weights also to errors applied
  - difficult to apply further hadronization
- real events all have weight = 1 !!!
- Hit & Miss method:

MC for function  $f(x)$ :  
get random number:  
 $R1$  in  $(0,1)$  and  $R2$  in  $(0,1)$   
calculate  $x = R1$   
reject event if:  $f_x < f_{\max} R2$

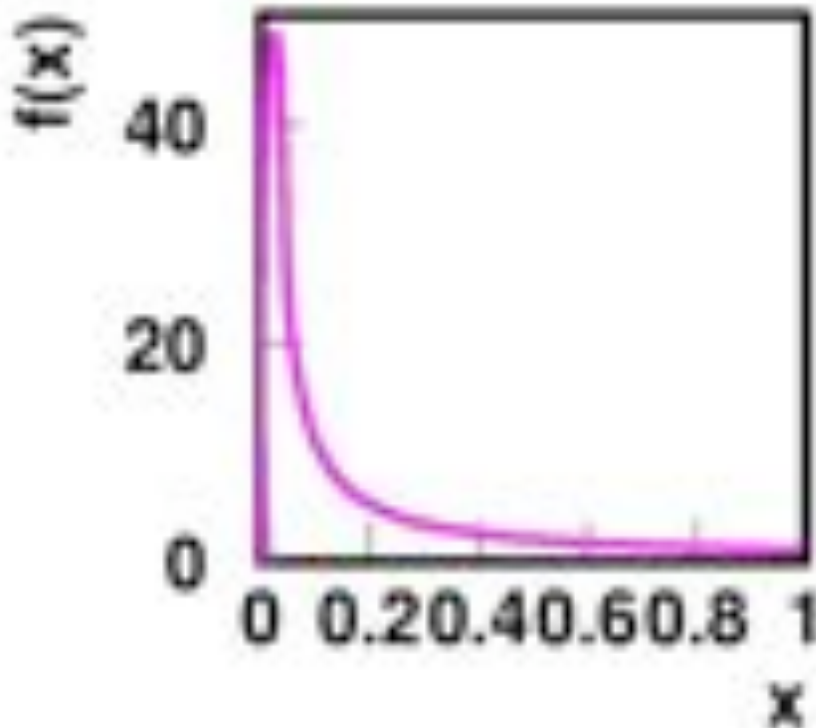
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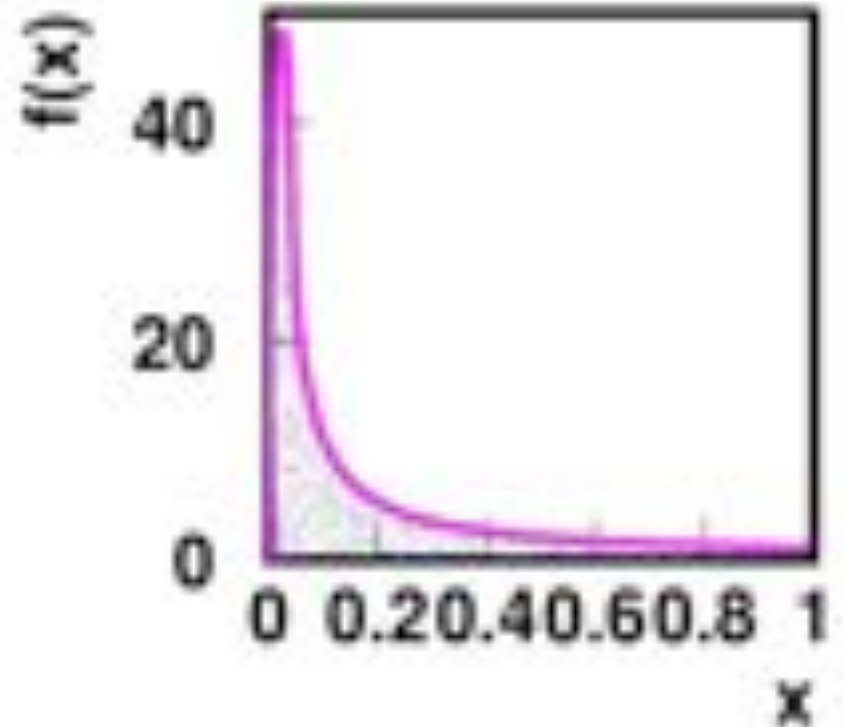
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calculate  $x = R1$   
reject event if:  $f_x < f_{\max} R2$



# MC method: advantage of hit & miss

- integration  $\rightarrow$  weighting events
  - large fluctuations from large weights:
  - weights also to errors applied
  - difficult to apply further hadronization
- real events all have weight = 1 !!!
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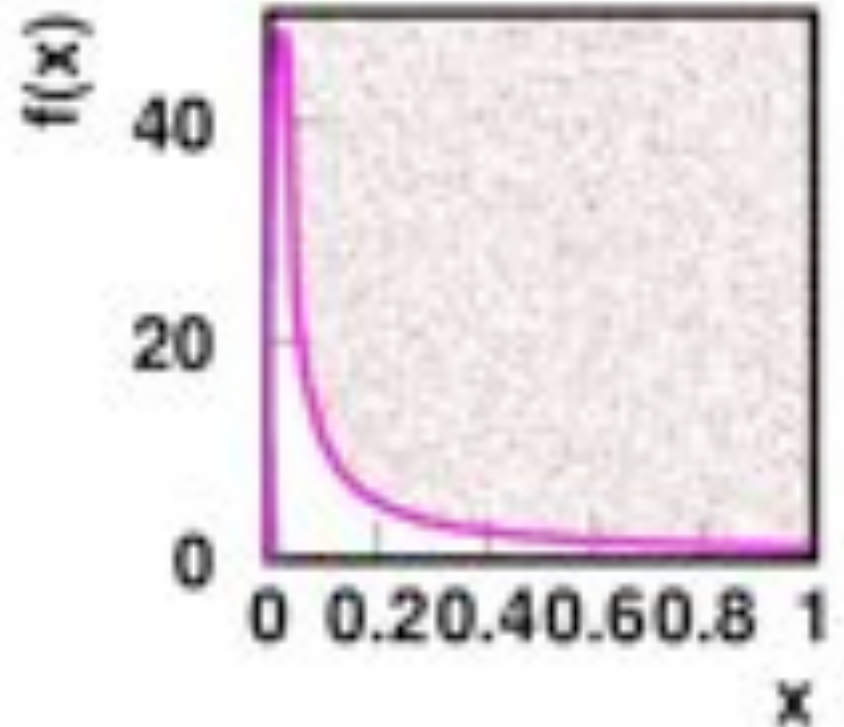
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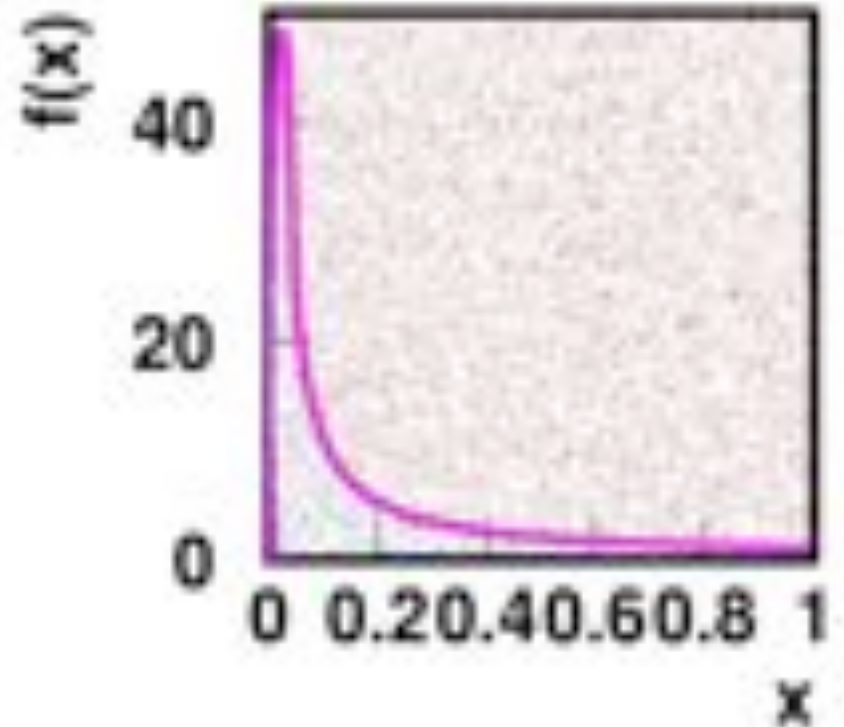




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- BUT: Hit & Miss method inefficient for peaked  $f(x)$

# MC method: do even better ...

- Importance sampling

MC for function  $f(x)$

approximate  $f(x) \sim g(x)$

with  $g(x) > f(x)$  simple and integrable  
generate  $x$  according to  $g(x)$ :

$$\int_{x_{min}}^x g(x') dx' = R1 \int_{x_{min}}^{x_{max}} g(x') dx'$$

example:  $f(x) = 1/x^{0.7}$

$$g(x) = 1/x$$

$$x = x_{min} \cdot \left( \frac{x_{max}}{x_{min}} \right)^{R1}$$

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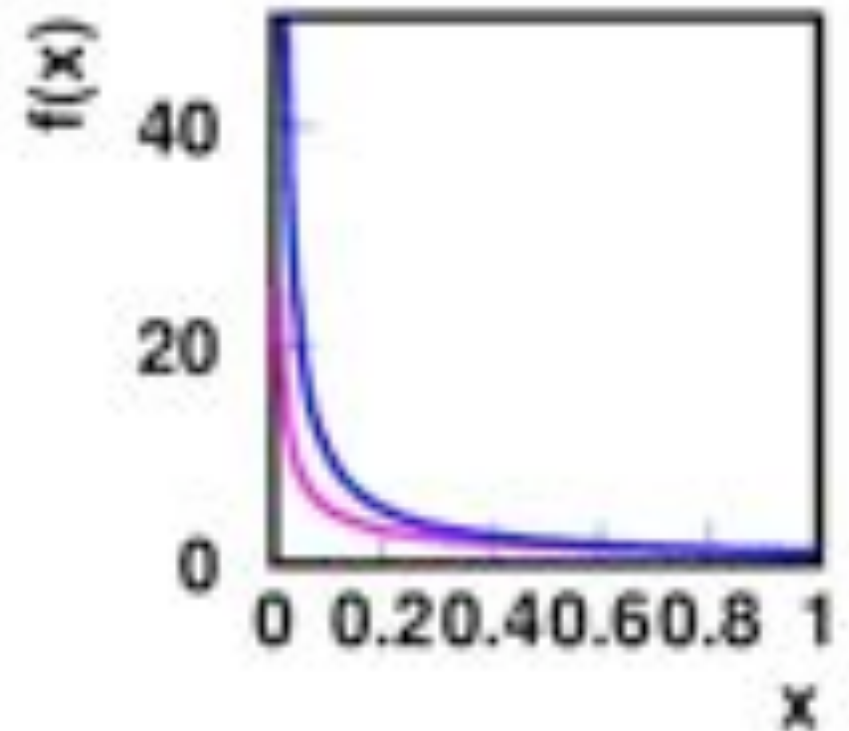
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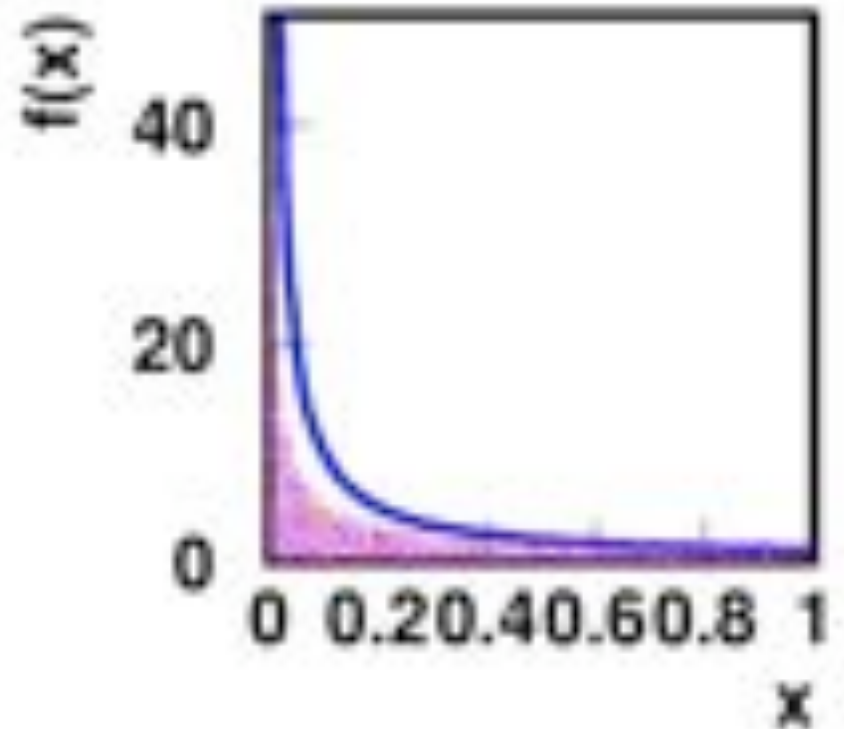
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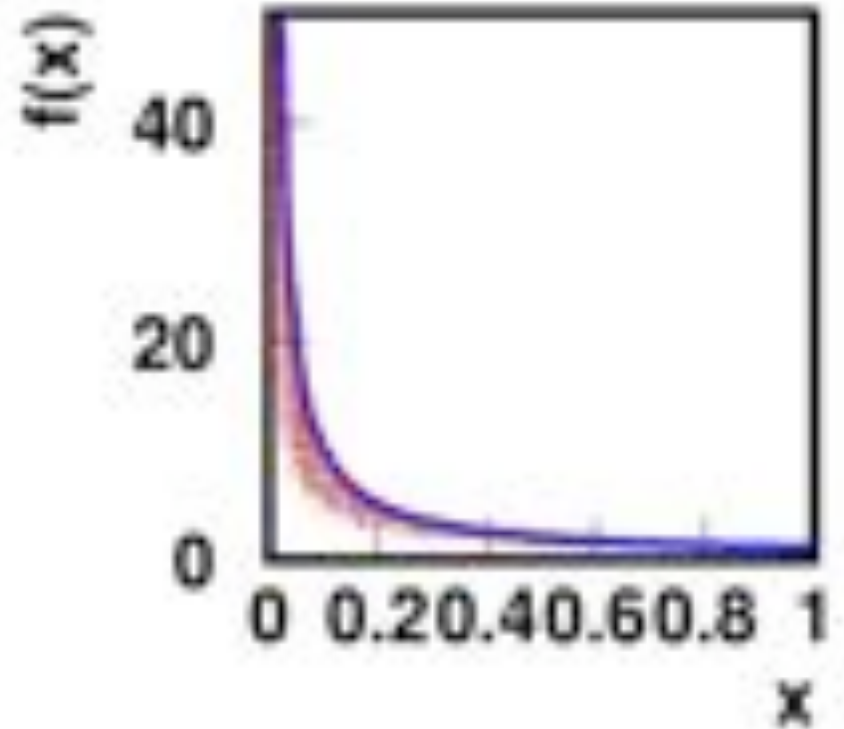
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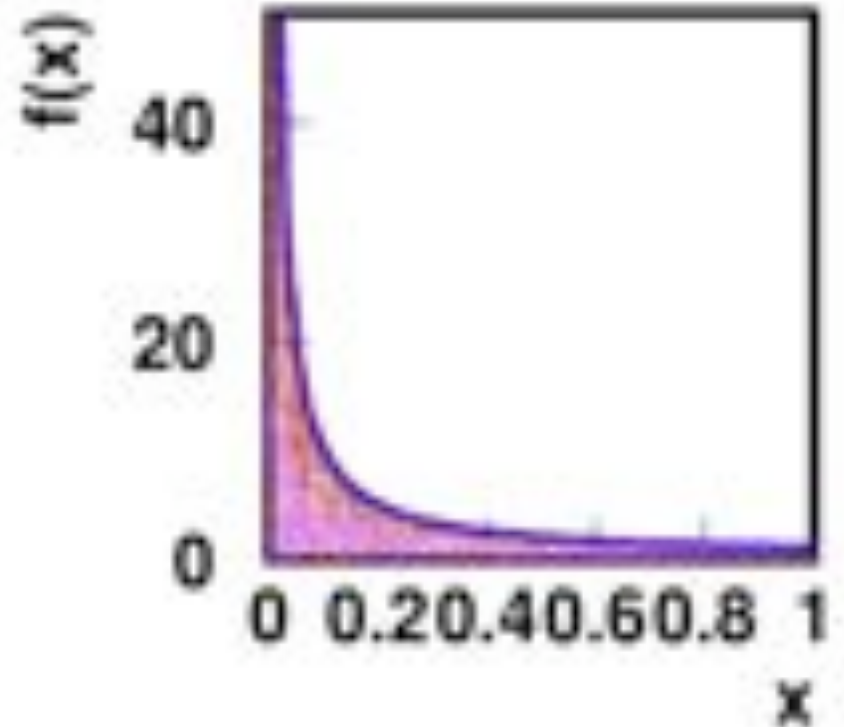
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reject event if:  $f(x) < g(x) R2$



- WOW !!!** very efficient even for peaked  $f(x)$

# Importance Sampling

- MC calculations most efficient for small weight fluctuations:

$$f(x)dx \rightarrow f(x) dG(x)/g(x)$$

- chose point according to  $g(x)$  instead of uniformly
- $f$  is divided by  $g(x) = dG(x)/dx$

- generate  $x$  according to:

$$R \int_a^b g(x') dx' = \int_a^x g(x') dx'$$

- relevant variance is now  $V(f/g)$ :

small if  $g(x) \sim f(x)$

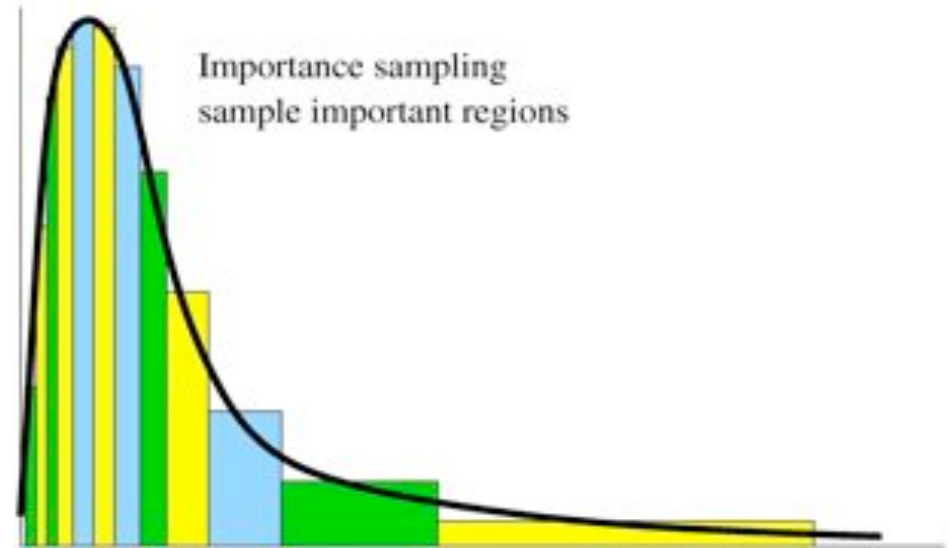
- how-to get  $g(x)$

(1)  $g(x)$  is probability:  $g(x) > 0$  and  $\int dG(x) = 1$

(2) integral  $\int dG(x)$  is known analytically

(3)  $G(x)$  can be inverted (solved for  $x$ )

(4)  $f(x)/g(x)$  is nearly constant, so that  $V(f/g)$  is small compared to  $V(f)$



We have the method,....

BUT

HOWTO

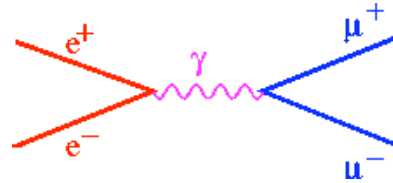
simulate the physics

?????



# Constructing a MC for $e^+e^-$ : the simple case

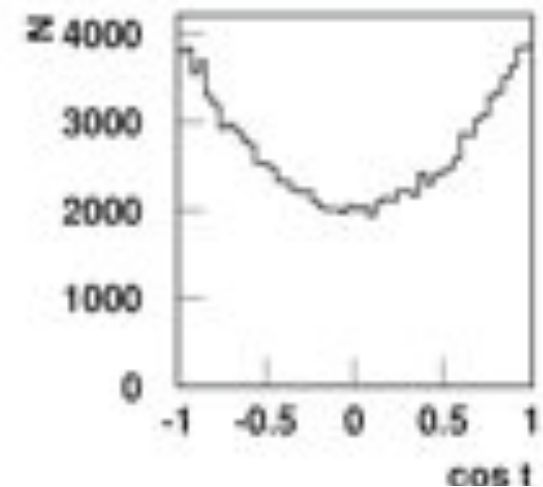
- process:  $e^+e^- \rightarrow \mu^+ \mu^-$



- $$\frac{d\sigma}{d\cos\theta d\phi} = \frac{\alpha_{em}^2}{4s} (1 + \cos^2\theta)$$

- goal: generate 4-momenta of  $\mu$ 's, need  $cm$  energy  $s$ ,  $\cos\theta$ ,  $\phi$

after 100000 events



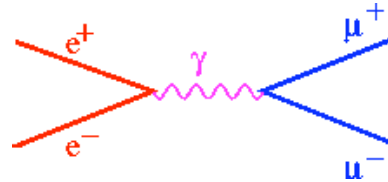
random number  $R1(0,1)$ :  $\phi = 2\pi R1$   
random number  $R2(0,1)$ :  $\cos\theta = -1 + 2R2$

for every  $R1, R2$  use weight with  
repeat many times

$$\frac{d\sigma}{d\cos\theta d\phi}$$

# Constructing a MC for $e^+e^-$ : the simple case

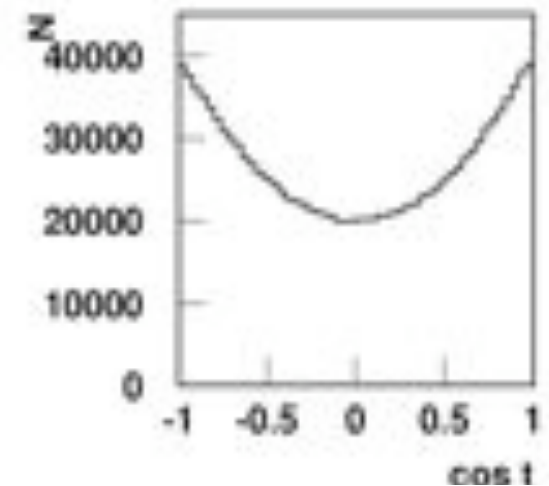
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after  $10^6$  events



random number  $R1(0,1)$ :  $\phi = 2\pi R1$   
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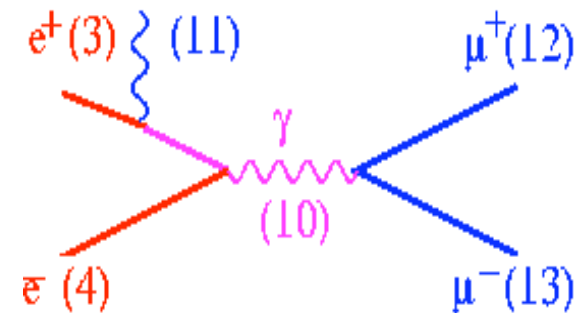
for every  $R1, R2$  use weight with  
repeat many times

$$\frac{d\sigma}{d\cos\theta d\phi}$$

# Example event: $e^+e^- \rightarrow \mu^+ \mu^-$

- example from PYTHIA: Event listing

I	particle/jet	KS	KF	orig	p_x	p_y	p_z	E	m
1	!e+!	21	-11	0	0.000	0.000	30.000	30.000	0.001
2	!e-!	21	11	0	0.000	0.000	-30.000	30.000	0.001
=====									
3	!e+!	21	-11	1	0.000	0.000	30.000	30.000	0.000
4	!e-!	21	11	2	0.000	0.000	-30.000	30.000	0.000
5	!e+!	21	-11	3	0.143	0.040	26.460	26.460	0.000
6	!e-!	21	11	4	0.000	0.000	-29.998	29.998	0.000
7	!Z0!	21	23	0	0.143	0.040	-3.539	56.458	56.347
8	!mu-!	21	13	7	-9.510	1.741	24.722	26.546	0.106
9	!mu+!	21	-13	7	9.653	-1.700	-28.261	29.913	0.106
=====									
10	(Z0)	11	23	7	0.143	0.040	-3.539	56.458	56.347
11	gamma	1	22	3	-0.143	-0.040	3.539	3.542	0.000
12	mu-	1	13	8	-9.510	1.741	24.722	26.546	0.106
13	mu+	1	-13	9	9.653	-1.700	-28.261	29.913	0.106
=====									
	sum:		0.00		0.000	0.000	0.000	60.000	60.000



- technicalities/advantages
  - can work in any frame
  - Lorentz-boost 4-vectors back and forth
  - can calculate any kinematic variable
  - history of event process

# Transition from Quarks to Hadrons

- **Independent Fragmentation** (Feynman & Field: Phys. Rev D15 (1977)2590, NPB 138 (1978) 1)
  - quarks fragment independently

# Transition from Quarks to Hadrons

- Independent Fragmentation
  - quarks fragment independently
  - not Lorentz invariant

PHYSICAL REVIEW D

VOLUME 27, NUMBER 1

1 JANUARY 1983

## Scaling violations in inclusive $e^+e^-$ annihilation spectra

C. Peterson,\* D. Schlatter, I. Schmitt,<sup>†</sup> and P. M. Zerwas<sup>‡</sup>

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

(Received 29 July 1982)

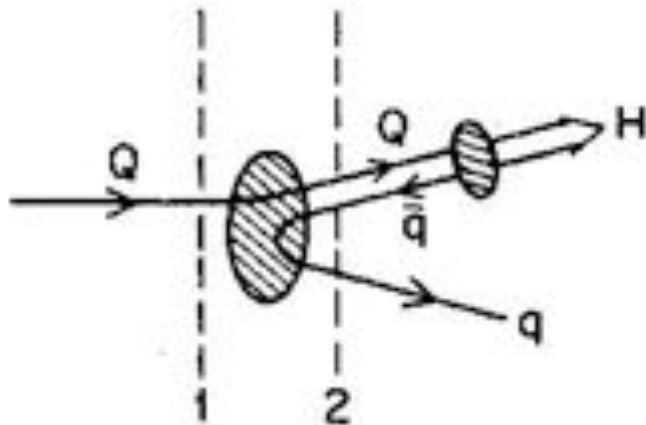


FIG. 3. The fragmentation of a heavy quark  $Q$  into a meson  $H(Q\bar{q})$ . Dashed lines are time slices used in the derivation of Eq. (3).

cussed in Ref. 18. The *gross features* of the amplitude for a fast moving heavy quark  $Q$  fragmentation into a hadron  $H=(Q\bar{q})$  and light quark  $q$  (Fig. 3) are determined by the value of the energy transfer  $\Delta E = E_H + E_q - E_Q$  in the breakup process,

$$\text{amplitude } (Q \rightarrow H + q) \propto \Delta E^{-1}. \quad (2)$$

Expanding the energies about the (transverse) particle masses ( $m_H \simeq m_Q$  for simplicity),

$$\begin{aligned} \Delta E &= (m_Q^2 + z^2 P^2)^{1/2} + (m_q^2 + (1-z)^2 P^2)^{1/2} \\ &\quad - (m_Q^2 + P^2)^{1/2} \\ &\propto 1 - (1/z) - (\epsilon_Q / (1-z)) \end{aligned} \quad (3)$$

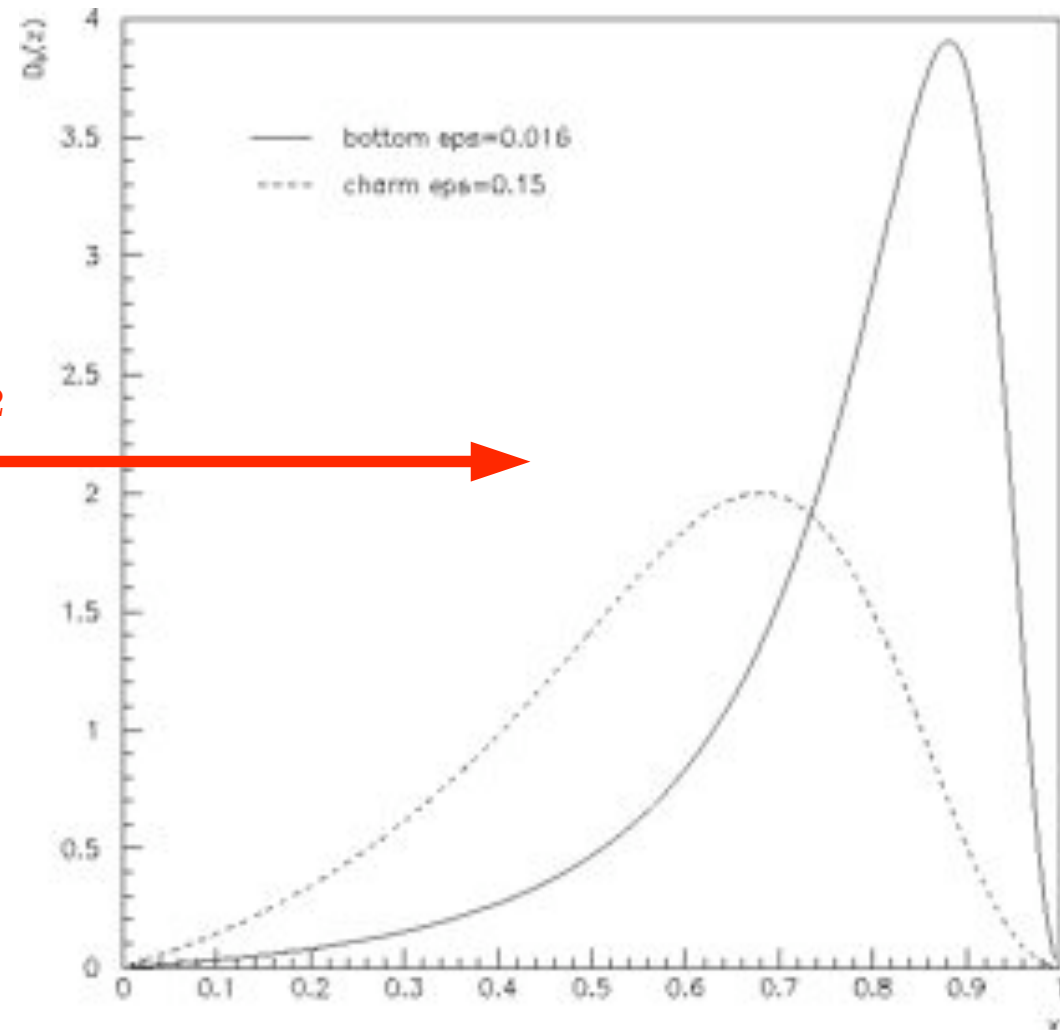
and taking a factor  $z^{-1}$  for longitudinal phase space, we suggest the following ansatz for the fragmentation function of heavy quarks  $Q$

$$D_Q^H(z) = \frac{N}{z[1 - (1/z) - \epsilon_Q / (1-z)]^2}. \quad (4)$$

# Heavy Quark Fragmentation

- transition from heavy quark to observable hadron by fragmentation function FF
- **Peterson FF:** (C. Peterson, D.Schlatter,I.Schmitt,P.Zerwas, PRD27 (1983) 105)

$$D_Q(z) = \frac{N}{z} \left[ 1 - \frac{1}{z} - \frac{\epsilon_Q}{1-z} \right]^{-2}$$



# Light Quark FF

- parametrisations by:

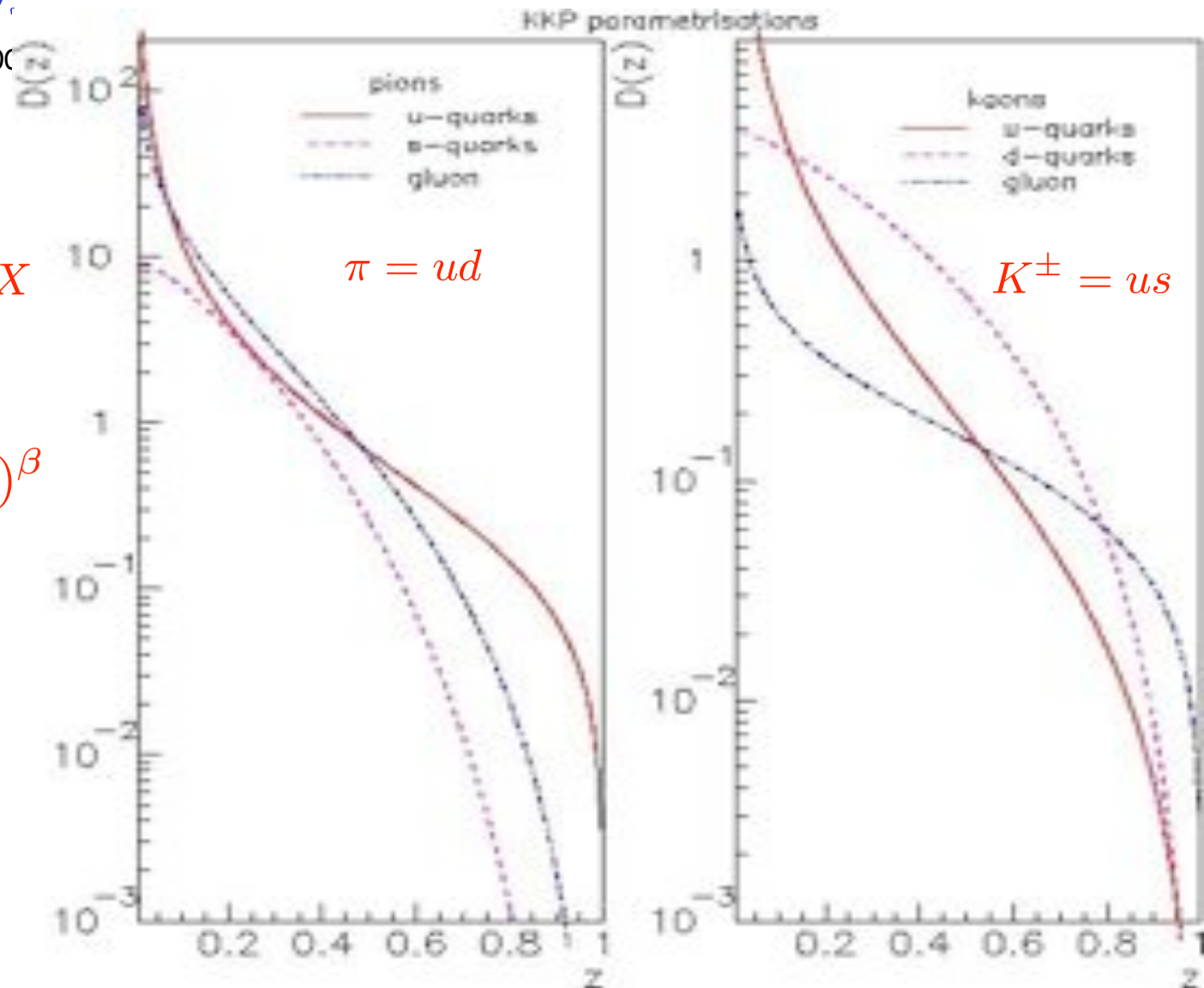
Kniehl, Kramer & Poetter NPB582(2000) 514

- use

$$e^+e^- \rightarrow (\gamma, Z) \rightarrow h + X$$

- starting distribution:

$$D_Q(z) = N z^\alpha (1-z)^\beta$$



# Transition from Quarks to Hadrons

- **Independent Fragmentation**

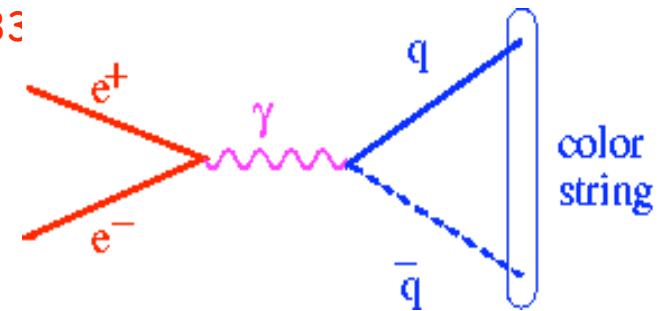
- quarks fragment independently
- gluons are split:  $g \rightarrow q\bar{q}$
- not Lorentz invariant

- **Lund String Fragmentation** (Andersson, Gustafson, Peterson ZPC 1, 105 (1979),

Andersson, Gustafson, Ingelman, Sjostrand Phys. Rep 97 (1983) 33

- use concept of local parton-hadron duality

$$e^+e^- \rightarrow q\bar{q}$$
$$\frac{d\sigma}{d\cos\theta d\phi} = \frac{\alpha_{em}^2}{\xi_S} (1 + \cos^2\theta)$$



linear confinement potential:  $V(r) \sim -1/r + \kappa r$   
with  $\kappa \sim 1 \text{ GeV/fm}$

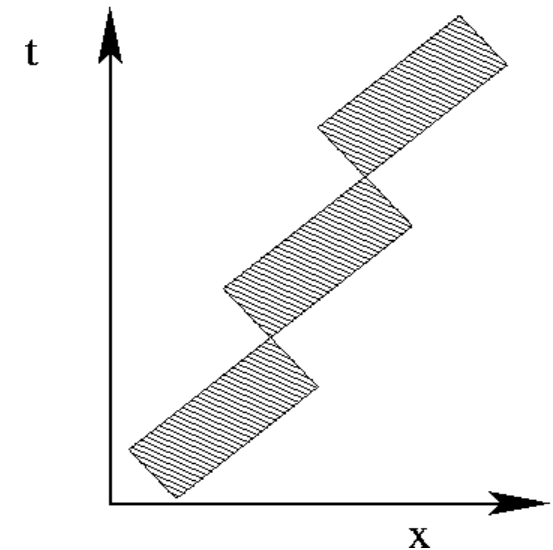
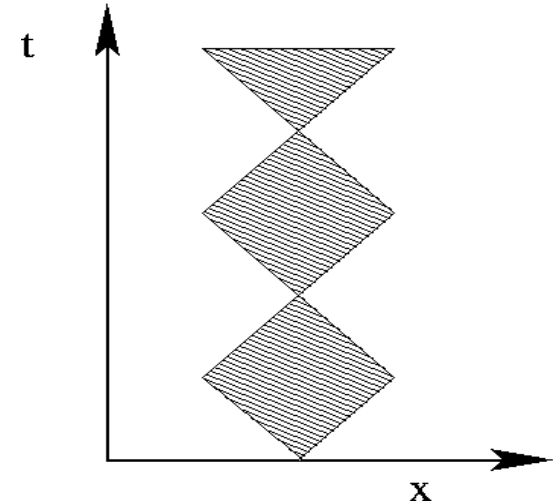
$q\bar{q}$  connected via color flux tube of transverse size of hadrons ( $\sim 1 \text{ fm}$ )  
color tube: uniform along its length  $\rightarrow$  linearly rising potential

**$\rightarrow$  Lund string fragmentation**



# Lund string fragmentation

- in a color neutral  $q\bar{q}$ -pair, a color force is created in between
- color lines of the force are concentrated in a narrow tube connecting  $q$  and  $\bar{q}$ , with a string tension of:  
 $\kappa \sim 1 \text{ GeV/fm} \sim 0.2 \text{ GeV}^2$
- as  $q$  and  $\bar{q}$  are moving apart in  $q\bar{q}$  rest frame, they are de-accelerated by string tension, accelerated back etc ... (periodic oscillation)
- viewed in a moving system, the string is boosted



# Hadronization: particle mass and decays

- **particle masses**

- taken from PDG, where known, otherwise from constituent masses

- **particle widths**

- in hard scattering production process short lived particles ( $\rho, \Delta$ ) have nominal mass, without mass broadening

- in hadronization use Breit-Wigner:

$$\mathcal{P}(m)dm \propto \frac{1}{(m - m_0)^2 + \Gamma^2/4}$$

- **lifetimes**

- related to widths ... but for practical purpose separated

- **decays**

- taken from PDG, where known

- assume momentum distribution given by phase space **only**

- exceptions, like  $\omega, \phi \rightarrow \pi^+ \pi^- \pi^0$ , or  $D \rightarrow K\pi, D^* \rightarrow K\pi\pi$  and some semileptonic decays use matrix elements

# The first MC steps ...

## Monte Carlo source code of JETSET, fits on 1 page

T.Sjostrand, B. Soderberg LU-TP 78-18

```

SUBROUTINE JETSET(N)
COMMON /JET/ K(100:2), P(100:3)
COMMON /DATA/ MESO(9:2), CH1(4:2), PHAS(19)
COMMON /PAR/ P0, P01, SIGMA, C12, EREG, WFIN, IFLBEG
COMMON /DATA/ MESO(9:2), CH1(4:2), PHAS(19)
IFLBN=(10-IFLBE)/5
W=2.*EREG
I=0
IPI=0
C 1 FLAVOUR AND PT FOR FIRST QUARK
IFL=IABS(IFLBE)
P1=SIGMA*GRT(-ALOG(RANF(0)))
PH1=4.2832*W*P1
P1=PT1+COO(PH1)
P1=PT1+SIN(PH1)
DO 100 J=1
C 2 FLAVOUR AND PT FOR NEXT ANTIQUARK
IFL2=IABS(IFLBE)
P2=SIGMA*GRT(-ALOG(RANF(0)))
PH2=4.2832*W*P2
P2=PT2+COO(PH2)
P2=PT2+SIN(PH2)
C 3 MESON FORMED, SPIN ADDED AND FLAVOUR MIXED
K(1:1)=MESO(1)+IFL(1)+IFL2(1)
ISPIN=INT(P1+P2)
K(1:2)=K(1:1)+ISPIN*(1:1)
IF(K(1:1).LE.6) GOTO 110
TRJ=IABS(IFL)
K(1:1)=K(1:1)+2*ISPIN
K(1:2)=K(1:2)+INT(THI+CHI*(K(1:1)+INT(THI+CHI*(K(1:1)+2)))
C 4 MESON MASS FROM TABLE, PT FROM CONSTITUENTS
P(1:1)=P1+P2
P(1:2)=P1+P2
P(1:3)=P1+P2
P(1:4)=P1+P2
P(1:5)=P1+P2
P(1:6)=P1+P2
P(1:7)=P1+P2
P(1:8)=P1+P2
P(1:9)=P1+P2
P(1:10)=P1+P2
P(1:11)=P1+P2
P(1:12)=P1+P2
P(1:13)=P1+P2
P(1:14)=P1+P2
P(1:15)=P1+P2
P(1:16)=P1+P2
P(1:17)=P1+P2
P(1:18)=P1+P2
P(1:19)=P1+P2
P(1:20)=P1+P2
P(1:21)=P1+P2
P(1:22)=P1+P2
P(1:23)=P1+P2
P(1:24)=P1+P2
P(1:25)=P1+P2
P(1:26)=P1+P2
P(1:27)=P1+P2
P(1:28)=P1+P2
P(1:29)=P1+P2
P(1:30)=P1+P2
P(1:31)=P1+P2
P(1:32)=P1+P2
P(1:33)=P1+P2
P(1:34)=P1+P2
P(1:35)=P1+P2
P(1:36)=P1+P2
P(1:37)=P1+P2
P(1:38)=P1+P2
P(1:39)=P1+P2
P(1:40)=P1+P2
P(1:41)=P1+P2
P(1:42)=P1+P2
P(1:43)=P1+P2
P(1:44)=P1+P2
P(1:45)=P1+P2
P(1:46)=P1+P2
P(1:47)=P1+P2
P(1:48)=P1+P2
P(1:49)=P1+P2
P(1:50)=P1+P2
P(1:51)=P1+P2
P(1:52)=P1+P2
P(1:53)=P1+P2
P(1:54)=P1+P2
P(1:55)=P1+P2
P(1:56)=P1+P2
P(1:57)=P1+P2
P(1:58)=P1+P2
P(1:59)=P1+P2
P(1:60)=P1+P2
P(1:61)=P1+P2
P(1:62)=P1+P2
P(1:63)=P1+P2
P(1:64)=P1+P2
P(1:65)=P1+P2
P(1:66)=P1+P2
P(1:67)=P1+P2
P(1:68)=P1+P2
P(1:69)=P1+P2
P(1:70)=P1+P2
P(1:71)=P1+P2
P(1:72)=P1+P2
P(1:73)=P1+P2
P(1:74)=P1+P2
P(1:75)=P1+P2
P(1:76)=P1+P2
P(1:77)=P1+P2
P(1:78)=P1+P2
P(1:79)=P1+P2
P(1:80)=P1+P2
P(1:81)=P1+P2
P(1:82)=P1+P2
P(1:83)=P1+P2
P(1:84)=P1+P2
P(1:85)=P1+P2
P(1:86)=P1+P2
P(1:87)=P1+P2
P(1:88)=P1+P2
P(1:89)=P1+P2
P(1:90)=P1+P2
P(1:91)=P1+P2
P(1:92)=P1+P2
P(1:93)=P1+P2
P(1:94)=P1+P2
P(1:95)=P1+P2
P(1:96)=P1+P2
P(1:97)=P1+P2
P(1:98)=P1+P2
P(1:99)=P1+P2
P(1:100)=P1+P2
C 5 RANDOM CHOICE OF 8+10+21 MESON(8+P) AVAILABLE GIVES E AND P1
I=IABS(RANF(0))
IF(RANF(0).LT.C12) I=1+I*(5.73)
P(1:1)=I*(W+P1)/(1+I)
P(1:2)=I*(W+P2)/(1+I)
P(1:3)=I*(W+P3)/(1+I)
P(1:4)=I*(W+P4)/(1+I)
P(1:5)=I*(W+P5)/(1+I)
P(1:6)=I*(W+P6)/(1+I)
P(1:7)=I*(W+P7)/(1+I)
P(1:8)=I*(W+P8)/(1+I)
P(1:9)=I*(W+P9)/(1+I)
P(1:10)=I*(W+P10)/(1+I)
P(1:11)=I*(W+P11)/(1+I)
P(1:12)=I*(W+P12)/(1+I)
P(1:13)=I*(W+P13)/(1+I)
P(1:14)=I*(W+P14)/(1+I)
P(1:15)=I*(W+P15)/(1+I)
P(1:16)=I*(W+P16)/(1+I)
P(1:17)=I*(W+P17)/(1+I)
P(1:18)=I*(W+P18)/(1+I)
P(1:19)=I*(W+P19)/(1+I)
P(1:20)=I*(W+P20)/(1+I)
P(1:21)=I*(W+P21)/(1+I)
P(1:22)=I*(W+P22)/(1+I)
P(1:23)=I*(W+P23)/(1+I)
P(1:24)=I*(W+P24)/(1+I)
P(1:25)=I*(W+P25)/(1+I)
P(1:26)=I*(W+P26)/(1+I)
P(1:27)=I*(W+P27)/(1+I)
P(1:28)=I*(W+P28)/(1+I)
P(1:29)=I*(W+P29)/(1+I)
P(1:30)=I*(W+P30)/(1+I)
P(1:31)=I*(W+P31)/(1+I)
P(1:32)=I*(W+P32)/(1+I)
P(1:33)=I*(W+P33)/(1+I)
P(1:34)=I*(W+P34)/(1+I)
P(1:35)=I*(W+P35)/(1+I)
P(1:36)=I*(W+P36)/(1+I)
P(1:37)=I*(W+P37)/(1+I)
P(1:38)=I*(W+P38)/(1+I)
P(1:39)=I*(W+P39)/(1+I)
P(1:40)=I*(W+P40)/(1+I)
P(1:41)=I*(W+P41)/(1+I)
P(1:42)=I*(W+P42)/(1+I)
P(1:43)=I*(W+P43)/(1+I)
P(1:44)=I*(W+P44)/(1+I)
P(1:45)=I*(W+P45)/(1+I)
P(1:46)=I*(W+P46)/(1+I)
P(1:47)=I*(W+P47)/(1+I)
P(1:48)=I*(W+P48)/(1+I)
P(1:49)=I*(W+P49)/(1+I)
P(1:50)=I*(W+P50)/(1+I)
P(1:51)=I*(W+P51)/(1+I)
P(1:52)=I*(W+P52)/(1+I)
P(1:53)=I*(W+P53)/(1+I)
P(1:54)=I*(W+P54)/(1+I)
P(1:55)=I*(W+P55)/(1+I)
P(1:56)=I*(W+P56)/(1+I)
P(1:57)=I*(W+P57)/(1+I)
P(1:58)=I*(W+P58)/(1+I)
P(1:59)=I*(W+P59)/(1+I)
P(1:60)=I*(W+P60)/(1+I)
P(1:61)=I*(W+P61)/(1+I)
P(1:62)=I*(W+P62)/(1+I)
P(1:63)=I*(W+P63)/(1+I)
P(1:64)=I*(W+P64)/(1+I)
P(1:65)=I*(W+P65)/(1+I)
P(1:66)=I*(W+P66)/(1+I)
P(1:67)=I*(W+P67)/(1+I)
P(1:68)=I*(W+P68)/(1+I)
P(1:69)=I*(W+P69)/(1+I)
P(1:70)=I*(W+P70)/(1+I)
P(1:71)=I*(W+P71)/(1+I)
P(1:72)=I*(W+P72)/(1+I)
P(1:73)=I*(W+P73)/(1+I)
P(1:74)=I*(W+P74)/(1+I)
P(1:75)=I*(W+P75)/(1+I)
P(1:76)=I*(W+P76)/(1+I)
P(1:77)=I*(W+P77)/(1+I)
P(1:78)=I*(W+P78)/(1+I)
P(1:79)=I*(W+P79)/(1+I)
P(1:80)=I*(W+P80)/(1+I)
P(1:81)=I*(W+P81)/(1+I)
P(1:82)=I*(W+P82)/(1+I)
P(1:83)=I*(W+P83)/(1+I)
P(1:84)=I*(W+P84)/(1+I)
P(1:85)=I*(W+P85)/(1+I)
P(1:86)=I*(W+P86)/(1+I)
P(1:87)=I*(W+P87)/(1+I)
P(1:88)=I*(W+P88)/(1+I)
P(1:89)=I*(W+P89)/(1+I)
P(1:90)=I*(W+P90)/(1+I)
P(1:91)=I*(W+P91)/(1+I)
P(1:92)=I*(W+P92)/(1+I)
P(1:93)=I*(W+P93)/(1+I)
P(1:94)=I*(W+P94)/(1+I)
P(1:95)=I*(W+P95)/(1+I)
P(1:96)=I*(W+P96)/(1+I)
P(1:97)=I*(W+P97)/(1+I)
P(1:98)=I*(W+P98)/(1+I)
P(1:99)=I*(W+P99)/(1+I)
P(1:100)=I*(W+P100)/(1+I)
C 6 IF UNSTABLE, DECAY CHAIN INTO STABLE PARTICLES
DO 100 I=1,N
IF(I.EQ.2) CALL DECAY(IP,2)
IF(I.EQ.3) CALL DECAY(IP,3)
IF(I.EQ.4) CALL DECAY(IP,4)
IF(I.EQ.5) CALL DECAY(IP,5)
IF(I.EQ.6) CALL DECAY(IP,6)
IF(I.EQ.7) CALL DECAY(IP,7)
IF(I.EQ.8) CALL DECAY(IP,8)
IF(I.EQ.9) CALL DECAY(IP,9)
IF(I.EQ.10) CALL DECAY(IP,10)
IF(I.EQ.11) CALL DECAY(IP,11)
IF(I.EQ.12) CALL DECAY(IP,12)
IF(I.EQ.13) CALL DECAY(IP,13)
IF(I.EQ.14) CALL DECAY(IP,14)
IF(I.EQ.15) CALL DECAY(IP,15)
IF(I.EQ.16) CALL DECAY(IP,16)
IF(I.EQ.17) CALL DECAY(IP,17)
IF(I.EQ.18) CALL DECAY(IP,18)
IF(I.EQ.19) CALL DECAY(IP,19)
IF(I.EQ.20) CALL DECAY(IP,20)
IF(I.EQ.21) CALL DECAY(IP,21)
IF(I.EQ.22) CALL DECAY(IP,22)
IF(I.EQ.23) CALL DECAY(IP,23)
IF(I.EQ.24) CALL DECAY(IP,24)
IF(I.EQ.25) CALL DECAY(IP,25)
IF(I.EQ.26) CALL DECAY(IP,26)
IF(I.EQ.27) CALL DECAY(IP,27)
IF(I.EQ.28) CALL DECAY(IP,28)
IF(I.EQ.29) CALL DECAY(IP,29)
IF(I.EQ.30) CALL DECAY(IP,30)
IF(I.EQ.31) CALL DECAY(IP,31)
IF(I.EQ.32) CALL DECAY(IP,32)
IF(I.EQ.33) CALL DECAY(IP,33)
IF(I.EQ.34) CALL DECAY(IP,34)
IF(I.EQ.35) CALL DECAY(IP,35)
IF(I.EQ.36) CALL DECAY(IP,36)
IF(I.EQ.37) CALL DECAY(IP,37)
IF(I.EQ.38) CALL DECAY(IP,38)
IF(I.EQ.39) CALL DECAY(IP,39)
IF(I.EQ.40) CALL DECAY(IP,40)
IF(I.EQ.41) CALL DECAY(IP,41)
IF(I.EQ.42) CALL DECAY(IP,42)
IF(I.EQ.43) CALL DECAY(IP,43)
IF(I.EQ.44) CALL DECAY(IP,44)
IF(I.EQ.45) CALL DECAY(IP,45)
IF(I.EQ.46) CALL DECAY(IP,46)
IF(I.EQ.47) CALL DECAY(IP,47)
IF(I.EQ.48) CALL DECAY(IP,48)
IF(I.EQ.49) CALL DECAY(IP,49)
IF(I.EQ.50) CALL DECAY(IP,50)
IF(I.EQ.51) CALL DECAY(IP,51)
IF(I.EQ.52) CALL DECAY(IP,52)
IF(I.EQ.53) CALL DECAY(IP,53)
IF(I.EQ.54) CALL DECAY(IP,54)
IF(I.EQ.55) CALL DECAY(IP,55)
IF(I.EQ.56) CALL DECAY(IP,56)
IF(I.EQ.57) CALL DECAY(IP,57)
IF(I.EQ.58) CALL DECAY(IP,58)
IF(I.EQ.59) CALL DECAY(IP,59)
IF(I.EQ.60) CALL DECAY(IP,60)
IF(I.EQ.61) CALL DECAY(IP,61)
IF(I.EQ.62) CALL DECAY(IP,62)
IF(I.EQ.63) CALL DECAY(IP,63)
IF(I.EQ.64) CALL DECAY(IP,64)
IF(I.EQ.65) CALL DECAY(IP,65)
IF(I.EQ.66) CALL DECAY(IP,66)
IF(I.EQ.67) CALL DECAY(IP,67)
IF(I.EQ.68) CALL DECAY(IP,68)
IF(I.EQ.69) CALL DECAY(IP,69)
IF(I.EQ.70) CALL DECAY(IP,70)
IF(I.EQ.71) CALL DECAY(IP,71)
IF(I.EQ.72) CALL DECAY(IP,72)
IF(I.EQ.73) CALL DECAY(IP,73)
IF(I.EQ.74) CALL DECAY(IP,74)
IF(I.EQ.75) CALL DECAY(IP,75)
IF(I.EQ.76) CALL DECAY(IP,76)
IF(I.EQ.77) CALL DECAY(IP,77)
IF(I.EQ.78) CALL DECAY(IP,78)
IF(I.EQ.79) CALL DECAY(IP,79)
IF(I.EQ.80) CALL DECAY(IP,80)
IF(I.EQ.81) CALL DECAY(IP,81)
IF(I.EQ.82) CALL DECAY(IP,82)
IF(I.EQ.83) CALL DECAY(IP,83)
IF(I.EQ.84) CALL DECAY(IP,84)
IF(I.EQ.85) CALL DECAY(IP,85)
IF(I.EQ.86) CALL DECAY(IP,86)
IF(I.EQ.87) CALL DECAY(IP,87)
IF(I.EQ.88) CALL DECAY(IP,88)
IF(I.EQ.89) CALL DECAY(IP,89)
IF(I.EQ.90) CALL DECAY(IP,90)
IF(I.EQ.91) CALL DECAY(IP,91)
IF(I.EQ.92) CALL DECAY(IP,92)
IF(I.EQ.93) CALL DECAY(IP,93)
IF(I.EQ.94) CALL DECAY(IP,94)
IF(I.EQ.95) CALL DECAY(IP,95)
IF(I.EQ.96) CALL DECAY(IP,96)
IF(I.EQ.97) CALL DECAY(IP,97)
IF(I.EQ.98) CALL DECAY(IP,98)
IF(I.EQ.99) CALL DECAY(IP,99)
IF(I.EQ.100) CALL DECAY(IP,100)
C 7 FLAVOUR AND PT OF QUARK FORMED IN PAIR WITH ANTIQUARK ABOVE
IFL=IFL2
P1=P2
PH1=PH2
P1=PT1+COO(PH1)
P1=PT1+SIN(PH1)
C 8 IF ENOUGH E+P1 LEFT, GO TO 2
W=(1-I)*W
IF(W.GT.WFIN.AND.I.LE.95) GOTO 100
N=N+1
RETURN
END

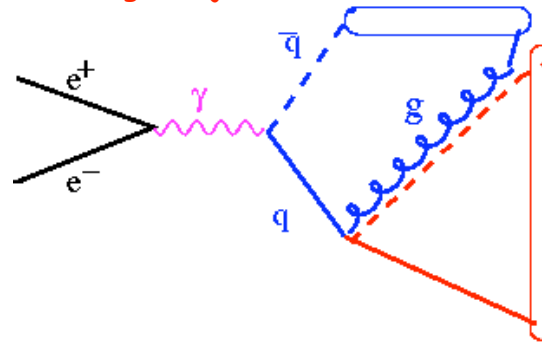
SUBROUTINE DECAY(IP,2)
COMMON /JET/ K(100:2), P(100:3)
COMMON /DATA/ MESO(9:2), CH1(4:2), PHAS(19)
COMMON /PAR/ P0, P01, SIGMA, C12, EREG, WFIN, IFLBEG
COMMON /DATA/ MESO(9:2), CH1(4:2), PHAS(19)
DIMENSION U(1: 8E12)
I=IABS(RANF(0))
I2=I*200*(100-2)-7)
DO 100 J=1,8E12
IF(I2.EQ.0) GOTO 100
N=I2*P0/P01
DO 110 I=1,N
K(1:1)=I
K(1:2)=I*P0/P01
K(1:3)=I*P0/P01
K(1:4)=I*P0/P01
K(1:5)=I*P0/P01
K(1:6)=I*P0/P01
K(1:7)=I*P0/P01
K(1:8)=I*P0/P01
K(1:9)=I*P0/P01
K(1:10)=I*P0/P01
K(1:11)=I*P0/P01
K(1:12)=I*P0/P01
K(1:13)=I*P0/P01
K(1:14)=I*P0/P01
K(1:15)=I*P0/P01
K(1:16)=I*P0/P01
K(1:17)=I*P0/P01
K(1:18)=I*P0/P01
K(1:19)=I*P0/P01
K(1:20)=I*P0/P01
K(1:21)=I*P0/P01
K(1:22)=I*P0/P01
K(1:23)=I*P0/P01
K(1:24)=I*P0/P01
K(1:25)=I*P0/P01
K(1:26)=I*P0/P01
K(1:27)=I*P0/P01
K(1:28)=I*P0/P01
K(1:29)=I*P0/P01
K(1:30)=I*P0/P01
K(1:31)=I*P0/P01
K(1:32)=I*P0/P01
K(1:33)=I*P0/P01
K(1:34)=I*P0/P01
K(1:35)=I*P0/P01
K(1:36)=I*P0/P01
K(1:37)=I*P0/P01
K(1:38)=I*P0/P01
K(1:39)=I*P0/P01
K(1:40)=I*P0/P01
K(1:41)=I*P0/P01
K(1:42)=I*P0/P01
K(1:43)=I*P0/P01
K(1:44)=I*P0/P01
K(1:45)=I*P0/P01
K(1:46)=I*P0/P01
K(1:47)=I*P0/P01
K(1:48)=I*P0/P01
K(1:49)=I*P0/P01
K(1:50)=I*P0/P01
K(1:51)=I*P0/P01
K(1:52)=I*P0/P01
K(1:53)=I*P0/P01
K(1:54)=I*P0/P01
K(1:55)=I*P0/P01
K(1:56)=I*P0/P01
K(1:57)=I*P0/P01
K(1:58)=I*P0/P01
K(1:59)=I*P0/P01
K(1:60)=I*P0/P01
K(1:61)=I*P0/P01
K(1:62)=I*P0/P01
K(1:63)=I*P0/P01
K(1:64)=I*P0/P01
K(1:65)=I*P0/P01
K(1:66)=I*P0/P01
K(1:67)=I*P0/P01
K(1:68)=I*P0/P01
K(1:69)=I*P0/P01
K(1:70)=I*P0/P01
K(1:71)=I*P0/P01
K(1:72)=I*P0/P01
K(1:73)=I*P0/P01
K(1:74)=I*P0/P01
K(1:75)=I*P0/P01
K(1:76)=I*P0/P01
K(1:77)=I*P0/P01
K(1:78)=I*P0/P01
K(1:79)=I*P0/P01
K(1:80)=I*P0/P01
K(1:81)=I*P0/P01
K(1:82)=I*P0/P01
K(1:83)=I*P0/P01
K(1:84)=I*P0/P01
K(1:85)=I*P0/P01
K(1:86)=I*P0/P01
K(1:87)=I*P0/P01
K(1:88)=I*P0/P01
K(1:89)=I*P0/P01
K(1:90)=I*P0/P01
K(1:91)=I*P0/P01
K(1:92)=I*P0/P01
K(1:93)=I*P0/P01
K(1:94)=I*P0/P01
K(1:95)=I*P0/P01
K(1:96)=I*P0/P01
K(1:97)=I*P0/P01
K(1:98)=I*P0/P01
K(1:99)=I*P0/P01
K(1:100)=I*P0/P01
C 2 IN THREE-PARTICLE DECAY CHOICE OF INVARIANT MASS OF PRODUCTS 2+3
IF(I2.EQ.2) GOTO 100
I2=I2*200*(100-2)-7)
DO 110 I=1,N
K(1:1)=I
K(1:2)=I*P0/P01
K(1:3)=I*P0/P01
K(1:4)=I*P0/P01
K(1:5)=I*P0/P01
K(1:6)=I*P0/P01
K(1:7)=I*P0/P01
K(1:8)=I*P0/P01
K(1:9)=I*P0/P01
K(1:10)=I*P0/P01
K(1:11)=I*P0/P01
K(1:12)=I*P0/P01
K(1:13)=I*P0/P01
K(1:14)=I*P0/P01
K(1:15)=I*P0/P01
K(1:16)=I*P0/P01
K(1:17)=I*P0/P01
K(1:18)=I*P0/P01
K(1:19)=I*P0/P01
K(1:20)=I*P0/P01
K(1:21)=I*P0/P01
K(1:22)=I*P0/P01
K(1:23)=I*P0/P01
K(1:24)=I*P0/P01
K(1:25)=I*P0/P01
K(1:26)=I*P0/P01
K(1:27)=I*P0/P01
K(1:28)=I*P0/P01
K(1:29)=I*P0/P01
K(1:30)=I*P0/P01
K(1:31)=I*P0/P01
K(1:32)=I*P0/P01
K(1:33)=I*P0/P01
K(1:34)=I*P0/P01
K(1:35)=I*P0/P01
K(1:36)=I*P0/P01
K(1:37)=I*P0/P01
K(1:38)=I*P0/P01
K(1:39)=I*P0/P01
K(1:40)=I*P0/P01
K(1:41)=I*P0/P01
K(1:42)=I*P0/P01
K(1:43)=I*P0/P01
K(1:44)=I*P0/P01
K(1:45)=I*P0/P01
K(1:46)=I*P0/P01
K(1:47)=I*P0/P01
K(1:48)=I*P0/P01
K(1:49)=I*P0/P01
K(1:50)=I*P0/P01
K(1:51)=I*P0/P01
K(1:52)=I*P0/P01
K(1:53)=I*P0/P01
K(1:54)=I*P0/P01
K(1:55)=I*P0/P01
K(1:56)=I*P0/P01
K(1:57)=I*P0/P01
K(1:58)=I*P0/P01
K(1:59)=I*P0/P01
K(1:60)=I*P0/P01
K(1:61)=I*P0/P01
K(1:62)=I*P0/P01
K(1:63)=I*P0/P01
K(1:64)=I*P0/P01
K(1:65)=I*P0/P01
K(1:66)=I*P0/P01
K(1:67)=I*P0/P01
K(1:68)=I*P0/P01
K(1:69)=I*P0/P01
K(1:70)=I*P0/P01
K(1:71)=I*P0/P01
K(1:72)=I*P0/P01
K(1:73)=I*P0/P01
K(1:74)=I*P0/P01
K(1:75)=I*P0/P01
K(1:76)=I*P0/P01
K(1:77)=I*P0/P01
K(1:78)=I*P0/P01
K(1:79)=I*P0/P01
K(1:80)=I*P0/P01
K(1:81)=I*P0/P01
K(1:82)=I*P0/P01
K(1:83)=I*P0/P01
K(1:84)=I*P0/P01
K(1:85)=I*P0/P01
K(1:86)=I*P0/P01
K(1:87)=I*P0/P01
K(1:88)=I*P0/P01
K(1:89)=I*P0/P01
K(1:90)=I*P0/P01
K(1:91)=I*P0/P01
K(1:92)=I*P0/P01
K(1:93)=I*P0/P01
K(1:94)=I*P0/P01
K(1:95)=I*P0/P01
K(1:96)=I*P0/P01
K(1:97)=I*P0/P01
K(1:98)=I*P0/P01
K(1:99)=I*P0/P01
K(1:100)=I*P0/P01
C 3 TWO-PARTICLE DECAY IN CH, TWICE TO SIMULATE THREE-PARTICLE DECAY
DO 100 J=1,8E12
I2=I2*200*(100-2)-7)
DO 110 I=1,N
K(1:1)=I
K(1:2)=I*P0/P01
K(1:3)=I*P0/P01
K(1:4)=I*P0/P01
K(1:5)=I*P0/P01
K(1:6)=I*P0/P01
K(1:7)=I*P0/P01
K(1:8)=I*P0/P01
K(1:9)=I*P0/P01
K(1:10)=I*P0/P01
K(1:11)=I*P0/P01
K(1:12)=I*P0/P01
K(1:13)=I*P0/P01
K(1:14)=I*P0/P01
K(1:15)=I*P0/P01
K(1:16)=I*P0/P01
K(1:17)=I*P0/P01
K(1:18)=I*P0/P01
K(1:19)=I*P0/P01
K(1:20)=I*P0/P01
K(1:21)=I*P0/P01
K(1:22)=I*P0/P01
K(1:23)=I*P0/P01
K(1:24)=I*P0/P01
K(1:25)=I*P0/P01
K(1:26)=I*P0/P01
K(1:27)=I*P0/P01
K(1:28)=I*P0/P01
K(1:29)=I*P0/P01
K(1:30)=I*P0/P01
K(1:31)=I*P0/P01
K(1:32)=I*P0/P01
K(1:33)=I*P0/P01
K(1:34)=I*P0/P01
K(1:35)=I*P0/P01
K(1:36)=I*P0/P01
K(1:37)=I*P0/P01
K(1:38)=I*P0/P01
K(1:39)=I*P0/P01
K(1:40)=I*P0/P01
K(1:41)=I*P0/P01
K(1:42)=I*P0/P01
K(1:43)=I*P0/P01
K(1:44)=I*P0/P01
K(1:45)=I*P0/P01
K(1:46)=I*P0/P01
K(1:47)=I*P0/P01
K(1:48)=I*P0/P01
K(1:49)=I*P0/P01
K(1:50)=I*P0/P01
K(1:51)=I*P0/P01
K(1:52)=I*P0/P01
K(1:53)=I*P0/P01
K(1:54)=I*P0/P01
K(1:55)=I*P0/P01
K(1:56)=I*P0/P01
K(1:57)=I*P0/P01
K(1:58)=I*P0/P01
K(1:59)=I*P0/P01
K(1:60)=I*P0/P01
K(1:61)=I*P0/P01
K(1:62)=I*P0/P01
K(1:63)=I*P0/P01
K(1:64)=I*P0/P01
K(1:65)=I*P0/P01
K(1:66)=I*P0/P01
K(1:67)=I*P0/P01
K(1:68)=I*P0/P01
K(1:69)=I*P0/P01
K(1:70)=I*P0/P01
K(1:71)=I*P0/P01
K(1:72)=I*P0/P01
K(1:73)=I*P0/P01
K(1:74)=I*P0/P01
K(1:75)=I*P0/P01
K(1:76)=I*P0/P01
K(1:77)=I*P0/P01
K(1:78)=I*P0/P01
K(1:79)=I*P0/P01
K(1:80)=I*P0/P01
K(1:81)=I*P0/P01
K(1:82)=I*P0/P01
K(1:83)=I*P0/P01
K(1:84)=I*P0/P01
K(1:85)=I*P0/P01
K(1:86)=I*P0/P01
K(1:87)=I*P0/P01
K(1:88)=I*P0/P01
K(1:89)=I*P0/P01
K(1:90)=I*P0/P01
K(1:91)=I*P0/P01
K(1:92)=I*P0/P01
K(1:93)=I*P0/P01
K(1:94)=I*P0/P01
K(1:95)=I*P0/P01
K(1:96)=I*P0/P01
K(1:97)=I*P0/P01
K(1:98)=I*P0/P01
K(1:99)=I*P0/P01
K(1:100)=I*P0/P01
C 4 DECAY PRODUCTS LORENTZ TRANSFORMED TO LAB SYSTEM
DO 100 J=1,8E12
I2=I2*200*(100-2)-7)
DO 110 I=1,N
K(1:1)=I
K(1:2)=I*P0/P01
K(1:3)=I*P0/P01
K(1:4)=I*P0/P01
K(1:5)=I*P0/P01
K(1:6)=I*P0/P01
K(1:7)=I*P0/P01
K(1:8)=I*P0/P01
K(1:9)=I*P0/P01
K(1:10)=I*P0/P01
K(1:11)=I*P0/P01
K(1:12)=I*P0/P01
K(1:13)=I*P0/P01
K(1:14)=I*P0/P01
K(1:15)=I*P0/P01
K(1:16)=I*P0/P01
K(1:17)=I*P0/P01
K(1:18)=I*P0/P01
K(1:19)=I*P0/P01
K(1:20)=I*P0/P01
K(1:21)=I*P0/P01
K(1:22)=I*P0/P01
K(1:23)=I*P0/P01
K(1:24)=I*P0/P01
K(1:25)=I*P0/P01
K(1:26)=I*P0/P01
K(1:27)=I*P0/P01
K(1:28)=I*P0/P01
K(1:29)=I*P0/P01
K(1:30)=I*P0/P01
K(1:31)=I*P0/P01
K(1:32)=I*P0/P01
K(1:33)=I*P0/P01
K(1:34)=I*P0/P01
K(1:35)=I*P0/P01
K(1:36)=I*P0/P01
K(1:37)=I*P0/P01
K(1:38)=I*P0/P01
K(1:39)=I*P0/P01
K(1:40)=I*P0/P01
K(1:41)=I*P0/P01
K(1:42)=I*P0/P01
K(1:43)=I*P0/P01
K(1:44)=I*P0/P01
K(1:45)=I*P0/P01
K(1:46)=I*P0/P01
K(1:47)=I*P0/P01
K(1:48)=I*P0/P01
K(1:49)=I*P0/P01
K(1:50)=I*P0/P01
K(1:51)=I*P0/P01
K(1:52)=I*P0/P01
K(1:53)=I*P0/P01
K(1:54)=I*P0/P01
K(1:55)=I*P0/P01
K(1:56)=I*P0/P01
K(1:57)=I*P0/P01
K(1:58)=I*P0/P01
K(1:59)=I*P0/P01
K(1:60)=I*P0/P01
K(1:61)=I*P0/P01
K(1:62)=I*P0/P01
K(1:63)=I*P0/P01
K(1:64)=I*P0/P01
K(1:65)=I*P0/P01
K(1:66)=I*P0/P01
K(1:67)=I*P0/P01
K(1:68)=I*P0/P01
K(1:69)=I*P0/P01
K(1:70)=I*P0/P01
K(1:71)=I*P0/P01
K(1:72)=I*P0/P01
K(1:73)=I*P0/P01
K(1:74)=I*P0/P01
K(1:75)=I*P0/P01
K(1:76)=I*P0/P01
K(1:77)=I*P0/P01
K(1:78)=I*P0/P01
K(1:79)=I*P0/P01
K(1:80)=I*P0/P01
K(1:81)=I*P0/P01
K(1:82)=I*P0/P01
K(1:83)=I*P0/P01
K(1:84)=I*P0/P01
K(1:85)=I*P0/P01
K(1:86)=I*P0/P01
K(1:87)=I*P0/P01
K(1:88)=I*P0/P01
K(1:89)=I*P0/P01
K(1:90)=I*P0/P01
K(1:91)=I*P0/P01
K(1:92)=I*P0/P01
K(1:93)=I*P0/P01
K(1:94)=I*P0/P01
K(1:95)=I*P0/P01
K(1:96)=I*P0/P01
K(1:97)=I*P0/P01
K(1:98)=I*P0/P01
K(1:99)=I*P0/P01
K(1:100)=I*P0/P01
SUBROUTINE LISTINI
COMMON /JET/ K(100:2), P(100:3)
COMMON /DATA/ CHA1(19), CHA2(19), CHA3(12)
WRITE(4,110)
DO 100 I=1,N
IF(K(I:1).GT.0) C1=CHA1(K(I:1))
IF(K(I:1).LE.0) C1=K(I:1)
C2=CHA2(K(I:1))
C3=CHA3(K(I:1))
C4=CHA4(K(I:1))
IF(K(I:1).GT.0) WRITE(4,120) I, C1, C2, C3, (P(I:1), J=1:5)
100 IF(K(I:1).LE.0) WRITE(4,130) I, C1, C2, C3, (P(I:1
```



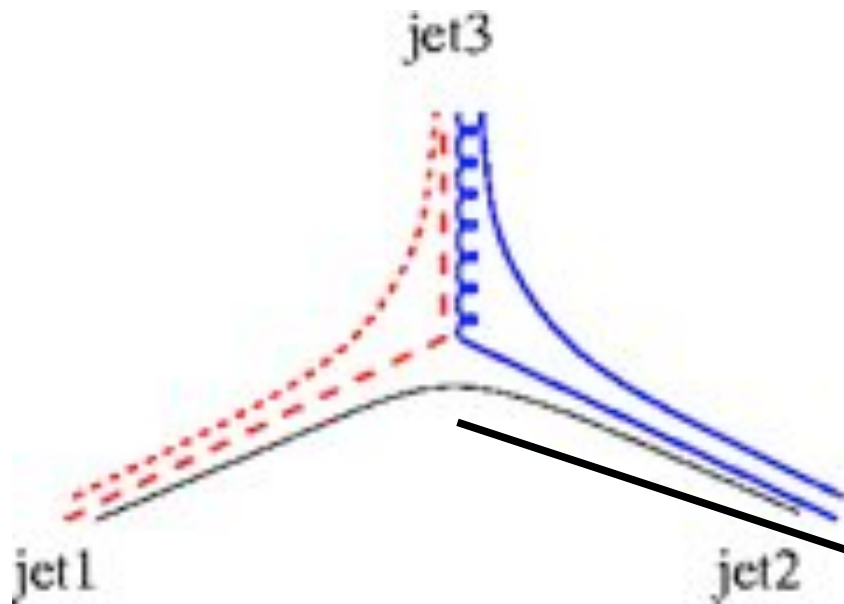
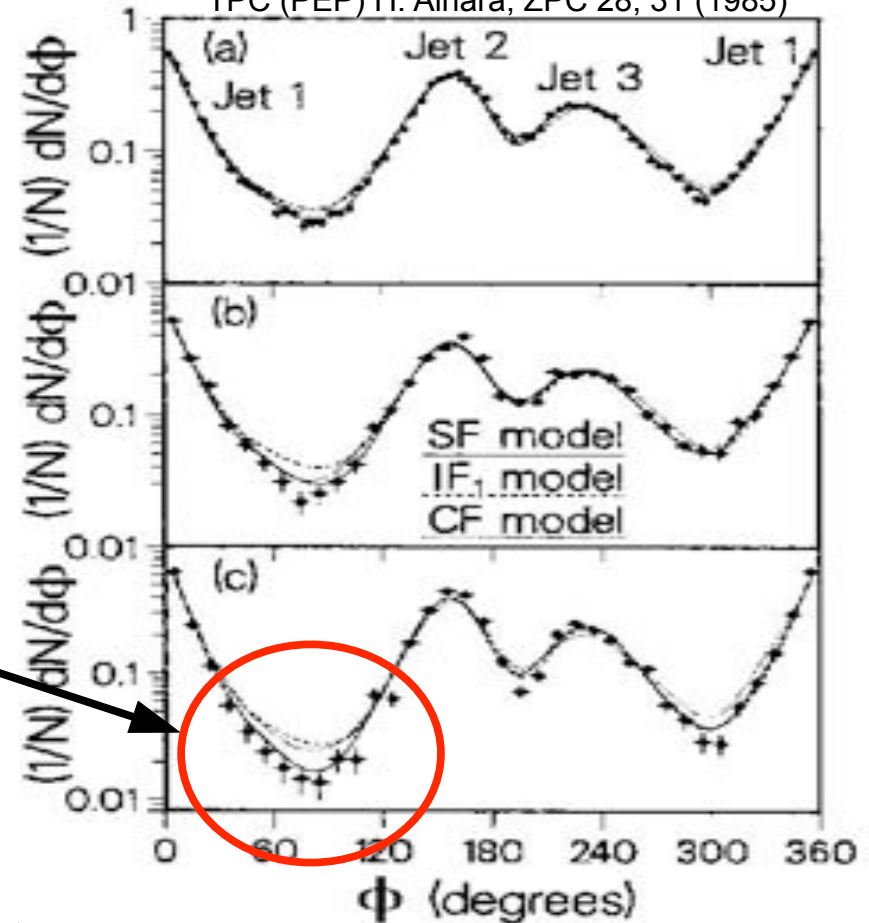
# Gluons in Lund String Fragmentation

How to find the gluon jets, Andersson, Gustafson, Sjostrand, PLB 94,211 (1980)

- process  $e^+e^- \rightarrow q\bar{q}g$
- watch out color flow !!!
- gluons act as kinks on strings
- string effect seen in experiment



TPC (PEP) H. Aihara, ZPC 28, 31 (1985)

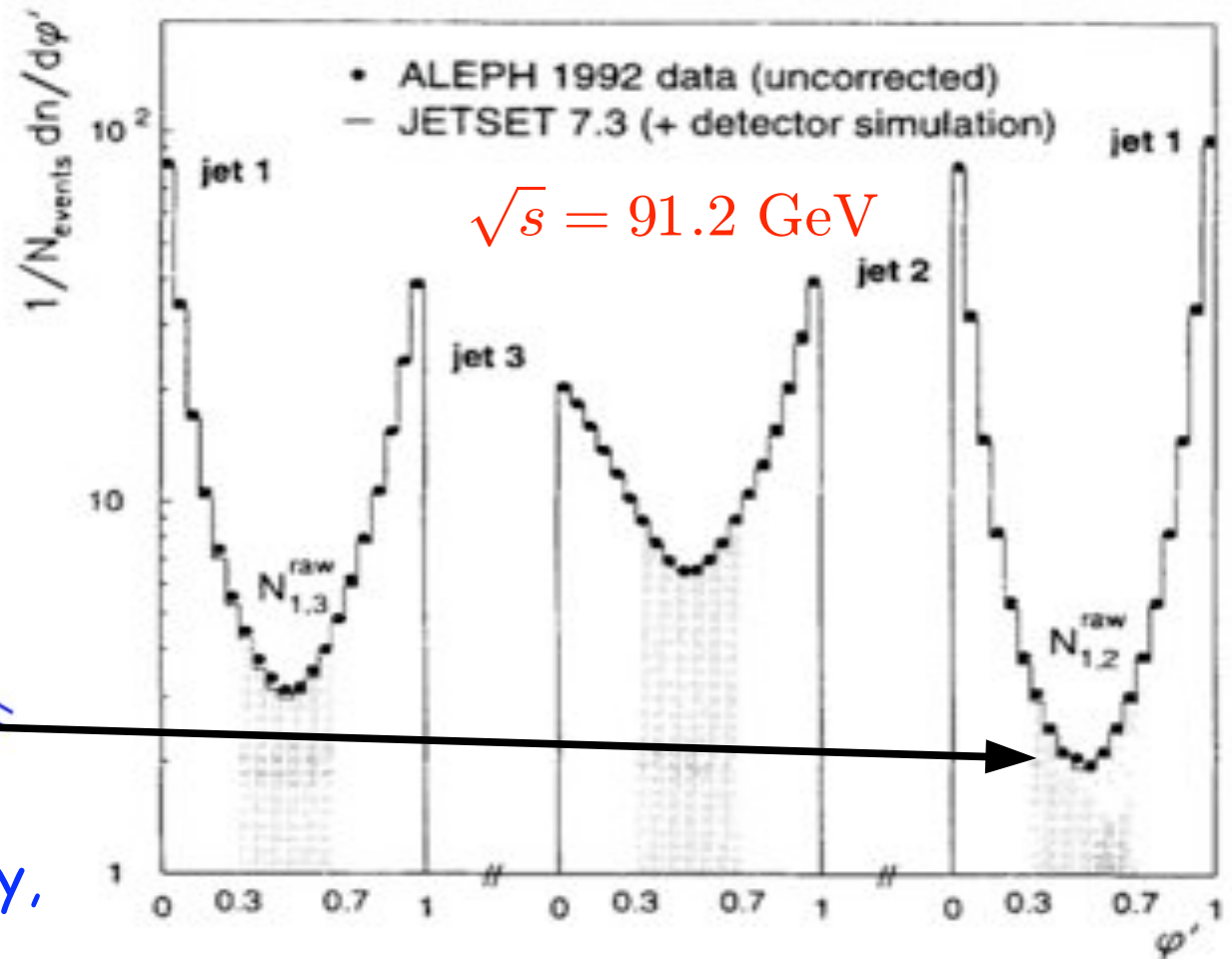
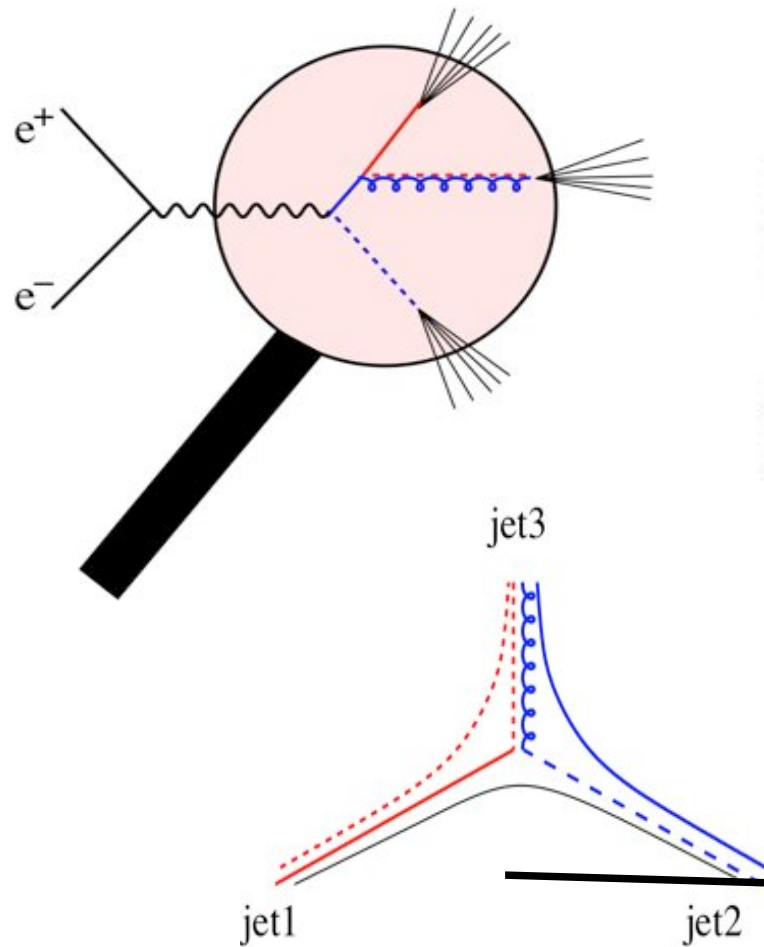


**The Lund Model does describe it !!!**

# ... and with more precise data ...

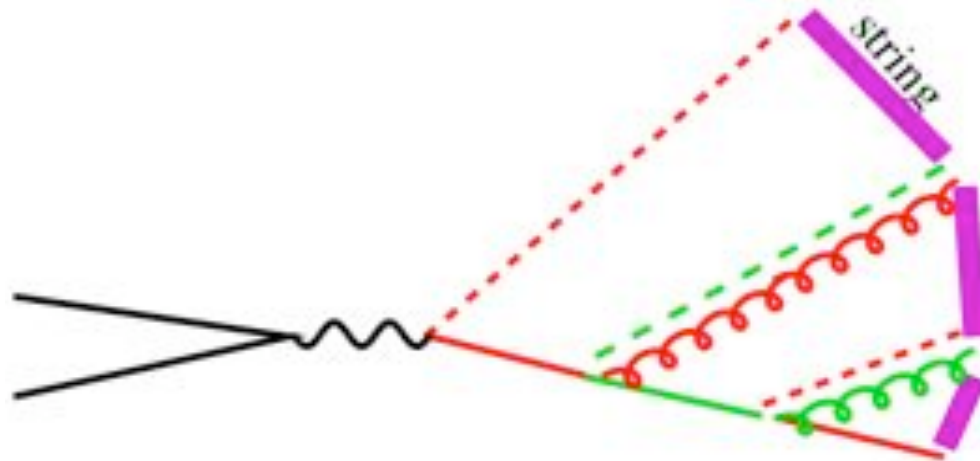
How to find the gluon jets, Andersson, Gustafson, Sjostrand, PLB 94,211 (1980)

ALEPH Collaboration / Physics Reports 294 (1998) 1-165



- jets ordered by energy, highest is quark (~94%), lowest is gluon (~70%)

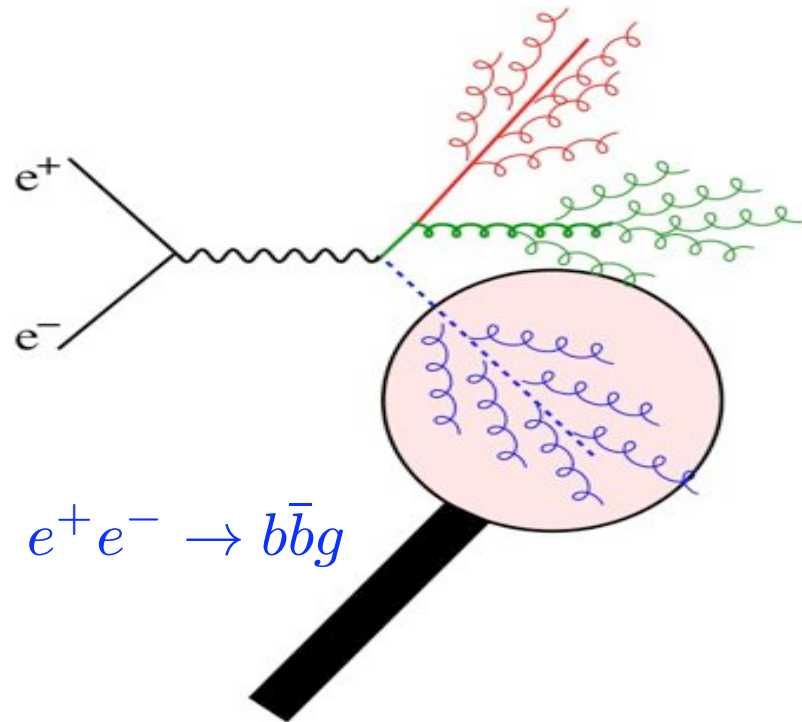
# Models for jet evolution



- Parton Showering

- Color field of Lund string interpreted in terms of gluons
- successive parton radiation, with splitting function
- ordering introduced explicitly:
  - virtuality, pt or angular ordered
- need to take care of recoil
- implemented in JETSET/PYTHIA/HERWIG

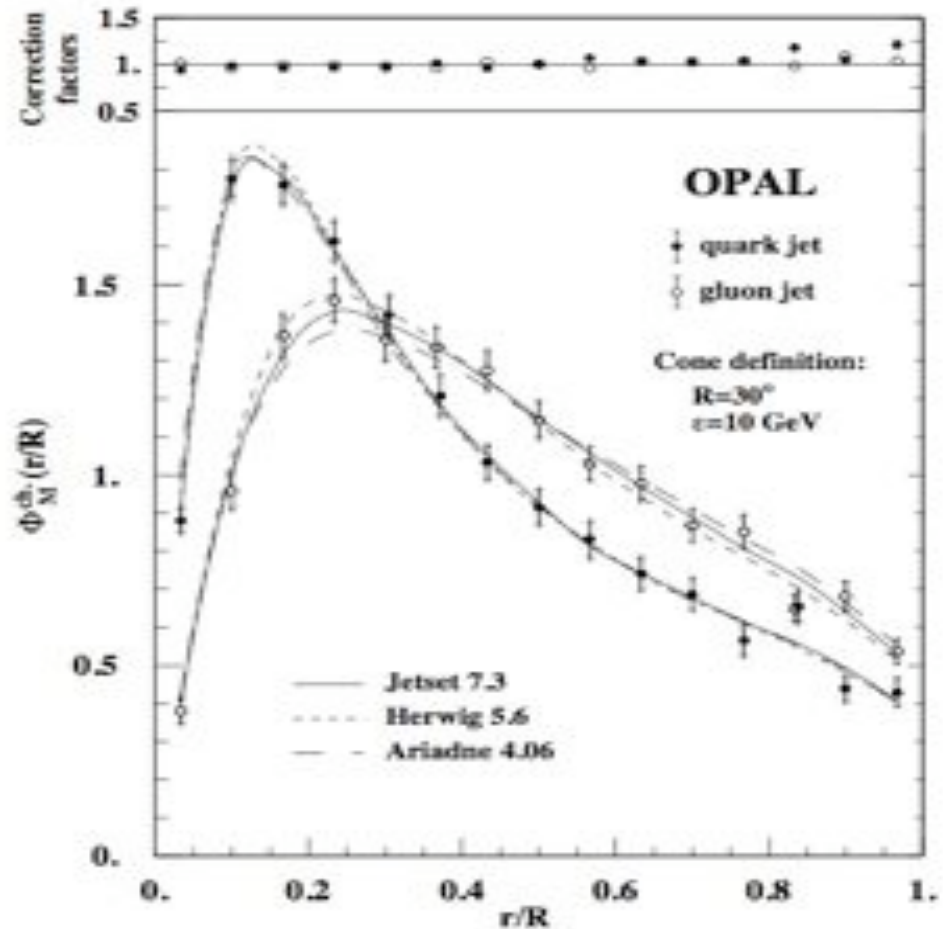
# Jet evolution: q and g jets



- select 3 jets, highest E-jet with secondary vertex (b-jet)
- 2 lower E jets are enriched gluon-jets
- use MC for corrections to true gluon

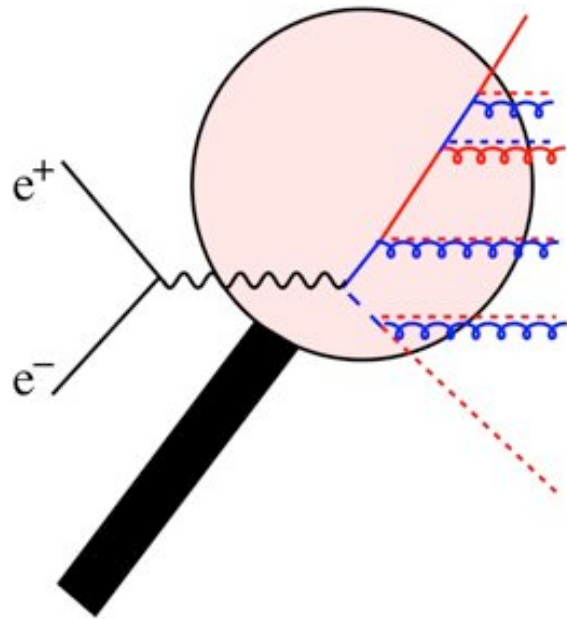
→ gluon jets are wider ...  
 → MC's with parton shower and CDM describe jet shapes

OPAL Collaboration Z.Phys.C68:179-202,1995.



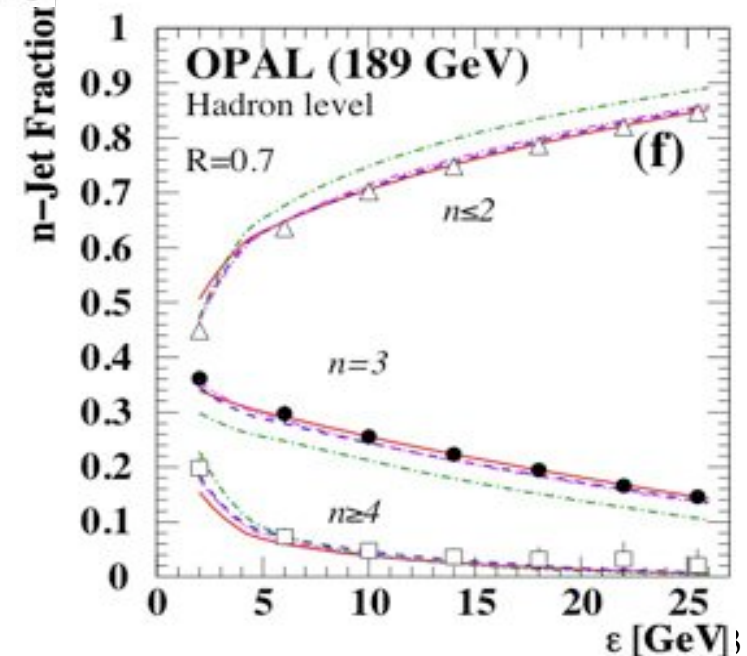
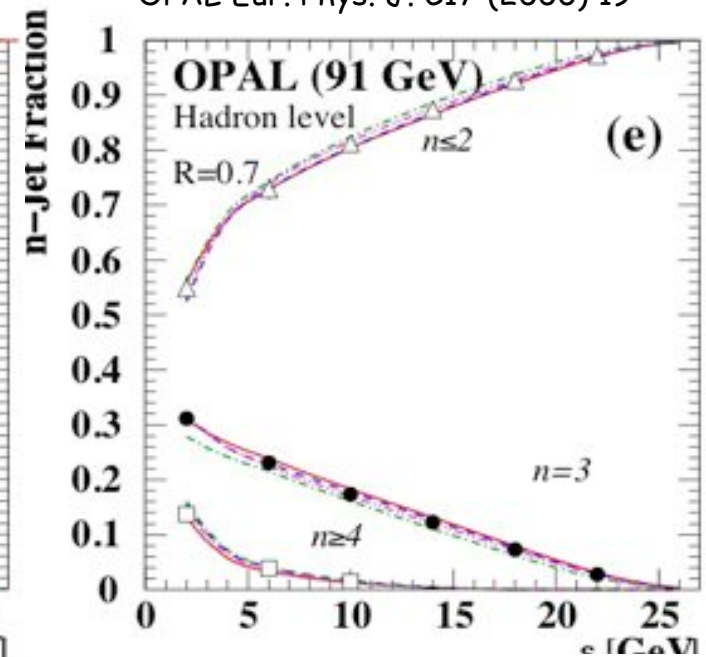
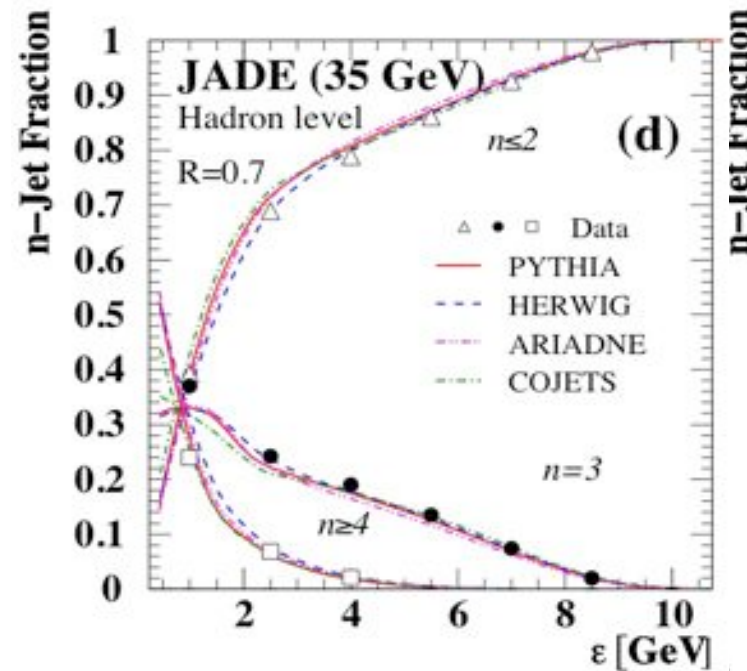


# Multijet production



- using cone jet algorithm
- shower MCs are able to reproduce multi jets rates from low to highest CM energy

OPAL Eur. Phys. J. C17 (2000) 19

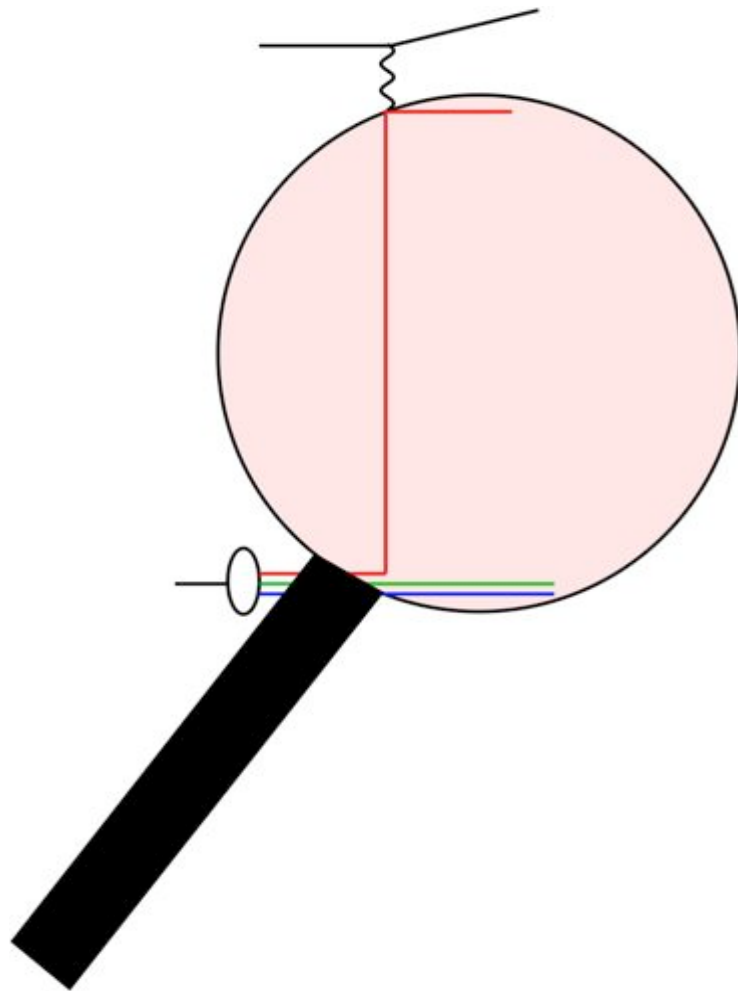


Wait .....

Let's go one step back ...

Let's look at ep first !

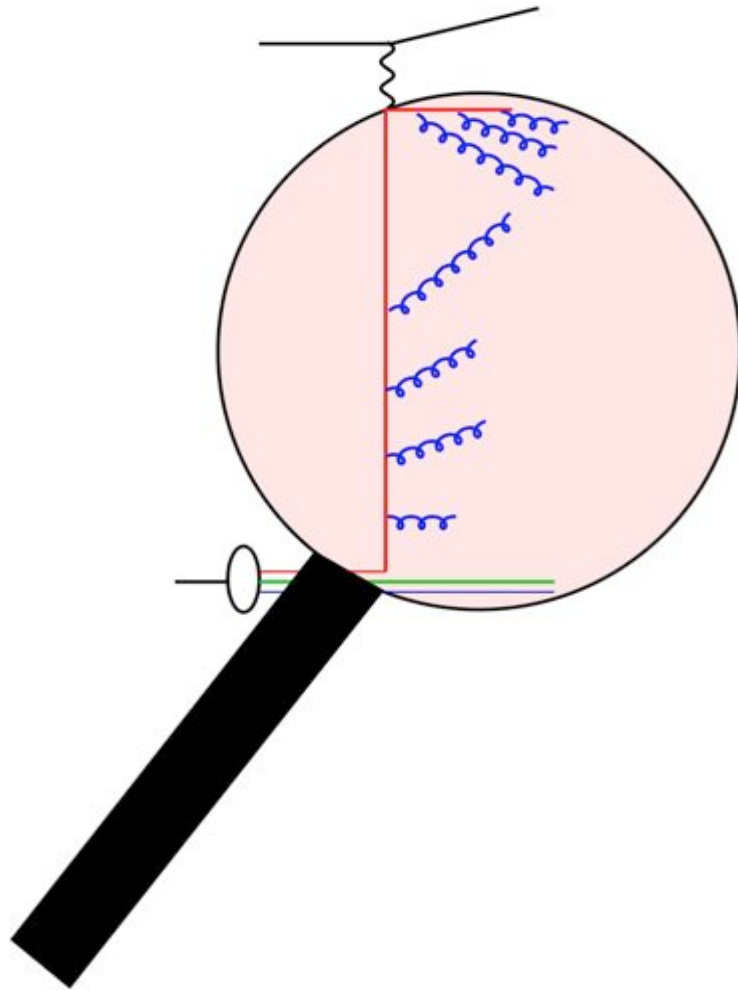
# The fun with ep scattering



- Deep Inelastic Scattering is a incoherent sum of  $e^+ q \rightarrow e + q$
- only 50 % of p momentum carried by quarks
- need a large gluon component
- partonic part convoluted with parton density function  $f_i(x)$

$$\sigma(e^+ p \rightarrow e^+ X) = \sum_i f_i(x, \dots) \sigma(e^+ q_i \rightarrow e^+ q_i)$$

# The fun with ep scattering

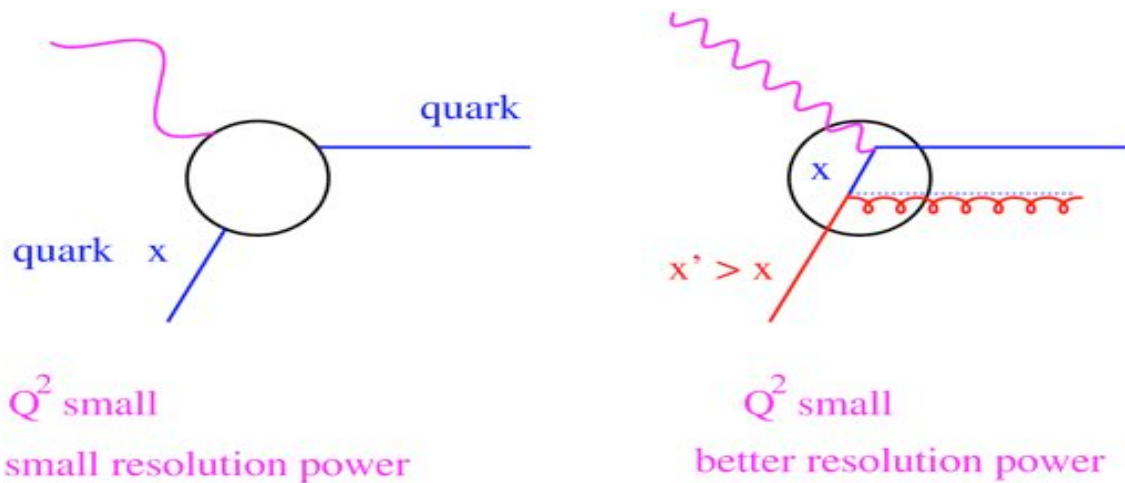


- Deep Inelastic Scattering is a incoherent sum of  $e^+q \rightarrow e + q$
- only 50 % of p momentum carried by quarks
- need a large gluon component
- partonic part convoluted with parton density function  $f_i(x)$
- BUT we know, PDF depends on resolution scale  $Q^2$

$$\sigma(e^+p \rightarrow e^+X) = \sum_i f_i(x, Q^2) \sigma(e^+q_i \rightarrow e^+q_i)$$

# $F_2(x, Q^2)$ : DGLAP evolution equation

- QPM:  $F_2$  is independent of  $Q^2$
- $Q^2$  dependence of structure function: **D**okshitzer **G**ribov **L**ipatov **A**ltarelli **P**arisi



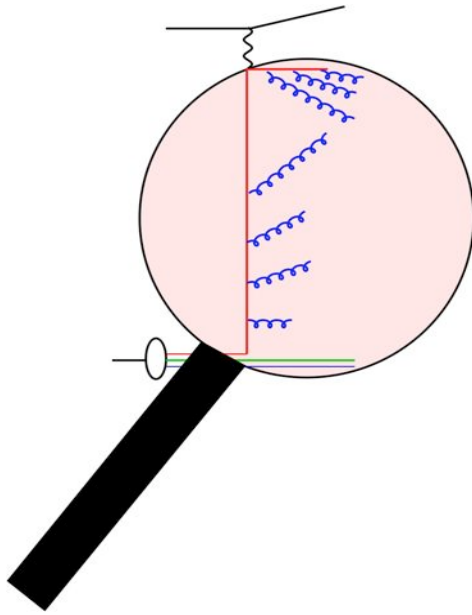
→ Probability to find parton at small  $x$  increases with  $Q^2$

$$F_2 = \left| \begin{array}{c} \text{Diagram 1} \\ \text{OPM} \end{array} \right|^2 + \left| \begin{array}{c} \text{Diagram 2} \\ \text{QCDC} \end{array} \right|^2 + \left| \begin{array}{c} \text{Diagram 3} \\ \text{BGF} \end{array} \right|^2$$

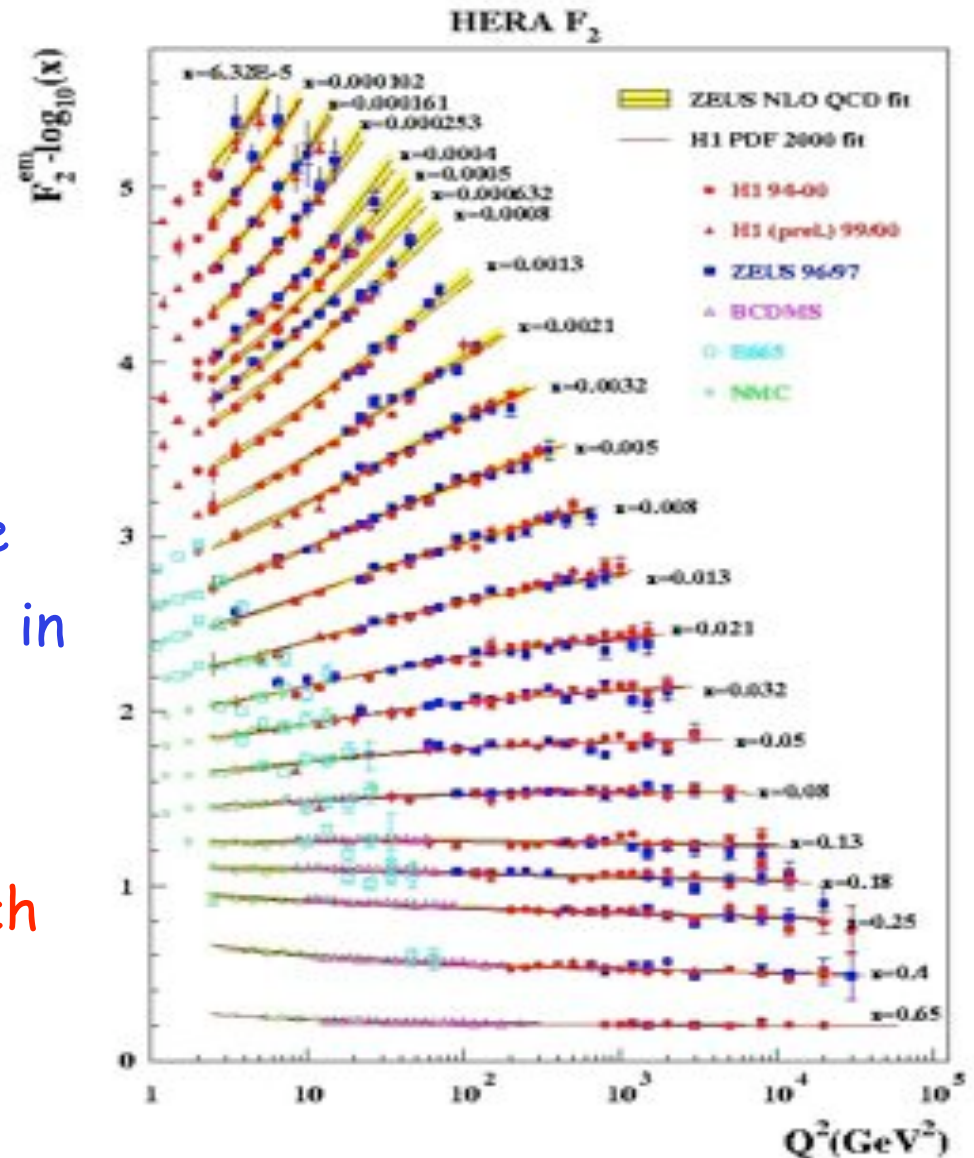
→ Test of theory:  $Q^2$  evolution of  $F_2(x, Q^2)$  !!!!!

# The fun with ep scattering

$$\sigma(e^+p \rightarrow e^+X) = \sum f_i(x, Q^2) \sigma(e^+q_i \rightarrow e^+q_i)$$

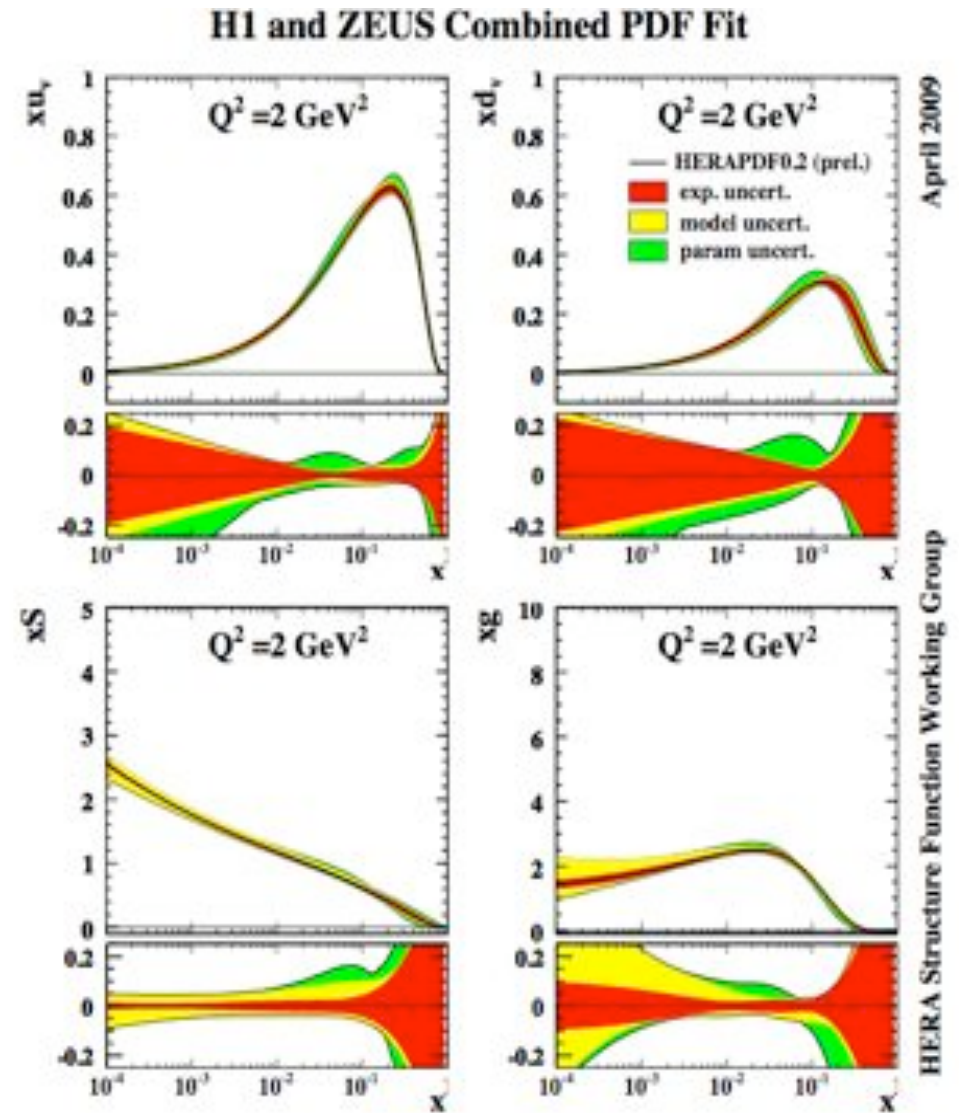
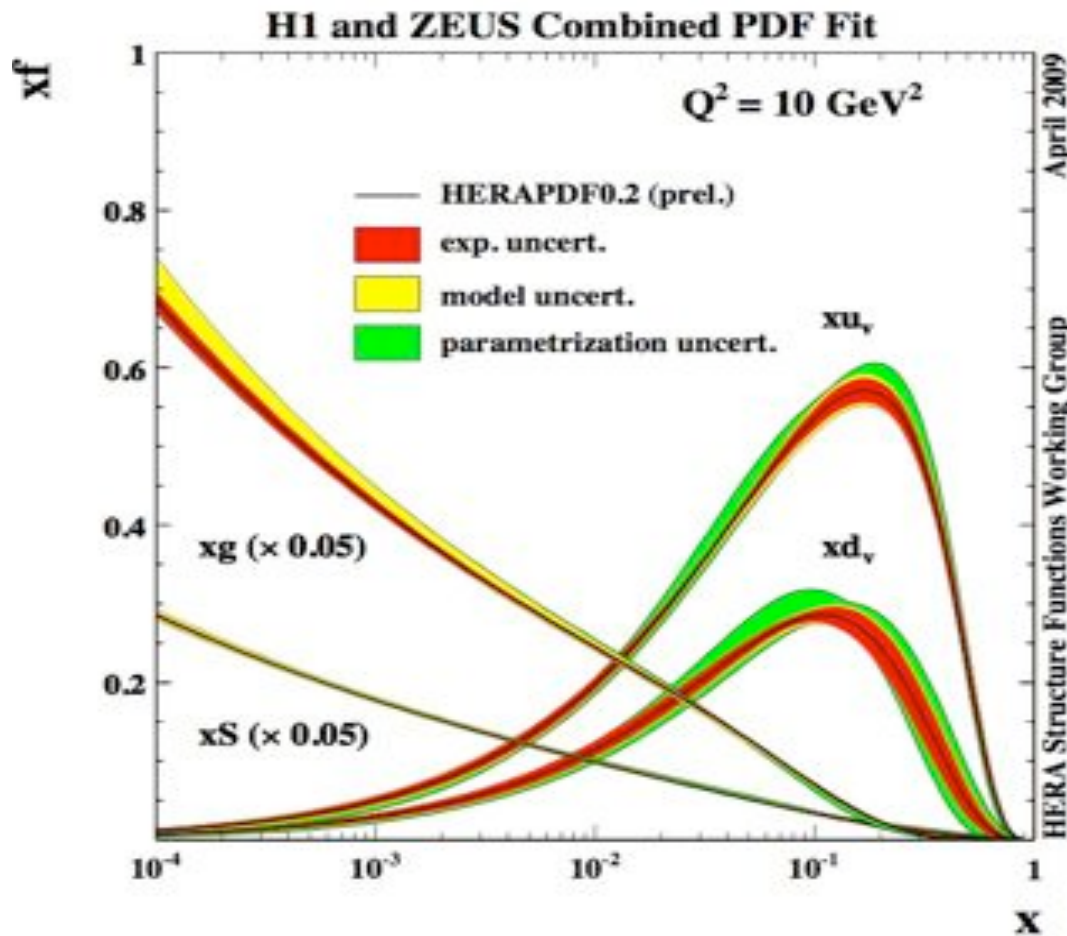


- perfect description of precise measurements of **HUGE** range in  $x$  and  $Q^2$
- Theory works well.....
- ➔ extract parton densities, which are universal
- ➔ to be used at LHC.....



# The proton PDFs ...

- quark and gluon PDFs



→ Very large gluon density, even at small resolution scales  $Q^2$

A simple  
ep  
Monte Carlo event  
generator ...



# The DIS process $ep \rightarrow epX$

- cross section  $\frac{d\sigma(ep \rightarrow e' X)}{dy dQ^2} = \frac{4\pi\alpha^2}{yQ^4} \left( \left(1 - y + \frac{y^2}{2}\right) F_2^p(x, Q^2) - \frac{y^2}{2} F_L^p(x, Q^2) \right)$

with  $F_2^p(x, Q^2) = \sum_f e_f^2 (xq_f(x, Q^2) + x\bar{q}_f(x, Q^2))$

using the PDFs from before.....

# The DIS process $ep \rightarrow epX$

- cross section  $\frac{d\sigma(ep \rightarrow e'X)}{dy dQ^2} = \frac{4\pi\alpha^2}{yQ^4} \left( \left(1 - y + \frac{y^2}{2}\right) F_2^p(x, Q^2) - \frac{y^2}{2} F_L^p(x, Q^2) \right)$

with  $F_2^p(x, Q^2) = \sum_f e_f^2 (xq_f(x, Q^2) + x\bar{q}_f(x, Q^2))$

using the PDFs from before.....

- generate  $y$  with  $g(y)=1/y$ , and  $Q^2$  with  $g(Q^2)=1/Q^2$  (why not  $1/Q^4$  ?):

$$y = y_{min} \left( \frac{y_{max}}{y_{min}} \right)^{R_1} Q^2 = Q_{min}^2 \left( \frac{Q_{max}^2}{Q_{min}^2} \right)^{R_2}$$

$$\sigma(ep \rightarrow e'X) = \frac{1}{N} \sum_{i=1}^N \frac{\frac{d\sigma}{dy_i dQ_i^2}}{g(y_i)g(Q_i^2)} \int g(y)dy \int g(Q^2)dQ^2$$

# The DIS process $ep \rightarrow epX$

- cross section  $\frac{d\sigma(ep \rightarrow e'X)}{dy dQ^2} = \frac{4\pi\alpha^2}{yQ^4} \left( \left(1 - y + \frac{y^2}{2}\right) F_2^p(x, Q^2) - \frac{y^2}{2} F_L^p(x, Q^2) \right)$

with  $F_2^p(x, Q^2) = \sum_f e_f^2 (xq_f(x, Q^2) + x\bar{q}_f(x, Q^2))$

using the PDFs from before.....

- generate  $y$  with  $g(y)=1/y$ , and  $Q^2$  with  $g(Q^2)=1/Q^2$  (why not  $1/Q^4$  ?):

$$y = y_{min} \left( \frac{y_{max}}{y_{min}} \right)^{R_1} \quad Q^2 = Q_{min}^2 \left( \frac{Q_{max}^2}{Q_{min}^2} \right)^{R_2}$$

$$\sigma(ep \rightarrow e'X) = \frac{1}{N} \sum_{i=1}^N \frac{\frac{d\sigma}{dy_i dQ_i^2}}{g(y_i)g(Q_i^2)} \int g(y)dy \int g(Q^2)dQ^2$$

- calculate x-section with:

$$\sigma(ep \rightarrow e'X) = \frac{1}{N} \sum_{i=1}^N y_i Q_i^2 \frac{d\sigma}{dy_i dQ_i^2} \log \left( \frac{y_{max}}{y_{min}} \right) \log \left( \frac{Q_{max}^2}{Q_{min}^2} \right)$$

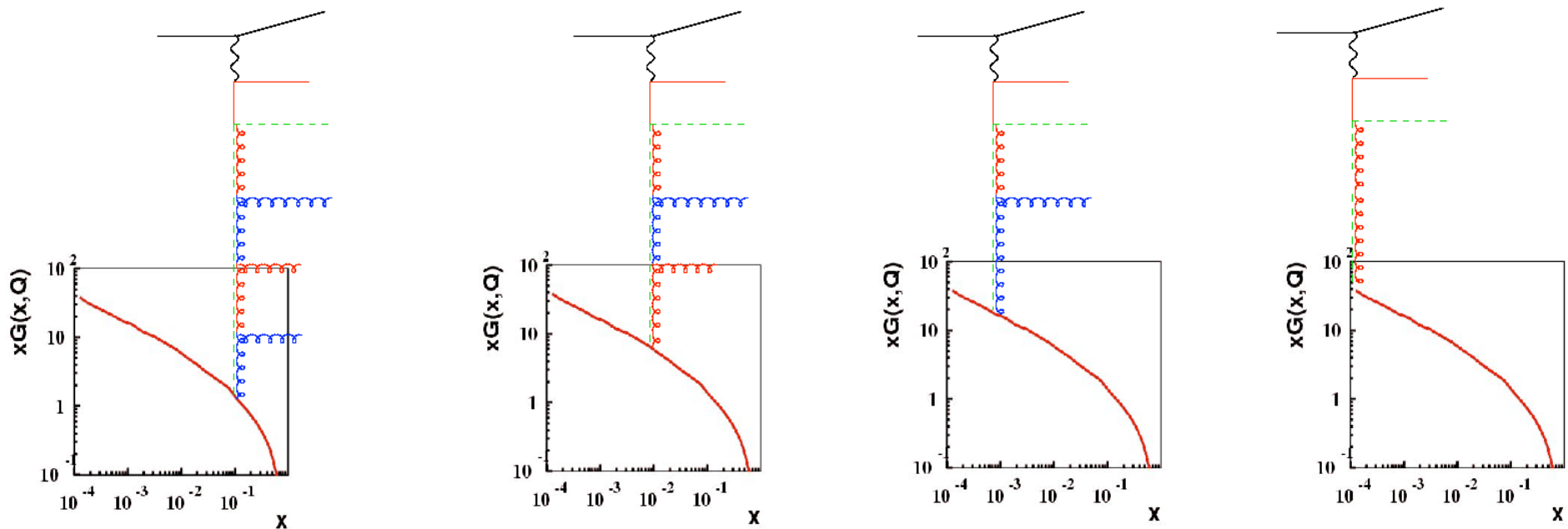
- calculate 4-momenta of scattered electron and virtual photon

We have now an event  
generator for the  
total cross section.

What about the final  
states ?

# DGLAP evolution equation... again...

- for fixed  $x$  and  $Q^2$  chains with different branchings contribute
- iterative procedure, **spacelike** parton showering



$$f(x, t) = \sum_{k=1}^{\infty} f_k(x_k, t_k) + f_0(x, t_0) \Delta_s(t)$$

# Parton Showers from evolution eq.

$$f(x, t) = f(x, t_0) \Delta_s(t) + \int \frac{dz}{z} \int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')} \tilde{P}(z) f\left(\frac{x}{z}, t'\right)$$

- solve integral equation via **explicit** iteration:

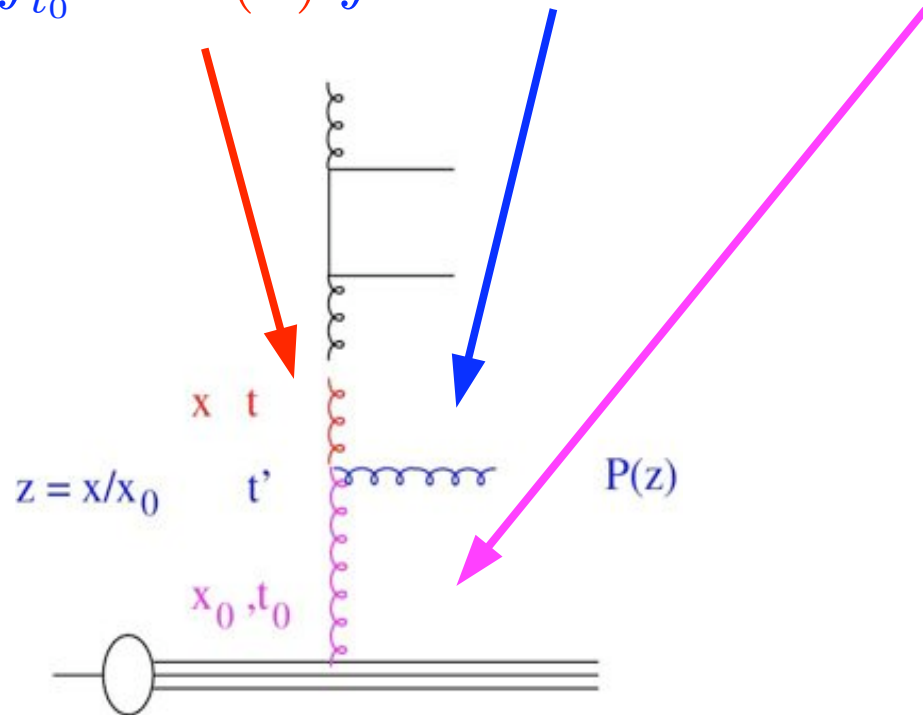
$$f_0(x, t) = f(x, t_0) \Delta(t)$$

from  $t'$  to  $t$   
w/o branching

branching at  $t'$

from  $t_0$  to  $t'$   
w/o branching

$$f_1(x, t) = f(x, t_0) \Delta(t) + \int_{t_0}^t \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} \tilde{P}(z) f(x/z, t_0) \Delta(t')$$



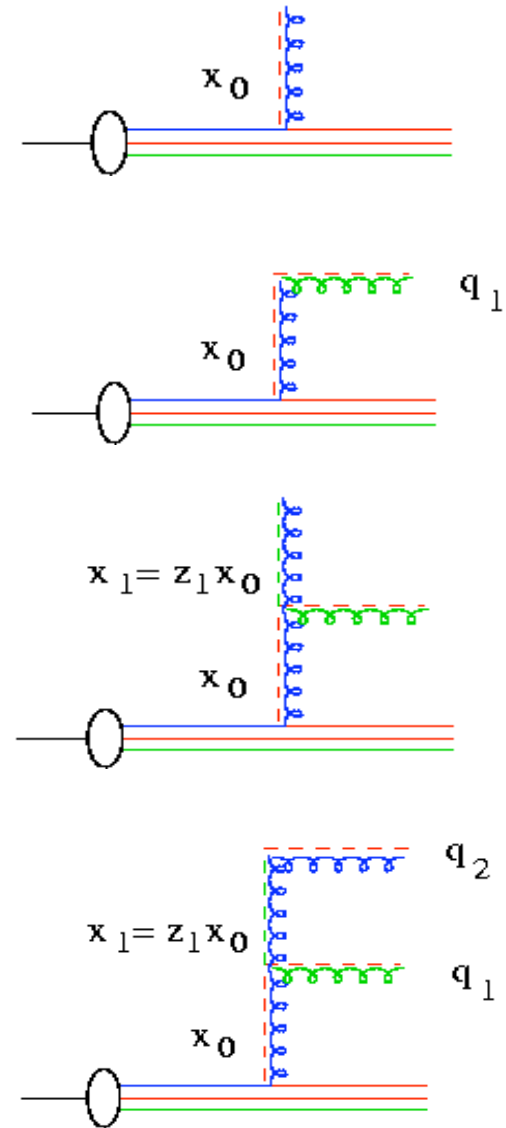
# Parton Shower

- **Sudakov form factor**
  - gives probability for **no-branching** between  $q_0$  and  $q$
  - sums virtual contributions to all orders (via unitarity)
    - **virtual (parton loop)** and
    - **real (non-resolvable)** parton emissions
- **Sudakov form factor** particularly suited for Monte Carlo approach
  - need to specify scale of hard process (matrix element)  $Q \sim p_T$
  - need to specify cutoff scale  $Q_0 \sim 1 \text{ GeV}$
- Evolution equation with **Sudakov form factor** recovers exactly evolution equation (with  $_+$  prescription)

# Parton showers for the initial state

## spacelike ( $Q < 0$ ) parton shower evolution

- starting from hadron (fwd evolution)  
or from hard scattering (bwd evolution)
- select  $q_1$  from Sudakov form factor
- select  $z_1$  from splitting function
- select  $q_2$  from Sudakov form factor
- select  $z_2$  from splitting function
- stop evolution if  $q_2 > Q_{\text{hard}}$

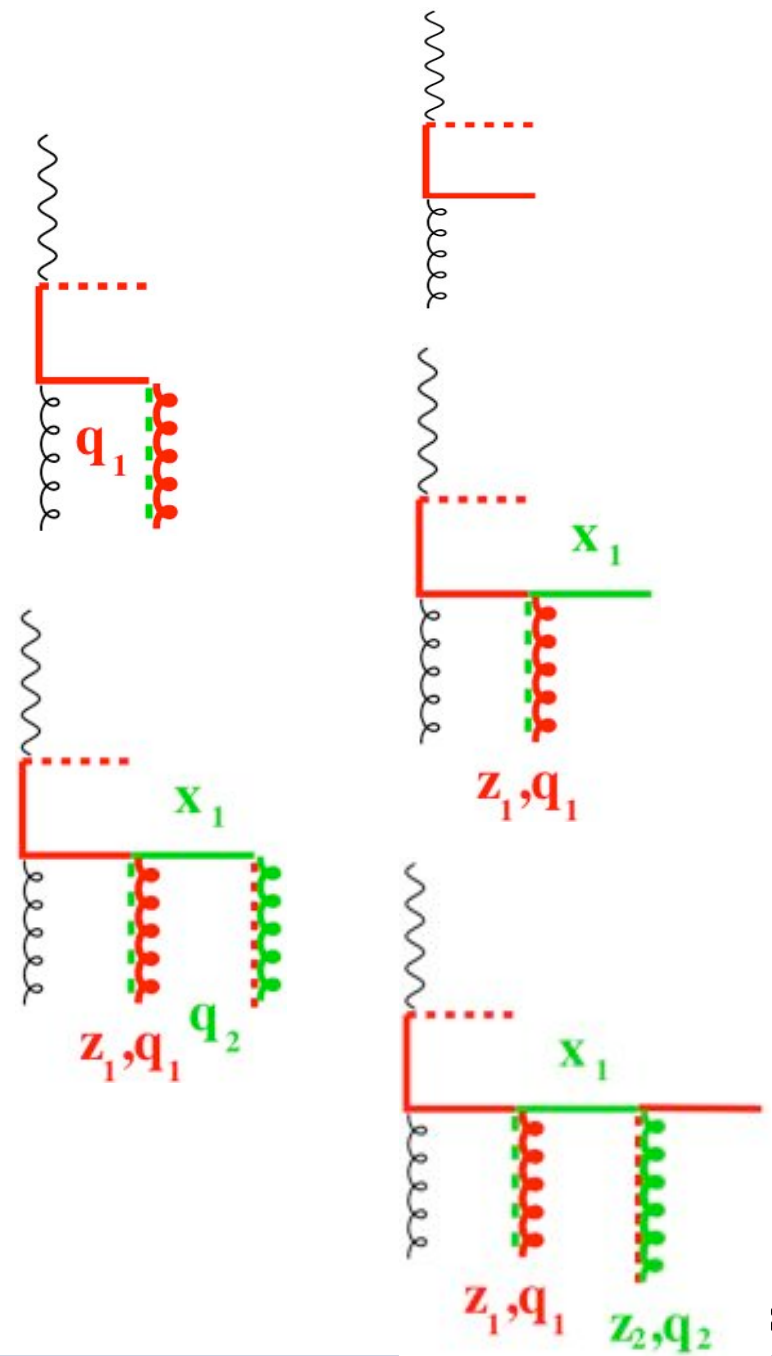




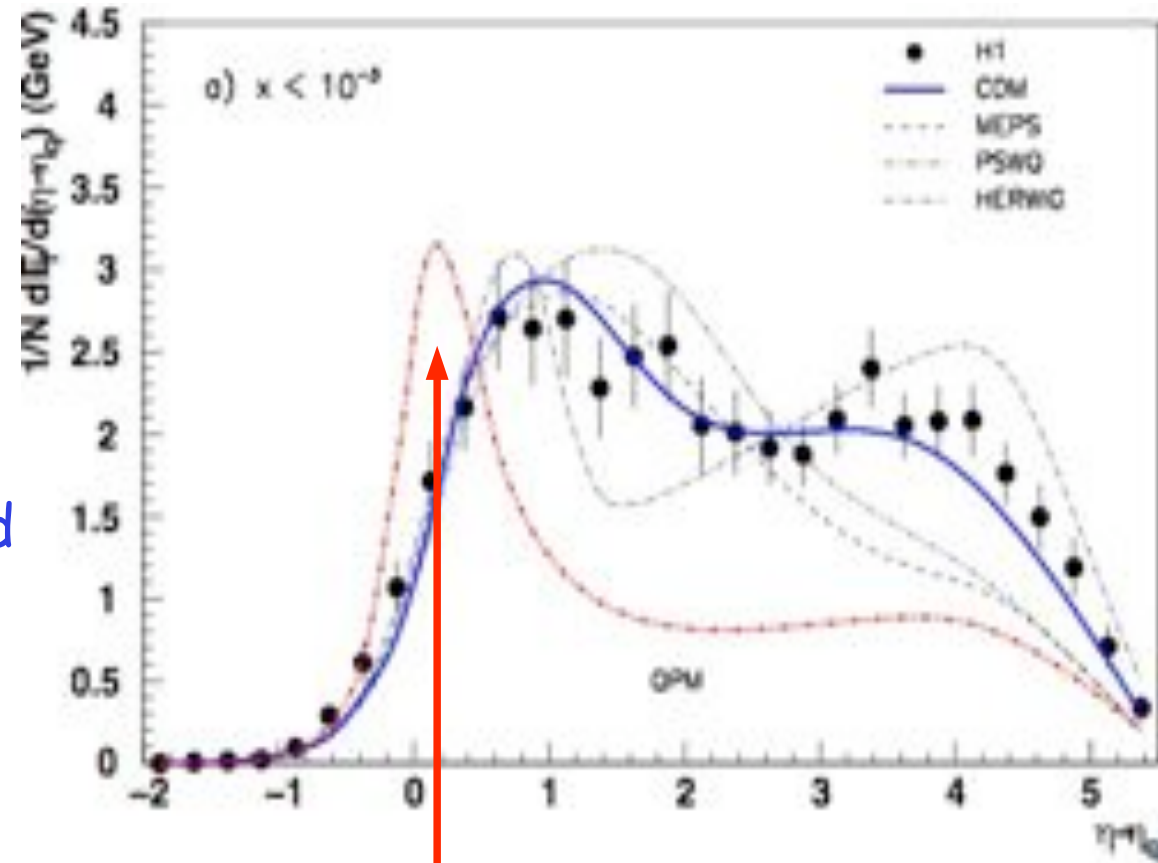
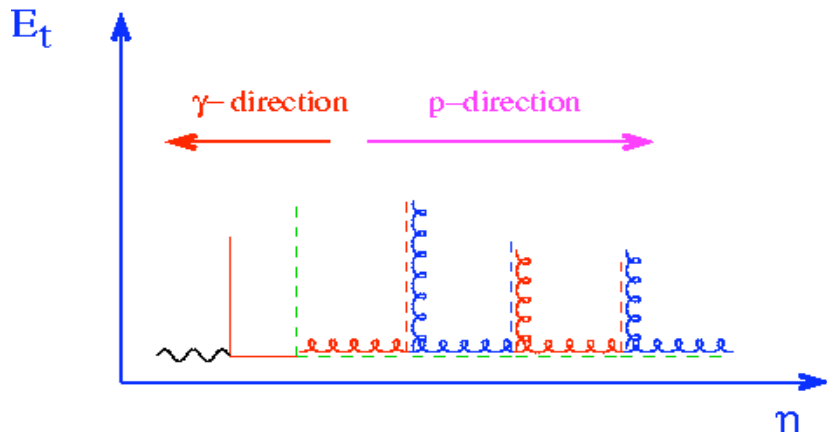
# Parton Showers for the final state

timelike parton shower evolution

- starting with hard scattering
- select  $q_1$  from Sudakov form factor
- select  $z_1$  from splitting function
- select  $q_2$  from Sudakov form factor
- select  $z_2$  from splitting function
- stop evolution if  $q_2 < q_0$



# Hadronic final state: Energy flow

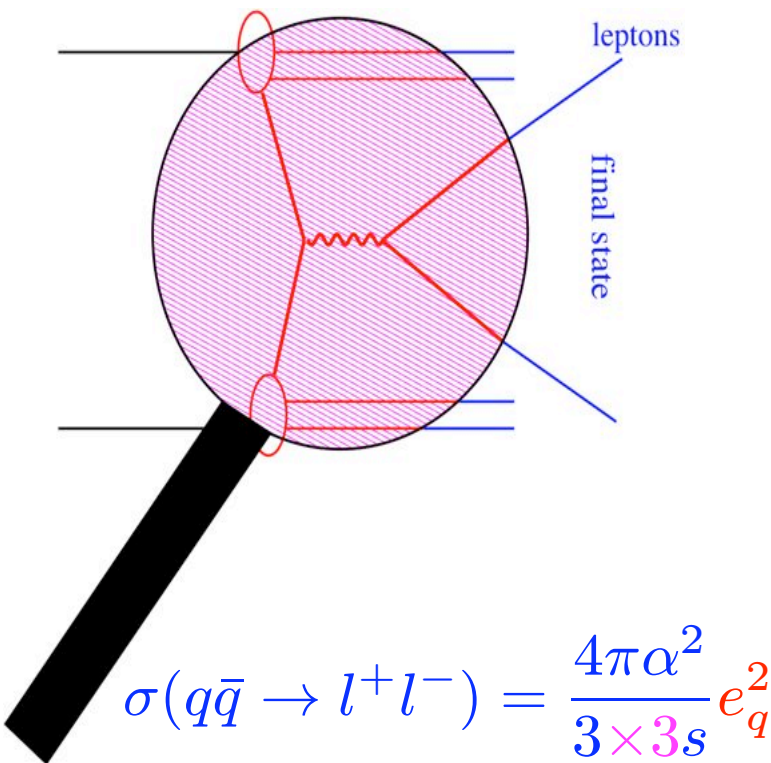


- $E_+$  flow in DIS at small  $x$  and forward angle (p-direction):
- QPM is not enough
- clearly parton showers or higher order contributions needed

leading jet direction

And ....  
What about pp ?

# Rotating the diagrams



$$\sigma(q\bar{q} \rightarrow l^+l^-) = \frac{4\pi\alpha^2}{3 \times 3s} e_q^2$$

## Factorization:

- hard process:  $q\bar{q} \rightarrow l^+l^-$
- parton densities: prob to find parton with  $x$  at scale  $Q^2$  in proton:  $q(x, Q^2), g(x, Q^2)$ ...

Measurement of Z0 and Drell-Yan production cross-section using dimuons in anti-p p collisions at  $\sqrt{s} = 1.8$ -TeV. CDF Collaboration F. Abe et al. Phys.Rev.D59:052002,1999.

## Drell-Yan differential cross-section

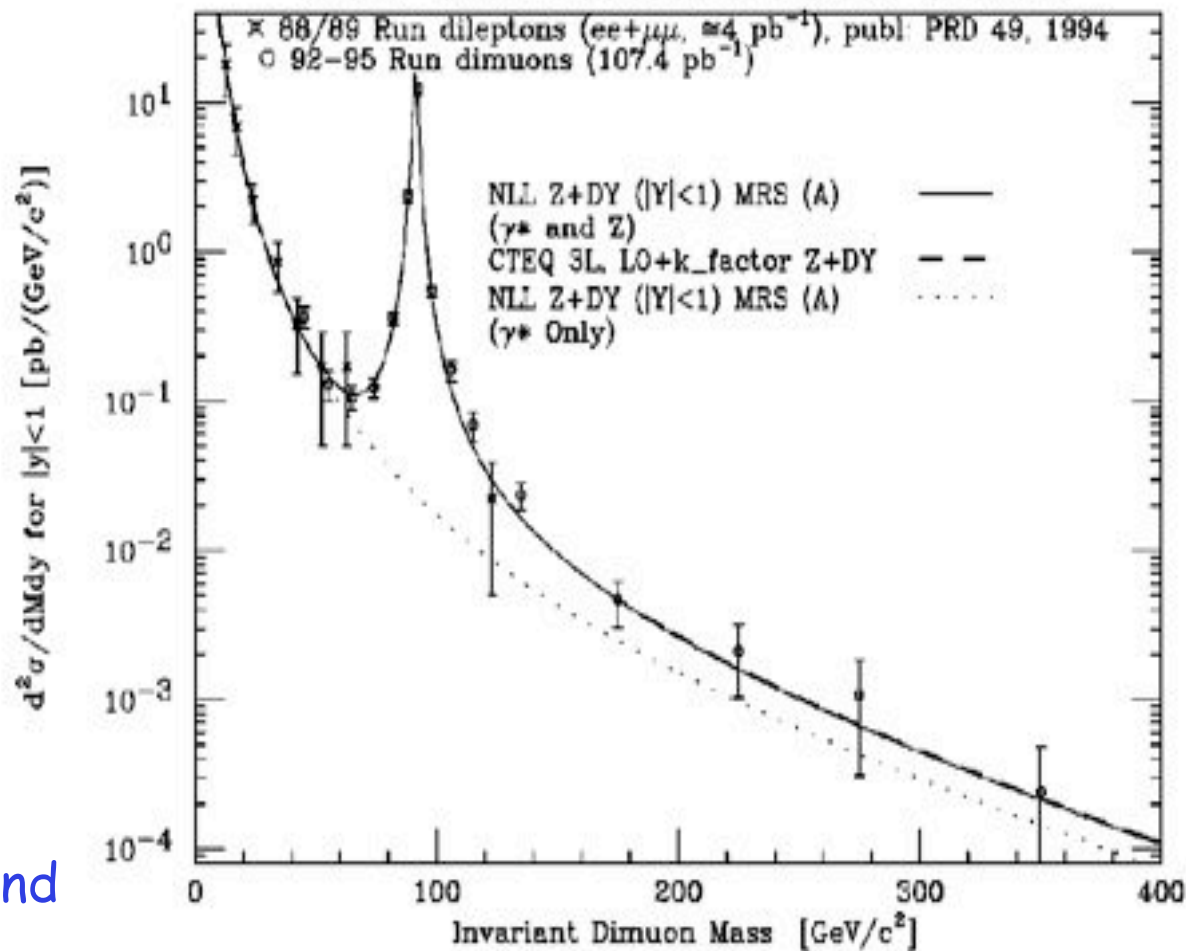


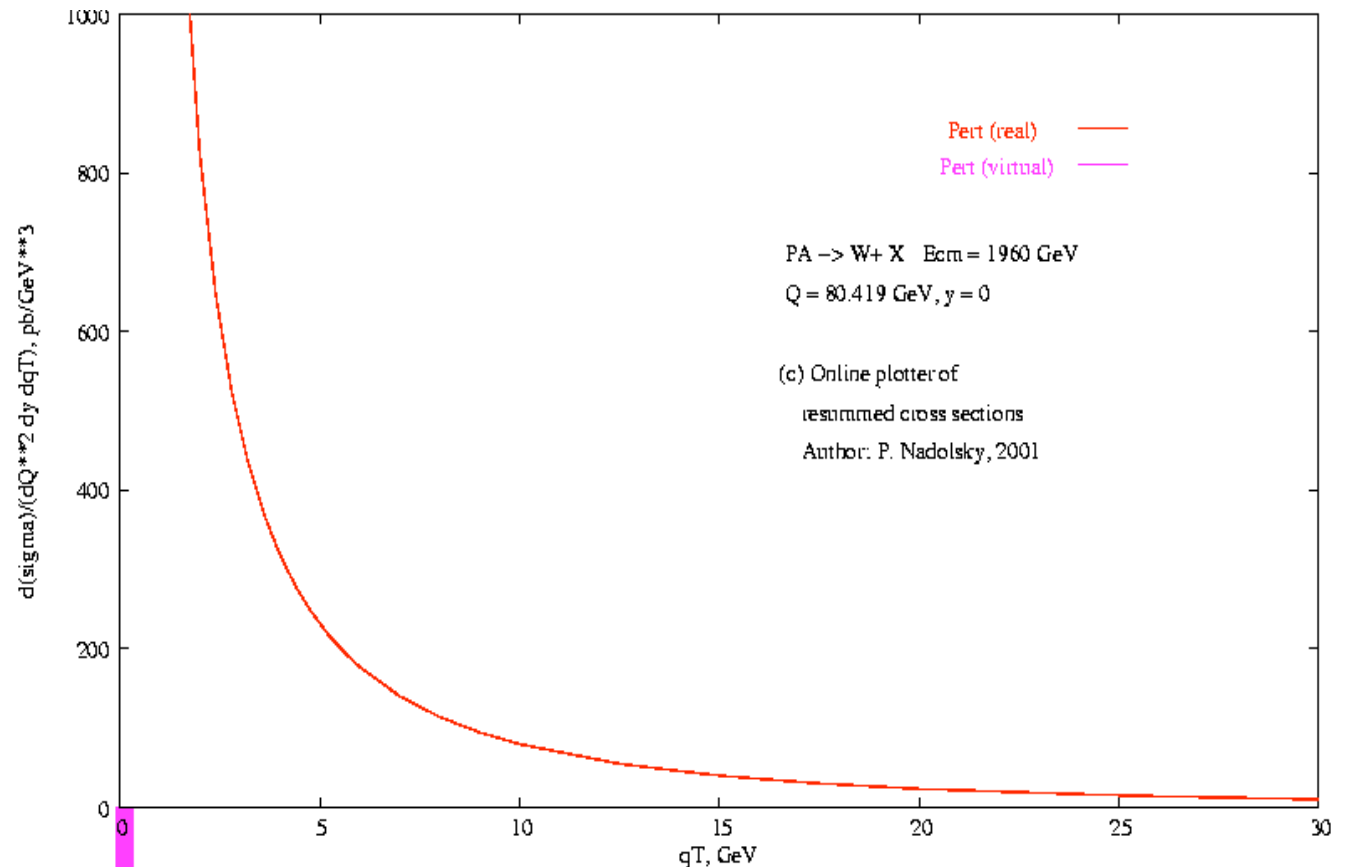
FIG. 8. Drell-Yan dimuon production cross section extracted

# Transverse Momentum of W/Z

Perturbative calculation

$$\mathcal{O}(\alpha_s), \mathcal{O}(\alpha_s^2)$$

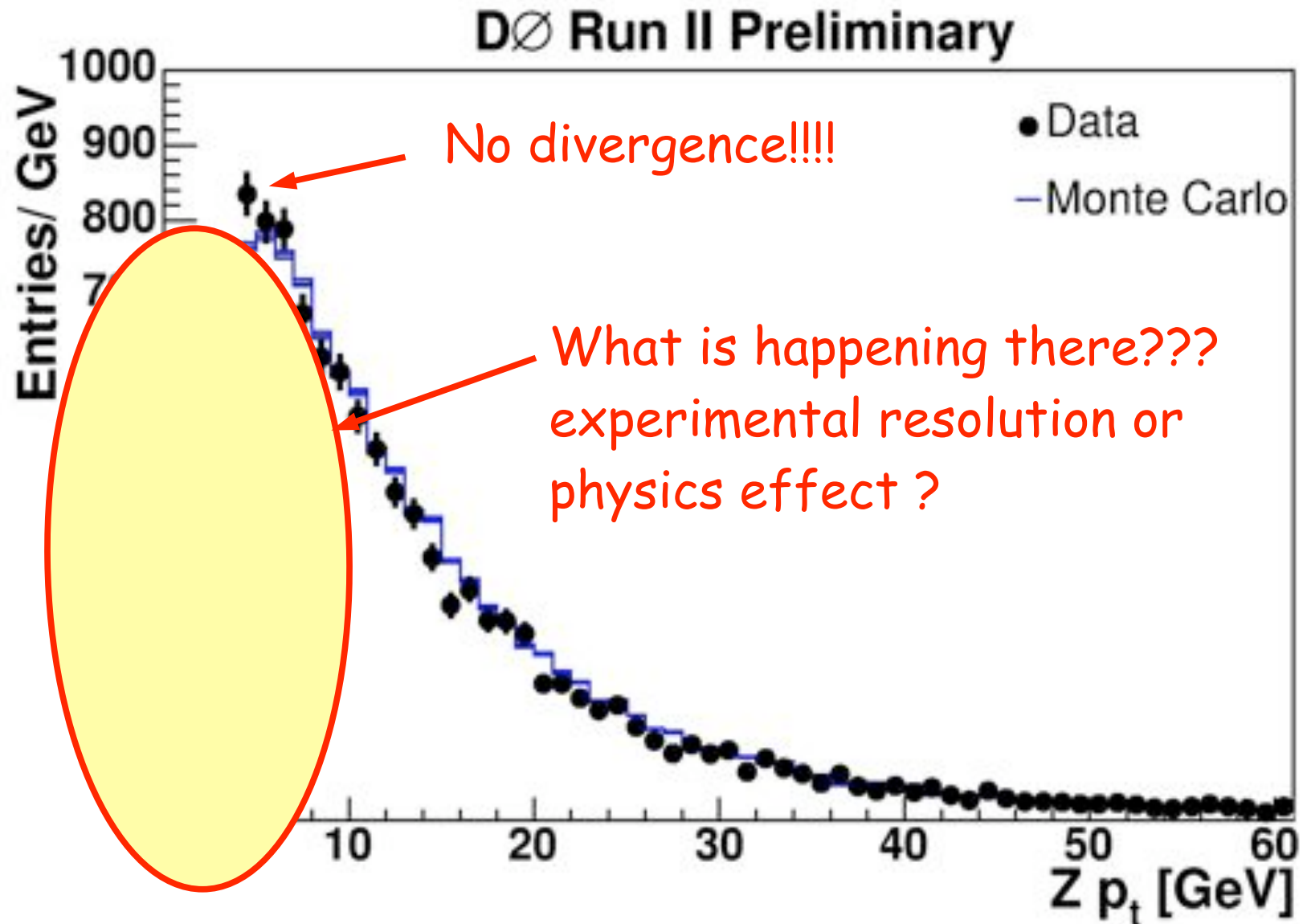
diverges for small  $p_\perp$



<http://hep.pa.msu.edu/wwwlegacy/>

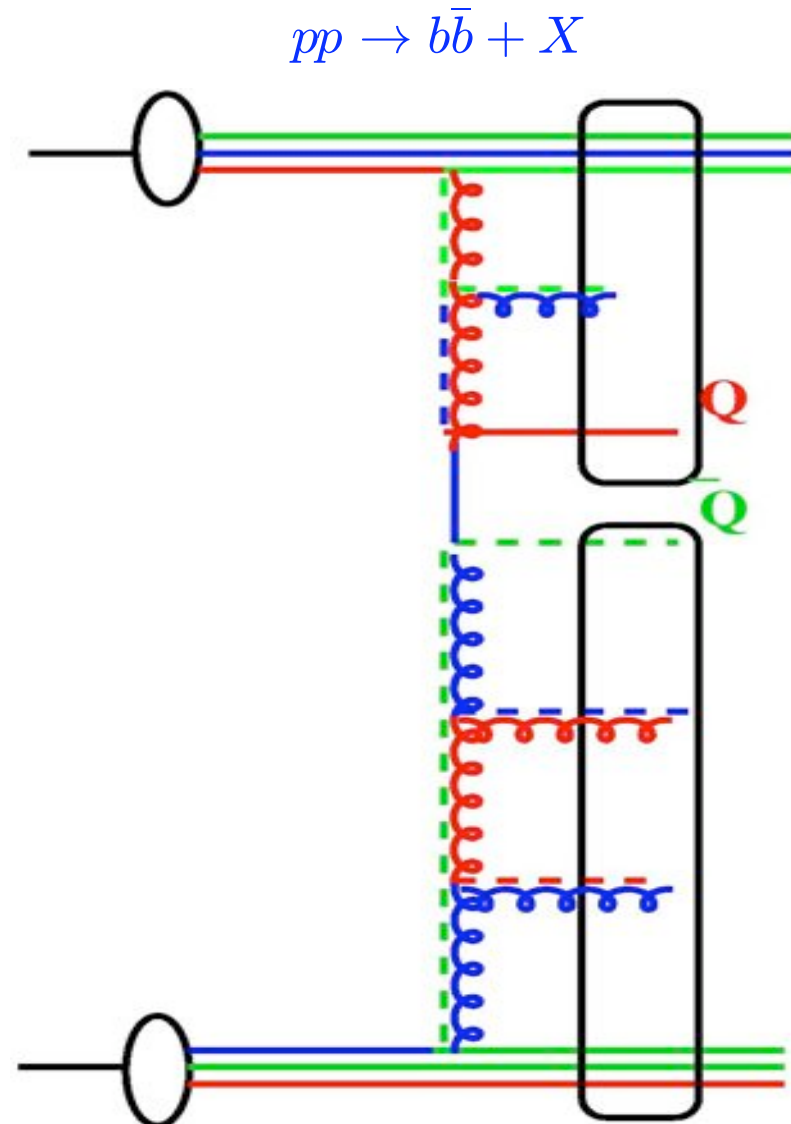
- Need a  $p_T$  cut to avoid divergency
- or include virtual corrections and calculate at NLO .....
-

# BUT: the data .....



# Color Flow in pp

- quarks carry color
- anti-quarks carry anticolor
- gluons carry color - anticolor
  - connect to color singlet systems
  - watch out  $pp$  or  $p\bar{p}$

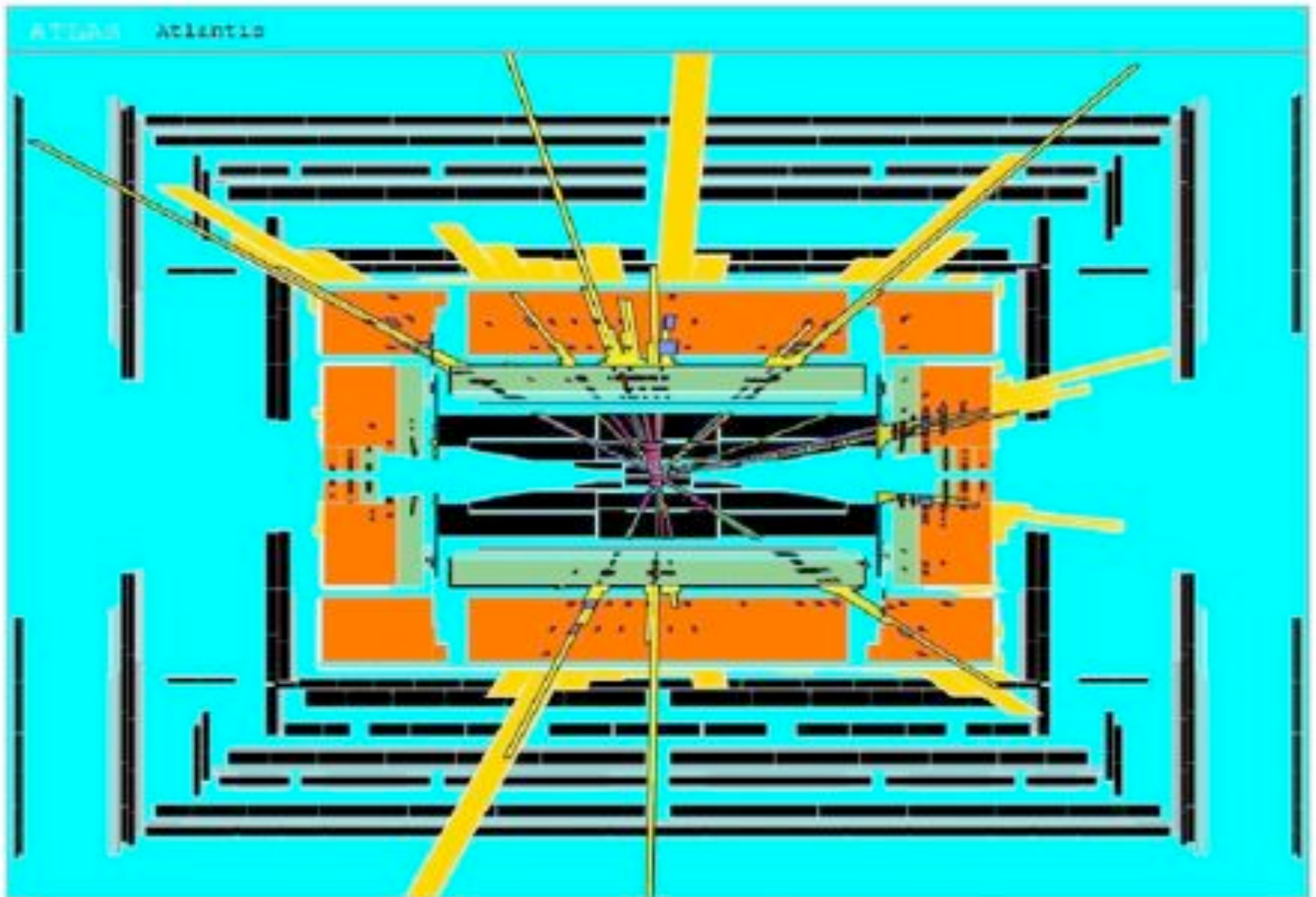




Can we now tell which type of event this is ?

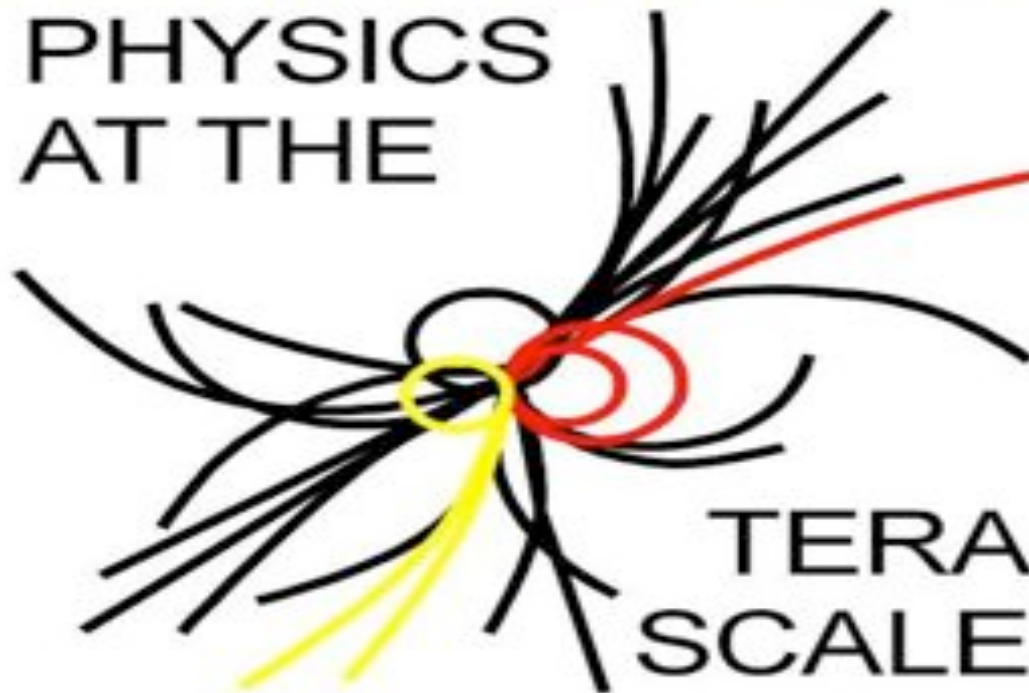


# What was this ?



# The Analysis Centre in the

PHYSICS  
AT THE



TERA  
SCALE

**Helmholtz Alliance**

- Areas
  - Monte Carlo (user support, tuning, development ...)
  - Parton Distribution Functions
  - Statistics Tools
  - Collaborative tools (web based infos etc )

# Monte Carlo group activities

- Development of Monte Carlo generators
  - Tuning of MC generators
  - PDF4MC
  - User support
- Training (schools, seminars)
  - MC schools in spring 2008, 2009
- link to MC group page

**Monte Carlo School**  
**PHYSICS AT THE TERASCALE**  
Strategic Helmholtz Alliance

PHYSICS AT THE TERASCALE Helmholtz Alliance

21-24 April 2008,  
DESY Hamburg

**Topics:**

- Monte Carlo techniques and physics (L. Lönnblad)
- NLO Calculations (NIN)
- NLO and parton showers (M. Dineale)
- Monte Carlo event generators
  - CASCADE (H. Jung)
  - HERWIG (S. Gieseke, P. Richardson)
  - PYTHIA (T. Sjöstrand)
  - SHERPA (F. Krauss)
- Exercises (L. Sonnenschein et al.)

The school covers Monte Carlo techniques and applications in NLO calculations as well as full hadron level Monte Carlo event generators. Predictions coming from different generators will be compared in practical exercises and first steps for comparison with measurements will be shown in tutorials.

Registration deadline: 15.03.2008  
Please register via the school webpage.

Organizing Committee: Hanses Jung, J. Kitz, A. Koutikov, K. Kutak, Serguei Levonian

<http://www.terascale.de/mcs2008>

# Monte Carlo group activities

If you are interested to do your

- diploma/masters thesis
- PhD
- postdoc

please get in contact with us...

There are plenty of possibilities and positions to do interesting physics with MC simulations ..... and help to find **extra dimensions** or **SUSY** or **new phenomena** in **QCD**

**Monte Carlo School**  
**PHYSICS AT THE TERASCALE**  
Strategic Helmholtz Alliance  
20-24 April 2009,  
DESY Hamburg

**Topics:**

- Monte Carlo techniques and standard physics (S. Dittus)
- Automated matrix element calculations (M. Dittus)
- Monte Carlo event generators
  - Herwig++ (T. Sjöstrand)
  - PYTHIA6 (C. Pascaud)
  - Sherpa (S. Höche)
  - MadGraph (J. Alwall)
  - Whizard (S. Alirol)
  - Higgs (P. Marziani)
  - SHERPA (S. Höche)
- Exercises

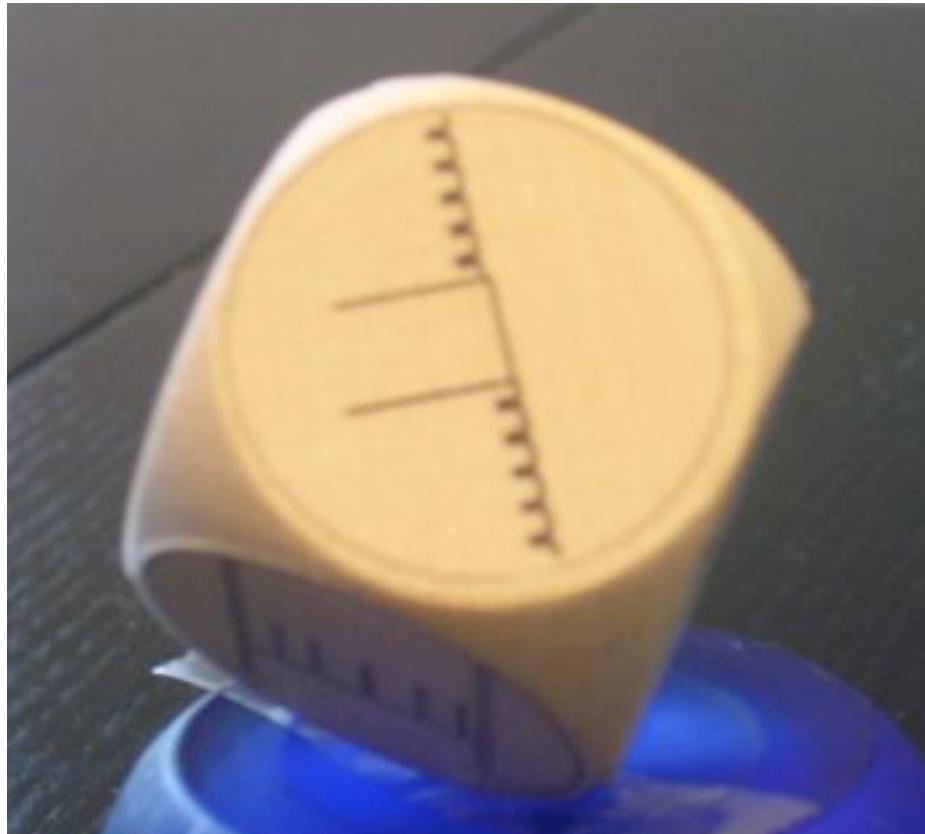
The school covers Monte Carlo techniques and automated calculation of matrix elements. The focus of the school is on Monte Carlo event generators for simulation of processes beyond the standard model. The simulation of QCD and electroweak "background" will also be covered. In practical exercises, BSM signal processes as well as standard-model background will be simulated and analyzed.

Registration deadline: 11.07.2009  
Please register via the school webpage.

http://www.terascale.de/school2009

# Typical Monte Carlo toys ...

The necessary tool for a true Monte Carlo event generator:



# Conclusion

- **Monte Carlo event generators** are needed to calculate multi-parton cross sections
- **Monte Carlo method** is a well defined procedure
  - parton shower are essential
  - hadronization is needed to compare with measurements
- **MC approach** extended from simple  $e+e^-$  processes to
  - $ep$  processes
  - $pp$  processes and **heavy Ion** processes
- proper **Monte Carlos** are essential for any measurement

**Monte Carlo event generators  
contain all our physics knowledge !!!!!**

# List of available MC programs

- HERA Monte Carlo workshop: [www.desy.de/~heramc](http://www.desy.de/~heramc)
- **ARIADNE**  
A program for simulation of QCD cascades implementing the color dipole model
- **CASCADE**  
is a full hadron level Monte Carlo generator for ep and pp scattering at small x build according to the CCFM evolution equation, including the basic QCD processes as well as Higgs and associated W/Z production
- **HERWIG**  
General purpose generator for Hadron Emission Reactions With Interfering Gluons; based on matrix elements, parton showers including color coherence within and between jets, and a cluster model for hadronization.
- **JETSET**  
The Lund string model for hadronization of parton systems.

# List of available MC programs

- **LDCMC**

A program which implements the Linked Dipole Chain (LDC) model for deeply inelastic scattering within the framework of ARIADNE. The LDC model is a reformulation of the CCFM model.

- **PHOJET**

Multi-particle production in high energy hadron-hadron, photon-hadron, and photon-photon interactions (hadron = proton, antiproton, neutron, or pion).

- **PYTHIA**

General purpose generator for  $e^+e^-$  pp and ep-interactions, based on LO matrix elements, parton showers and Lund hadronization.

- **RAPGAP**

A full Monte Carlo suited to describe Deep Inelastic Scattering, including diffractive DIS and LO direct and resolved processes. Also applicable for photo-production and partially for pp scattering.



# Literature & References

- F. James Rep. Prog. Phys., Vol 43, 1145 (1980)
- Glen Cowan STATISTICAL DATA ANALYSIS. Clarendon, 1998.
- Particle Data Book S. Eidelman et al., Physics Letters B592, 1 (2004)  
section on: Mathematical Tools (<http://pdg.lbl.gov/>)
- Michael J. Hurben Buffons Needle  
(<http://www.angelfire.com/wa/hurben/buff.html>)
- J. Woller (Univ. of Nebraska-Lincoln) *Basics of Monte Carlo Simulations*  
(<http://www.chem.unl.edu/zeng/joy/mclab/mcintro.html>)
- Hardware Random Number Generators:
  - A Fast and Compact Quantum Random Number Generator  
(<http://arxiv.org/abs/quant-ph/9912118>)
  - Quantum Random Number Generator  
(<http://www.idquantique.com/products/quantis.htm>)
  - Hardware random number generator (<http://en.wikipedia.org/wiki/>)
- Monte Carlo Tutorials  
(<http://www.cooper.edu/engineering/chemechem/MMC/tutor.html>)
- History of Monte Carlo Method  
(<http://www.geocities.com/CollegePark/Quad/2435/history.html>)

# Literature & References (cont'd)

- T. Sjostrand et al  
PYTHIA/JETSET manual - The Lund Monte Carlos  
<http://www.thep.lu.se/tf2/staff/torbjorn/Pythia.html>
- H. Jung  
[RAPGAP manual](http://www.desy.de/~jung/rapgap.html)  
<http://www.desy.de/~jung/rapgap.html>  
[CASCADE manual](http://www.desy.de/~jung/cascade.html)  
<http://www.desy.de/~jung/cascade.html>
- V. Barger and R. J.N. Phillips  
Collider Physics  
Addison-Wesley Publishing Comp. (1987)
- R.K. Ellis, W.J. Stirling and B.R. Webber  
QCD and collider physics  
Cambridge University Press (1996)

# General literature

- Many new books are available in DESY library **NEW ... ask at the desk there ...**
- Statistische und numerische Methoden der Datenanalyse  
V. Blobel & E. Lohrmann
- STATISTICAL DATA ANALYSIS. Glen Cowan.
- Particle Data Book S. Eidelman et al., Physics Letters B592, 1 (2004)  
(<http://pdg.lbl.gov/>)
- Applications of pQCD R.D. Field Addison-Wesley 1989
- Collider Physics V.D. Barger & R.J.N. Phillips Addison-Wesley 1987
- Deep Inelastic Scattering. R. Devenish & A. Cooper-Sarkar, Oxford 2
- Handbook of pQCD G. Sterman et al
- Quarks and Leptons, F. Halzen & A.D. Martin, J.Wiley 1984
- QCD and collider physics R.K. Ellis & W.J. Stirling & B.R. Webber Cambridge 1996
- QCD: High energy experiments and theory G. Dissertori, I. Knowles, M. Schmelling Oxford 2003

# Backup Slides

# W & Z cross sections

- Basic process: Drell - Yan

$$p + p \rightarrow l^+ + l^- + X$$

- Factorize process:

- $q + \bar{q} \rightarrow \gamma^* \rightarrow l^+ + l^-$

- and then

- $q + \bar{q} \rightarrow \gamma^*$

- $q + \bar{q} \rightarrow Z_0$

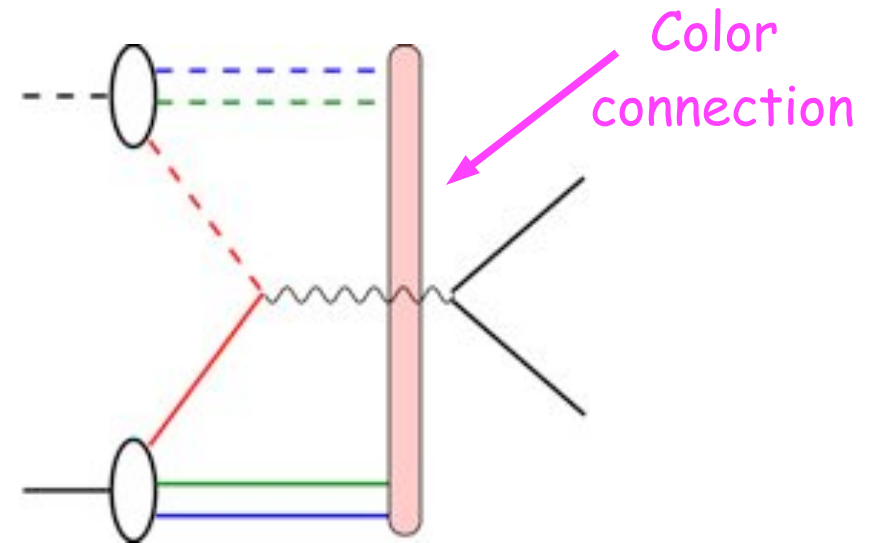
- $q + \bar{q}' \rightarrow W^\pm$

- and then

- $\gamma^* \rightarrow l^+ + l^-$

- $Z_0 \rightarrow l^+ + l^-$

- $W^\pm \rightarrow l + \nu$



- We need

- PDFs

- hard scattering

- Decays

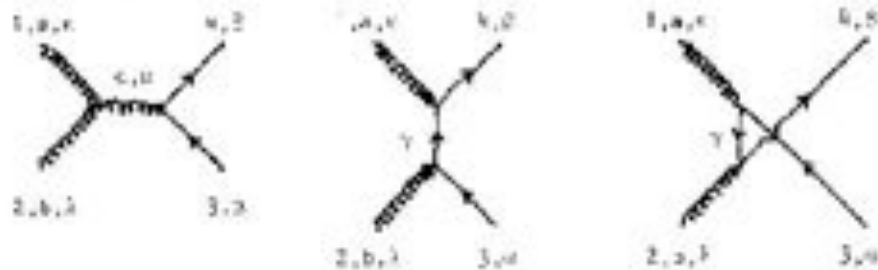
- remnant treatment

# Color Flow in pp

The Lund Monte Carlo For High P(T) Physics H.U. Bengtsson  
Comput.Phys.Commun.31:323,1984.

Process:  $gg \rightarrow q, \bar{q}$

Diagrams:



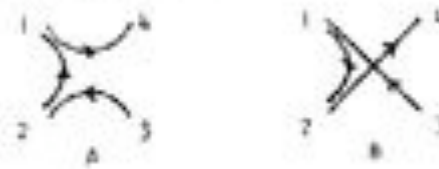
Amplitudes:

$$s: g^2 f^{abc} T_{ar}^c \bar{u}_r^b(q_4) \frac{t_1^a t_2^b \gamma^a}{s} C_{ab\mu} (q_1, q_2, -q_1 - q_2) u_1^c(q_3)$$

$$t: -ig^2 T_{ar}^b T_{cb}^a \bar{u}_r^c(q_4) \epsilon_1 \frac{q_1 - q_4}{t} \epsilon_2 u_1^a(q_3)$$

$$u: -ig^2 T_{ar}^a T_{cb}^b \bar{u}_r^c(q_4) \epsilon_2 \frac{q_2 - q_4}{u} \epsilon_1 u_1^a(q_3)$$

Colour flows:



String configurations:



Colour factors: A:  $T_{ar}^b T_{cb}^a$ ; B:  $T_{ar}^a T_{cb}^b$

Amplitudes:

$$A: -ig^2 \bar{u}_r^c(q_4) \left[ \epsilon_1 \frac{q_1 - q_4}{t} \epsilon_2 - \frac{t_1^a t_2^b \gamma^a}{s} C_{ab\mu} (q_1, q_2, -q_1 - q_2) \right] u_1^c(q_3)$$

$$B: -ig^2 \bar{u}_r^c(q_4) \left[ \epsilon_2 \frac{q_2 - q_4}{u} \epsilon_1 + \frac{t_1^a t_2^b \gamma^a}{s} C_{ab\mu} (q_1, q_2, -q_1 - q_2) \right] u_1^c(q_3)$$

Cross-sections:

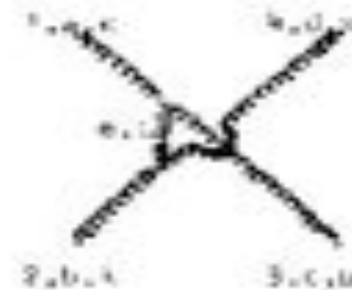
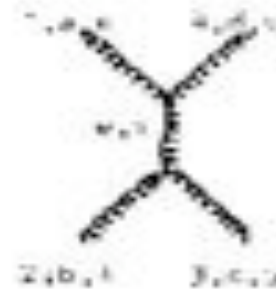
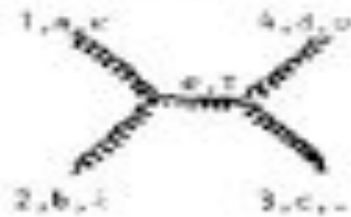
$$A: \frac{\pi\alpha_s^2}{s^2} \left[ \frac{u}{t} - 2 \frac{u^2}{t^2} \right]; \quad B: \frac{\pi\alpha_s^2}{s^2} \left[ \frac{t}{u} - 2 \frac{t^2}{u^2} \right]$$

# Color Flow in pp

The Lund Monte Carlo For High P(T) Physics H.U. Bengtsson  
Comput.Phys.Commun.31:323,1984.

Process:  $gg \rightarrow gg$

Diagrams:



Amplitudes:

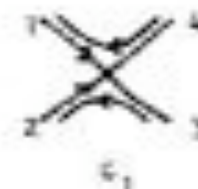
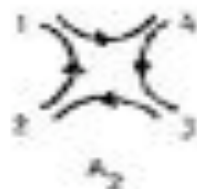
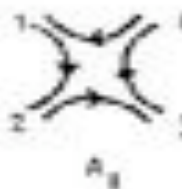
$$s: -ig^2 \frac{1}{s} f^{abc} f^{abd} \epsilon_1^a \epsilon_2^b \epsilon_3^c \epsilon_4^d C_{\lambda\mu\nu}(-q_1, -q_2, q_3, q_4) C_{\rho\sigma\tau}(q_3, q_4, -q_2, -q_1)$$

$$t: -ig^2 \frac{1}{t} f^{abc} f^{abd} \epsilon_1^a \epsilon_2^b \epsilon_3^c \epsilon_4^d C_{\lambda\mu\nu}(q_1, -q_2, q_3, -q_4) C_{\rho\sigma\tau}(-q_2, -q_3, q_1, q_4)$$

$$u: -ig^2 \frac{1}{u} f^{abc} f^{abd} \epsilon_1^a \epsilon_2^b \epsilon_3^c \epsilon_4^d C_{\lambda\mu\nu}(q_1, -q_2, q_1, -q_3) C_{\rho\sigma\tau}(-q_2, -q_4, q_2, -q_3)$$

$$4: -ig^2 f^{abc} f^{abd} (S_{\mu\nu} S_{\lambda\rho} - R_{\mu\nu} R_{\lambda\rho}) + ig^2 f^{abc} f^{abd} (S_{\mu\nu} S_{\lambda\rho} - R_{\mu\nu} R_{\lambda\rho})$$

Colour Flows:

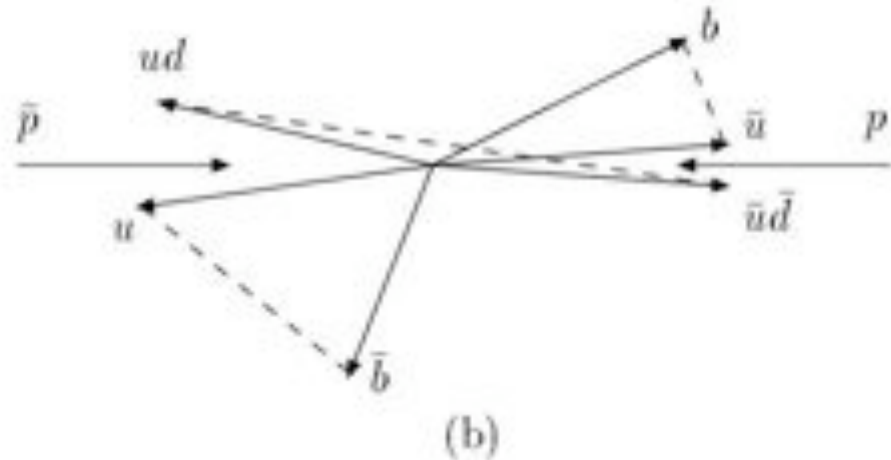
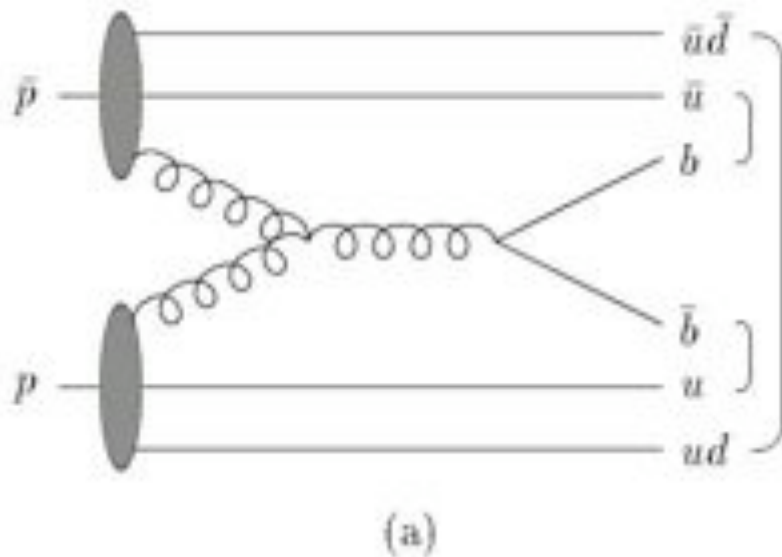


# Color Flow in

$$p\bar{p} \rightarrow b\bar{b} + X$$

B physics at the Tevatron: Run II and beyond  
E. Norrbin, hep-ph/0201071,p522

$$p\bar{p} \rightarrow b\bar{b} + X$$



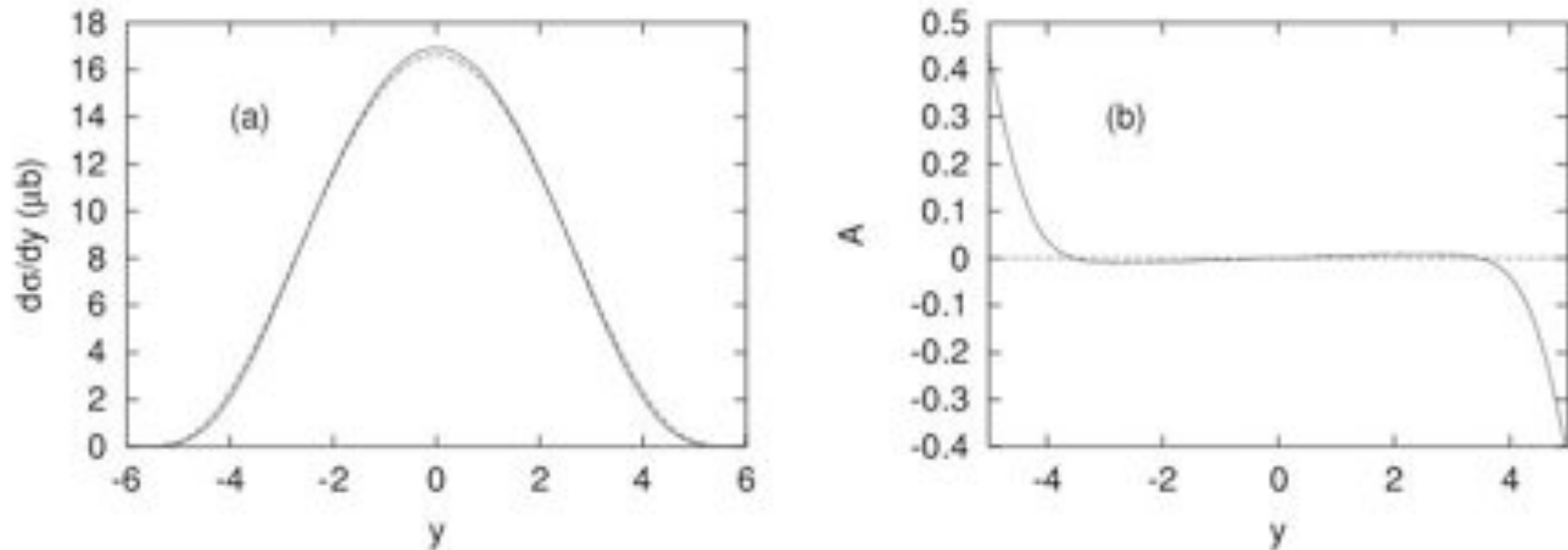
**Figure 9.45:** Example of a string configuration in a  $p\bar{p}$  collision. (a) Graph of the process, with brackets denoting the final color singlet subsystems. (b) Corresponding momentum space picture, with dashed lines denoting the strings.



# Beam - drag effect

- Due to color connection of produced b-quark with beam remnants, the rapidity distribution of b-quarks and B-hadrons is different.
- Asymmetry of  $B^0\bar{B}^0$

B physics at the Tevatron: Run II and beyond  
E. Norrbin, hep-ph/0201071,p525



**Figure 9.47:** Bottom production at the Tevatron. (a) Rapidity distribution of bottom quarks (full) and the B hadrons produced from them (dashed). (b) The asymmetry  $A = \frac{\sigma(B^0) - \sigma(\bar{B}^0)}{\sigma(B^0) + \sigma(\bar{B}^0)}$  as a function of rapidity. For simplicity, only pair production is included.

- **HowTo connect this to factorised fragmentation functions ?**

Is that now all ?

But with high parton  
densities,  
do we only have one  
interaction ?

# How many gluons are there ?

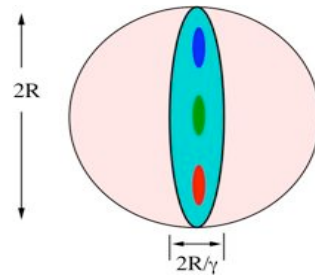
- number of gluons in long. phase space  $dx/x$ :

$$xg(x, \mu^2)dx/x$$

- occupation area:

nr of gluons  $\times$  (trans size)<sup>2</sup>

$$g(x, \mu^2) \frac{1}{\mu^2}$$



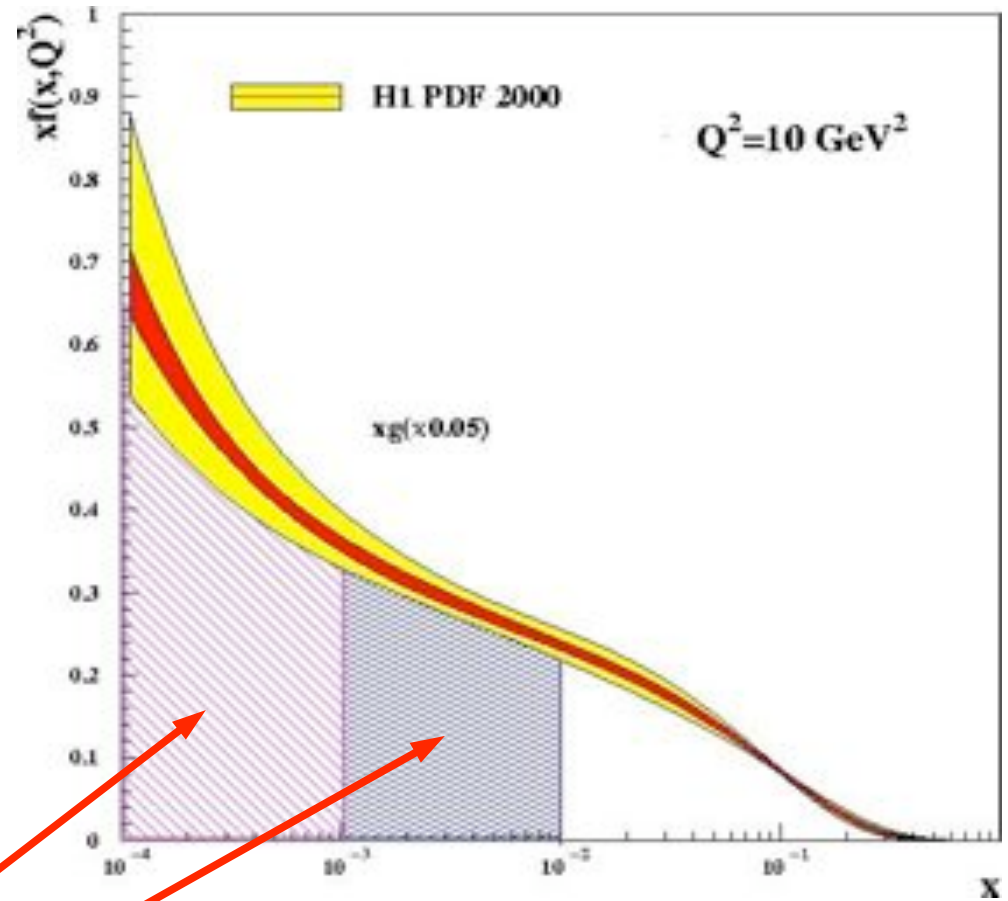
- saturation starts when:

$$\frac{\alpha_s(\mu^2)}{\mu^2} xg(x, \mu^2) \frac{dx}{x} \geq \pi R^2$$

- gluon density is very large: ~ 90 or 45 Gluons !!!!!

- with  $R \sim 1 \text{ GeV}^{-1}$  we obtain:

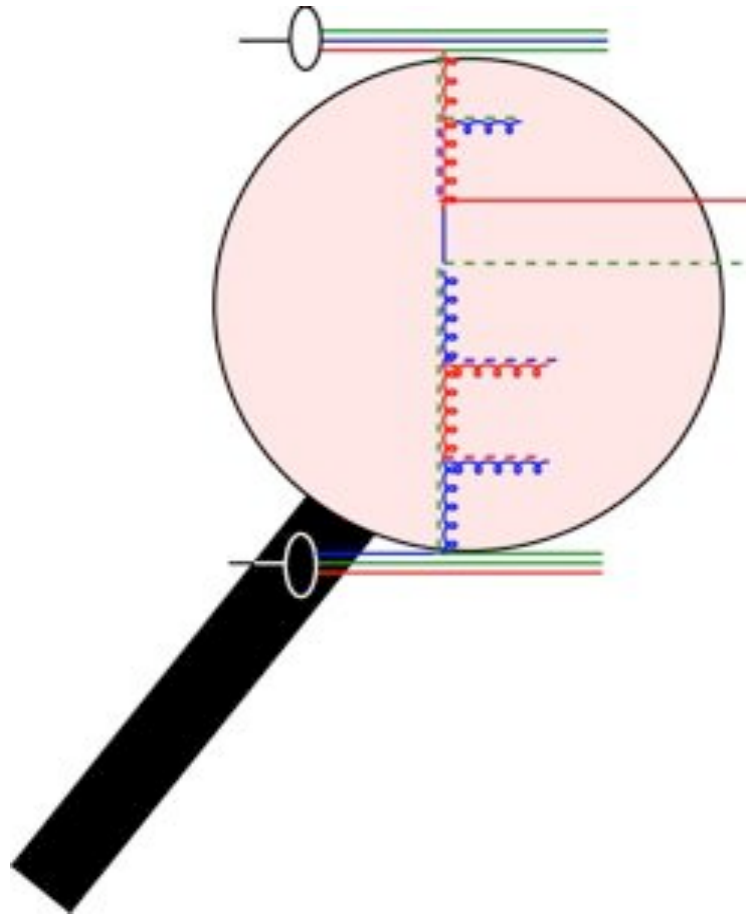
$$\frac{0.2}{10 \text{ GeV}^2} 100 \sim \pi R^2 \sim \pi \text{ !!!!!}$$



# Partonic Cross sections

- Cross section

$$\sigma(p_1 + p_2 \rightarrow j_1 + j_2 + X) = f(x_1, \mu^2) \otimes \hat{\sigma}(x_1 p_1 + x_2 p_2 \rightarrow j_1 + j_2) \otimes f(x_2, \mu^2)$$



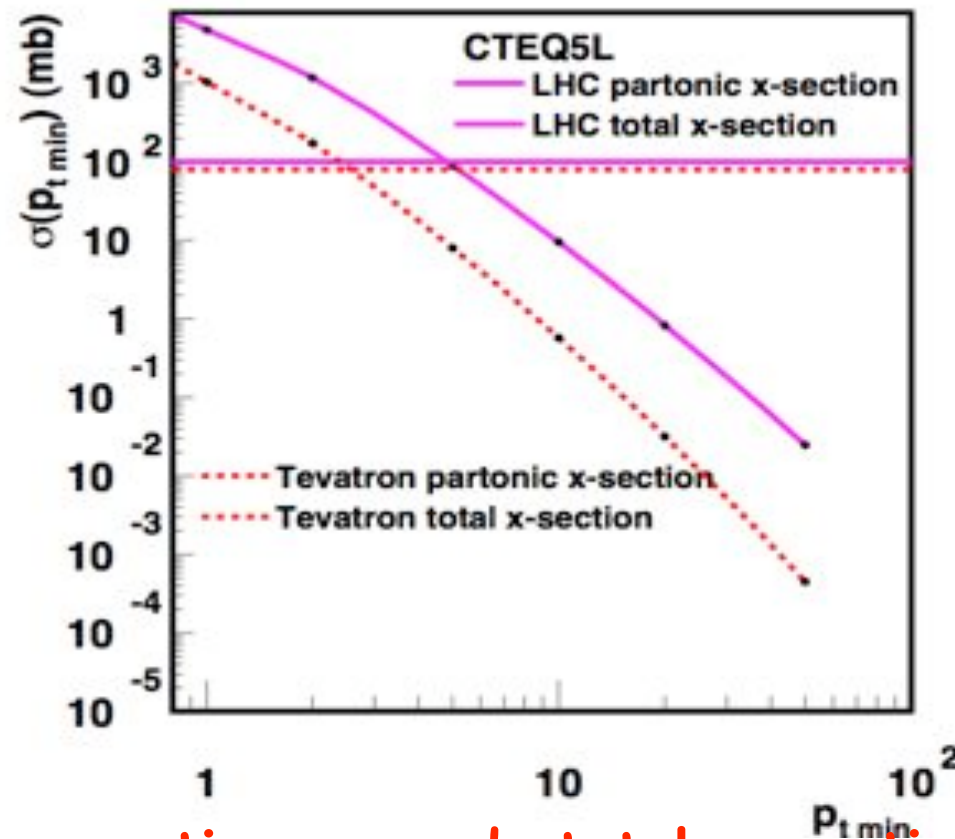
- partonic cross section diverges with  $p_{\perp}$
- calculate x-section as function of  $p_{\perp \min}$

$$\sigma_{\text{hard}}(p_{\perp \min}^2) = \int_{p_{\perp \min}^2} \frac{d\sigma_{\text{hard}}(p_{\perp}^2)}{dp_{\perp}^2} dp_{\perp}^2$$

# Partonic Cross sections

$$\sigma_{\text{hard}}(p_{\perp \text{min}}^2) = \int_{p_{\perp \text{min}}^2} \frac{d\sigma_{\text{hard}}(p_{\perp}^2)}{dp_{\perp}^2} dp_{\perp}^2$$

- Cross section at Tevatron/LHC



→ Partonic x-section exceeds total x-section !!!

→ with HERA PDFs .... at larger values of  $p_{t \text{min}}$  !!!!!

# Underlying event - Multiple Interaction

- Basic partonic perturbative cross section

$$\sigma_{\text{hard}}(p_{\perp\text{min}}^2) = \int_{p_{\perp\text{min}}^2} \frac{d\sigma_{\text{hard}}(p_{\perp}^2)}{dp_{\perp}^2} dp_{\perp}^2$$

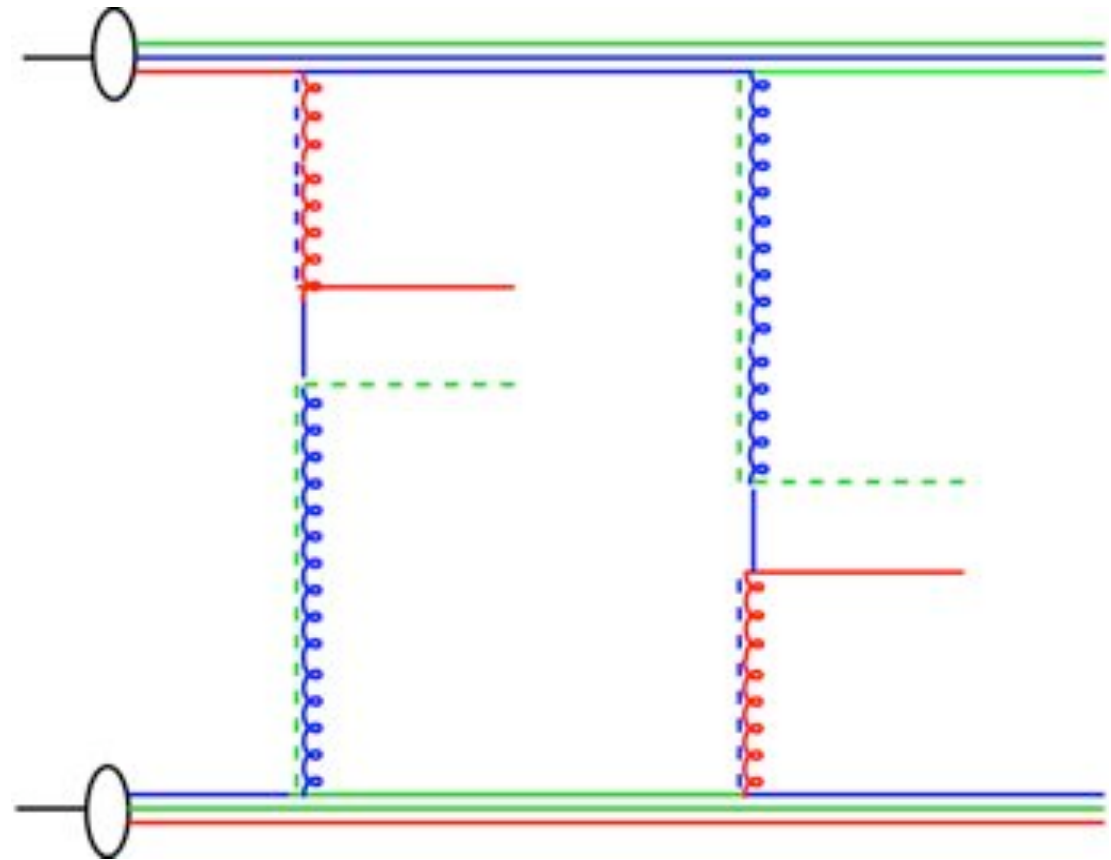
- diverges faster than  $1/p_{\perp\text{min}}^2$  as  $p_{\perp\text{min}} \rightarrow 0$  and exceeds eventually total inelastic (non-diffractive) cross section

HELP ....

HOW to solve this ?

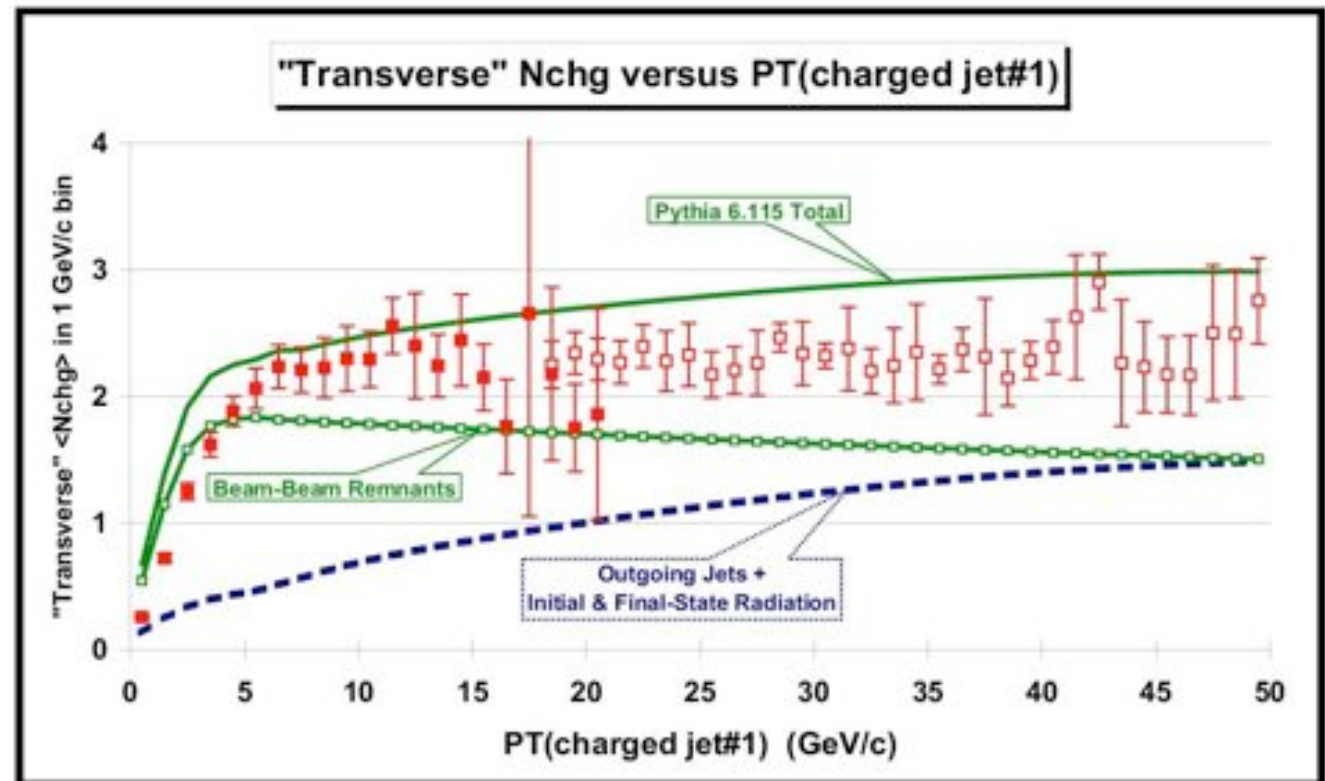
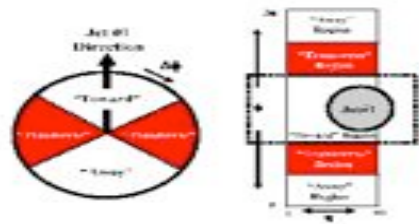
# Models for Multi-Parton Interaction

- The very simple model
  - add secondary interactions
  - first model by: T. Sjostrand,  
M. Ziji PRD 36 (1987) 2019



# Multiparton Interactions at TeVatron

CDF coll. PRD 65, 092002 (2002)



- Multiplicity distribution in region transverse to jet can only be described by adding multi-parton interactions (Remnant-Remnant Interactions)



# Tuning to pp data... Color flow in MI

- possible scenarios for color string connection in multiparton events
- to describe underlying events... need (CDF Tune A)

5 % quarks (default 33 %)

95 % gluons (default: 66%)

out of which 90 %  
(default 33 %) are

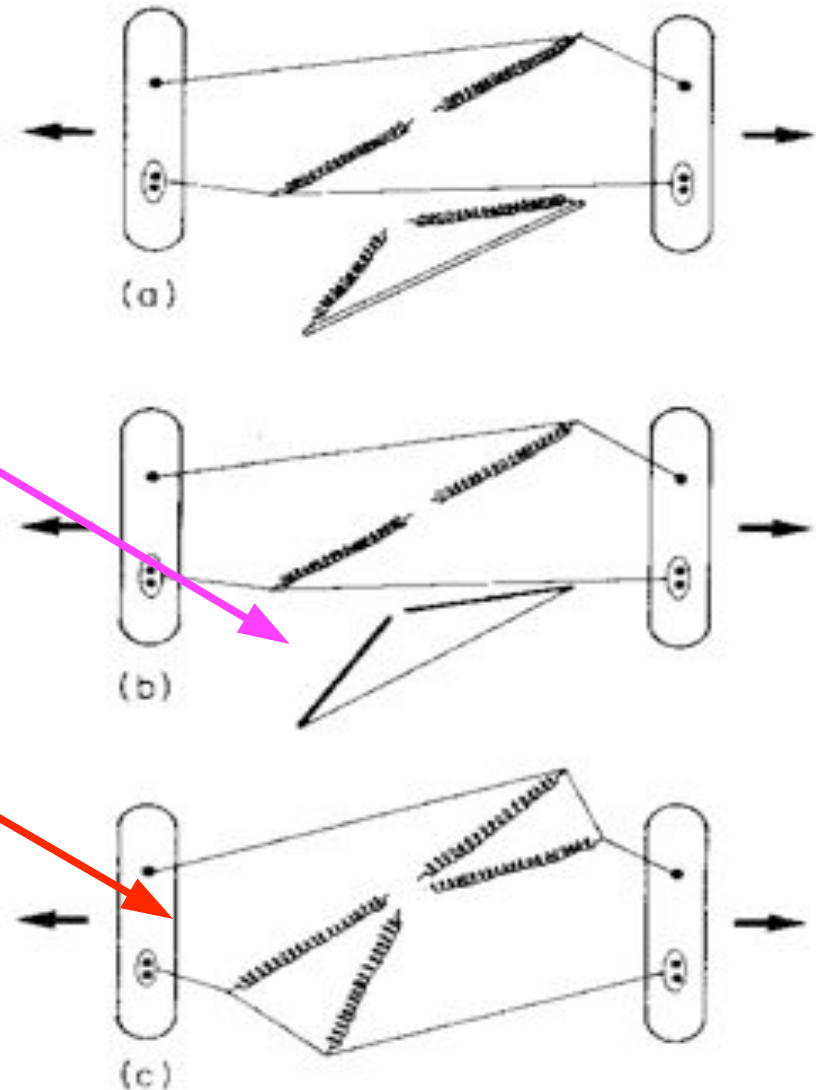
- smaller multiplicity  
with large transverse energy

- Are there good physics reasons for this mix ???

- Highly nontrivial to describe multiplicity AND transverse

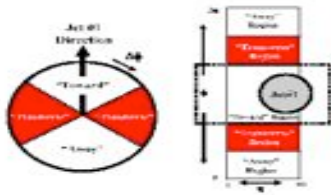
energy distributions ...

T. Sjostrand, M. Ziji  
PRD 36 (1987) 2019



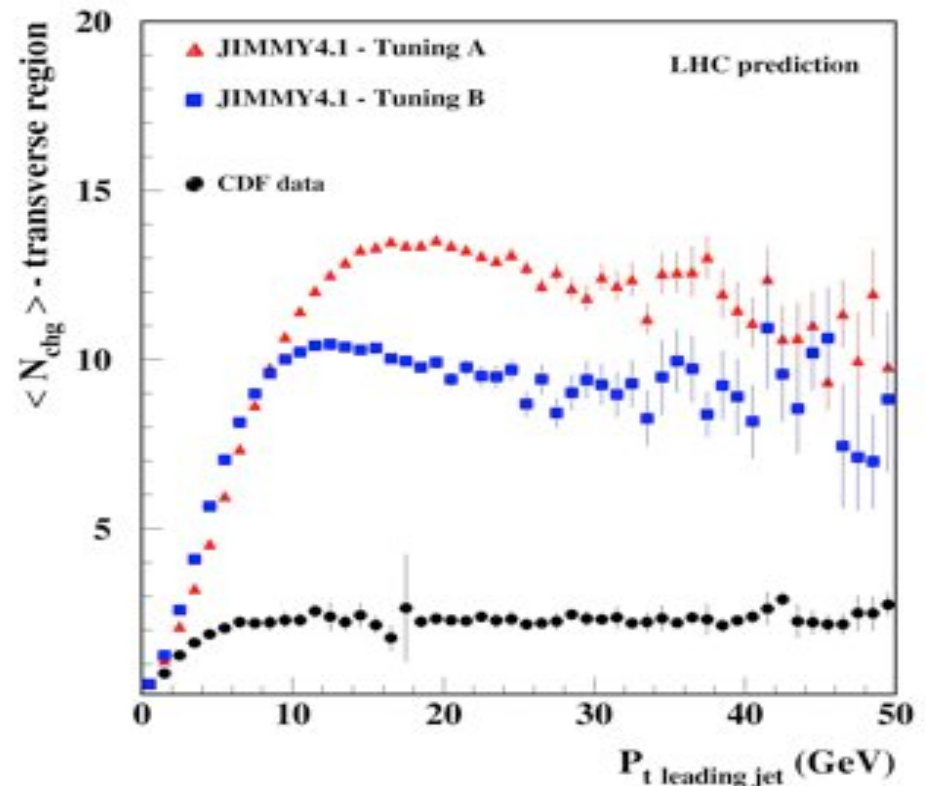
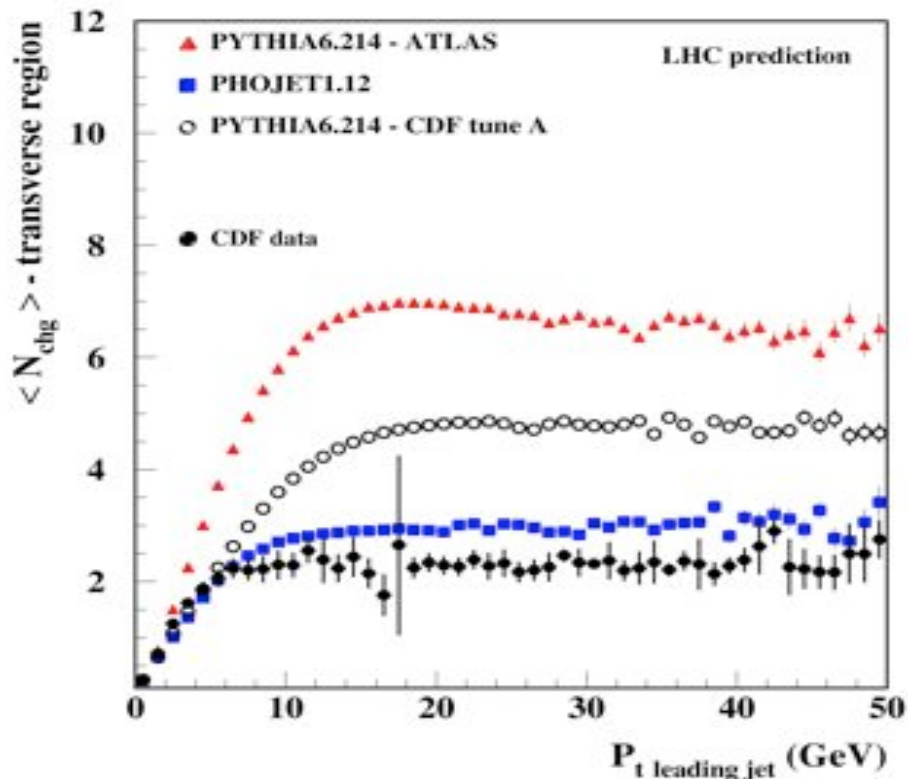
# Multiparton Interactions at LHC

C. Buttar et al in HERA – LHC workshop proceedings hep-ph/0601012

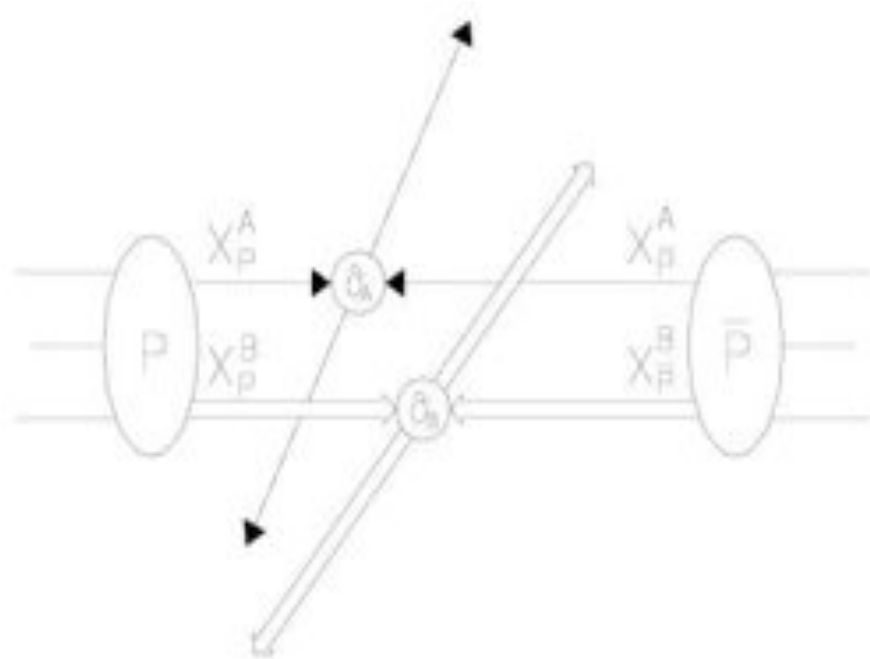


## Charged multiplicities in transverse region

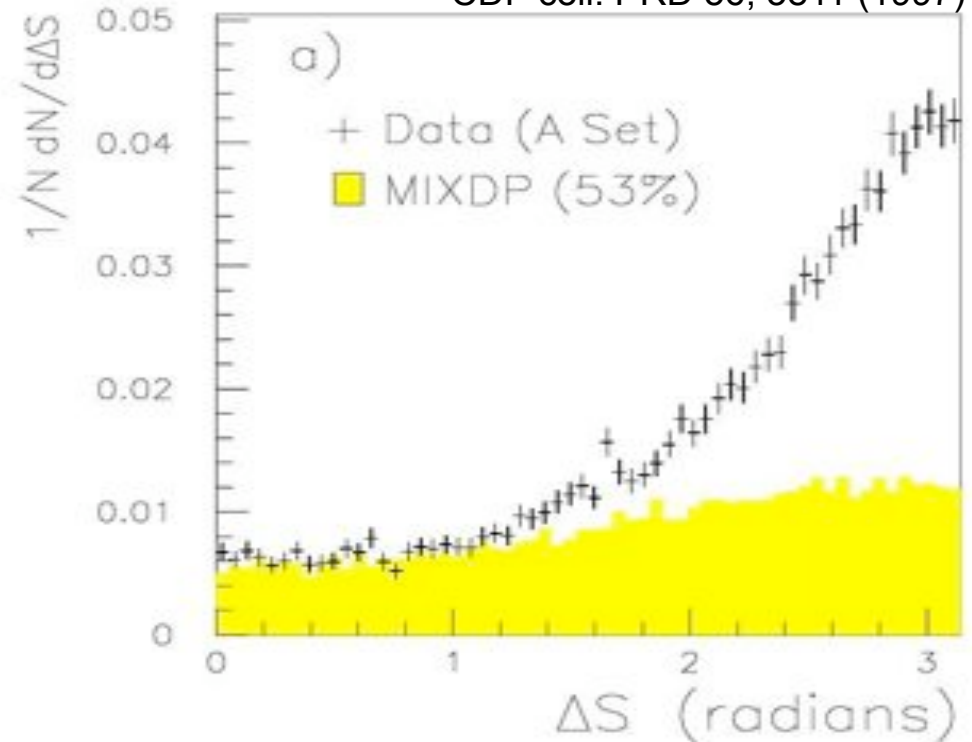
- Models tuned to TeVatron data
- give **HUGE** differences at LHC ...
- **better understand multiple interactions ...**



# Evidence for Multi-Parton Interactions



CDF coll. PRD 56, 3811 (1997)

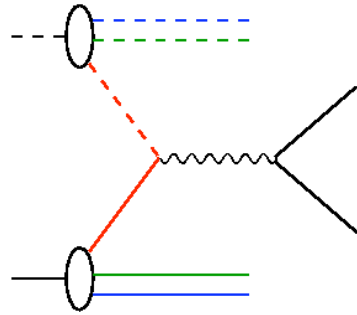


- look at  $\gamma + 3$  Jets  
with  $E_T^\gamma > 16\text{GeV}$   
 $E_T^{\text{Jets}} > 5\text{GeV}$
- angular correlation of jet/photon pairs  $\Delta S$
- compare to  $\gamma + 3$  Jets calculation
- **Need > 50 % double parton interaction to describe data**

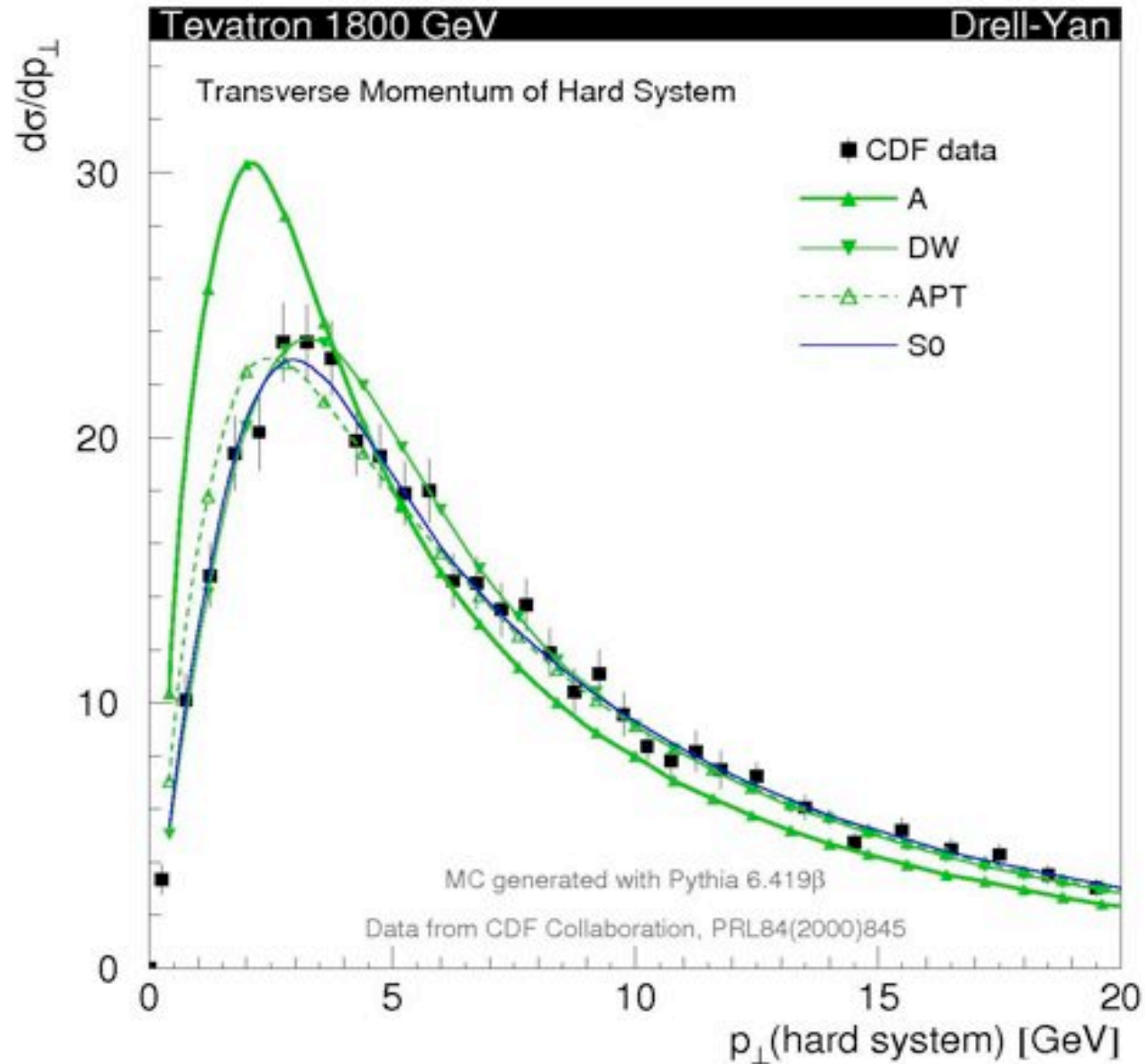
Does it matter  
for high pt  
physics ?

# Drell Yan process is affected ...

P. Skands, MPI@LHC, Perugia 2008



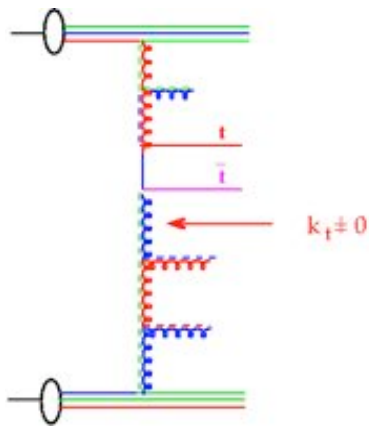
- $P_{\perp}$  of Drell Yan is affected by parton shower BUT also by the underlying events ....
- significant effects
- how to tune the truth ?



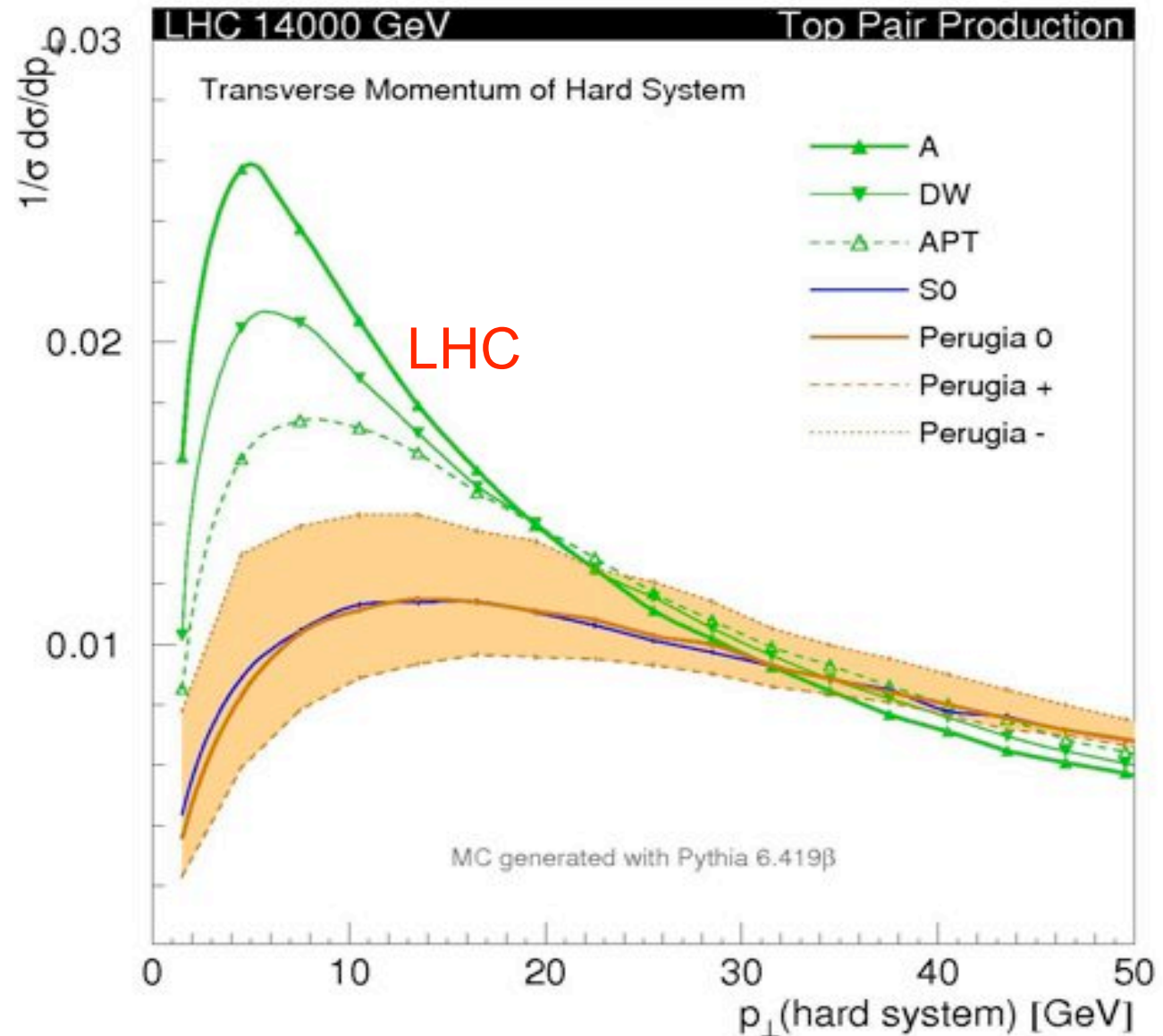
# $t\bar{t}$ is also affected ...

P. Skands, MPI@LHC, Perugia 2008

- $P_{\perp}$  of  $t\bar{t}$  is affected by parton shower BUT also by the underlying events ...



- note:  $p_{\perp}$  of the pair is plotted !!!
- HUGE effects**

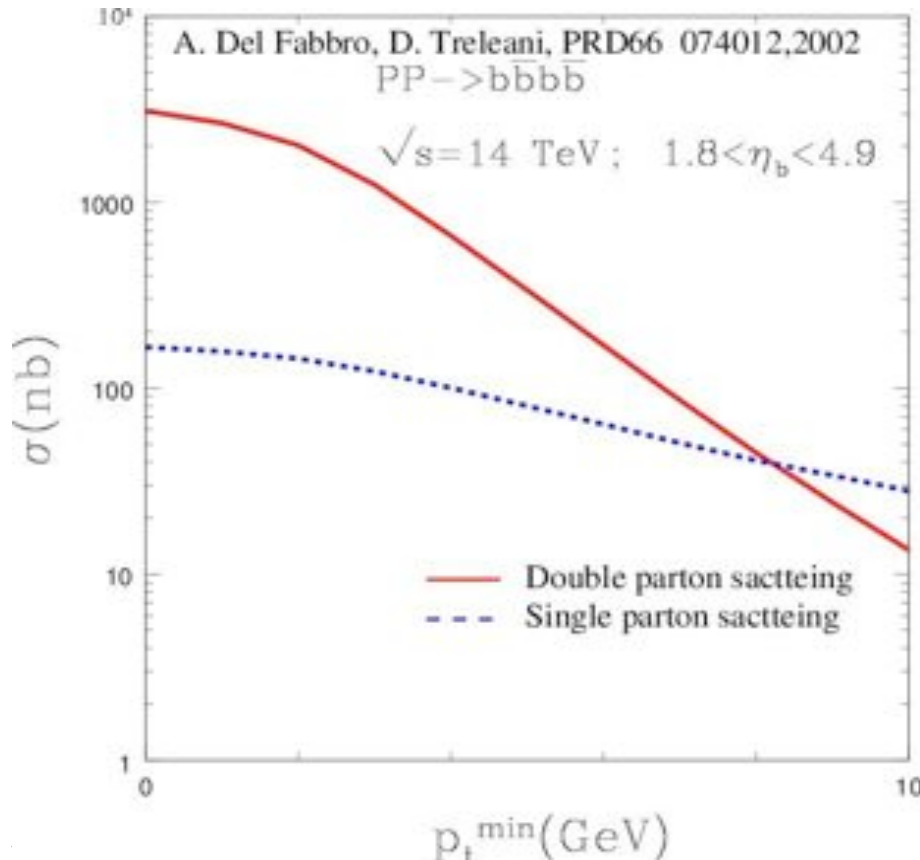


# Double-Parton Interactions at LHC

- xsection for  $p + p \rightarrow b\bar{b}b\bar{b}$ 
  - single parton exchange (SP)  
 $\sigma^{SP} \sim f^2 \hat{\sigma}(2 \rightarrow 4)$
  - double parton exchange (DP)  
 $\sigma^{DP} \sim f^4 \hat{\sigma}^2(2 \rightarrow 2)$

- PYTHIA predictions:  
 $\sigma^{DP} = 0.8 \dots 11.1 \mu b$

→ Depending on model for underlying event/multi-parton interactions...



# Multi-Parton Interactions at LHC

- Higgs:  $p + p \rightarrow W + H + X$

with  $W \rightarrow l\nu$ ,  $H \rightarrow b\bar{b}$

- Double parton scattering:

→  $p + p \rightarrow b\bar{b}X$

$p + p \rightarrow W + X$

$p + p \rightarrow W + b\bar{b} + X$

