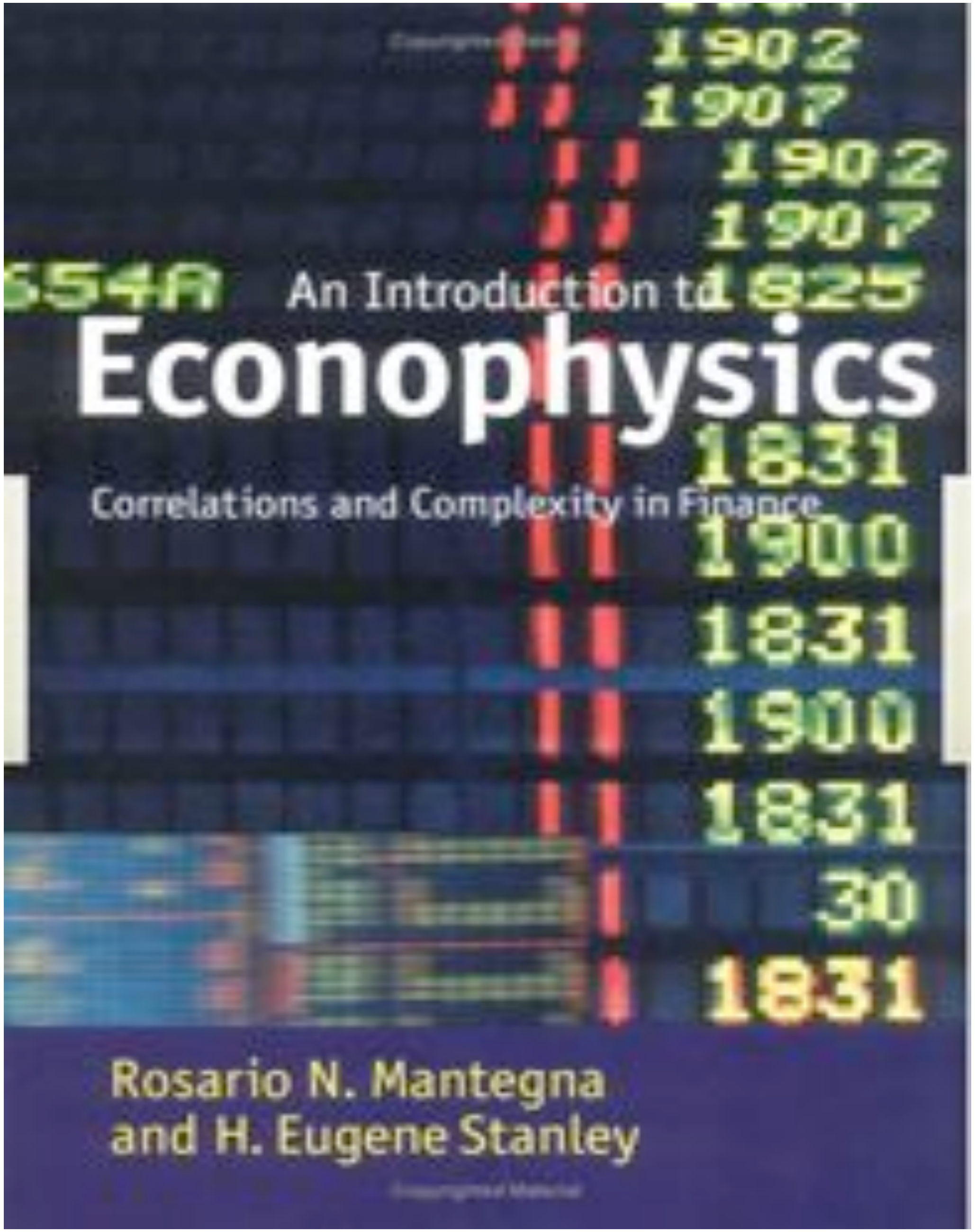
Simulations

H. Jung (DESY, University Antwerp)

LHC rap

From gambling ...

. . . . to


1902
1907
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654A
An Introduction to
1825
Econophysics
1831
Correlations and Complexity in Finance
1900
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1900
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30
1831
Rosario N. Mantegna
and H. Eugene Stanley

... via applications in risk management

Why do we need all that ?

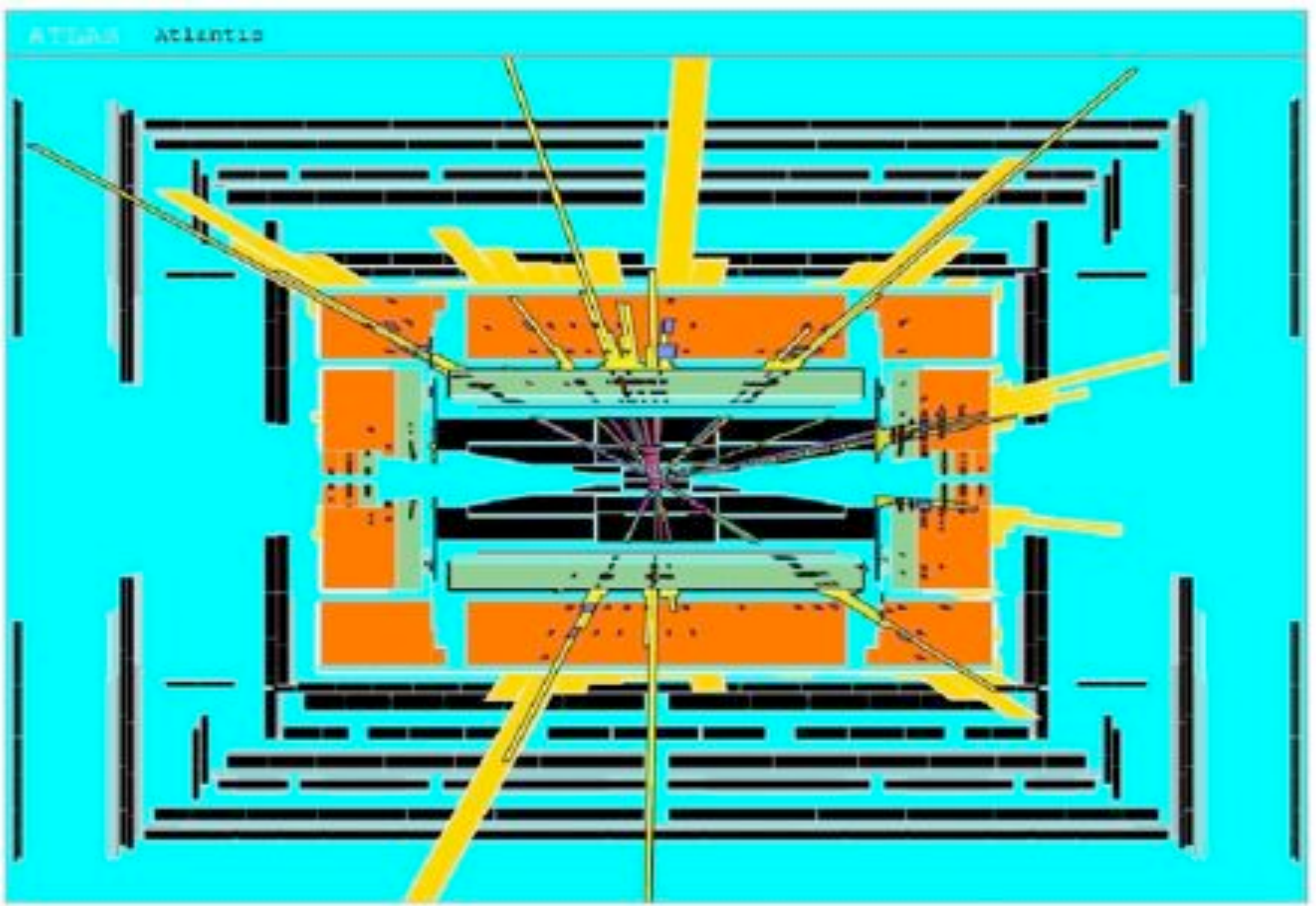
- because physics and life is more complicated than a simple equation, which can be solved analytically ....

- Monte Carlo techniques are
  - widely used
  - are of enormous advantages
  - can be used to simulate any complicated process
  - are now EVEN used in particle physics theory ...... !!!!! ....

… and in particle physics ?

Do we also need simulations ?

What is this ?



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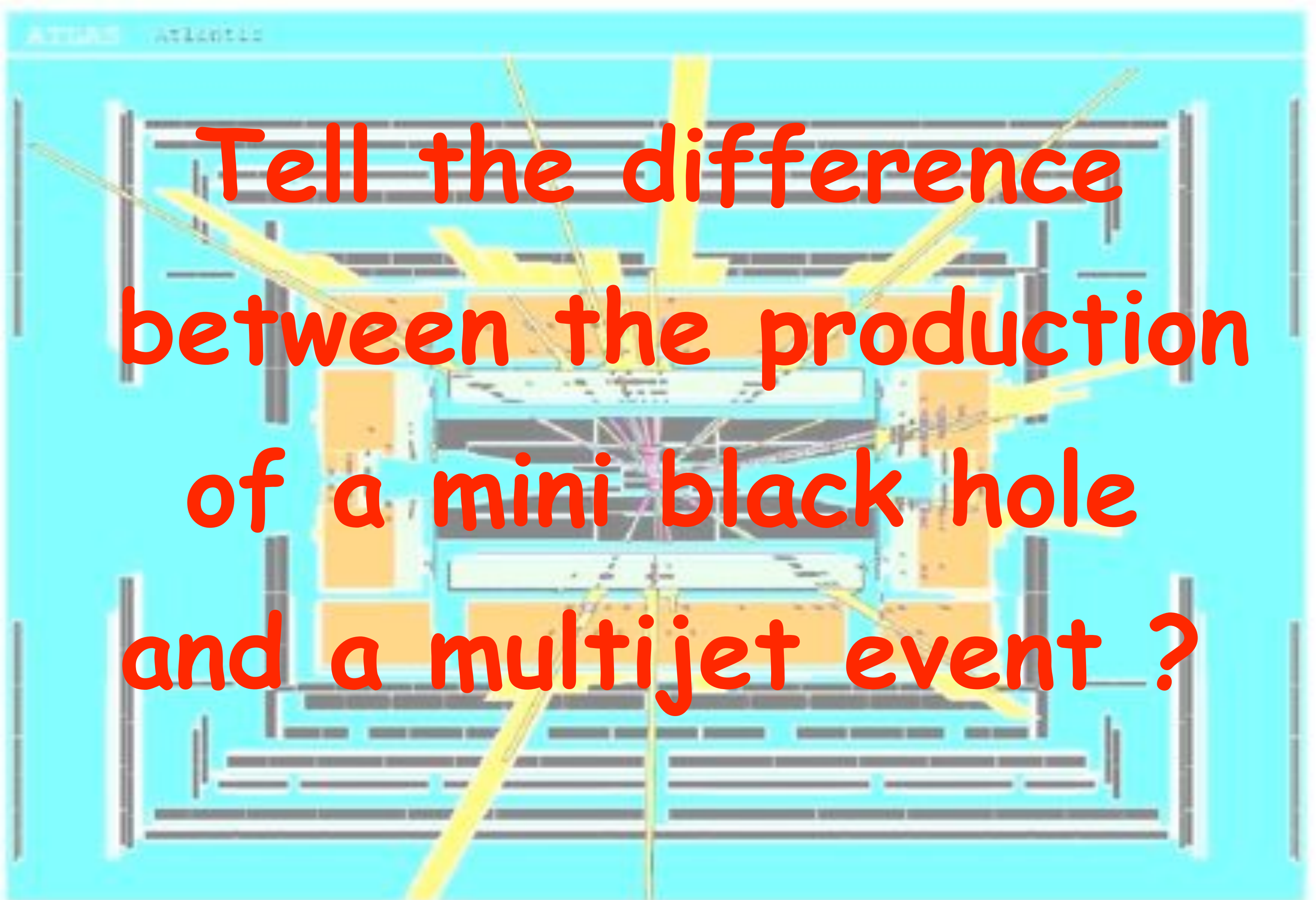
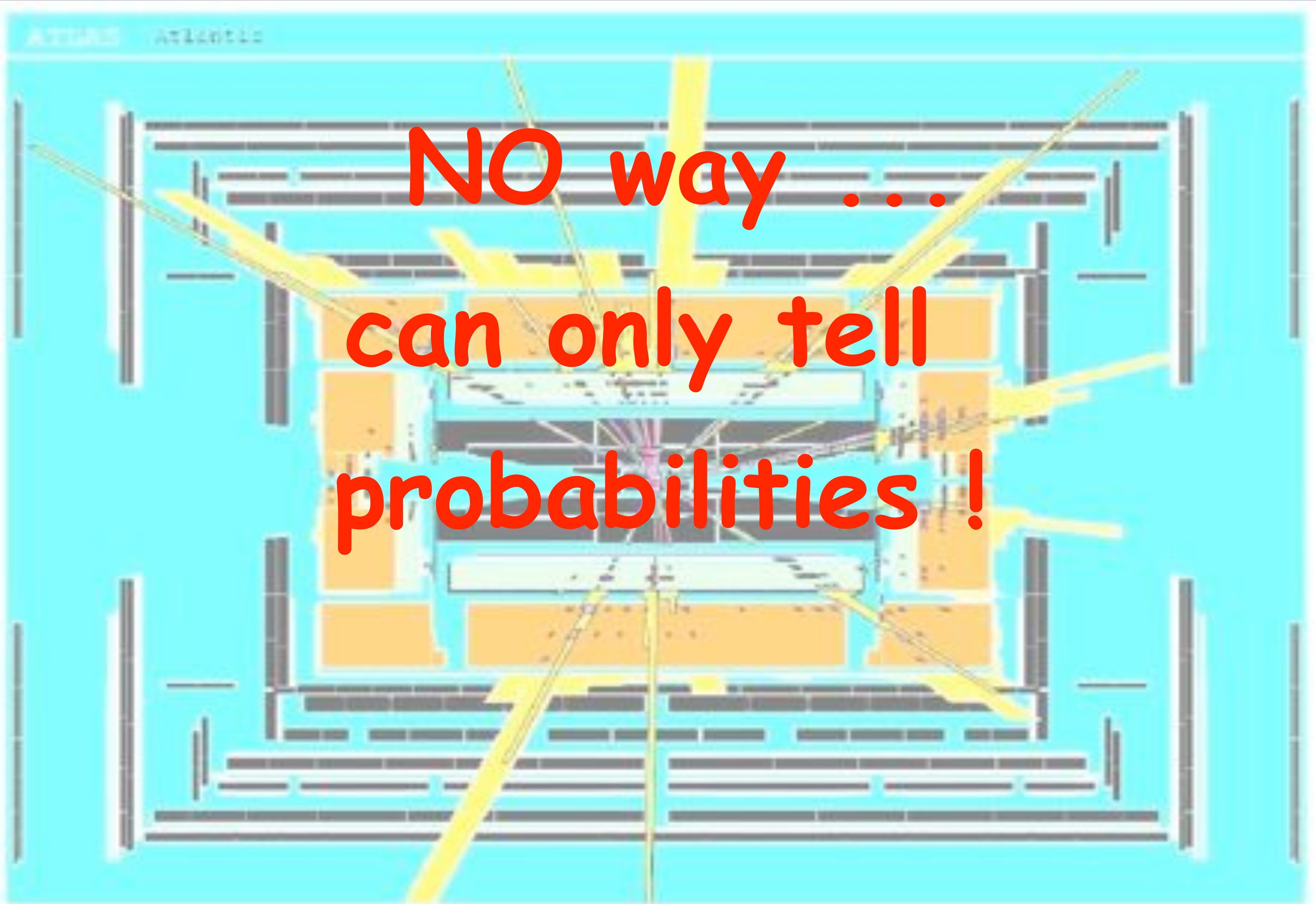


Tell the difference between the production of a mini black hole and a multijet event ?

What is this ?

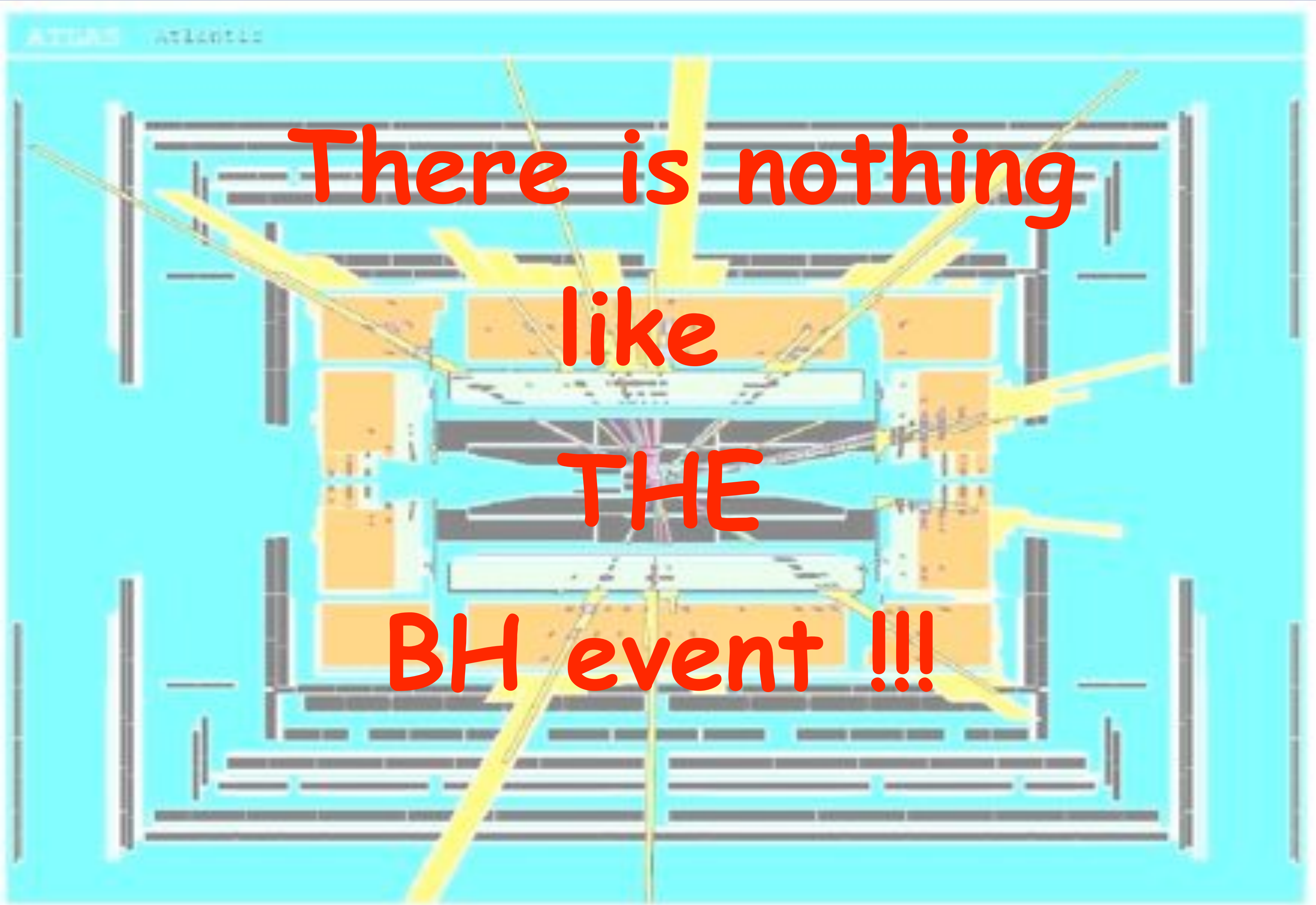


NO way ...

can only tell

probabilities !

What is this ?



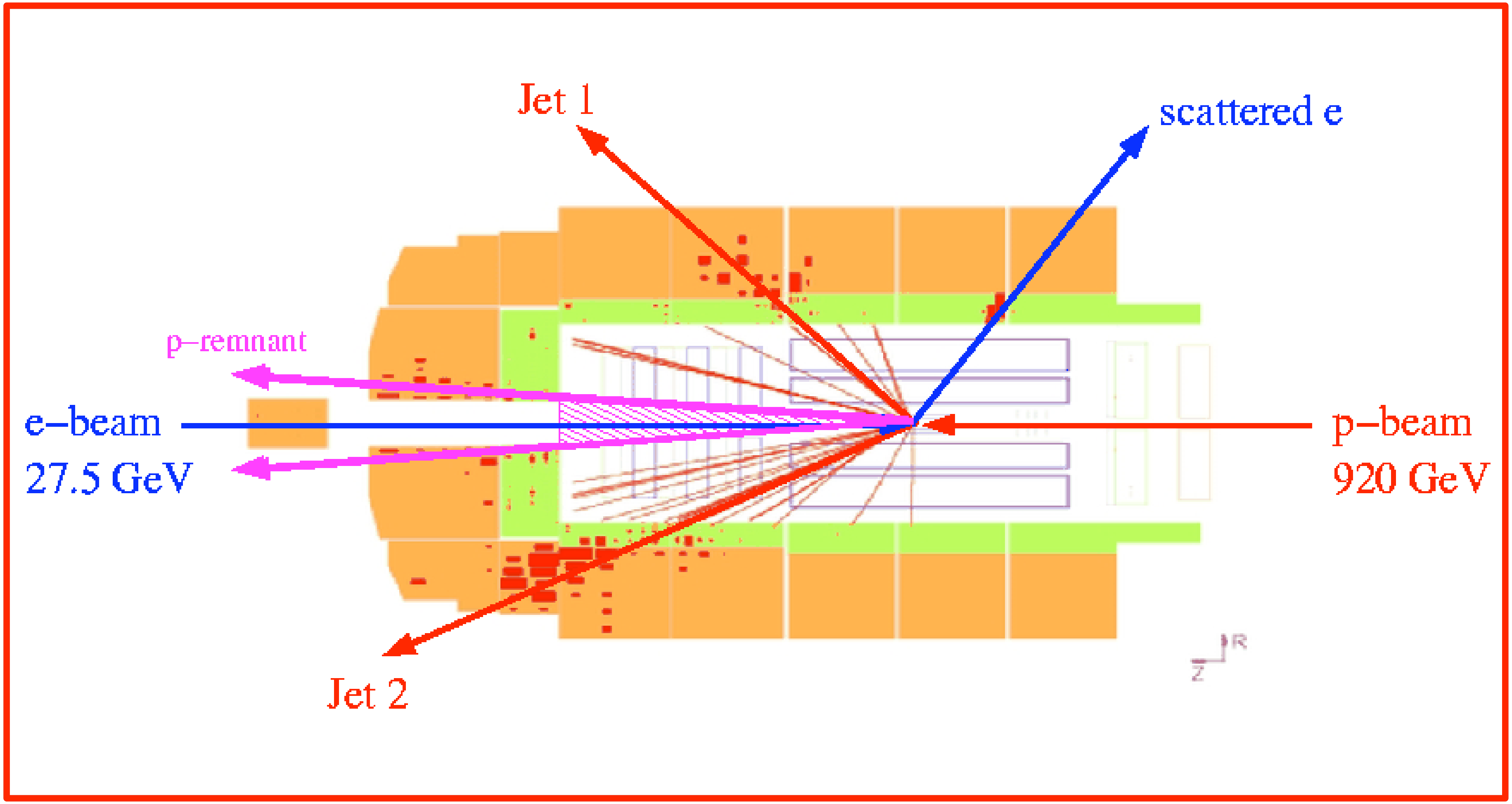
There is nothing

like

THE

BH event !!!

Events at HERA ...

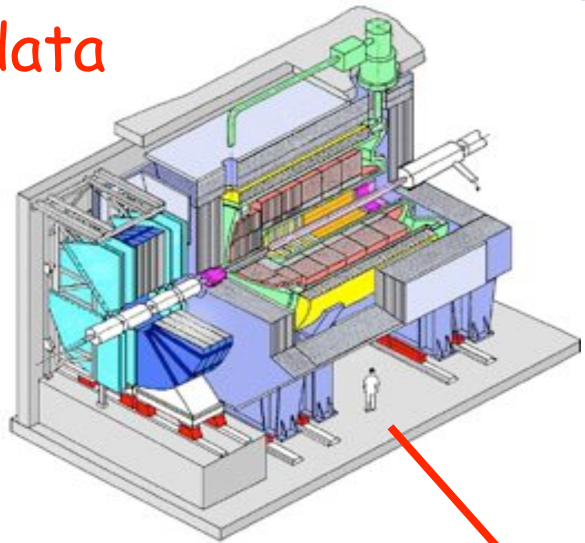


$\sqrt{s} \sim 318$ GeV

$x \sim 7.\ 10^{-5}$ at $Q^2 = 4$ GeV^2

From experiment to measurement

take data



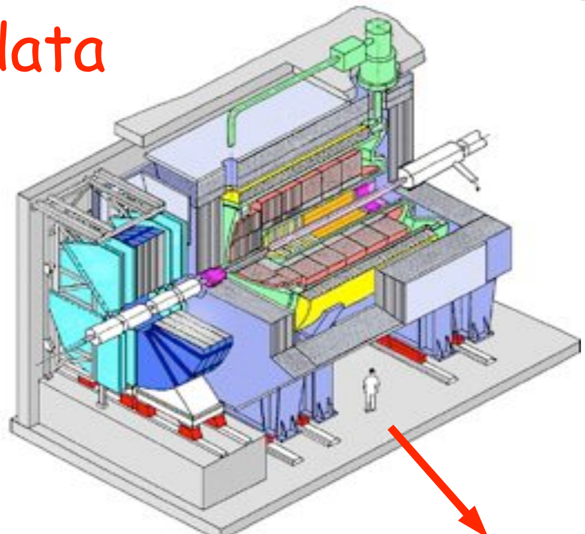
- run MC generator
- detector simulation

Compare measurement with simulation

Upppps ...... all measurements rely on proper MC generators and MC simulation !!!!

From experiment to measurement

take data



- run MC generator
- detector simulation

define visible x - section in kinematic variables

calculate factor C_{corr} to correct from detector to hadron level

$$\frac{d\sigma^{data}_{had}}{dx} = \frac{d\sigma^{data}_{det}}{dx} C_{corr} \text{ with } C_{corr} = \frac{\frac{d\sigma^{MC}_{had}}{dx}}{\frac{d\sigma^{MC}_{det}}{dx}}$$

visible x-section on hadron level

Upppps ...... all measurements rely on proper MC generators and MC simulation !!!!

Monte Carlo – different applications

- MC simulation of detector response
  - input: hadron level events - output: detector level events
  - Calorimeter ADC hits
  - Tracker hits
  - need knowledge of particle passage through matter, x-section ...
  - need knowledge of actual detector

Monte Carlo – different applications

- MC simulation of detector response
  - input: hadron level events - ou[illegible]tor level events
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  - need knowledge of actual detector

Not covered here

- multipurpose MC event generators:
  - x-section on parton level
  - including multi-parton (initial & final state) radiation
  - remnant treatment (proton remnant, electron remnant)
  - hadronization/fragmentation (more than simple fragmentation functions...)
- fixed order parton level ...... theorists like it !!!!!!!!!!!!!!!!!!!!!!!!
  - integration of multidimensional phase space

What this is about ...

From

$$e^+e^- \rightarrow e^+e^-$$

via

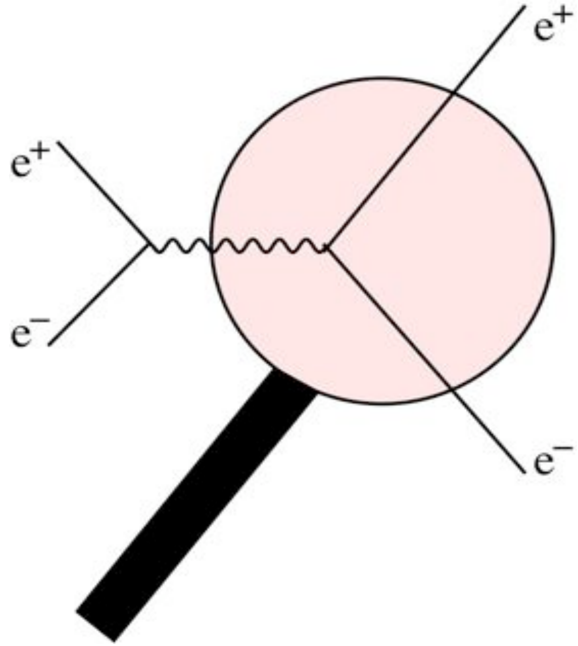
$$ep \rightarrow e'X$$

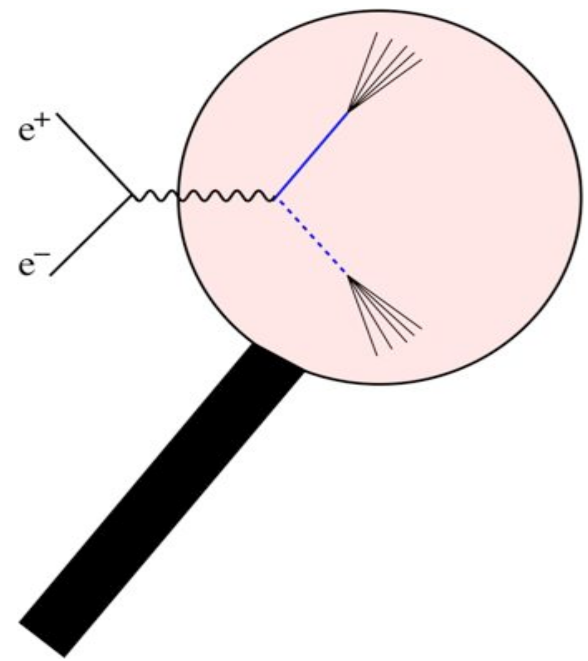
to

$$pp \rightarrow h + X$$

The easy case: $e^+e^- \rightarrow X$

- use $e^+e^- \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow q\bar{q} \rightarrow \text{hadrons}$





- cross sections can be calculated in QED: $\sigma(e^+e^- \rightarrow l^+l^-) = \frac{4\pi\alpha^2}{3s}$
- and for quarks $\sigma(e^+e^- \rightarrow q\bar{q}) = 3\,\frac{4\pi\alpha^2}{3s}e_q^2$

➔ but quarks carry color and fractional charge !!!!!

color (→ 3) charge (→ e_q^2)

The easy case: $e^+e^- \to X$

- measure ratio of hadronic / leptonic cross section

$$R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e- \to \mu^+\mu^-)}$$

$$= N_c \sum_i e_q^2 = 3 \sum_i e_q^2$$

- for 3 quarks:

$$R = 3\left[\left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2\right] = 2$$

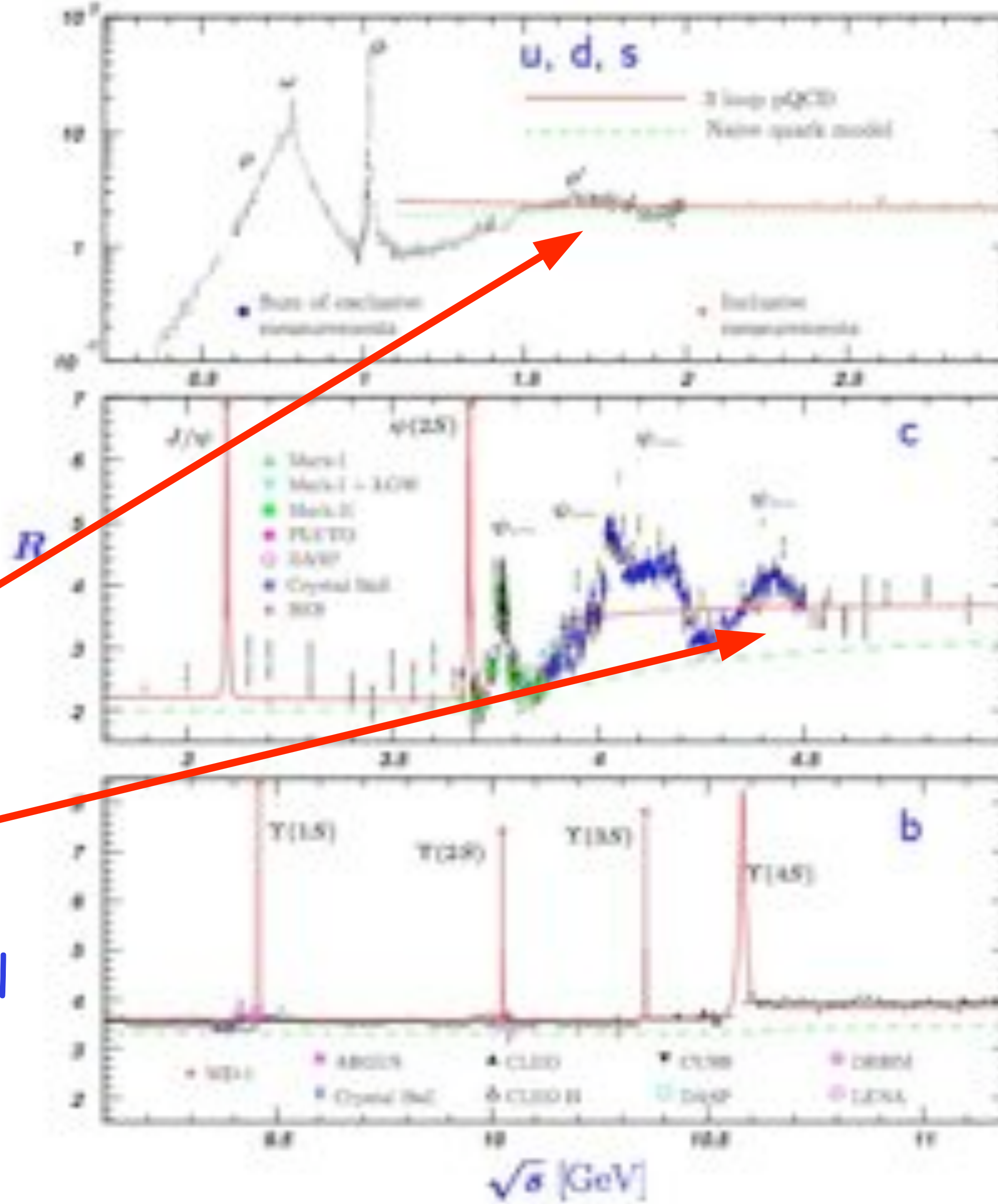
- including charm

$$R = 3\left[\frac{2}{3} + \left(\frac{2}{3}\right)^2\right] = 3.333$$

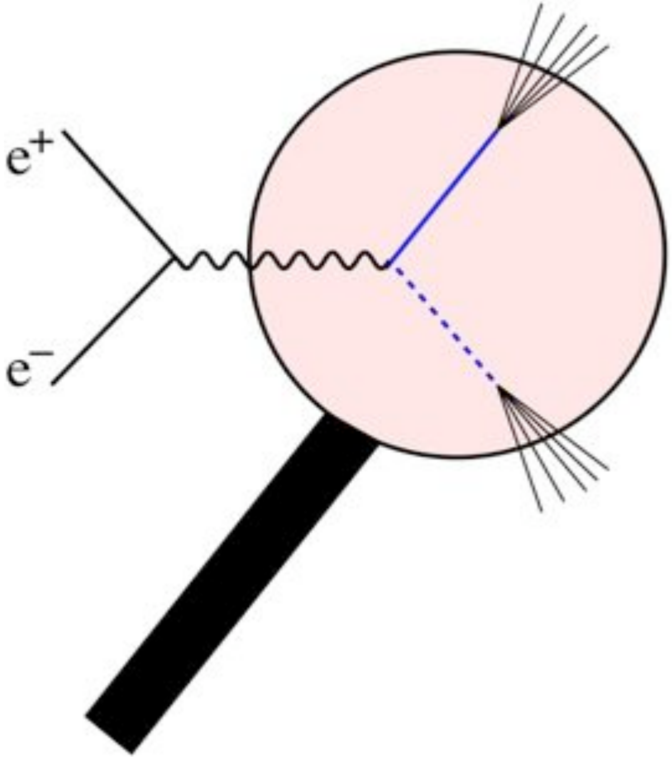
➔ "direct" observation of fractional charge of quarks and

➔ 3 different colors ....

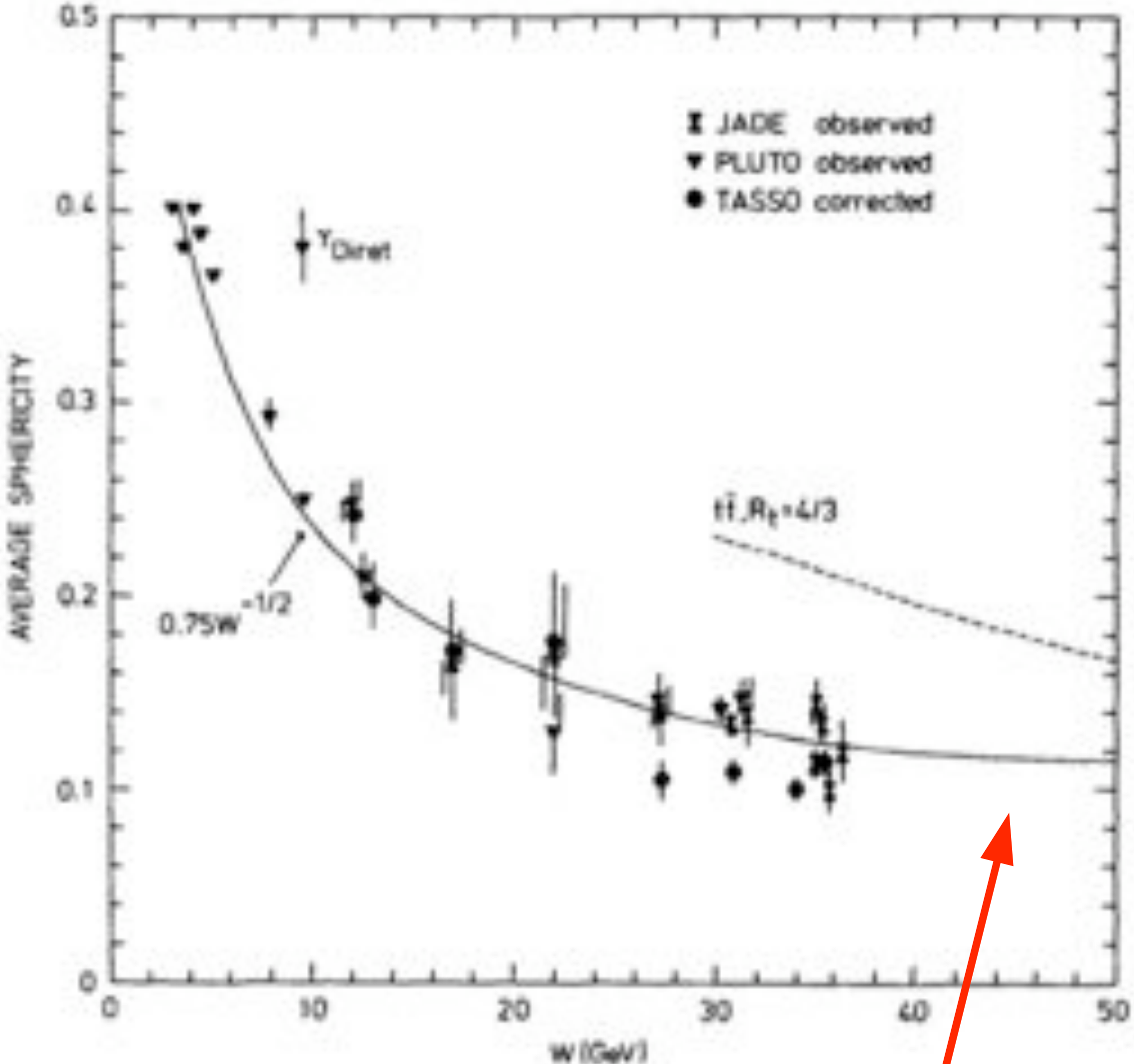
PDG 2008



The early steps: $e^+e^- \rightarrow$ hadrons



Soding,Wolf Experimental Evidence on QCD
Ann Rev Nucl Part Sci 1981, 31, 231



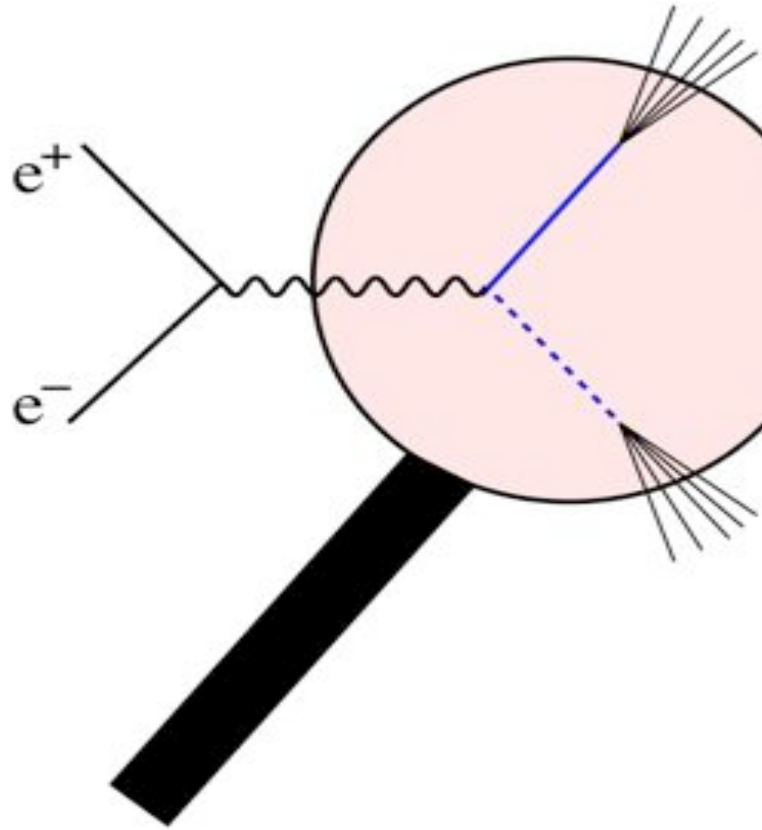
- sphericity: $S \sim 3/2 \langle \delta^2 \rangle$

  jet opening angle $\langle \delta \rangle = \langle P_t / P_{\|} \rangle$
- S ~ 0 for extreme jets

  S → 1 for spherical events

➔ evidence for 2 -jet structure

The early steps: $e^+e^- \rightarrow$ hadrons



➔ How to compare a detailed measurement with a theoretical prediction ?

Simulate these processes with Monte Carlo method !!!

Monte Carlo method

- Monte Carlo method
  - refers to any procedure that makes use of random numbers
  - uses probability statistics to solve the problem
- Monte Carlo methods are used in:
  - Simulation of natural phenomena
  - Simulation of experimental apparatus
  - Numerical analysis
- Random number:

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- Random number:

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  No such thing as a single random number

  A sequence of random numbers is a set of numbers that have nothing to do with the other numbers in a sequence

Going out to Monte Carlo

- Obtain true Random Numbers from Casino in Monte Carlo
- Puhhh... Going out every night ...

Random Numbers

- In a uniform distribution of random numbers in [0,1] every number has the same chance of showing up
- Note that 0.000000001 is just as likely as 0.5

To obtain random numbers:

- Use some chaotic system like roulette, lotto, 6-49, ...
- Use a process, inherently random, like radioactive decay
- Tables of a few million truly random numbers exist .....

(.....until a few years ago.....)

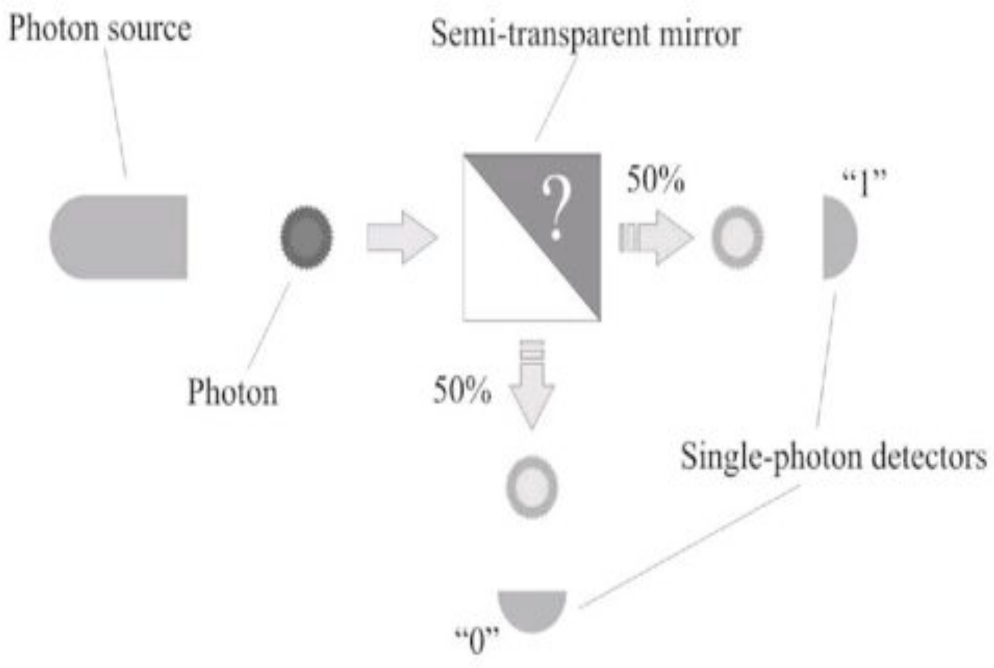
BUT not enough for most applications

➔ .... **we have true random number generators ...**

True Random Numbers

- Random numbers from classical physics: coin tossing
  - evolution of such a system can be predicted, once the initial condition is known... however it is a chaotic process ... extremely sensitive to initial conditions.
- Truly Random numbers used for
  - Cryptography
    - Confidentiality
    - Authentication
  - Scientific Calculation
  - Lotteries and gambling
    - do not allow to increase chance of winning by having a bias .... too bad

- Random numbers from quantum physics: intrinsic random
  - photons on a semi-transparent mirror



- Available and tested in MC generator by a summer student
- Generator is however very slow...

True Random Numbers

- atmospheric noise, which is quite easy to pick up with a normal radio: used by RANDOM.ORG
  (thanks to Albert)
- much more can be found on the web ....

Pseudo Random Numbers

Pseudo Random Numbers

- are a sequence of numbers generated by a computer algorithm, usually uniform in the range [0,1]
- more precisely: algo's generate integers between *0* and *M*, and then $r_n = I_n / M$
- A very early example: Middles Square (John van Neumann, 1946):

  generate a sequence, start with a number of 10 digits, square it, then take the middle 10 digits from the answer, as the next number etc.:

  $5772156649^2 = 33317792380594909291$

  Hmmmm, sequence is not random, since each number is determined from the previous, but it appears to be random
- this algorithm has problems .....

  BUT a more complex algo does not necessarily lead to better random sequences ....

  Better us an algo that is well understood

Congruential linear generator

- develop our own simple generator

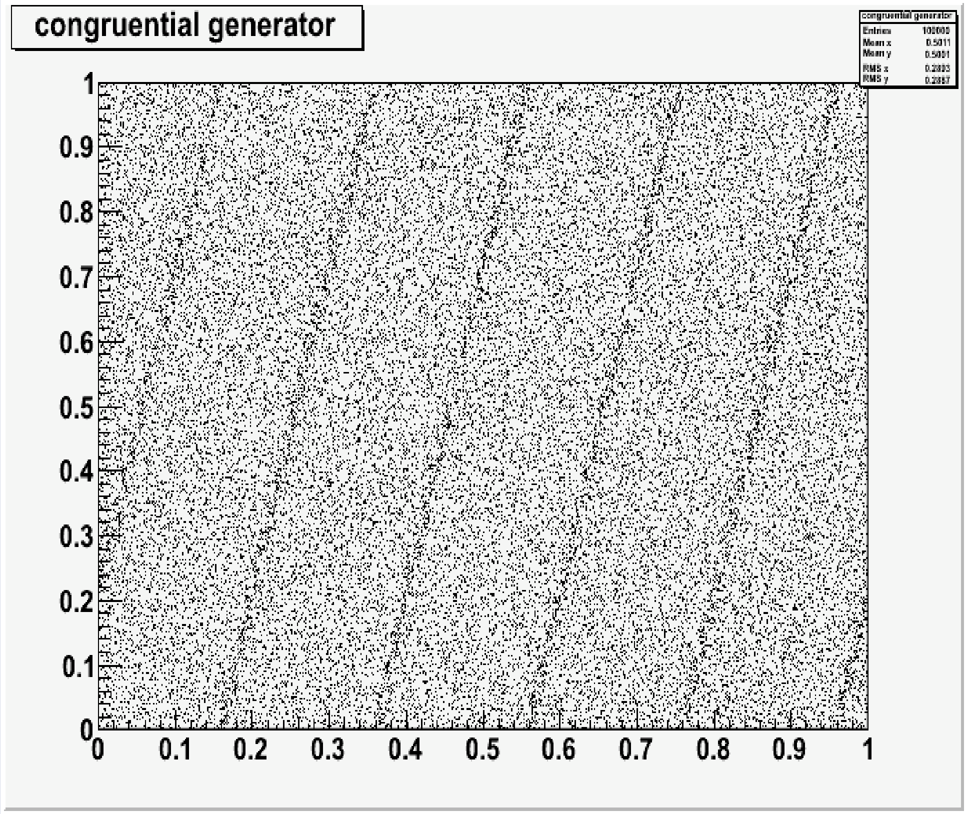
$$I_j = mod(aI_{j-1} + c, m)$$
$$R_j = \frac{I_j}{m}$$

- with seed I_0
- and multiplicative constant a and additive constant c
- modulus m

➔ maximal repetition period: $\mathcal{O}(m)$

Randomness tests

- Congruential generator

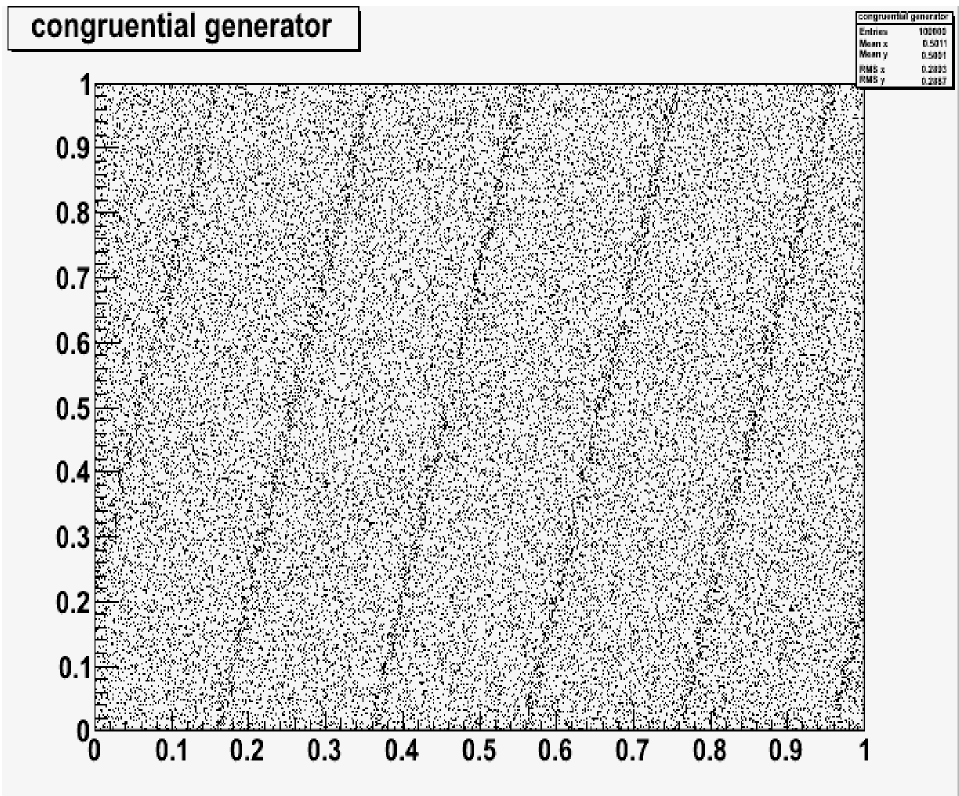


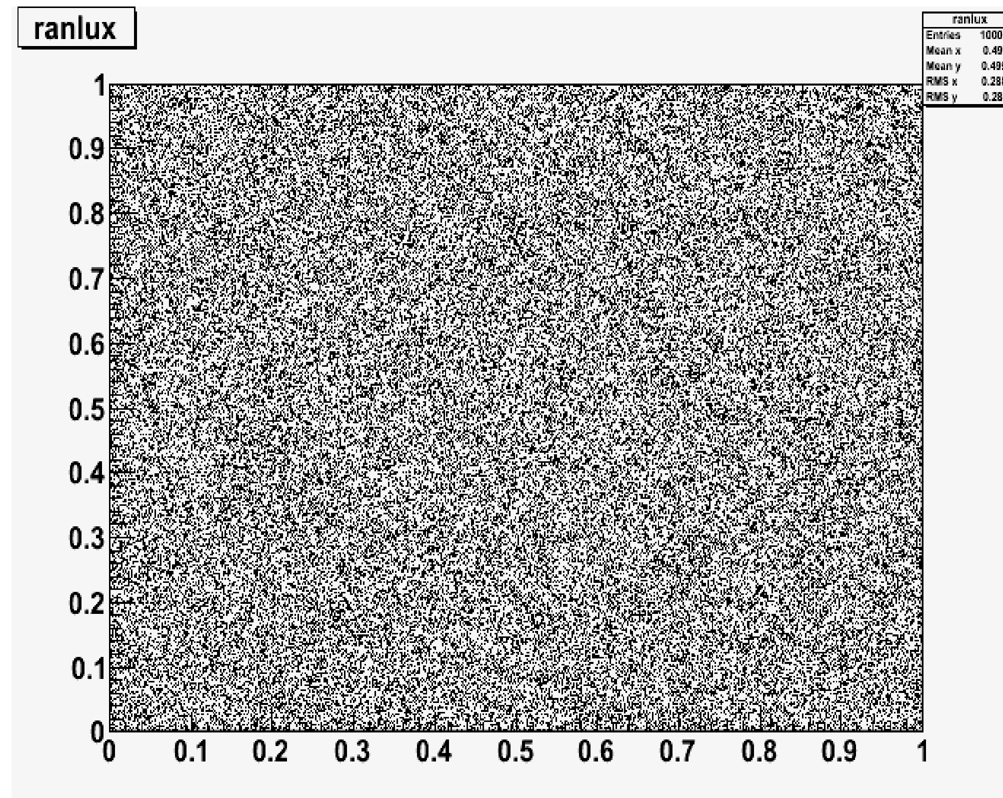
➔ Congruential generator is not bad ... but it could be better ....

Randomness tests

- Congruential generator
- RANLUX

M. Lüscher, A portable high-quality random number generator for lattice field theory simulations, Computer Physics Communications 79 (1994) 100
http://luscher.web.cern.ch/luscher/ranlux/index.html





➔ RANLUX much more sophisticated
Developed and used for QCD lattice calcs

Simulating Radioactive Decay

- radioactive decay is a truly random process
- $dN = - N \alpha \, dt$ i.e. $N = N_0 \, e^{-\alpha t}$
- probability of decay is constant ... independent of the age of the nuclei:

  probability that nucleus undergoes radioactive decay in time Δt is p:

  $p = \alpha \, \Delta t$ (for $\alpha \, \Delta t \ll 1$)
- Problem:

  consider a system initially having N_0 unstable nuclei.

  How does the number of parent nuclei, N, change with time ?
- Algorithm:

```
LOOP from t=0 to t, step Δt
  LOOP over each remaining parent nucleus
      Decide if nucleus decays:
      IF ( random # < α Δ t ) then
 reduce number of parents by 1
      ENDIF
  END LOOP over nuclei
  Plot or record N vrs t
END LOOP over time
END
```

The first simulation: radioactive decay

- implement algo into a small program
- show results after 3000 sec for:

  $N_0 = 100$, $\alpha = 0.01\ s^{-1}$

  $\Delta t = 1s$

  $N_0 = 5000$, $a = 0.03\ s^{-1}$

  $\Delta t = 1s$

- algo:

```
  alpha1 = 0.01
  N01 = 100
  deltat = 1
  do I=1,300
     it = it + 1
     do j = 1, N01
            x = RN1
            fr = deltat*alpha1
            if(x.lt.fr) then
c   reduce number of parents N01
               N01 = N01 - 1
            endif
c   fill for each time it number N01
        call hfill(400,real(it+0.3),0,1.) !
        enddo
```

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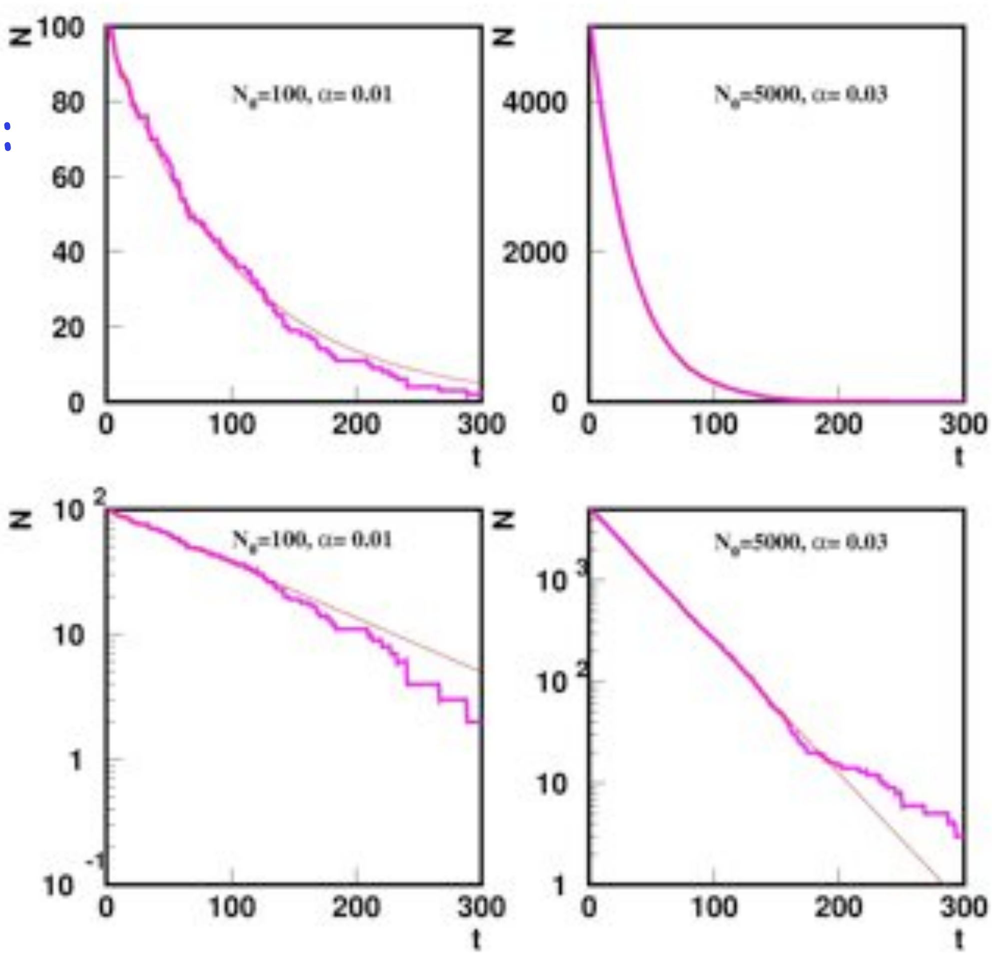
  $N_0 = 5000$, $a = 0.03\ s^{-1}$

  $\Delta t = 1s$

- MC experiment does not exactly reproduce theory ....
- results from MC experiment show statistical fluctuations ...
- ....as expected ....



Expectation values and variance

- Expectation value (defined as the average or mean value of function f):

$$E[f] = \int f(u) dG(u) = \left(\frac{1}{b-a} \int_a^b f(u) du \right) = \frac{1}{N} \sum_{i=1}^{N} f(u_i)$$

for uniformly distributed u in [a,b] *then* $dG(u) = du/(b-a)$

- rules for expectation values:

$$E[cx + y] = cE[x] + E[y]$$

- Variance

$$V[f] = \int (f - E[f])^2 \, dG = \left(\frac{1}{b-a} \int_a^b (f(u) - E[f])^2 \, du \right)$$

- rules for variance:

if x,y uncorrelated: $V[cx + y] = c^2 V[x] + V[y]$

if x,y correlated $V[cx + y] = c^2 V[x] + V[y] + 2cE[(y - E[y])(x - E[x])]$

Generating distributions

- From uniform distributions to distributions for any probability density function
  - use variable transformation
- linear p.d.f:

$$
\begin{aligned}
f(x) &= 2x \\
u(x) &= \int_0^x 2t\,dt = x^2 \\
x_j &= \sqrt{u_j}
\end{aligned}
$$

- 1/x distribution

$$
\begin{aligned}
f(x) &= \frac{1}{x} \\
u(x) &= \frac{\int_{x_{min}}^{x} \frac{1}{t}dt}{F_{max} - F_{min}} \\
x_j &= x_{min} \left(\frac{x_{max}}{x_{min}}\right)_j^u
\end{aligned}
$$

Generating distributions

- Brute Force or Hit & Miss method
  - use this if there is no easy way to find a analytic integrable function
  - find $c \leq \max f(x)$
  - reject if $f(x_i) < u_j \cdot c$
  - accept if $f(x_i) > u_j \cdot c$

- Improvements for Hit & Miss method by variable transformation
  - find $c \cdot g(x) > f(x)$
  - reject if $f(x) < u_j \cdot c \cdot g(x)$
  - accept if $f(x) > u_j \cdot c \cdot g(x)$

Monte Carlo technique: basics

- **Law of large numbers**

  chose N numbers u_i randomly, with probability density uniform in [a,b],

  evaluate $f(u_i)$ for each u_i :

$$\frac{1}{N}\sum_{i=1}^{N} f(u_i) \rightarrow \frac{1}{b-a}\int_a^b f(u)du$$

  for large enough N Monte Carlo estimate of integral converges to correct answer.

- Convergence

  is given with a certain probability ...

THIS is a mathematically serious and precise statement !!!!

Central Limit Theorem

- Central Limit Theorem

  for large N the sum of independent random variables is always normally (Gaussian) distributed:

$$f(x) = \frac{1}{s\sqrt{2\pi}} \exp\left[-\frac{(x-a)^2}{2s^2}\right]$$

$$\frac{\sum_i x_i - \sum_i \mu_i}{\sqrt{\sum_i \sigma_i^2}} \to N(0,1)$$

➔ independent on the original sub-distributions

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- example: take sum of uniformly distributed random numbers:

$$R_n = \sum_{i=1}^{n} R_i$$

$$E[R_1] = \int u du = 1/2,$$

$$V[R_1] = \int (u - 1/2)^2 du = 1/12$$

$$E[R_n] = n/2$$

$$V[R_n] = n/12$$

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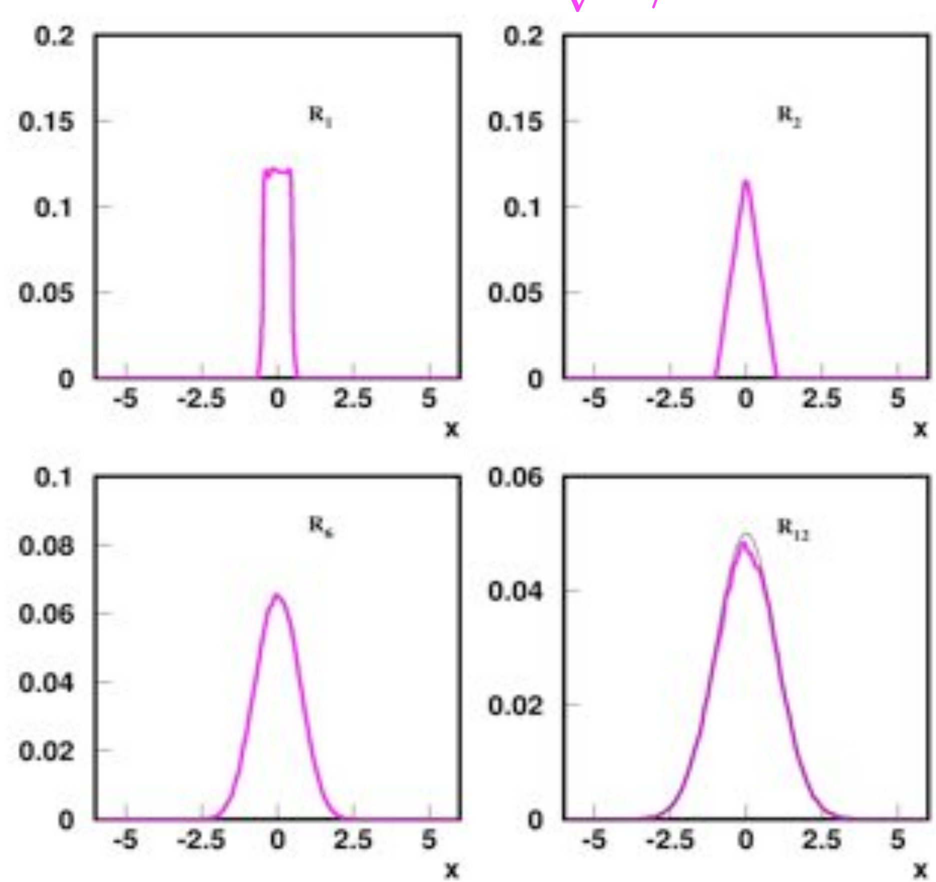
- for Gaussian with mean=0 and variance=1, take for n=12:

$$N(0,1) \rightarrow \frac{R_n - n/2}{\sqrt{n/12}}$$



Central Limit Theorem

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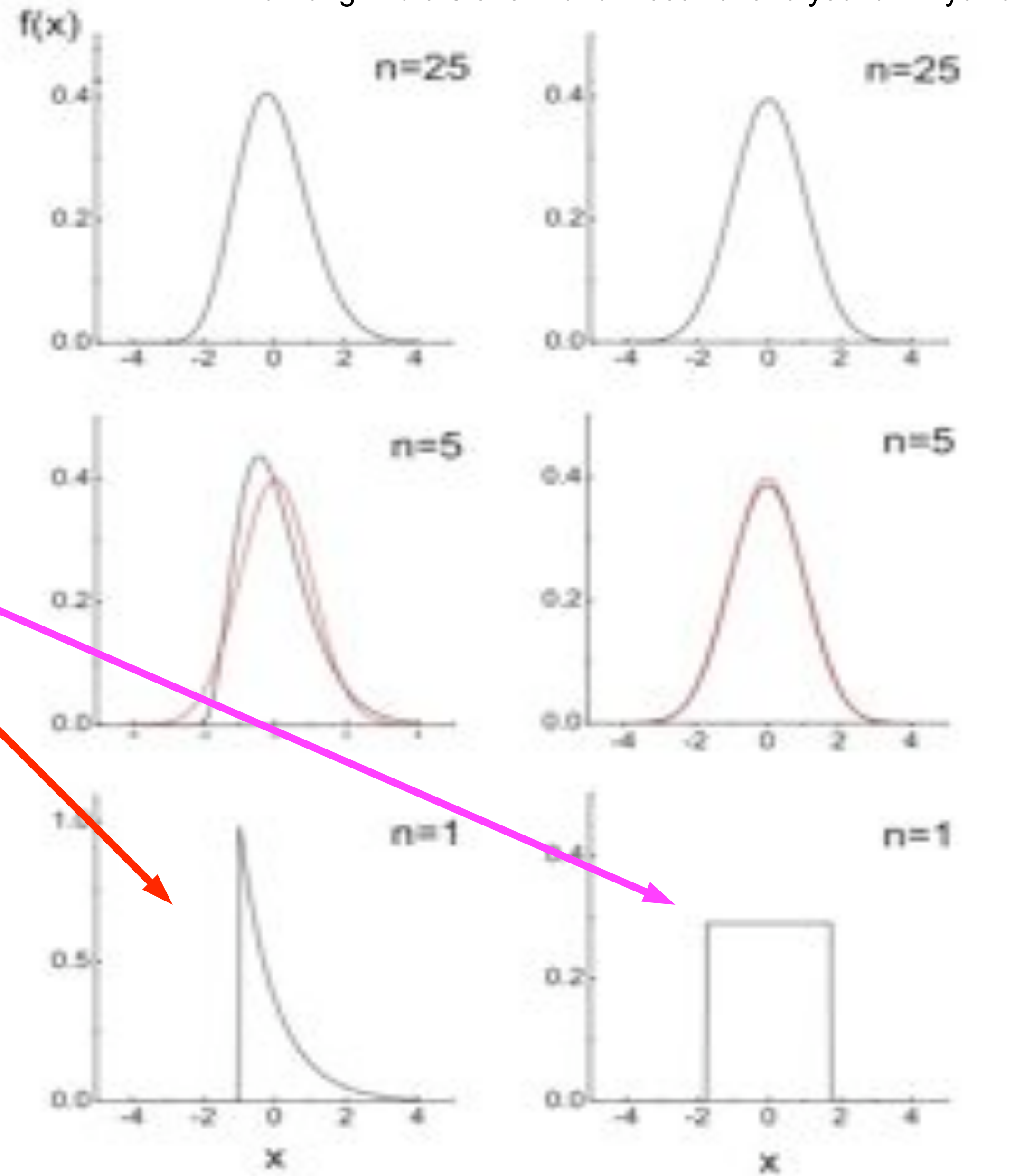
  for large N the sum of independent random variables is always normally (Gaussian) distributed

→ for any starting distribution

→ for uniform distribution

→ for exponential distribution

G. Bohm, G. Zech
Einfuhrung in die Statistik und Messwertanalyse fur Physiker



MC method: advantage of hit & miss

- integration ☛ weighting events

  large fluctuations from large weights

  weights also to errors applied

  difficult to apply further

  hadronization
- real events all have weight = 1 !!!
- Hit & Miss method:

MC for function f(x):
get random number:
R1 in (0,1) and R2 in (0,1)
calculate x = R1
reject event if: $f_x < f_{max}$ R2

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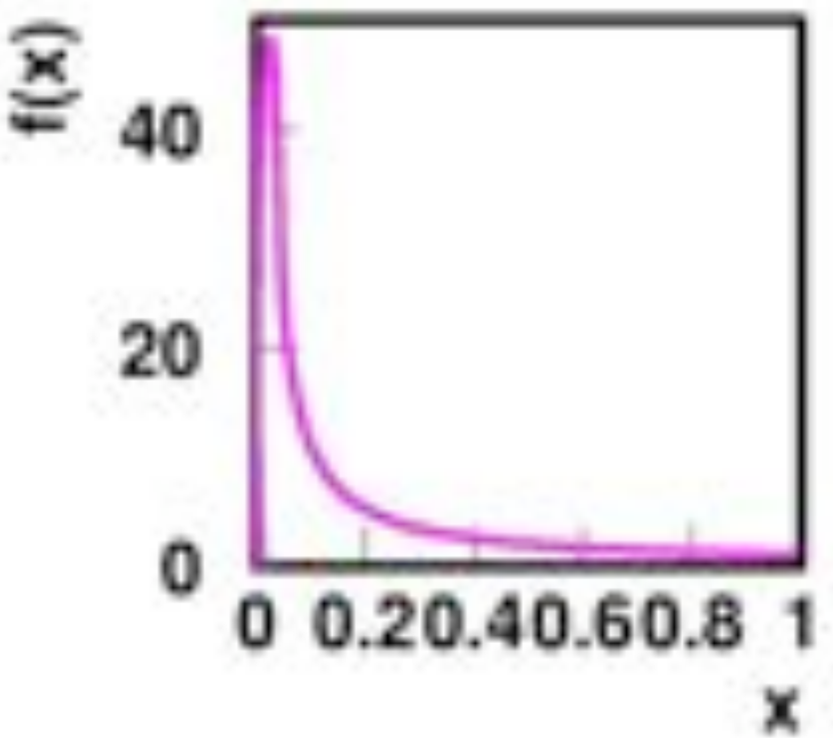

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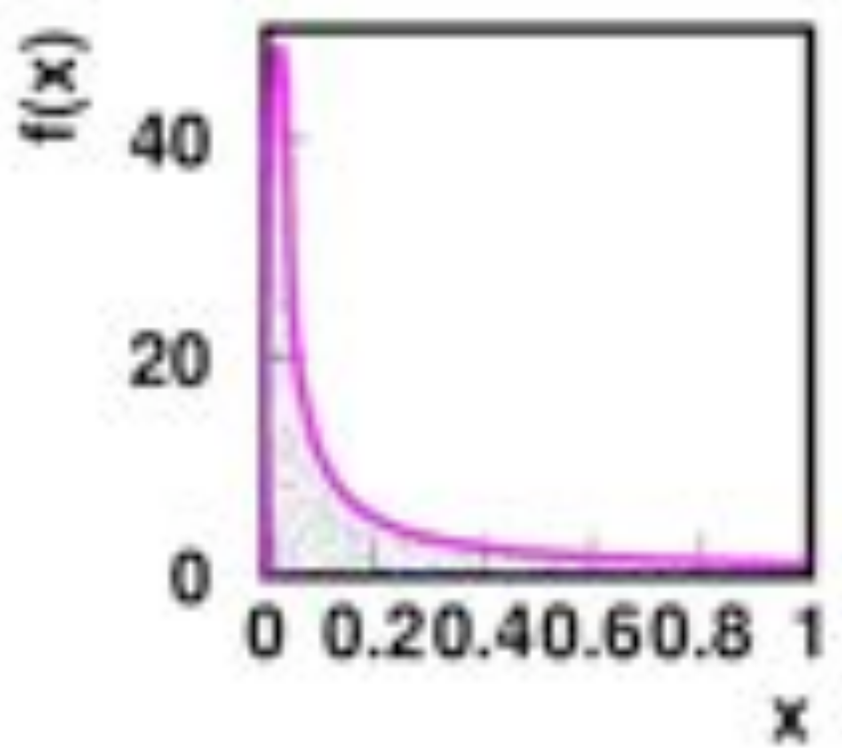

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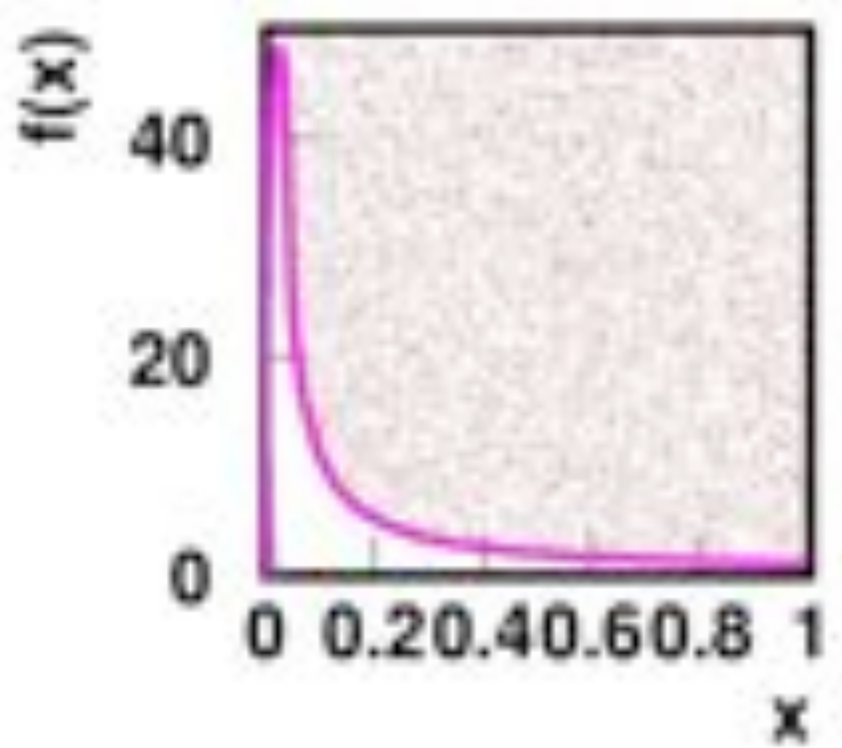

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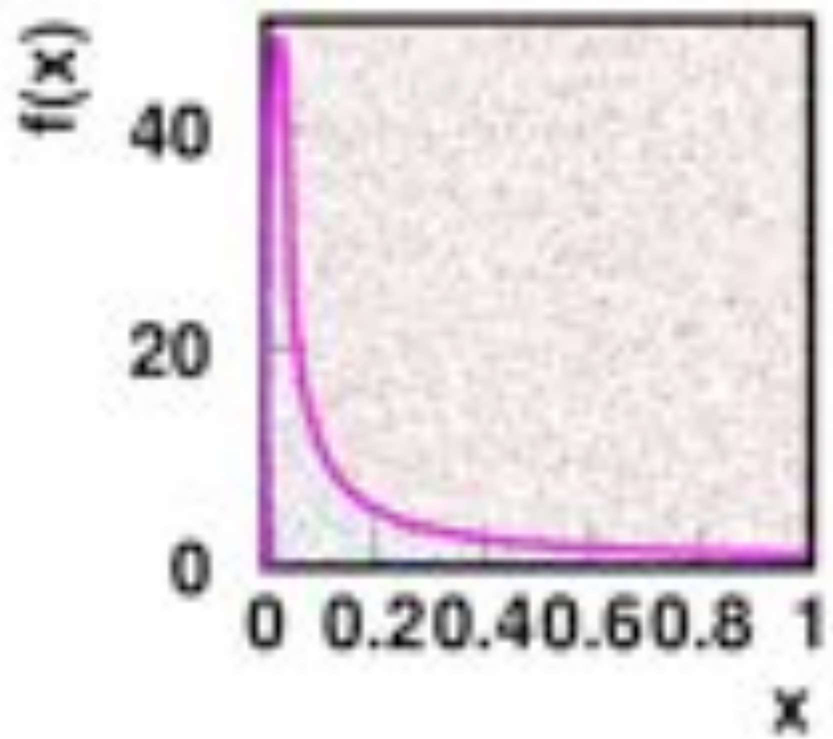

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reject event if: $f_x < f_{max}$ R2

- BUT: Hit & Miss method inefficient for peaked *f(x)*

MC method: do even better ...

- Importance sampling

MC for function f(x)

approximate f(x) ~ g(x)

with g(x) > f(x) simple and integrable

generate x according to g(x):

$$\int_{x_{min}}^{x} g(x')dx' = R1 \int_{x_{min}}^{x_{max}} g(x')dx'$$

example: $$f(x) = 1/x^{0.7}$$

$$g(x) = 1/x$$

$$x = x_{min} \cdot \left(\frac{x_{max}}{x_{min}}\right)^{R1}$$

reject event if: f(x) < g(x) R2

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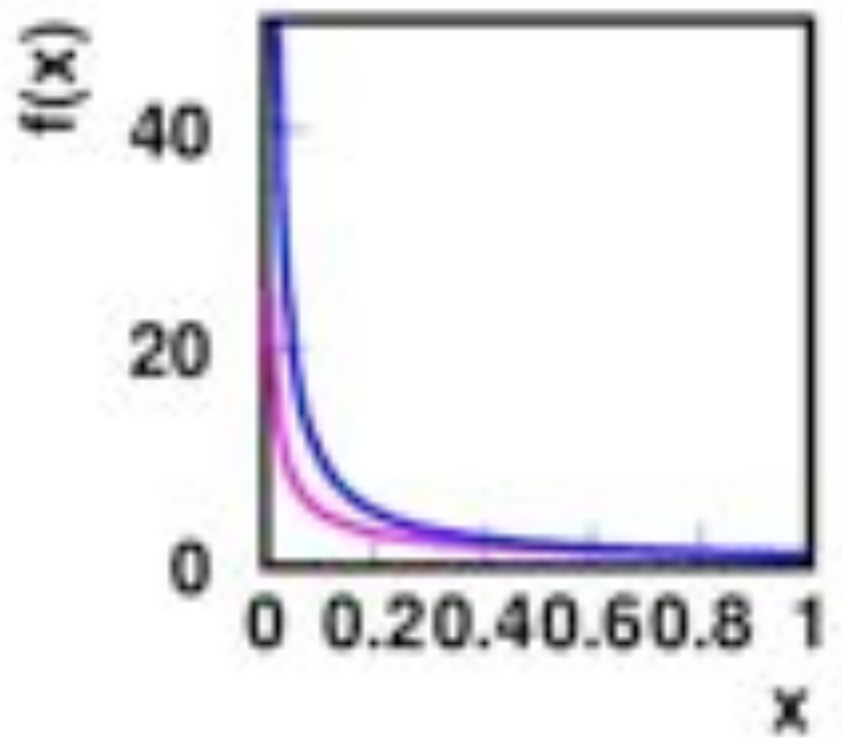
$$x = x_{min} \cdot \left(\frac{x_{max}}{x_{min}}\right)^{R1}$$

reject event if: f(x) < g(x) R2



MC method: do even better ...

- Importance sampling

MC for function f(x)
approximate f(x) ~ g(x)
with g(x) > f(x) simple and integrable
generate x according to g(x):

$$\int_{x_{min}}^{x} g(x')dx' = R1 \int_{x_{min}}^{x_{max}} g(x')dx'$$

example: $f(x) = 1/x^{0.7}$

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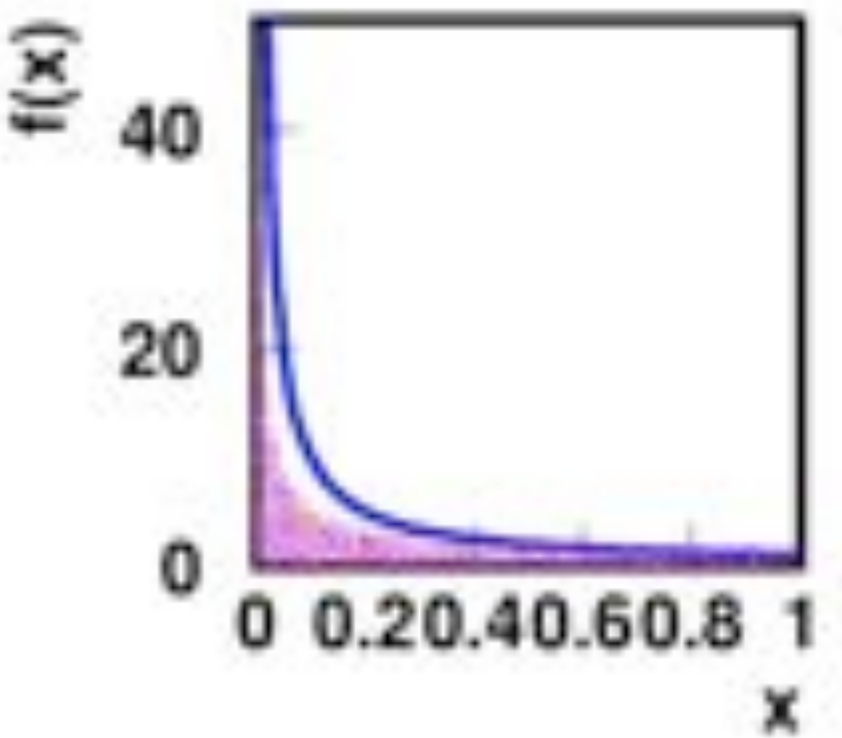

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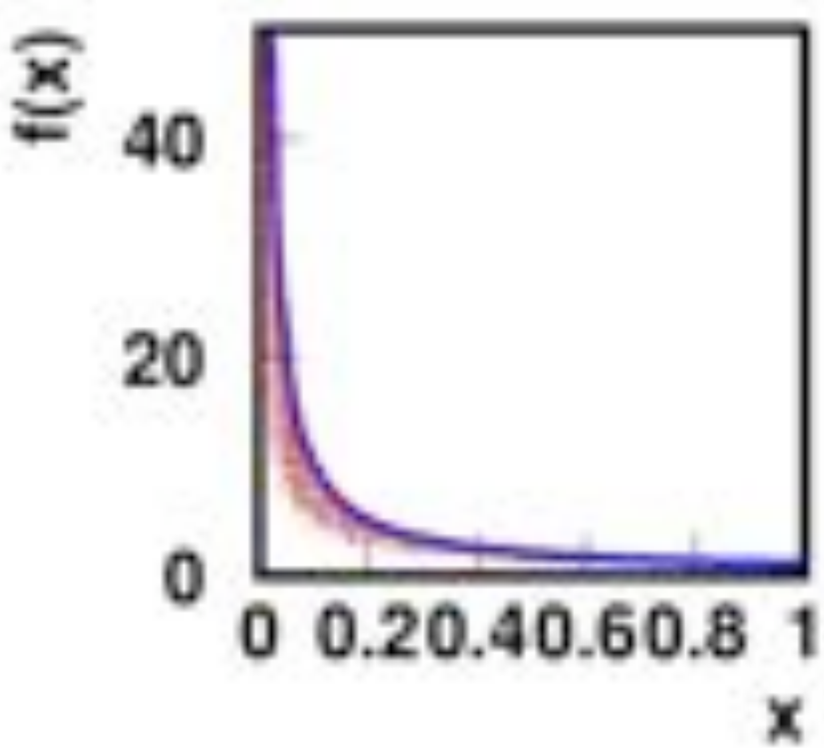

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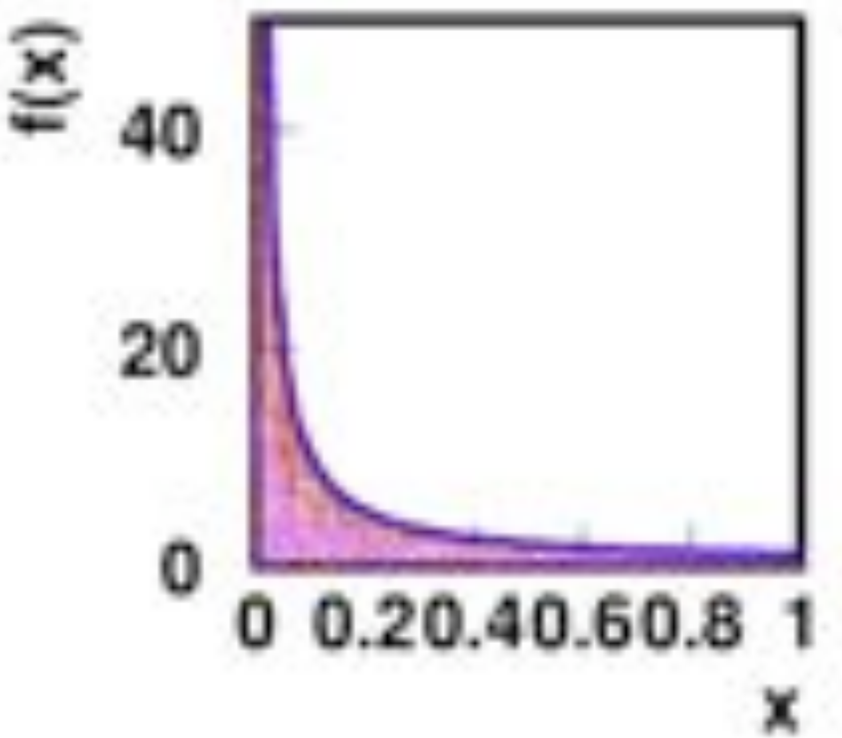


- WOW !!! very efficient even for peaked f(x)

Importance Sampling

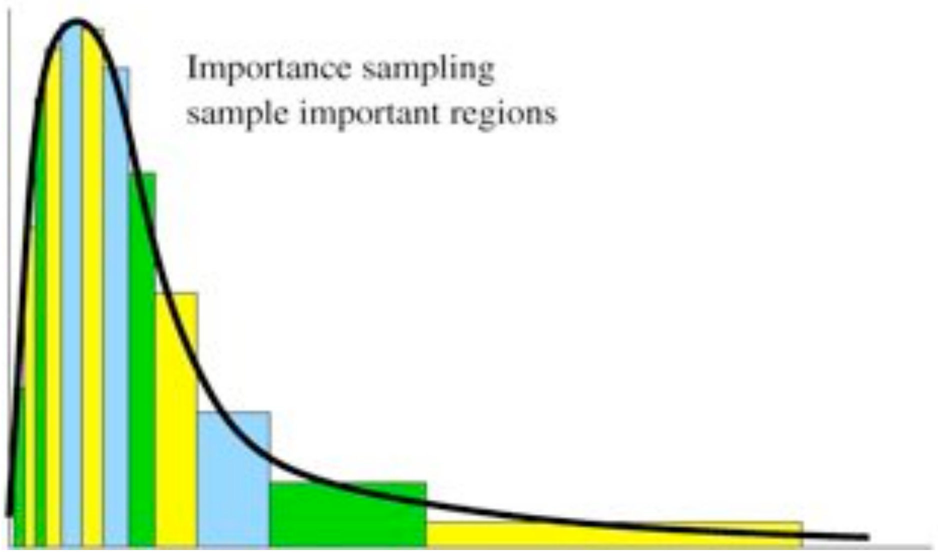
- MC calculations most efficient for small weight fluctuations:

  f(x)dx → f(x) dG(x)/g(x)
- chose point according to g(x) instead of uniformly
- f is divided by g(x) = dG(x)/dx
- generate *x* according to:

$$R \int_a^b g(x')dx' = \int_a^x g(x')dx'$$

- relevant variance is now V(f/g):

  small if g(x) ~ f(x)
- how-to get g(x)
  - (1) g(x) is probability: g(x) > 0 and ∫dG(x) = 1
  - (2) integral ∫dG(x) is known analytically
  - (3) *G(x)* can be inverted (solved for *x*)
  - (4) f(x)/g(x) is nearly constant, so that *V(f/g)* is small compared to V(f)



We have the method,....

BUT

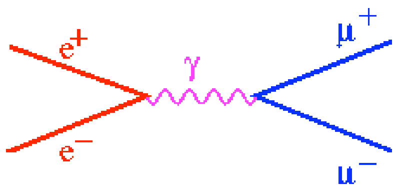
HOWTO

simulate the physics

????

Constructing a MC for e^+e^-: the simple case

- process: $e^+e^- \rightarrow \mu^+ \mu^-$

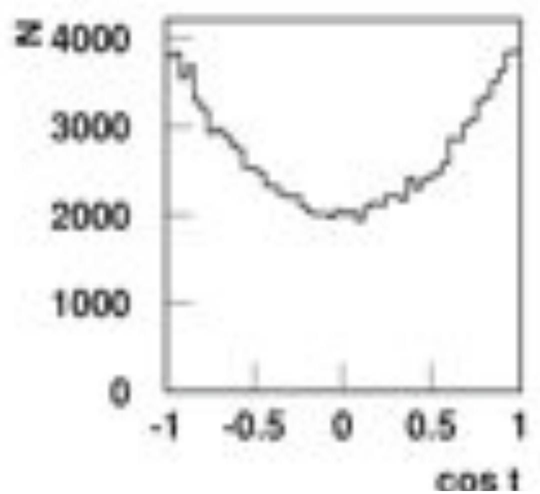


- $$\frac{d\sigma}{d\cos\theta d\phi} = \frac{\alpha_{em}^2}{4s}\left(1+\cos^2\theta\right)$$
- goal: generate 4-momenta of μ's, need *cm* energy s, cos θ, ϕ

random number R1(0,1): $\phi = 2\pi$ R1
random number R2(0,1): $\cos\theta = -1 + 2$ R2

for every R1, R2 use weight with $\frac{d\sigma}{d\cos\theta\, d\phi}$
repeat many times

after 100000 events



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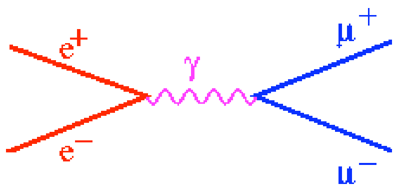

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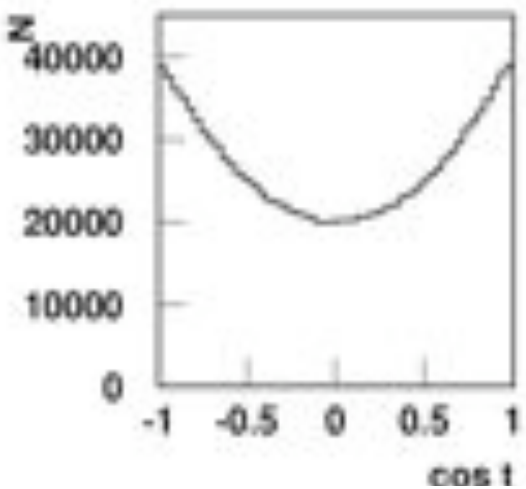
for every R1, R2 use weight with $\frac{d\sigma}{d\cos\theta\, d\phi}$
repeat many times

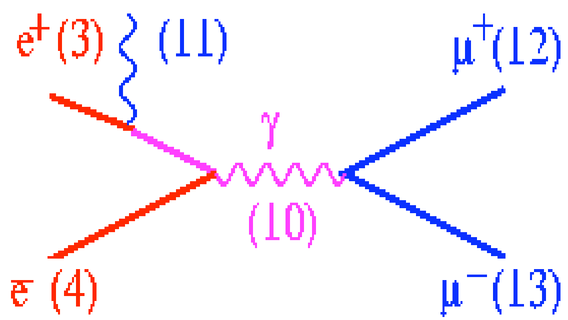
after 10^6 events



Example event: $e^+e^- \to \mu^+ \mu^-$

- example from PYTHIA: Event listing

I	particle/jet	KS	KF	orig	p_x	p_y	p_z	E	m
1	!e+!	21	-11	0	0.000	0.000	30.000	30.000	0.001
2	!e-!	21	11	0	0.000	0.000	-30.000	30.000	0.001
3	!e+!	21	-11	1	0.000	0.000	30.000	30.000	0.000
4	!e-!	21	11	2	0.000	0.000	-30.000	30.000	0.000
5	!e+!	21	-11	3	0.143	0.040	26.460	26.460	0.000
6	!e-!	21	11	4	0.000	0.000	-29.998	29.998	0.000
7	!Z0!	21	23	0	0.143	0.040	-3.539	56.458	56.347
8	!mu-!	21	13	7	-9.510	1.741	24.722	26.546	0.106
9	!mu+!	21	-13	7	9.653	-1.700	-28.261	29.913	0.106
10	(Z0)	11	23	7	0.143	0.040	-3.539	56.458	56.347
11	gamma	1	22	3	-0.143	-0.040	3.539	3.542	0.000
12	mu-	1	13	8	-9.510	1.741	24.722	26.546	0.106
13	mu+	1	-13	9	9.653	-1.700	-28.261	29.913	0.106
		sum:	0.00		0.000	0.000	0.000	60.000	60.000



- technicalities/advantages
  - ➔ can work in any frame
  - ➔ Lorentz-boost 4-vectors back and forth
  - ➔ can calculate any kinematic variable
  - ➔ history of event process

Transition from Quarks to Hadrons

- Independent Fragmentation (Feynman & Field: Phys. Rev D15 (1977)2590, NPB 138 (1978) 1)
  - quarks fragment independently

Transition from Quarks to Hadrons

- Independent Fragmentation
  - quarks fragment independently
  - not Lorentz invariant

PHYSICAL REVIEW D VOLUME 27, NUMBER 1 1 JANUARY 1983

Scaling violations in inclusive e^+e^- annihilation spectra

C. Peterson,* D. Schlatter, I. Schmitt,* and P. M. Zerwas‡
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 29 July 1982)

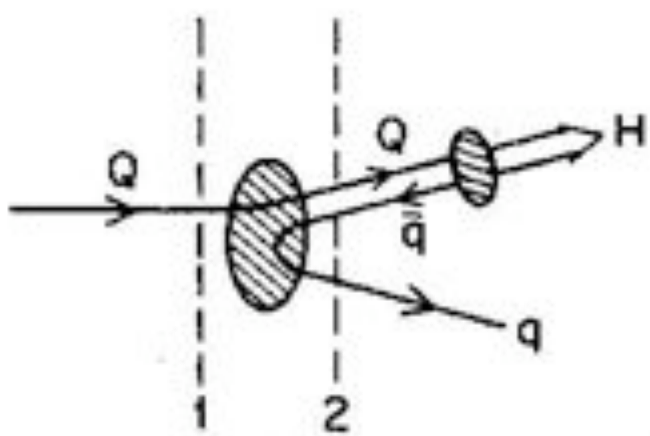


FIG. 3. The fragmentation of a heavy quark Q into a meson $H(Q\bar{q})$. Dashed lines are time slices used in the derivation of Eq. (3).

cussed in Ref. 18. The *gross features* of the amplitude for a fast moving heavy quark Q fragmentation into a hadron $H=(Q\bar{q})$ and light quark q (Fig. 3) are determined by the value of the energy transfer $\Delta E = E_H + E_q - E_Q$ in the breakup process,

$$\text{amplitude } (Q \to H + q) \propto \Delta E^{-1} . \quad (2)$$

Expanding the energies about the (transverse) particle masses ($m_H \simeq m_Q$ for simplicity),

$$\Delta E = (m_Q{}^2 + z^2P^2)^{1/2} + (m_q{}^2 + (1-z)^2P^2)^{1/2} - (m_Q{}^2 + P^2)^{1/2}$$
$$\propto 1 - (1/z) - (\epsilon_Q/1-z) \quad (3)$$

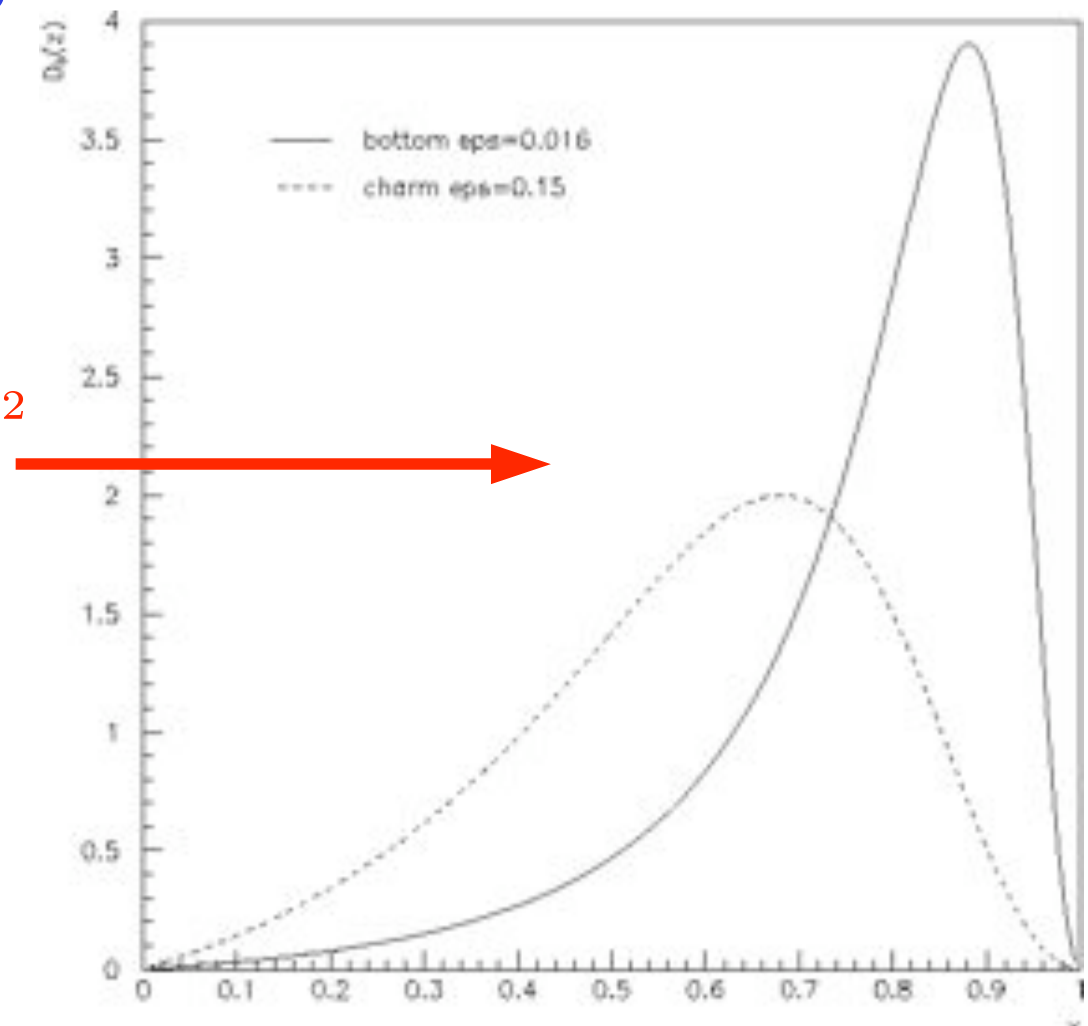
and taking a factor z^{-1} for longitudinal phase space, we suggest the following ansatz for the fragmentation function of heavy quarks Q

$$D_Q^H(z) = \frac{N}{z[1-(1/z)-\epsilon_Q/(1-z)]^2} . \quad (4)$$

Heavy Quark Fragmentation

- transition from heavy quark to observable hadron by fragmentation function FF
- Peterson FF: (C. Peterson, D.Schlatter,I.Schmitt,P.Zerwas, PRD27 (1983) 105)

$$D_Q(z) = \frac{N}{z}\left[1 - \frac{1}{z} - \frac{\epsilon_Q}{1-z}\right]^{-2}$$



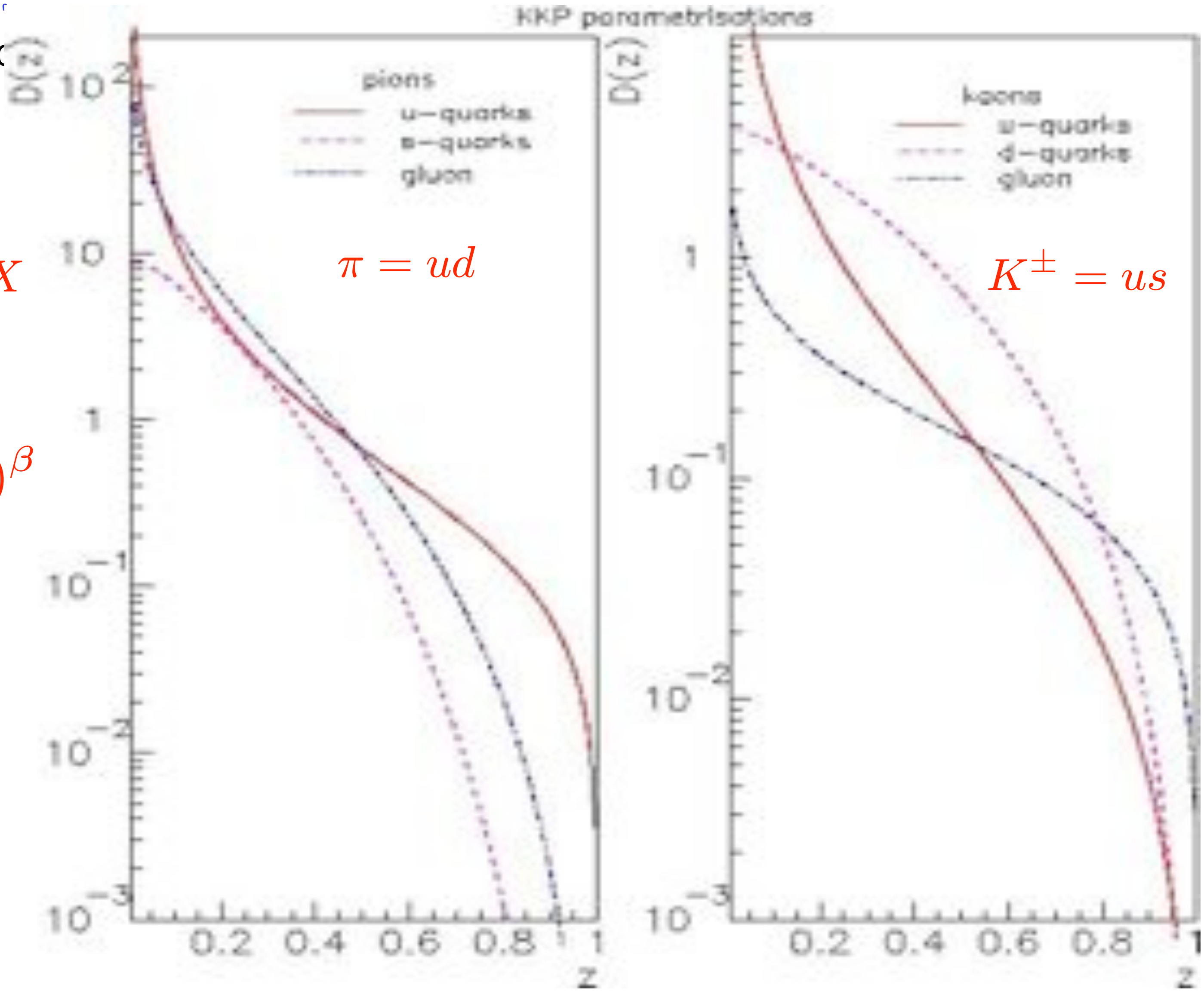
Light Quark FF

- parametrisations by:
  Kniehl,Kramer&Poetter NPB582(2000) 514
- use

  $$e^+e^- \to (\gamma, Z) \to h + X$$

- starting distribution:

  $$D_Q(z) = N z^\alpha (1-z)^\beta$$

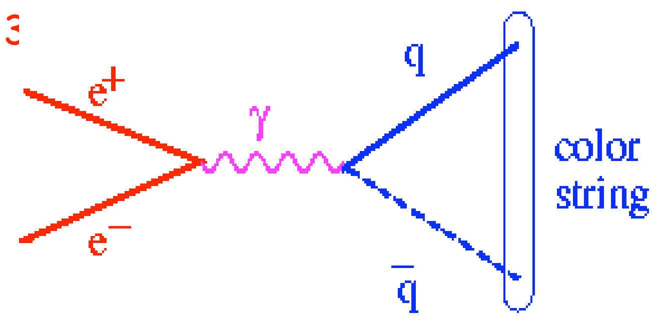


Transition from Quarks to Hadrons

- Independent Fragmentation
  - quarks fragment independently
  - gluon are split: $g \to q\bar{q}$
  - not Lorentz invariant
- Lund String Fragmentation (Andersson, Gustafson, Peterson ZPC 1, 105 (1979), Andersson, Gustafson, Ingelman, Sjostrand Phys. Rep 97 (1983) 33
  - use concept of local parton-hadron duality

$$e^+e^- \to q\bar{q}$$

$$\frac{d\sigma}{d\cos\theta d\phi} = \frac{\alpha_{em}^2}{4s}(1+\cos^2\theta)$$

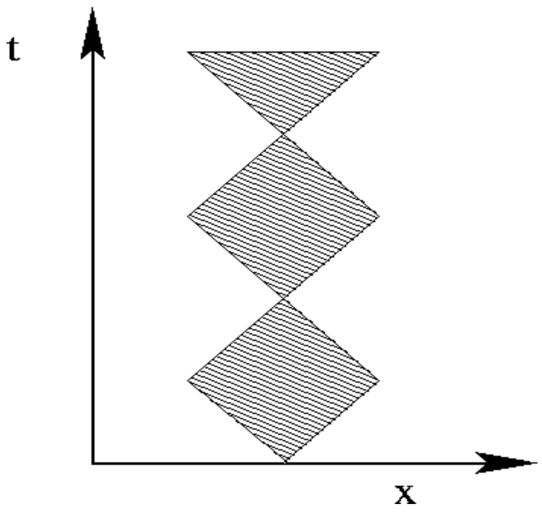


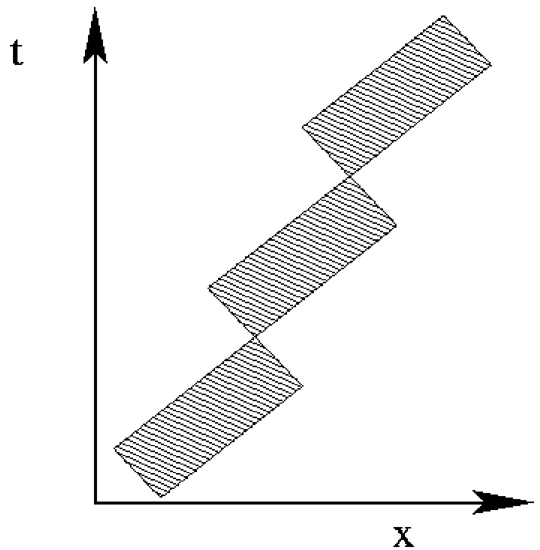
linear confinement potential: *V(r) ~ -1/r + κr*
with *κ ~ 1* GeV/fm
qq connected via color flux tube of transverse size of hadrons (~1 fm)
color tube: uniform along its length → linearly rising potential

→ Lund string fragmentation

Lund string fragmentation

- in a color neutral qq-pair, a color force is created in between
- color lines of the force are concentrated in a narrow tube connecting q and q, with a string tension of:
  $\kappa \sim 1 GeV/fm \sim 0.2\ GeV^2$
- as q and q are moving apart in qq rest frame, they are de-accelerated by string tension, accelerated back etc ... (periodic oscillation)
- viewed in a moving system, the string is boosted





Hadronization: particle mass and decays

- particle masses
  - ➔ taken from PDG, where known, otherwise from constituent masses
- particle widths
  - ➔ in hard scattering production process short lived particles (ρ,Δ) have nominal mass, without mass broadening
  - ➔ in hadronization use Breit-Wigner:

$$\mathcal{P}(m)dm \propto \frac{1}{(m-m_0)^2+\Gamma^2/4}$$

- lifetimes
  - ➔ related to widths ... but for practical purpose separated
- decays
  - ➔ taken from PDG, where known
  - ➔ assume momentum distribution given by phase space only
  - ➔ exceptions, like $\omega,\phi \to \pi^+\pi^-\pi^0$, or $D \to K\pi$, $D^* \to K\pi\pi$ and some semileptonic decays use matrix elements

The first MC steps ...

- Monte Carlo source code of JETSET, fits on 1 page

T.Sjostrand, B. Soderberg LU-TP 78-18

Appendix

Listings of the program components.

```
      SUBROUTINE JETGEN(N)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /PAR/ PUD, PS1, SIGMA, CX2, EBEG, WFIN, IFLBEG
      COMMON /DATA1/ MESO(9,2), CMIX(6,2), PMAS(19)
      IFLSGN=(10-IFLBEG)/5
      W=2.*EBEG
      I=0
      IPD=0
C 1 FLAVOUR AND PT FOR FIRST QUARK
      IFL1=IABS(IFLBEG)
      PT1=SIGMA*SQRT(-ALOG(RANF(0)))
      PHI1=6.2832*RANF(0)
      PX1=PT1*COS(PHI1)
      PY1=PT1*SIN(PHI1)
  100 I=I+1
C 2 FLAVOUR AND PT FOR NEXT ANTIQUARK
      IFL2=1+INT(RANF(0)/PUD)
      PT2=SIGMA*SQRT(-ALOG(RANF(0)))
      PHI2=6.2832*RANF(0)
      PX2=PT2*COS(PHI2)
      PY2=PT2*SIN(PHI2)
C 3 MESON FORMED, SPIN ADDED AND FLAVOUR MIXED
      K(I,1)=MESO(3*(IFL1-1)+IFL2,IFLSGN)
      ISPIN=INT(PS1+RANF(0))
      K(I,2)=1+9*ISPIN+K(I,1)
      IF(K(I,1).LE.6) GOTO 110
      TMIX=RANF(0)
      KM=K(I,1)-6+3*ISPIN
      K(I,2)=8+9*ISPIN+INT(TMIX+CMIX(KM,1))+INT(TMIX+CMIX(KM,2))
C 4 MESON MASS FROM TABLE, PT FROM CONSTITUENTS
  110 P(I,5)=PMAS(K(I,2))
      P(I,1)=PX1+PX2
      P(I,2)=PY1+PY2
      PMTS=P(I,1)**2+P(I,2)**2+P(I,5)**2
C 5 RANDOM CHOICE OF X=(E+PZ)MESON/(E+PZ)AVAILABLE GIVES E AND PZ
      X=RANF(0)
      IF(RANF(0).LT.CX2) X=1.-X**(1./3.)
      P(I,3)=(X*W-PMTS/(X*W))/2.
      P(I,4)=(X*W+PMTS/(X*W))/2.
C 6 IF UNSTABLE, DECAY CHAIN INTO STABLE PARTICLES
  120 IPD=IPD+1
      IF(K(IPD,2).GE.8) CALL DECAY(IPD,I)
      IF(IPD.LT.I.AND.I.LE.96) GOTO 120
C 7 FLAVOUR AND PT OF QUARK FORMED IN PAIR WITH ANTIQUARK ABOVE
      IFL1=IFL2
      PX1=-PX2
      PY1=-PY2
C 8 IF ENOUGH E+PZ LEFT, GO TO 2
      W=(1.-X)*W
      IF(W.GT.WFIN.AND.I.LE.95) GOTO 100
      N=I
      RETURN
      END
```

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```
      SUBROUTINE DECAY(IPD,I)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /DATA1/ MESO(9,2), CMIX(6,2), PMAS(19)
      COMMON /DATA2/ IDCD(12), CBR(29), KDP(29,3)
      DIMENSION U(3), BE(3)
C 1 DECAY CHANNEL CHOICE, GIVES DECAY PRODUCTS
      TBR=RANF(0)
      IDC=IDCD(K(IPD,2)-7)
  100 IDC=IDC+1
      IF(TBR.GT.CBR(IDC)) GOTO 100
      ND=(59+KDP(IDC,3))/20
      DO 110 IL=I+1,I+ND
      K(IL,1)=-IPD
      K(IL,2)=KDP(IDC,IL-I)
  110 P(IL,5)=PMAS(K(IL,2))
C 2 IN THREE-PARTICLE DECAY CHOICE OF INVARIANT MASS OF PRODUCTS 2+3
      IF(ND.EQ.2) GOTO 130
      SA=(P(IPD,5)+P(I+1,5))**2
      SB=(P(IPD,5)-P(I+1,5))**2
      SC=(P(I+2,5)+P(I+3,5))**2
      SD=(P(I+2,5)-P(I+3,5))**2
      TDU=(SA-SD)*(SB-SC)/(4.*SQRT(SB*SC))
      IF(K(IPD,2).GE.11) TDU=SQRT(SB*SC)*TDU**3
  120 SX=SC+(SB-SC)*RANF(0)
      TDF=SQRT((SX-SA)*(SX-SB)*(SX-SC)*(SX-SD))/SX
      IF(K(IPD,2).GE.11) TDF=SX*TDF**3
      IF(RANF(0)*TDU.GT.TDF) GOTO 120
      P(100,5)=SQRT(SX)
C 3 TWO-PARTICLE DECAY IN CM, TWICE TO SIMULATE THREE-PARTICLE DECAY
  130 DO 160 IL=1,ND-1
      ID=(IL-1)*100-(IL-2)*IPD
      I1=I+IL
      I2=(ND-IL-1)*100-(ND-IL-2)*(I+IL+1)
      PA=SQRT((P(ID,5)**2-(P(I1,5)+P(I2,5))**2)*
     &(P(ID,5)**2-(P(I1,5)-P(I2,5))**2))/(2.*P(ID,5))
  140 U(3)=2.*RANF(0)-1.
      PHI=6.2832*RANF(0)
      U(1)=SQRT(1.-U(3)**2)*COS(PHI)
      U(2)=SQRT(1.-U(3)**2)*SIN(PHI)
      TDA=1.-(U(1)*P(ID,1)+U(2)*P(ID,2)+U(3)*P(ID,3))**2/
     &(P(ID,1)**2+P(ID,2)**2+P(ID,3)**2)
      IF(K(IPD,2).GE.11.AND.IL.EQ.2.AND.RANF(0).GT.TDA) GOTO 140
      DO 150 J=1,3
      P(I1,J)=PA*U(J)
  150 P(I2,J)=-PA*U(J)
      P(I1,4)=SQRT(PA**2+P(I1,5)**2)
  160 P(I2,4)=SQRT(PA**2+P(I2,5)**2)
C 4 DECAY PRODUCTS LORENTZ TRANSFORMED TO LAB SYSTEM
      DO 190 IL=ND-1,1,-1
      ID=(IL-1)*100-(IL-2)*IPD
      DO 170 J=1,3
  170 BE(J)=P(ID,J)/P(ID,4)
      GA=P(ID,4)/P(ID,5)
      DO 190 I1=I+IL,I+ND
      BEP=BE(1)*P(I1,1)+BE(2)*P(I1,2)+BE(3)*P(I1,3)
      DO 180 J=1,3
  180 P(I1,J)=P(I1,J)+GA*(GA/(1.+GA)*BEP+P(I1,4))*BE(J)
  190 P(I1,4)=GA*(P(I1,4)+BEP)
      I=I+ND
      RETURN
      END
```

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```
      SUBROUTINE EDIT(N)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /EDPAR/ ITHROW, PZMIN, PMIN, THETA, PHI, BETA(3)
      REAL ROT(3,3), PR(3)
C 1 THROW AWAY NEUTRALS OR UNSTABLE OR WITH TOO LOW PZ OR P
      I1=0
      DO 110 I=1,N
      IF(ITHROW.GE.1.AND.K(I,2).GE.8) GOTO 110
      IF(ITHROW.GE.2.AND.K(I,2).GE.6) GOTO 110
      IF(ITHROW.GE.3.AND.K(I,2).EQ.1) GOTO 110
      IF(P(I,3).LT.PZMIN.OR.P(I,4)**2-P(I,5)**2.LT.PMIN**2) GOTO 110
      I1=I1+1
      K(I1,1)=IDIM(K(I,1),0)
      K(I1,2)=K(I,2)
      DO 100 J=1,5
  100 P(I1,J)=P(I,J)
  110 CONTINUE
      N=I1
C 2 ROTATE TO GIVE JET PRODUCED IN DIRECTION THETA, PHI
      IF(THETA.LT.1E-4) GOTO 140
      ROT(1,1)=COS(THETA)*COS(PHI)
      ROT(1,2)=-SIN(PHI)
      ROT(1,3)=SIN(THETA)*COS(PHI)
      ROT(2,1)=COS(THETA)*SIN(PHI)
      ROT(2,2)=COS(PHI)
      ROT(2,3)=SIN(THETA)*SIN(PHI)
      ROT(3,1)=-SIN(THETA)
      ROT(3,2)=0.
      ROT(3,3)=COS(THETA)
      DO 130 I=1,N
      DO 120 J=1,3
  120 PR(J)=P(I,J)
      DO 130 J=1,3
  130 P(I,J)=ROT(J,1)*PR(1)+ROT(J,2)*PR(2)+ROT(J,3)*PR(3)
C 3 OVERALL LORENTZ BOOST GIVEN BY BETA VECTOR
  140 IF(BETA(1)**2+BETA(2)**2+BETA(3)**2.LT.1E-8) RETURN
      GA=1./SQRT(1.-BETA(1)**2-BETA(2)**2-BETA(3)**2)
      DO 160 I=1,N
      BEP=BETA(1)*P(I,1)+BETA(2)*P(I,2)+BETA(3)*P(I,3)
      DO 150 J=1,3
  150 P(I,J)=P(I,J)+GA*(GA/(1.+GA)*BEP+P(I,4))*BETA(J)
  160 P(I,4)=GA*(P(I,4)+BEP)
      RETURN
      END
```

- 20 -

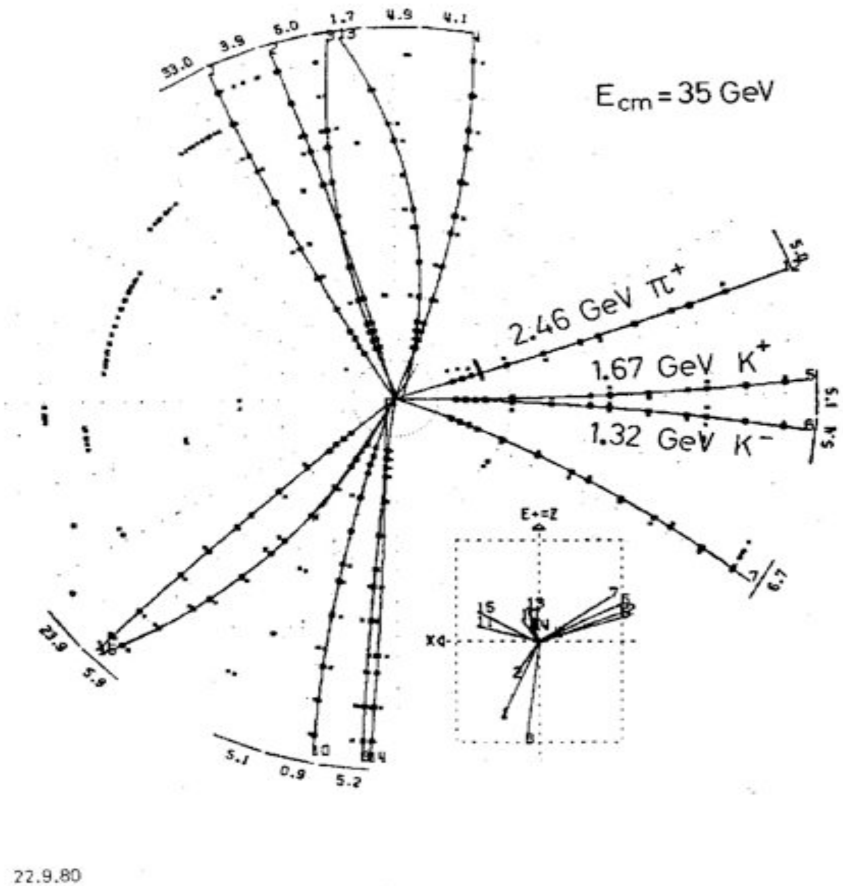
```
      SUBROUTINE LIST(N)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /DATA3/ CHA1(9), CHA2(19), CHA3(2)
      WRITE(6,110)
      DO 100 I=1,N
      IF(K(I,1).GT.0) C1=CHA1(K(I,1))
      IF(K(I,1).LE.0) IC1=-K(I,1)
      C2=CHA2(K(I,2))
      C3=CHA3((47-K(I,2))/20)
      IF(K(I,1).GT.0) WRITE(6,120) I, C1, C2, C3, (P(I,J), J=1,5)
  100 IF(K(I,1).LE.0) WRITE(6,130) I, IC1, C2, C3, (P(I,J), J=1,5)
      RETURN
  110 FORMAT(////T11,'I',T17,'ORI',T24,'PART',T32,'STAB',
     &T44,'PX',T56,'PY',T68,'PZ',T80,'E',T92,'M'/)
  120 FORMAT(10X,I2,4X,A2,1X,2(4X,A4),5(4X,F8.1))
  130 FORMAT(10X,I2,4X,1X,I2,2(4X,A4),5(4X,F8.1))
      END
```

```
      BLOCK DATA
      COMMON /PAR/ PUD, PS1, SIGMA, CX2, EBEG, WFIN, IFLBEG
      COMMON /EDPAR/ ITHROW, PZMIN, PMIN, THETA, PHI, BETA(3)
      COMMON /DATA1/ MESO(9,2), CMIX(6,2), PMAS(19)
      COMMON /DATA2/ IDCD(12), CBR(29), KDP(29,3)
      COMMON /DATA3/ CHA1(9), CHA2(19), CHA3(2)
      DATA PUD/0.4/, PS1/0.5/, SIGMA/350./, CX2/0.77/,
     &EBEG/10000./, WFIN/100./, IFLBEG/1/
      DATA ITHROW/1/, PZMIN/0./, PMIN/0./, THETA,PHI,BETA/5*0./
      DATA MESO/7,1,3,2,8,5,4,6,9,7,2,4,1,3,6,2,5,9/
      DATA CMIX/2*0.5,1.,2*0.5,1.,2*0.25,0.5,2*0.,1./
      DATA PMAS/0.,2*139.6,2*493.7,2*497.7,135.,548.8,957.6,
     &2*765.9,2*892.2,2*896.3,770.2,782.5,1019.6/
      DATA IDCD/0,1,6,11,12,13,15,17,19,21,22,25/
      DATA CBR/1.,0.381,0.681,0.918,0.969,1.,0.426,0.662,0.959,
     &0.980,1.,1.,1.,0.667,1.,0.667,1.,0.667,1.,0.667,1.,1.,
     &0.899,0.987,1.,0.486,0.837,0.984,1./
      DATA KDP/1,1,8,2,1,1,2,8,1,1,1,2,3,6,4,7,5,4,6,5,7,2,2,
     &1,2,4,6,2,1,1,1,8,3,2,1,3,8,17,18,1,8,8,2,8,3,8,3,8,2,8,
     &3,3,8,3,5,7,3,9,0,0,8,8,3,8,9,9,14*0,8,4*0,8,0/
      DATA CHA1/'UD','DU','US','SU','DS','SD','UU','DD','SS'/
      DATA CHA2/'GAMM','PI+','PI-','K+','K-','K0','KB0','PI0','ETA',
     &'ETAP','RHO+','RHO-','K*+','K*-','K*0','KB*0','RHO0','OMEG','PHI'/
      DATA CHA3/'    ','STAB'/
      END
```

DESY: Discovery of the Gluon

Gluon discovery in 1979

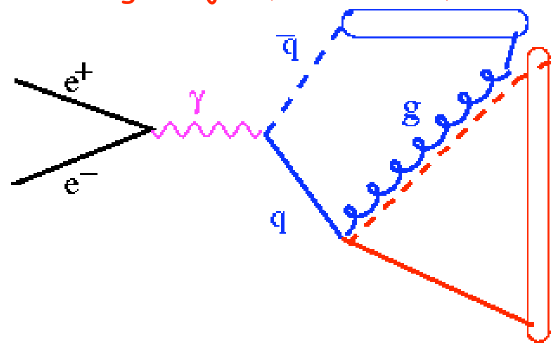
At the PETRA storage ring, the "gluon" was directly observed for the first time. For their discovery of the gluon in 1979, four DESY scientists received the Particle Physics Prize of the European Physical Society (EPS), considered the "European Nobel Prize in Physics", in 1995.

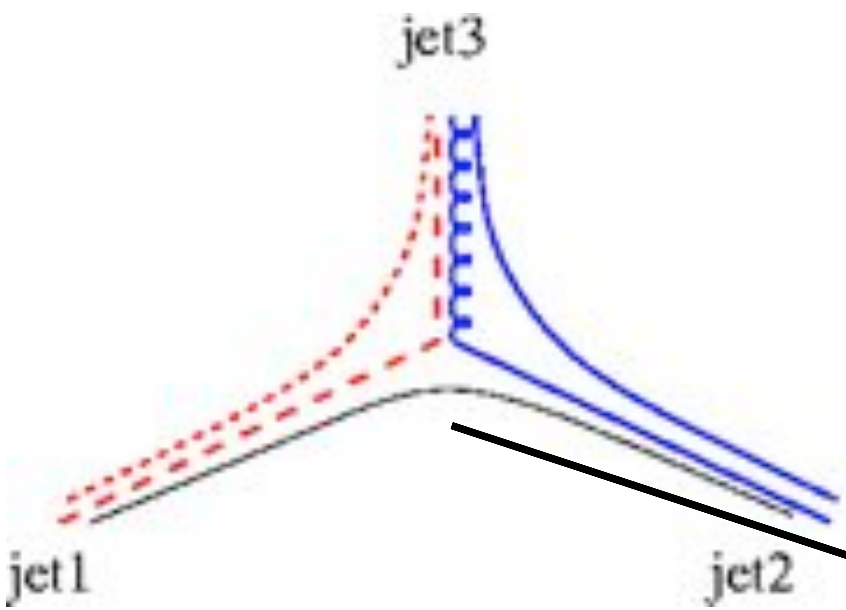


Gluons in Lund String Fragmentation

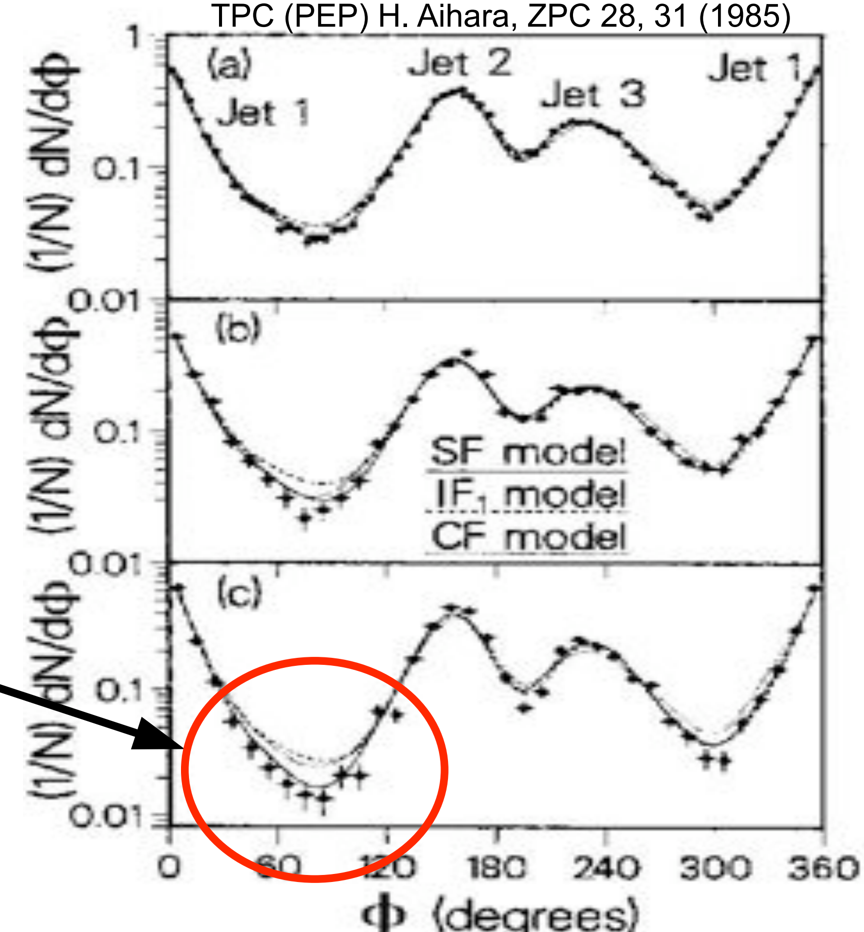
How to find the gluon jets, Andersson, Gustafson, Sjostrand, PLB 94,211 (1980)

- process $e^+e^- \to q\bar{q}g$
- watch out color flow !!!
- gluons act as kinks on strings
- string effect seen in experiment





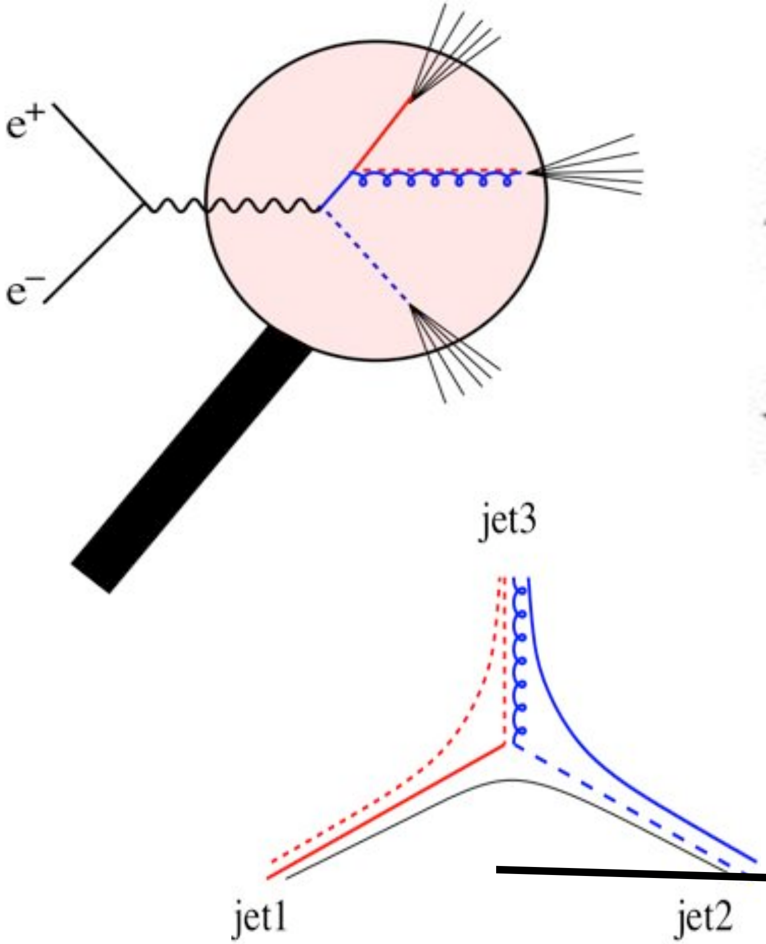
TPC (PEP) H. Aihara, ZPC 28, 31 (1985)

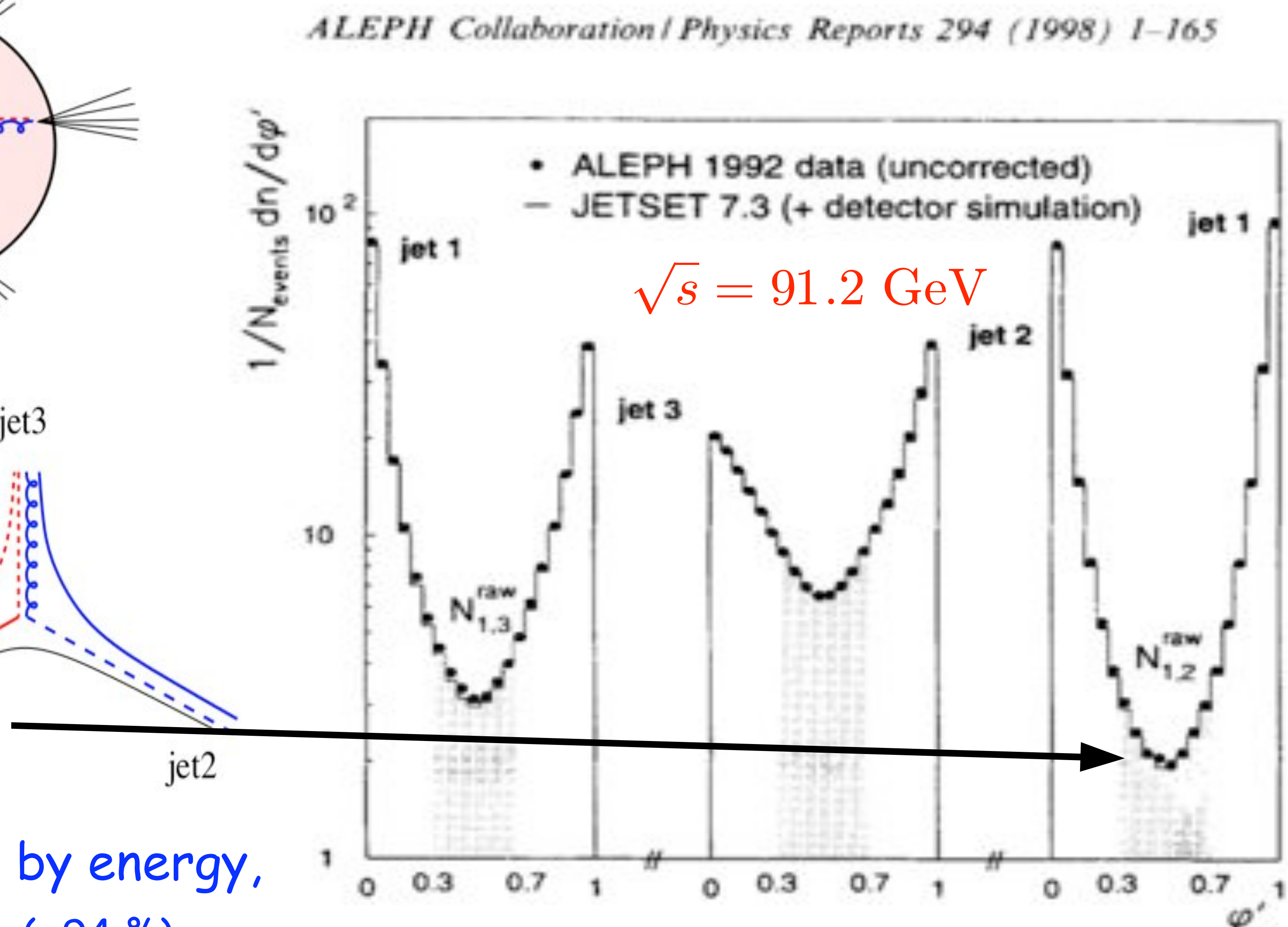


The Lund Model does describe it !!!

… and with more precise data …

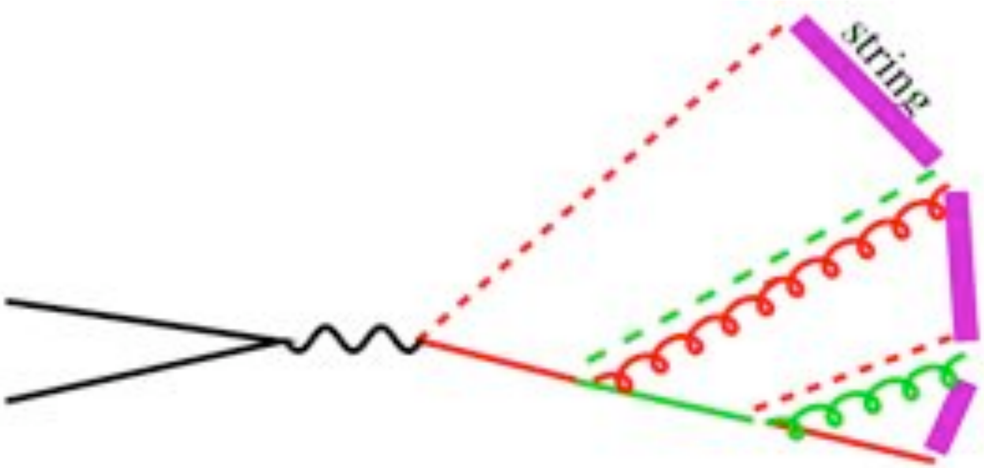
How to find the gluon jets, Andersson, Gustafson, Sjostrand, PLB 94,211 (1980)





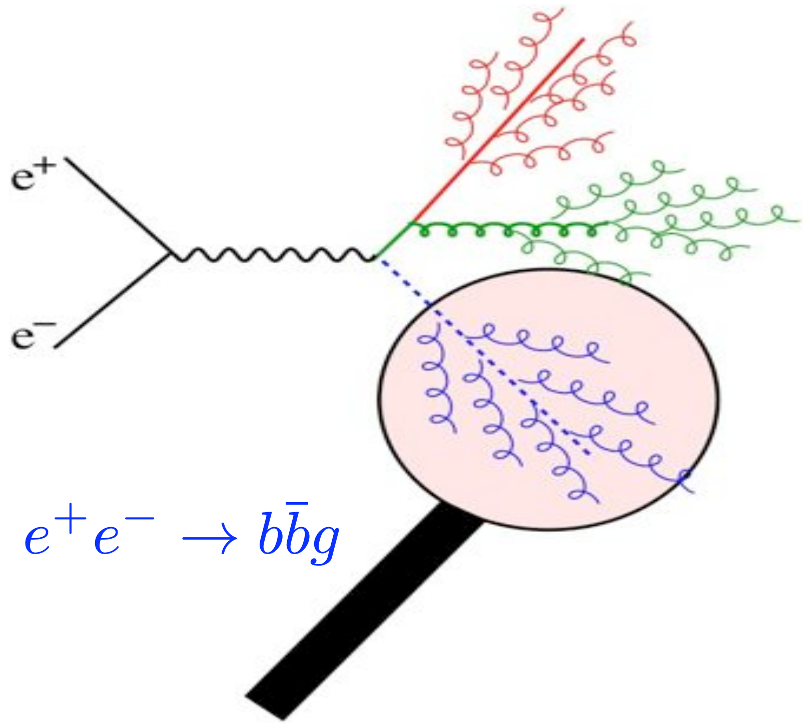
- *jets ordered by energy,*
  highest is quark (~94 %) ,
  lowest is gluon (~70%)

Models for jet evolution



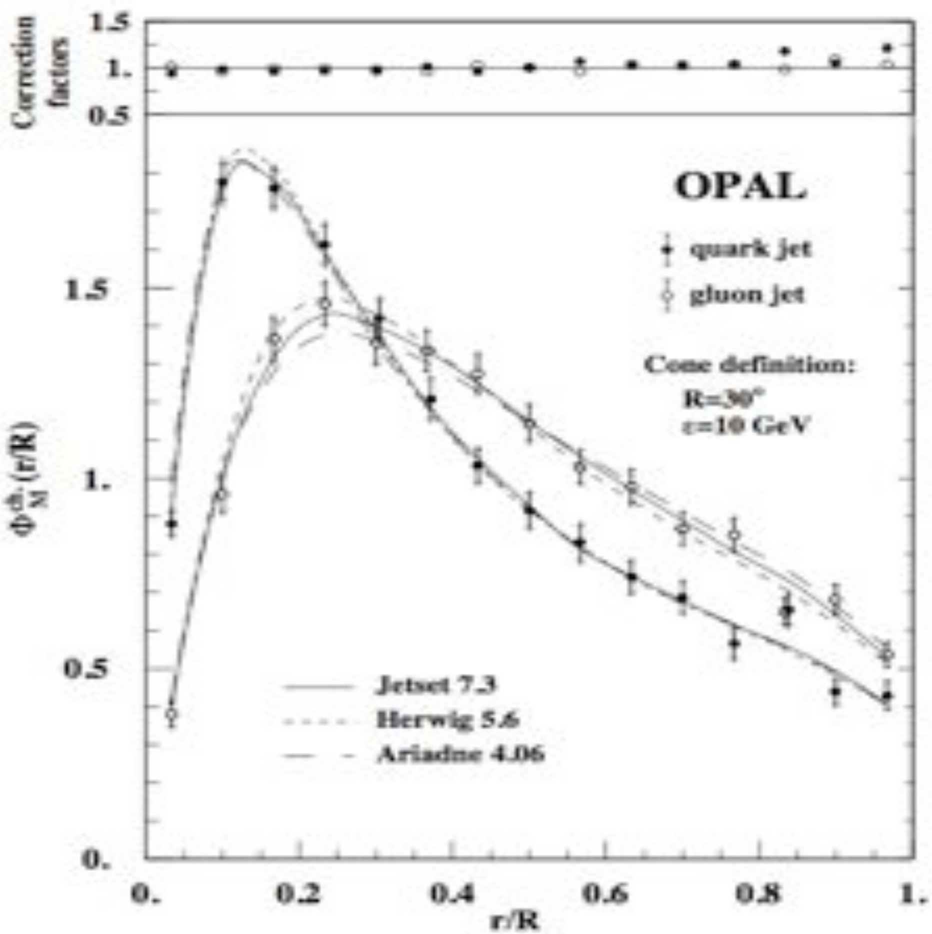
- Parton Showering
  - ➔ Color field of Lund string interpreted in terms of gluons
  - ➔ successive parton radiation, with splitting function
  - ➔ ordering introduced explicitly:
    - ➔ virtuality, pt or angular ordered
  - ➔ need to take care of recoil
  - ➔ implemented in JETSET/PYTHIA/HERWIG

Jet evolution: q and g jets



$e^+e^- \rightarrow b\bar{b}g$

OPAL Collaboration Z.Phys.C68:179-202,1995.

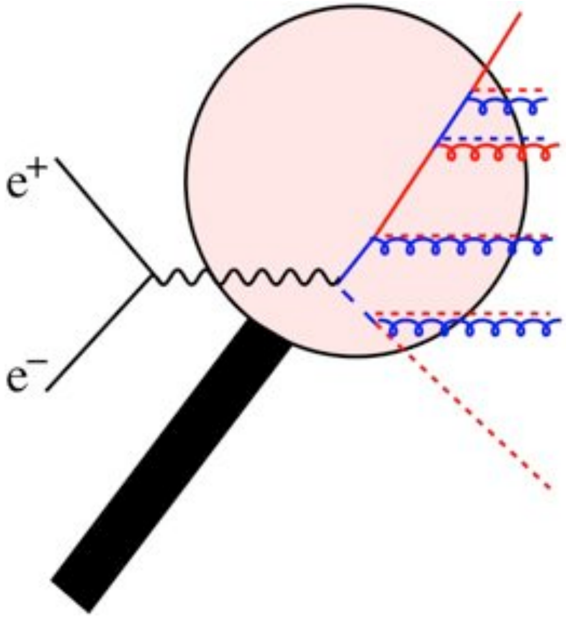


- select 3 jets, highest E-jet with secondary vertex (b-jet)
- 2 lower E jets are enriched gluon-jets
- use MC for corrections to true gluon

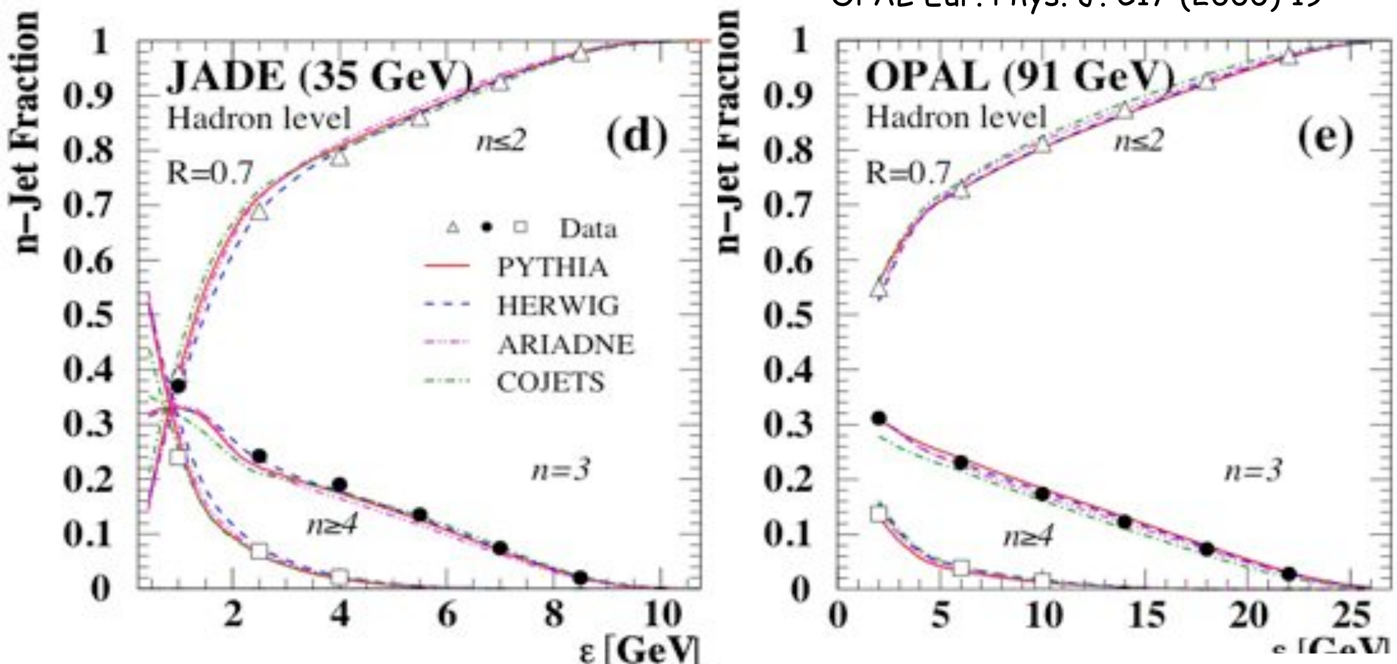
➔ gluon jets are wider ...

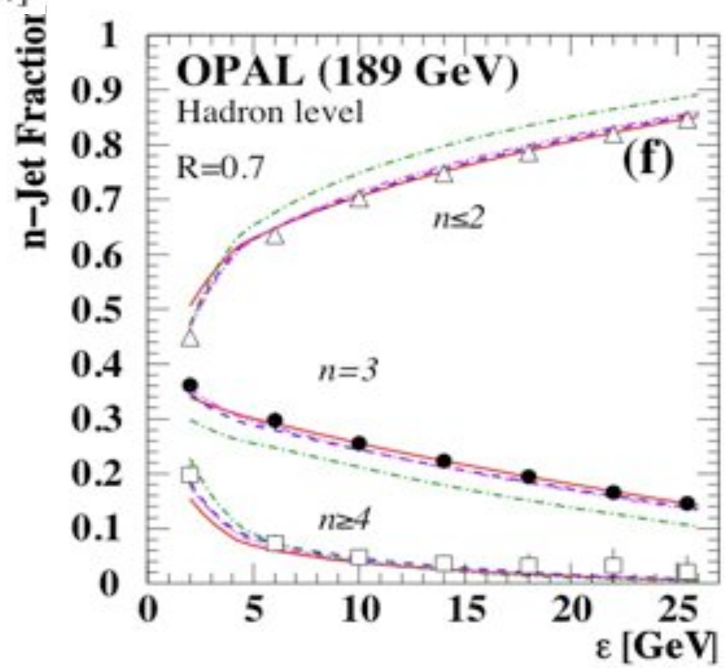
➔ MC's with parton shower and CDM describe jet shapes

Multijet production



OPAL Eur. Phys. J. C17 (2000) 19





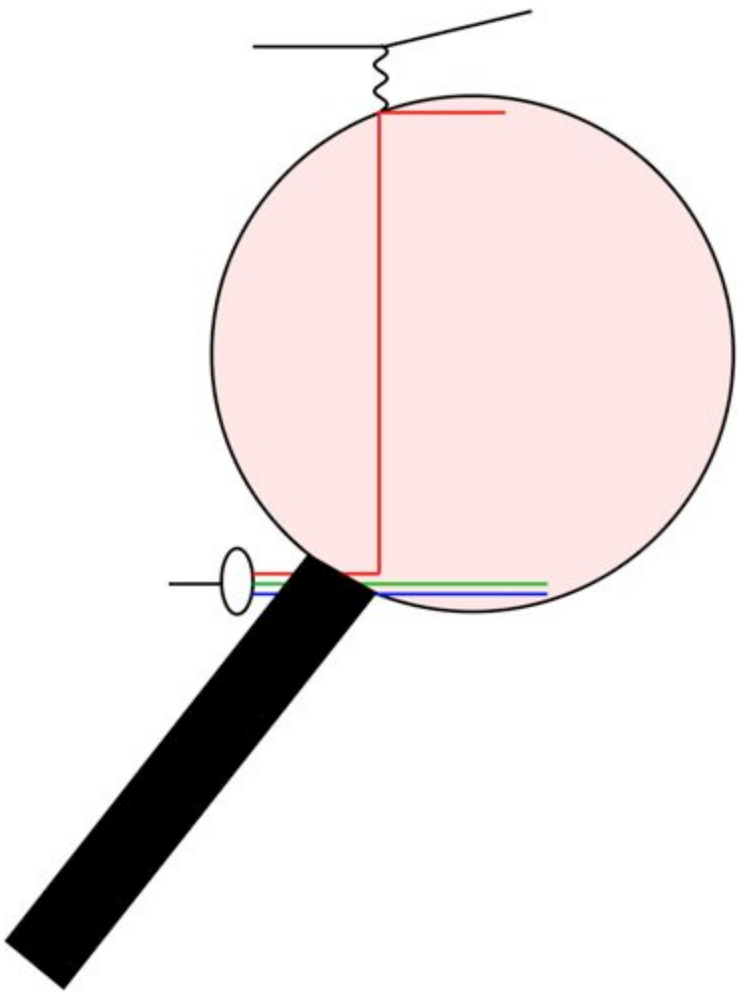
- using cone jet algorithm
- shower MCs are able to reproduce multi jets rates from low to highest CM energy

Wait .....

Let's go one step back ...

Let's look at ep first !

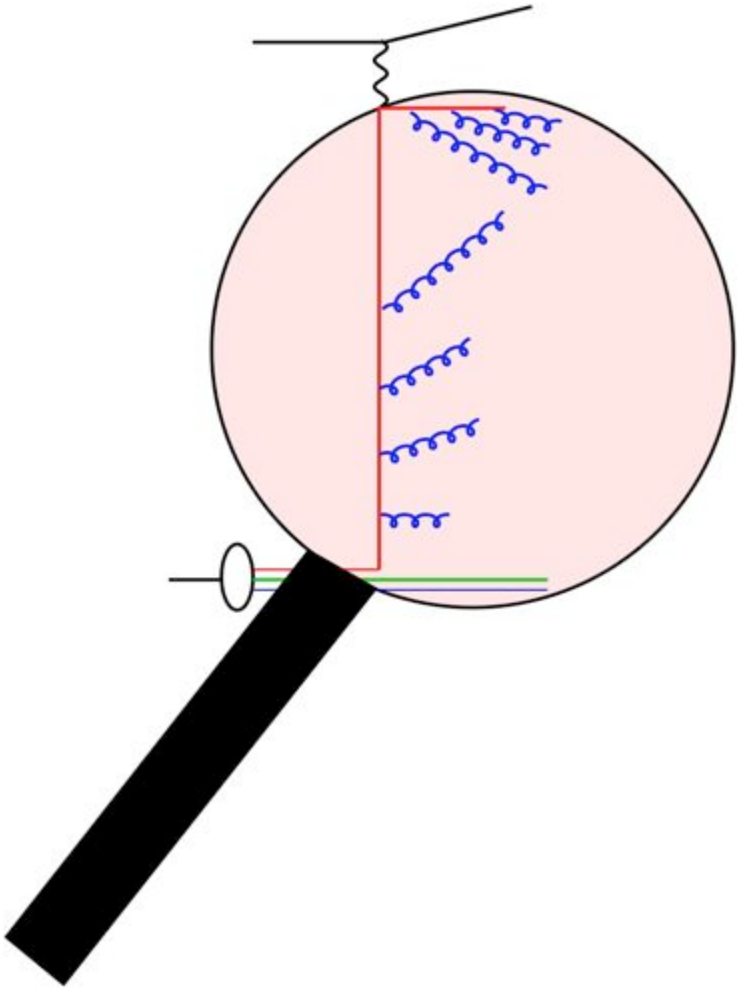
The fun with ep scattering



- Deep Inelastic Scattering is a incoherent sum of $e^+q \to e + q$
- only 50 % of p momentum carried by quarks
- need a large gluon component
- partonic part convoluted with parton density function $f_i(x)$

$$\sigma(e^+p \to e^+X) = \sum_i f_i(x, \quad)\sigma(e^+q_i \to e^+q_i)$$

The fun with ep scattering

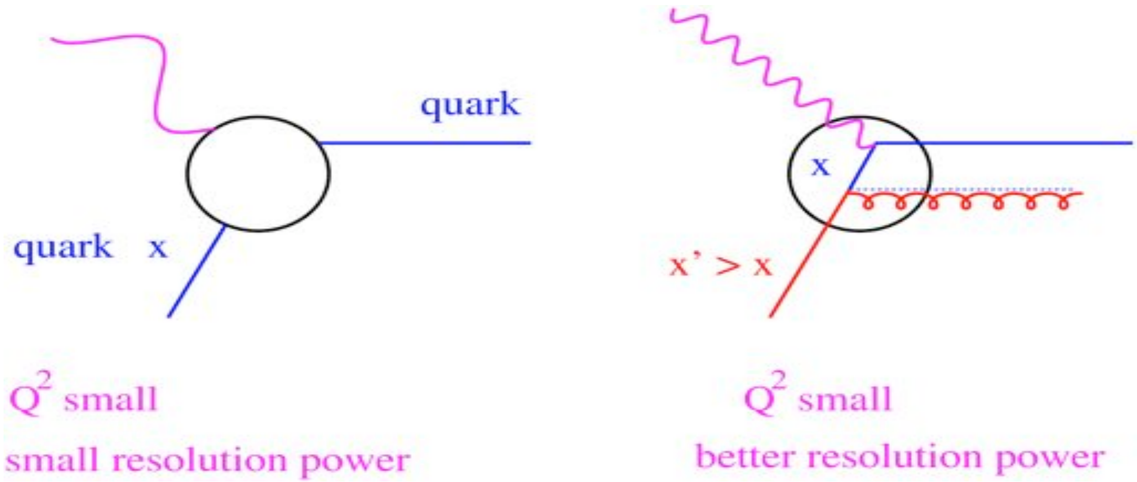


- Deep Inelastic Scattering is a incoherent sum of $e^+q \to e+q$
- only 50 % of p momentum carried by quarks
- need a large gluon component
- partonic part convoluted with parton density function $f_i(x)$
- BUT we know, PDF depends on resolution scale Q^2

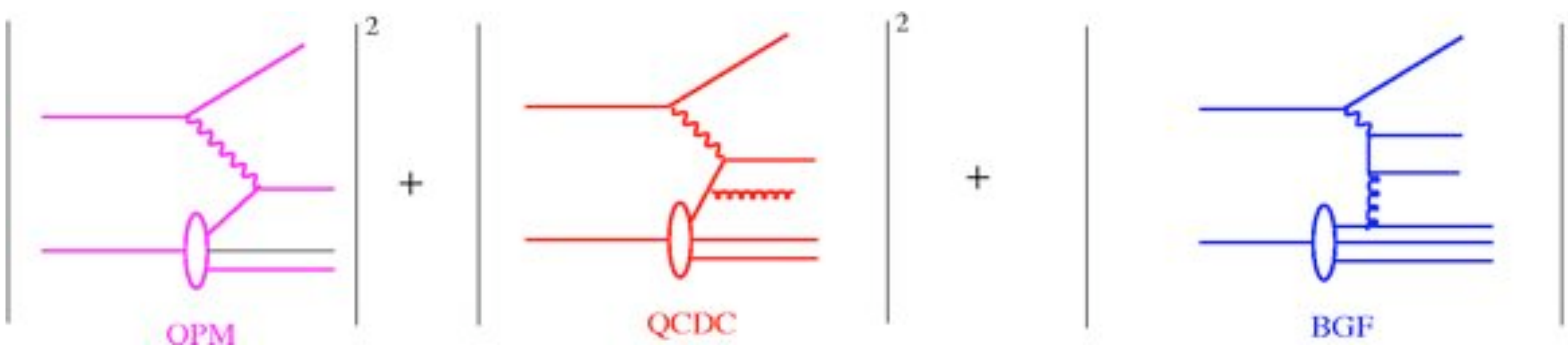
$$\sigma(e^+p \to e^+X) = \sum_i f_i(x,Q^2)\sigma(e^+q_i \to e^+q_i)$$

$F_2(x,Q^2)$: DGLAP evolution equation

- QPM: F_2 is independent of Q^2
- Q^2 dependence of structure function: **D**okshitzer **G**ribov **L**ipatov **A**ltarelli **P**arisi



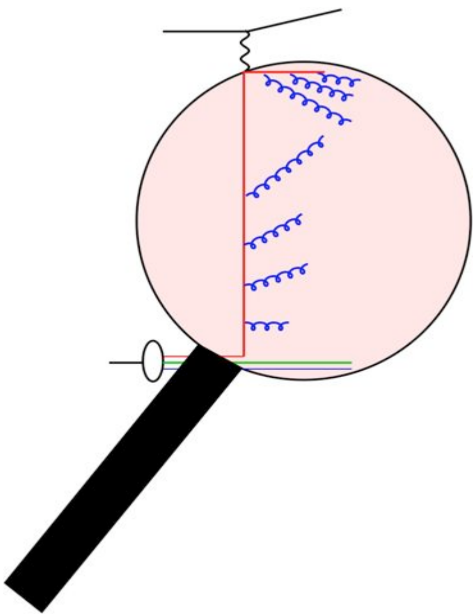
➔ Probability to find parton at small x increases with Q^2

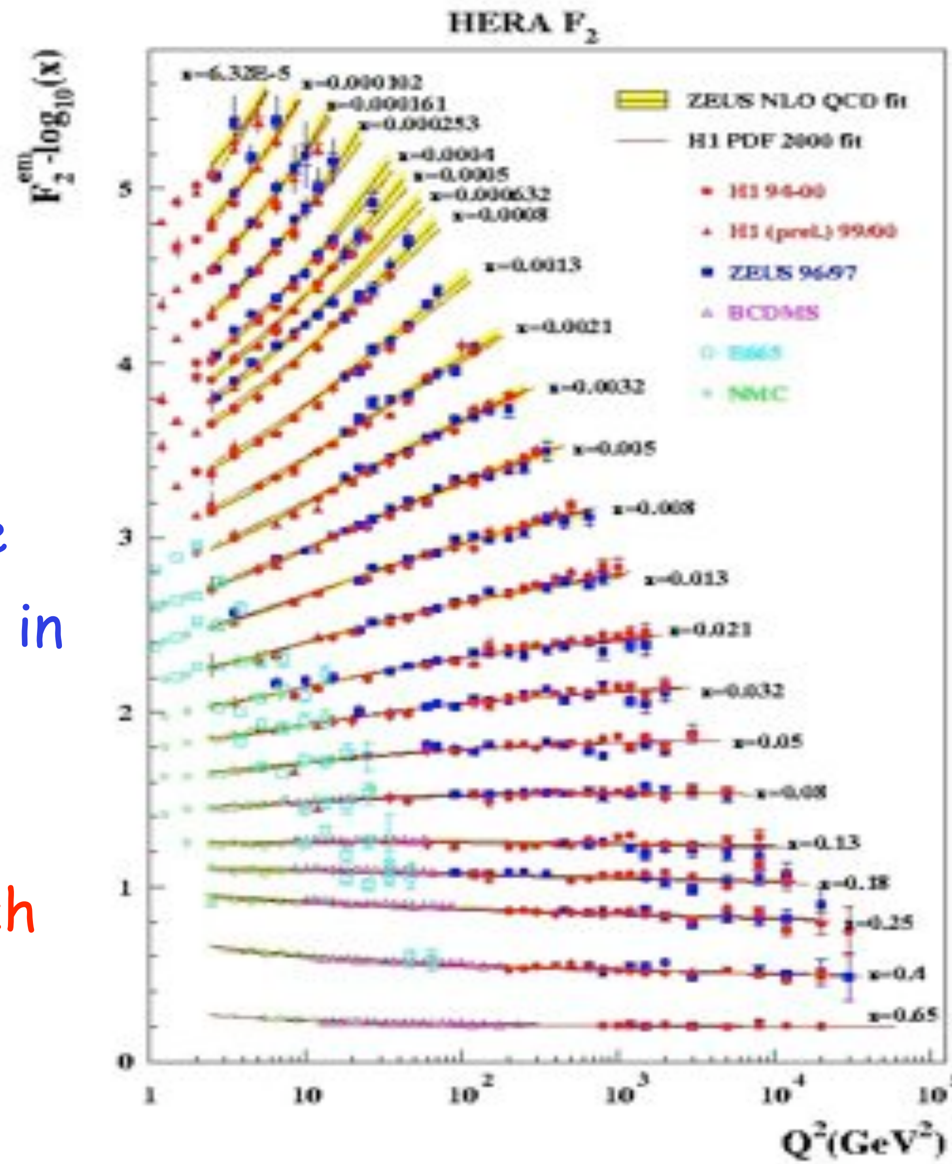
$F_2 =$ 

➔ **Test of theory: Q^2 evolution of $F_2(x,Q^2)$!!!!!**

The fun with ep scattering

$$\sigma(e^+p \to e^+X) = \sum f_i(x,Q^2)\sigma(e^+q_i \to e^+q_i)$$





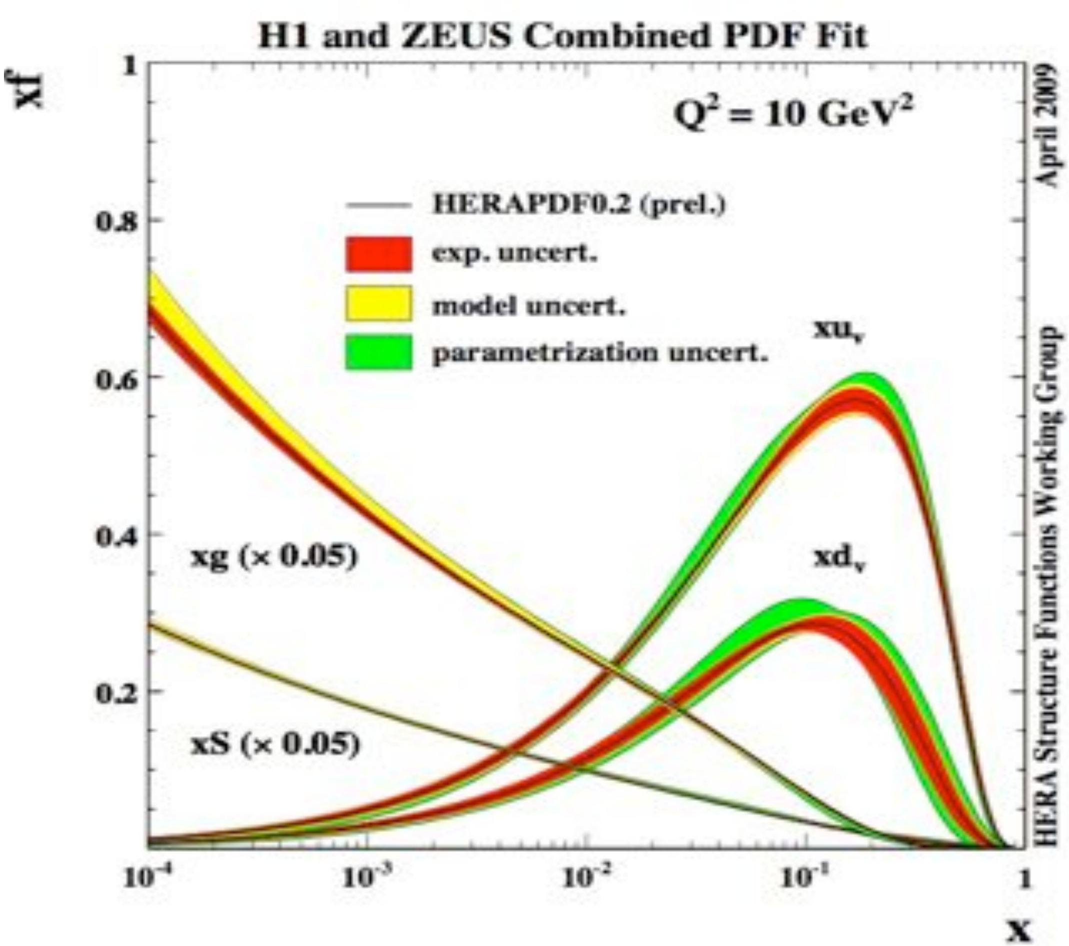
- perfect description of precise measurements of HUGE range in x and Q^2
- Theory works well.....

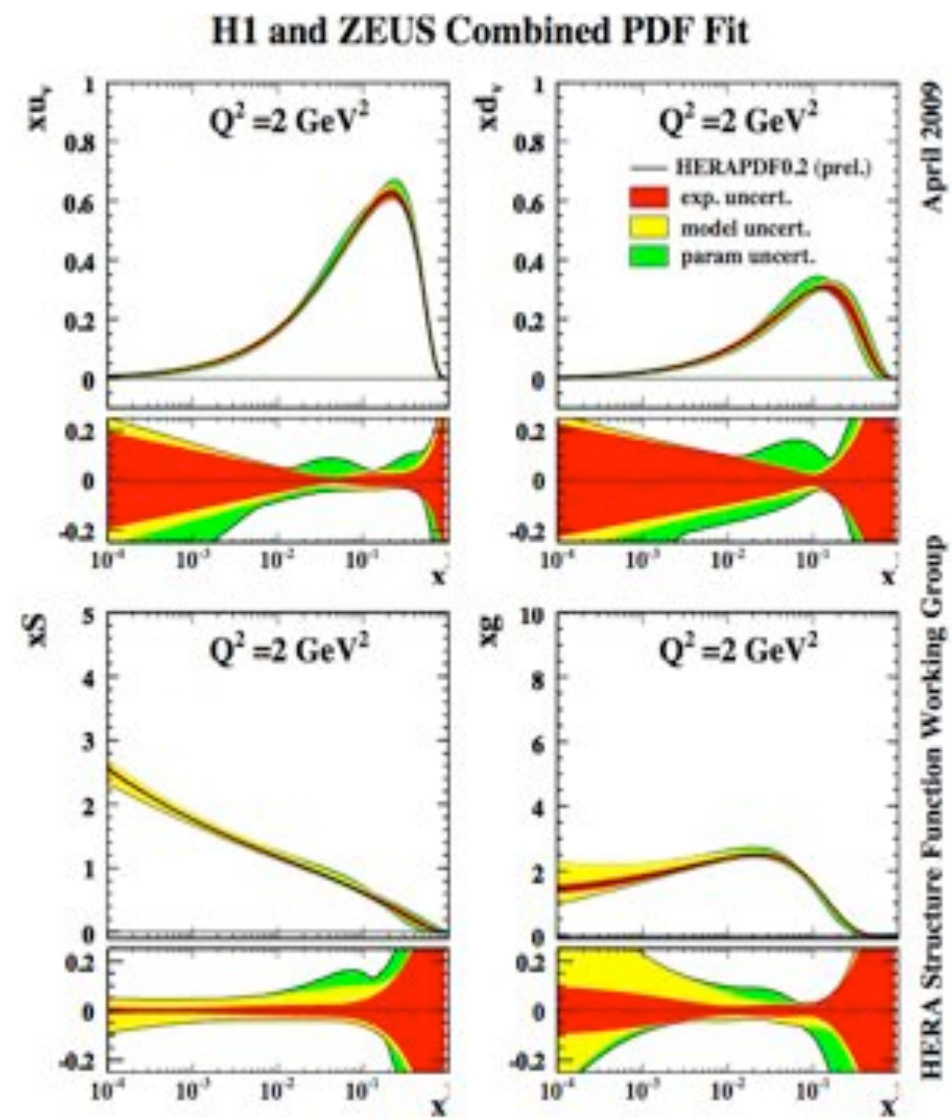
➔ extract parton densities, which are universal

➔ to be used at LHC.....

The proton PDFs ...

- quark and gluon PDFs





→ **Very large gluon density, even at small resolution scales Q^2**

A simple
ep
Monte Carlo event
generator ...

The DIS process ep → epX

- cross section $\frac{d\sigma(ep\to e'X)}{dy\,dQ^2} = \frac{4\pi\alpha^2}{yQ^4}\left(\left(1-y+\frac{y^2}{2}\right)F_2^p(x,Q^2) - \frac{y^2}{2}F_L^p(x,Q^2)\right)$

  with $F_2^p(x,Q^2) = \sum_f e_f^2\left(xq_f(x,Q^2) + x\bar{q}_f(x,Q^2)\right)$

  using the PDFs from before.....

The DIS process ep → epX

- cross section $\frac{d\sigma(ep\to e'X)}{dy\,dQ^2} = \frac{4\pi\alpha^2}{yQ^4}\left(\left(1-y+\frac{y^2}{2}\right)F_2^p(x,Q^2) - \frac{y^2}{2}F_L^p(x,Q^2)\right)$

  with $F_2^p(x,Q^2) = \sum_f e_f^2\left(xq_f(x,Q^2) + x\bar{q}_f(x,Q^2)\right)$

  using the PDFs from before.....

- generate y with g(y)=1/y, and Q^2 with $g(Q^2)=1/Q^2$ (why not $1/Q^4$?):

$$y = y_{min}\left(\frac{y_{max}}{y_{min}}\right)^{R_1} \quad Q^2 = Q^2_{min}\left(\frac{Q^2_{max}}{Q^2_{min}}\right)^{R_2}$$

$$\sigma(ep\to e'X) = \frac{1}{N}\sum_{i=1}^{N}\frac{\frac{d\sigma}{dy_i\,dQ_i^2}}{g(y_i)g(Q_i^2)}\int g(y)dy\int g(Q^2)dQ^2$$

The DIS process ep → epX

- cross section $\frac{d\sigma(ep\to e'X)}{dy\,dQ^2} = \frac{4\pi\alpha^2}{yQ^4}\left(\left(1-y+\frac{y^2}{2}\right)F_2^p(x,Q^2) - \frac{y^2}{2}F_L^p(x,Q^2)\right)$

  with $F_2^p(x,Q^2) = \sum_f e_f^2\left(xq_f(x,Q^2) + x\bar{q}_f(x,Q^2)\right)$

  using the PDFs from before.....

- generate y with g(y)=1/y, and Q^2 with $g(Q^2)=1/Q^2$ (why not $1/Q^4$?):

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$$\sigma(ep\to e'X) = \frac{1}{N}\sum_{i=1}^{N}\frac{\frac{d\sigma}{dy_i\,dQ_i^2}}{g(y_i)g(Q_i^2)}\int g(y)dy\int g(Q^2)dQ^2$$

- calculate x-section with:

$$\sigma(ep\to e'X) = \frac{1}{N}\sum_{i=1}^{N} y_iQ_i^2\frac{d\sigma}{dy_i\,dQ_i^2}\log\left(\frac{y_{max}}{y_{min}}\right)\log\left(\frac{Q^2_{max}}{Q^2_{min}}\right)$$

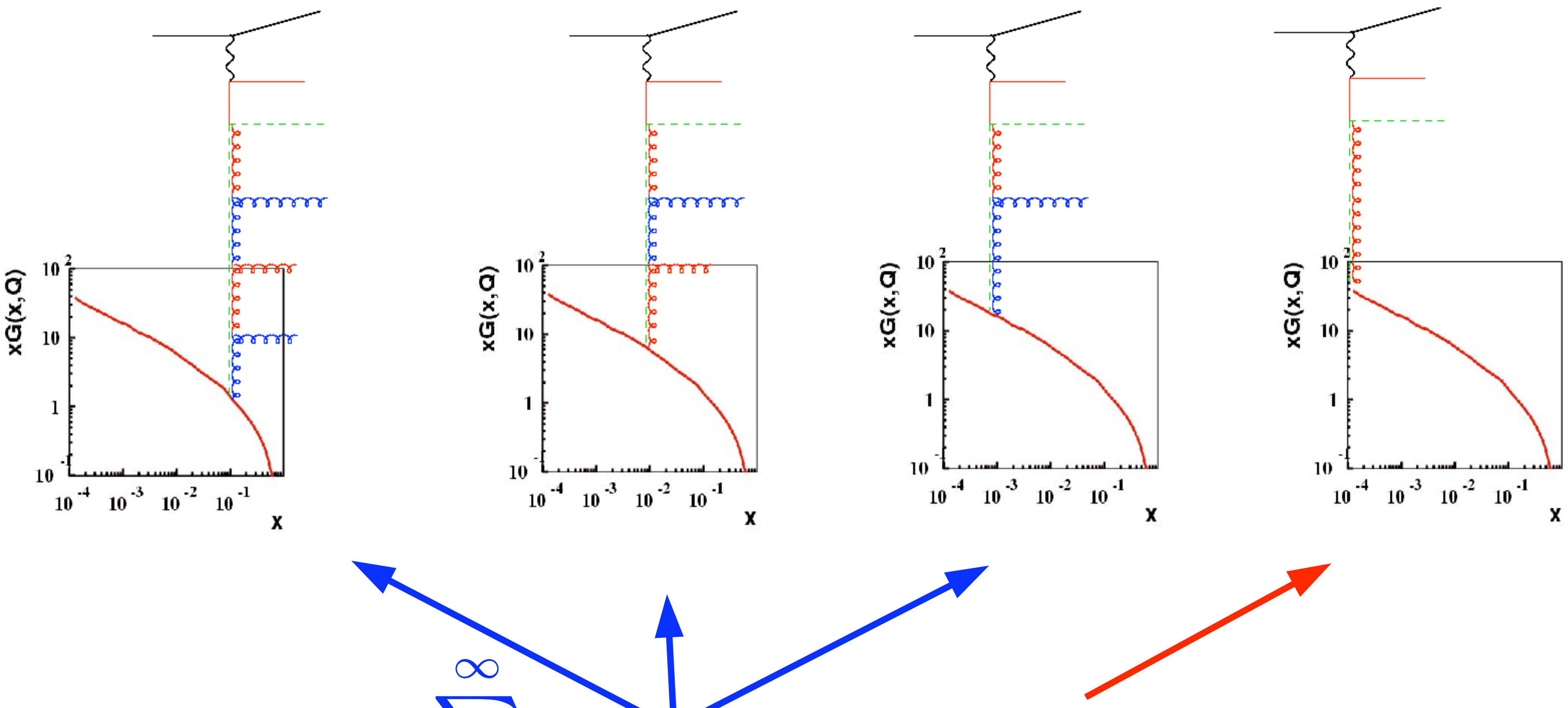
- calculate 4-momenta of scattered electron and virtual photon

We have now an event generator for the total cross section.

What about the final states ?

DGLAP evolution equation... again...

- for fixed x and Q^2 chains with different branchings contribute
- iterative procedure, spacelike parton showering



$$f(x,t) = \sum_{k=1}^{\infty} f_k(x_k, t_k) + f_0(x, t_0)\Delta_s(t)$$

Parton Showers from evolution eq.

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z} \int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')} \tilde{P}(z) f\left(\frac{x}{z}, t'\right)$$

- solve integral equation via explicit iteration:

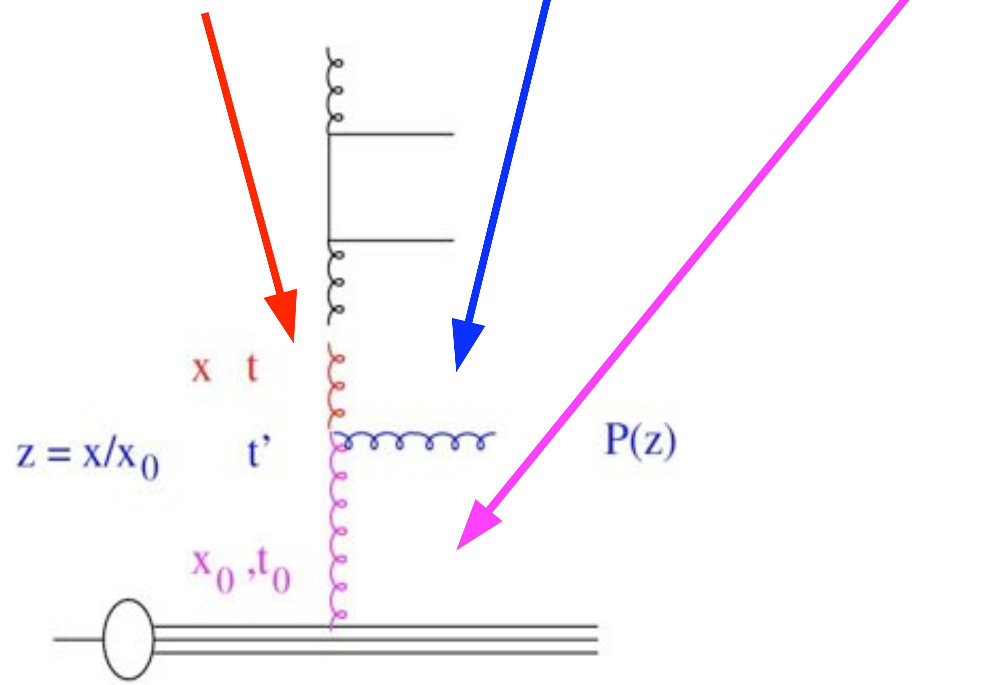
$$f_0(x,t) = f(x,t_0)\Delta(t)$$

$$f_1(x,t) = f(x,t_0)\Delta(t) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} \tilde{P}(z) f(x/z, t_0)\Delta(t')$$

from t' to t w/o branching

branching at t'

from t_0 to t' w/o branching



Parton Shower

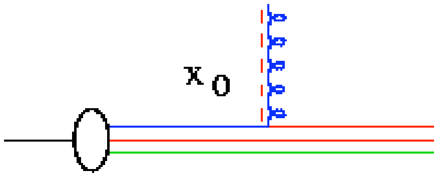
- Sudakov form factor
  - ➔ gives probability for no-branching between q_0 and q
  - ➔ sums virtual contributions to all orders (via unitarity)
    - ➔ virtual (parton loop) and
    - ➔ real (non-resolvable) parton emissions
- Sudakov form factor particularly suited form Monte Carlo approach
  - need to specify scale of hard process (matrix element) $Q \sim p_t$
  - need to specify cutoff scale $Q_0 \sim 1\ GeV$
- Evolution equation with Sudakov form factor recovers exactly evolution equation (with $_+$ prescription)

Parton showers for the initial state

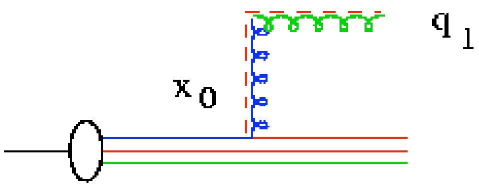
spacelike ($Q<0$) parton shower evolution

- starting from hadron (fwd evolution)

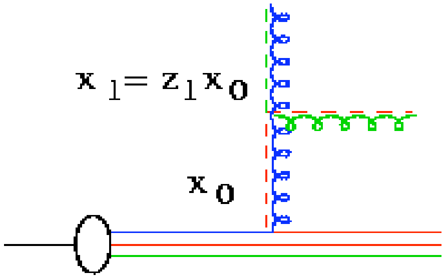
  or from hard scattering (bwd evolution)



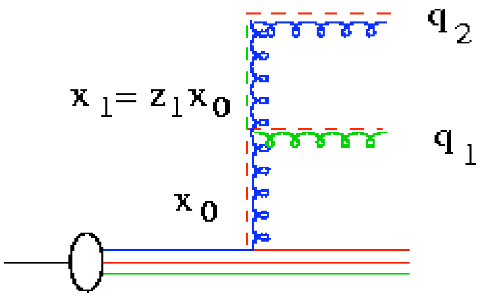
- select q_1 from Sudakov form factor



- select z_1 from splitting function



- select q_2 from Sudakov form factor

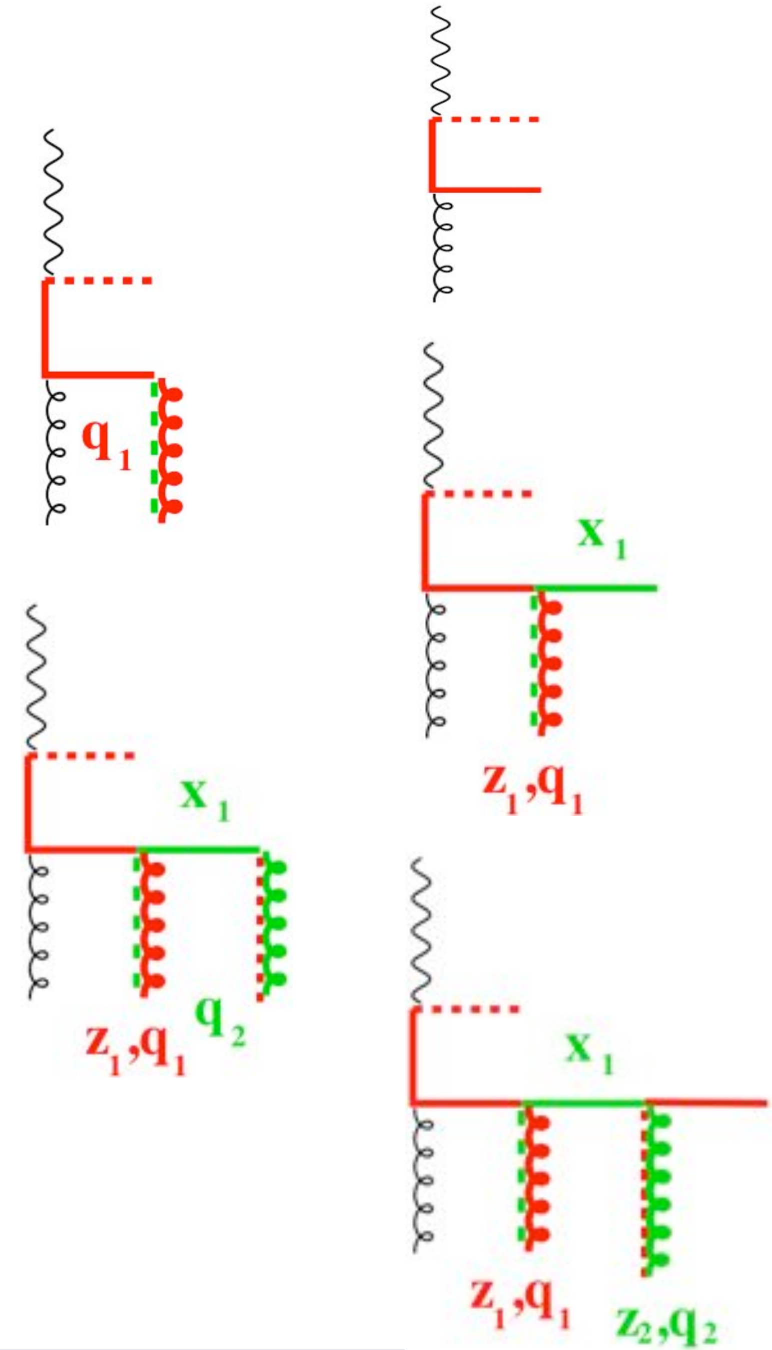


- select z_2 from splitting function
- stop evolution if $q_2 > Q_{hard}$

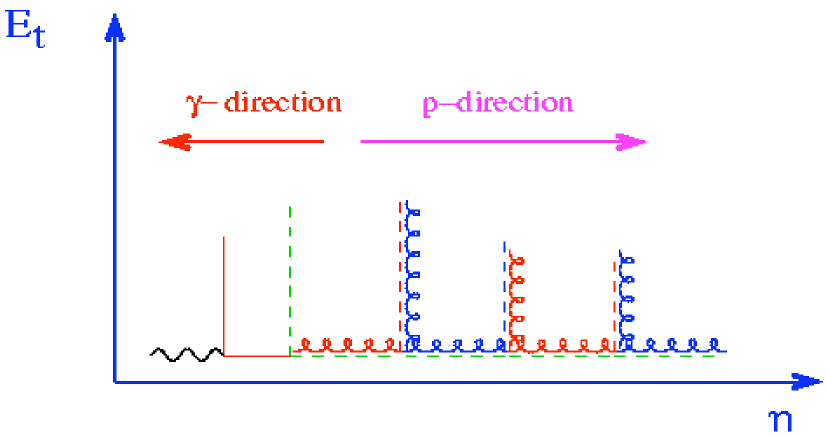
Parton Showers for the final state

timelike parton shower evolution

- starting with hard scattering
- select q_1 from Sudakov form factor
- select z_1 from splitting function
- select q_2 from Sudakov form factor
- select z_2 from splitting function
- stop evolution if $q_2 < q_0$



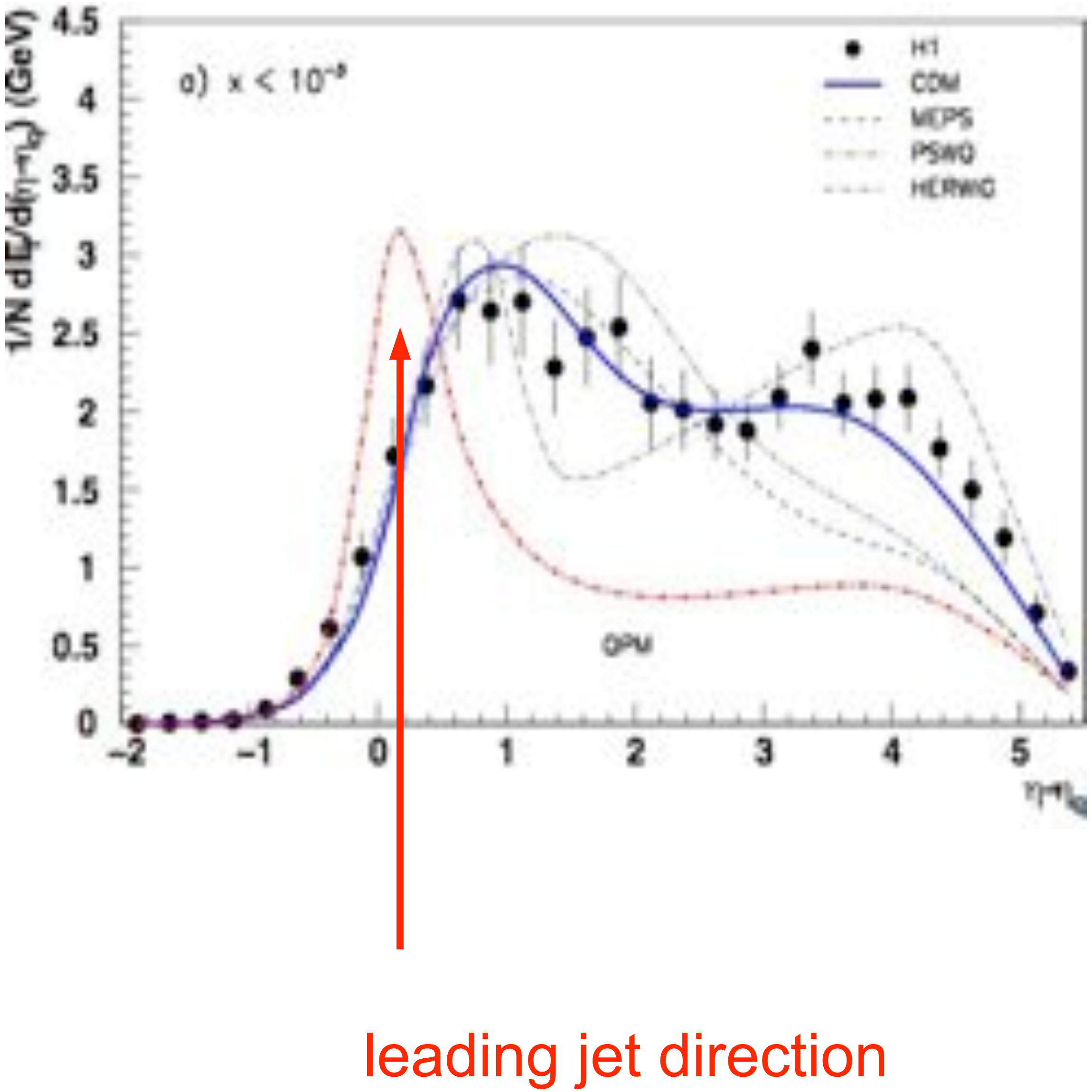
Hadronic final state: Energy flow



- E_t flow in DIS at small x and forward angle (p-direction):

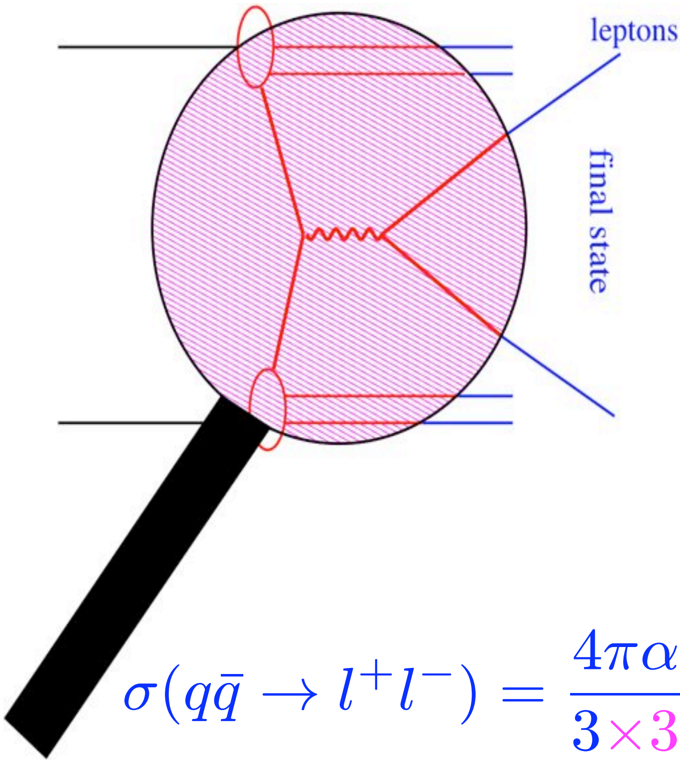
➔ QPM is not enough

➔ clearly parton showers or higher order contributions needed



And ....
What about pp ?

Rotating the diagrams



$$\sigma(q\bar{q} \to l^+l^-) = \frac{4\pi\alpha^2}{3\times 3s} e_q^2$$

Measurement of Z0 and Drell-Yan production cross-section using dimuons in anti-p p collisions at S**(1/2) = 1.8-TeV. CDF Collaboration F. Abe et al. Phys.Rev.D59:052002,1999.

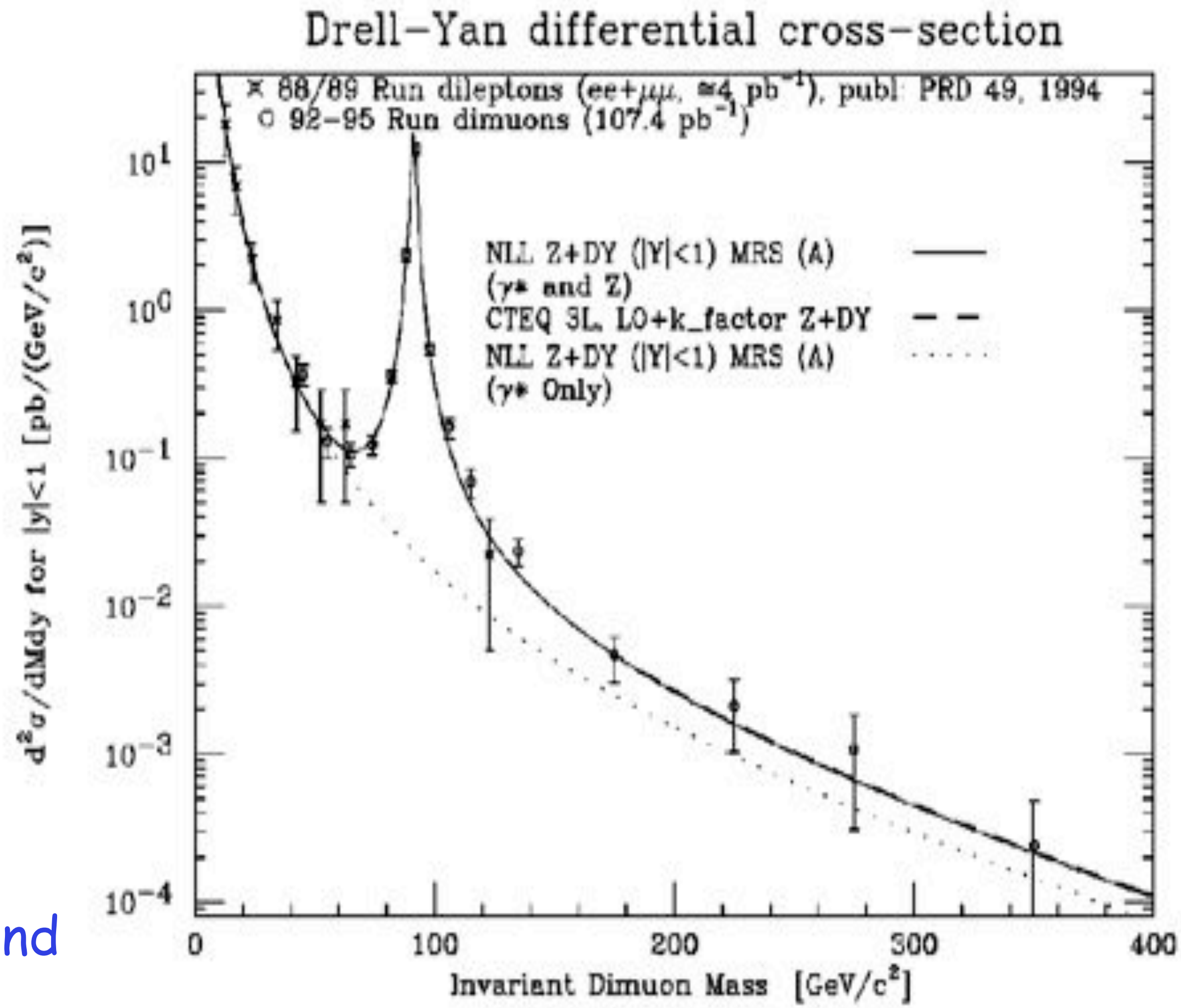


FIG. 8. Drell-Yan dimuon production cross section extracted

Factorization:

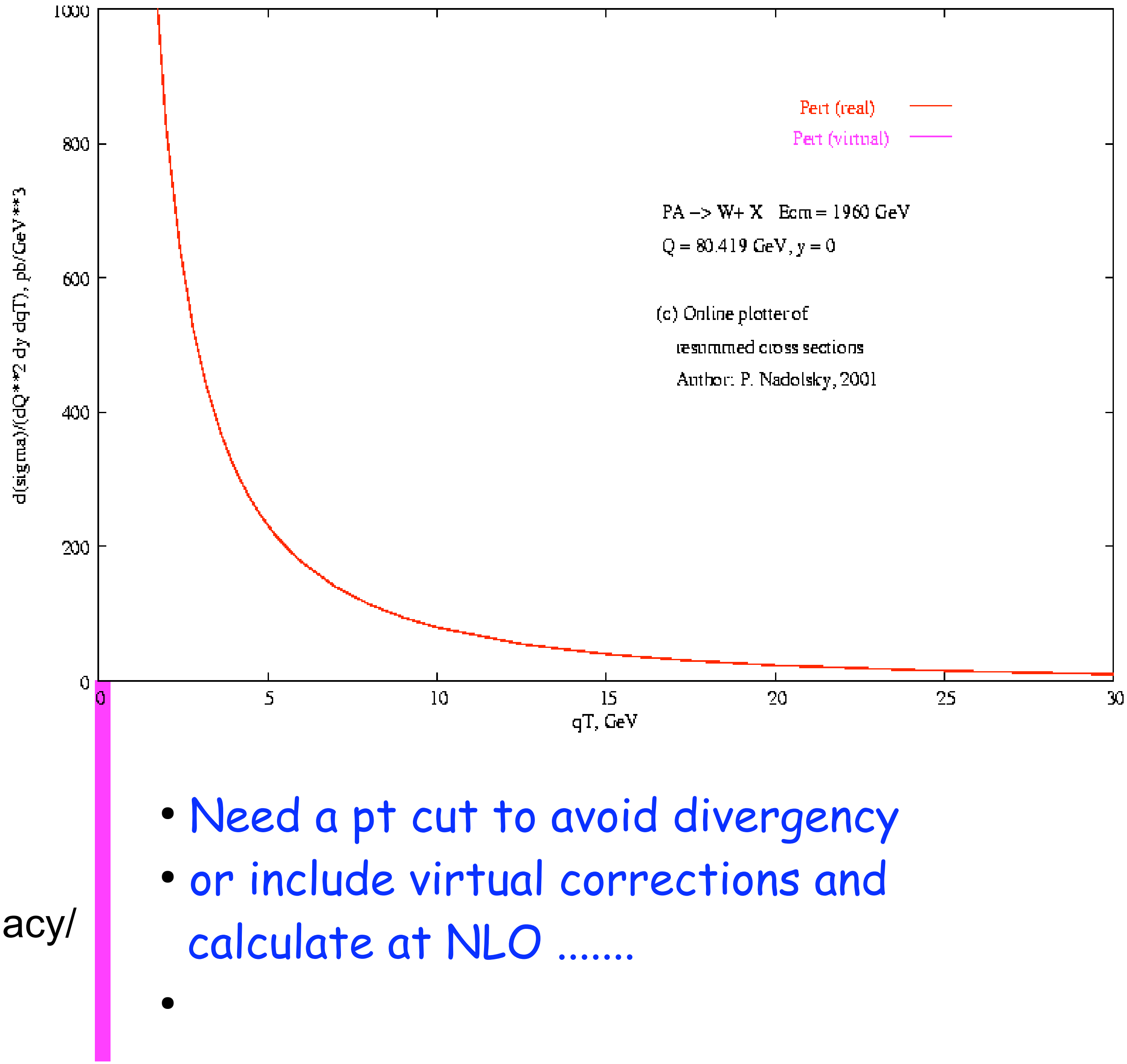
- hard process: $q\bar{q} \to l^+l^-$
- parton densities: prob to find parton with x at scale Q^2 in proton: $q(x,Q^2)$, $g(x,Q^2)$...

Transverse Momentum of W/Z

Perturbative calculation

$\mathcal{O}(\alpha_s)$, $\mathcal{O}(\alpha_s^2)$

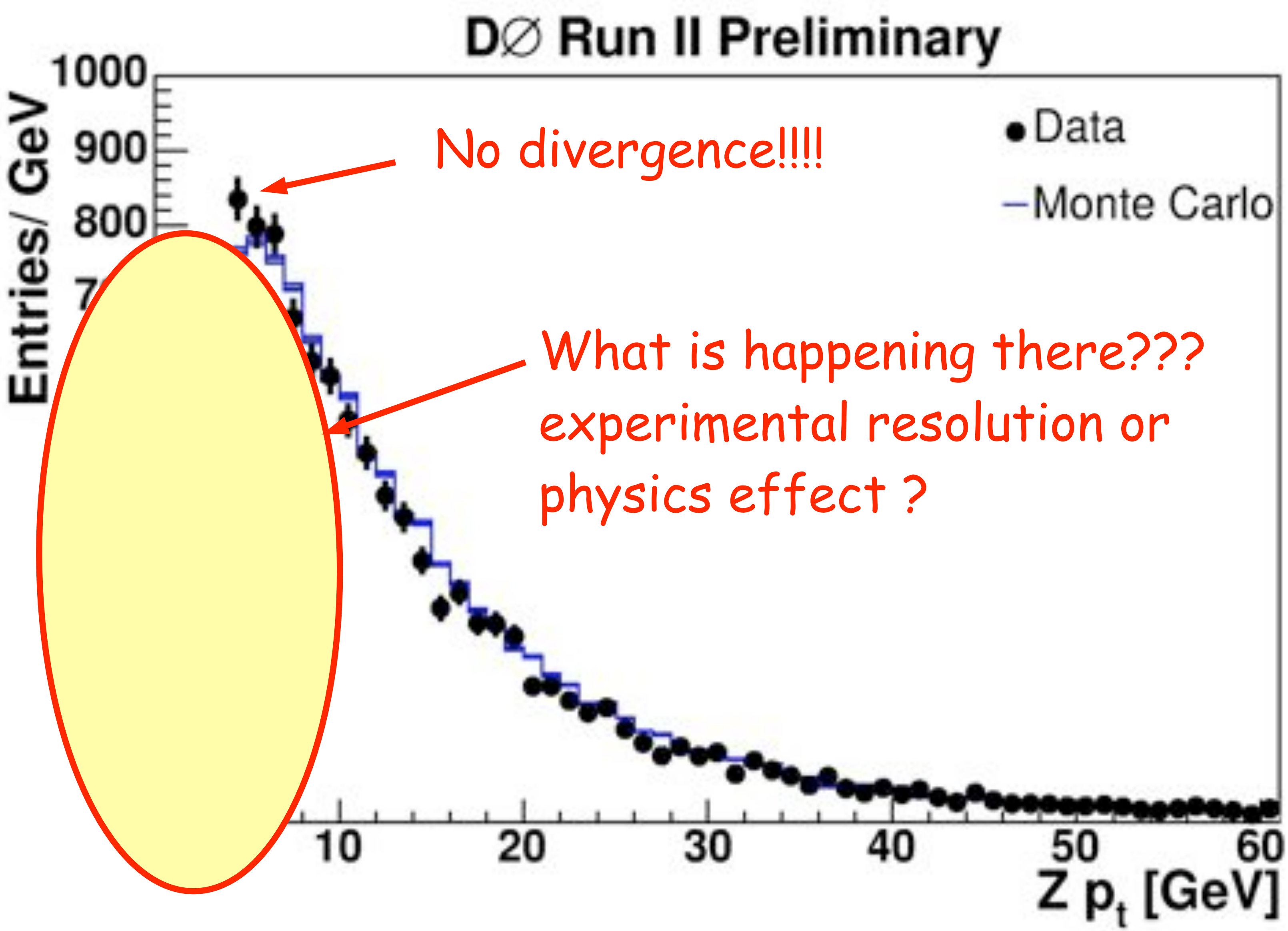
diverges for small p_t



http://hep.pa.msu.edu/wwwlegacy/

- Need a pt cut to avoid divergency
- or include virtual corrections and calculate at NLO ....
-

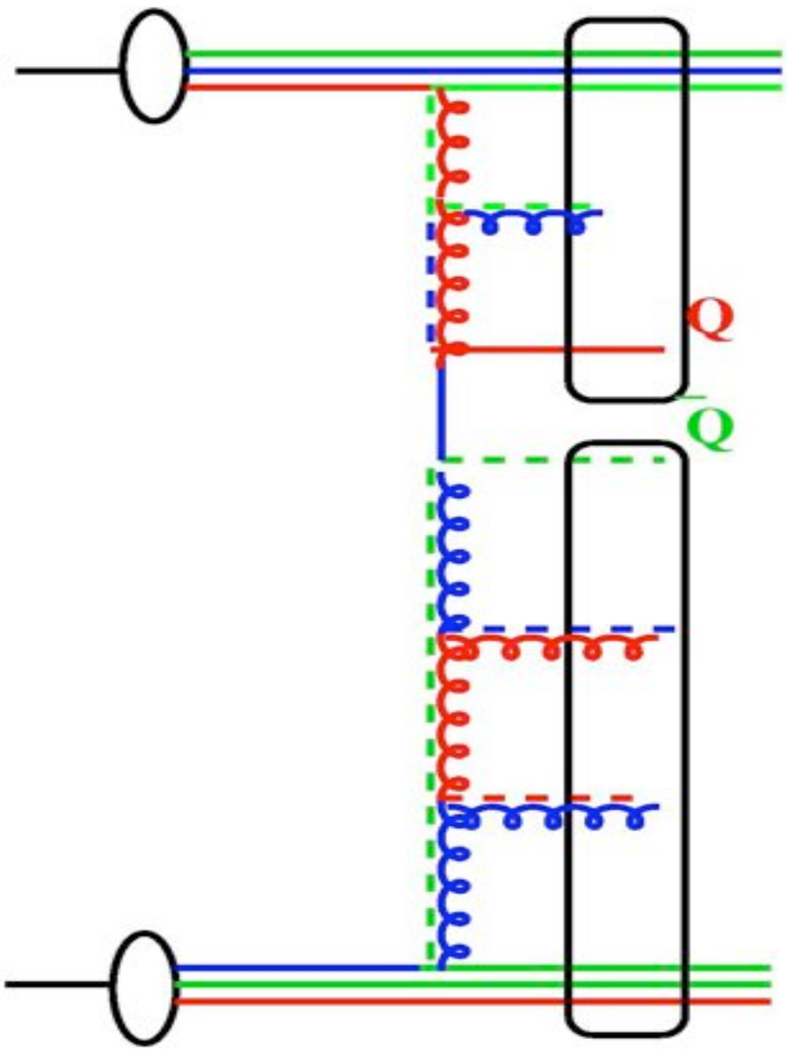
BUT: the data .....


DØ Run II Preliminary
Entries/ GeV
1000
900
800
● Data
— Monte Carlo
No divergence!!!!
What is happening there???
experimental resolution or
physics effect ?
10
20
30
40
50
60
Z p_t [GeV]

Color Flow in pp

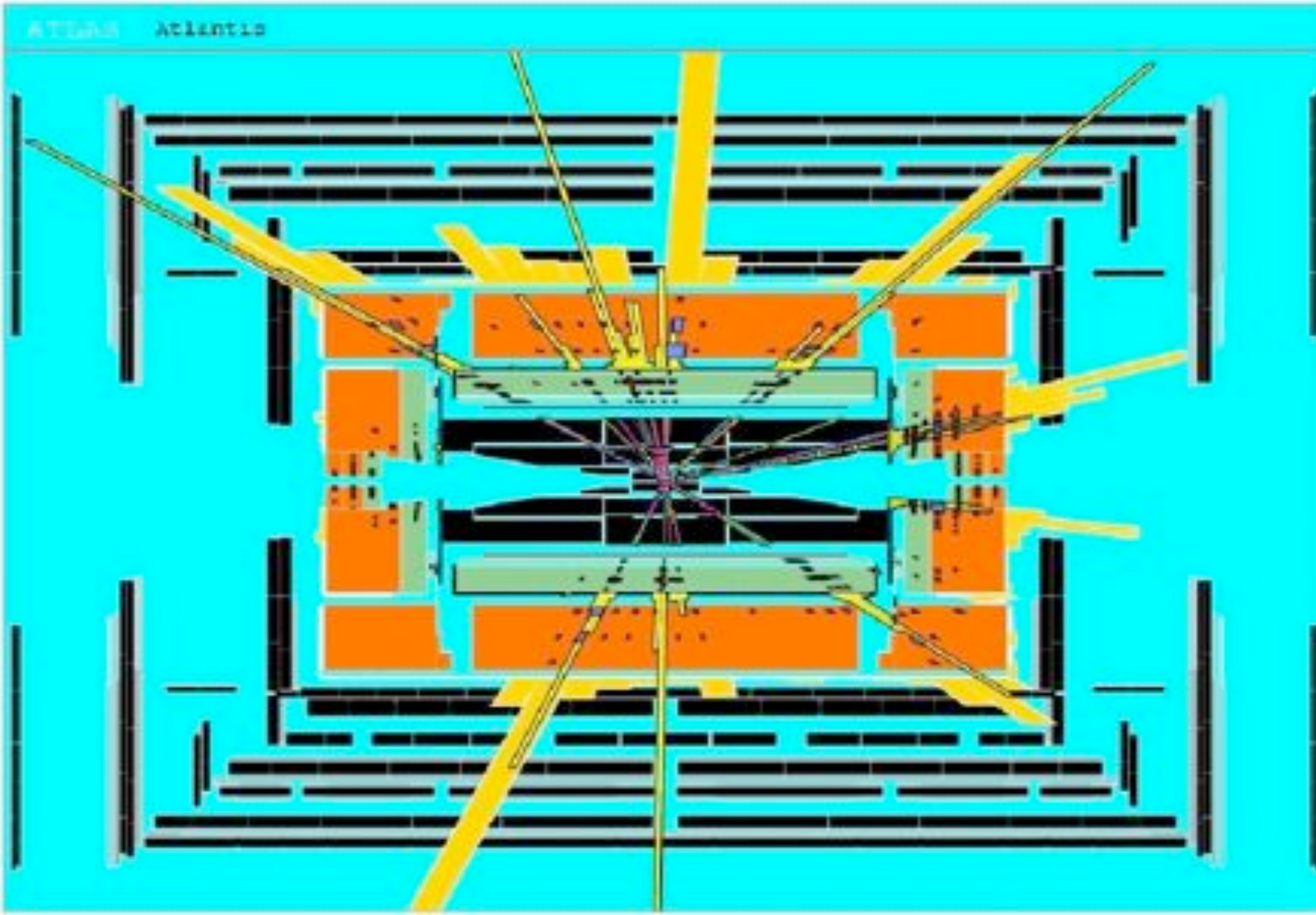
- quarks carry color
- anti-quarks carry anticolor
- gluons carry color - anticolor
  - connect to color singlet systems
  - watch out pp or $p\bar{p}$

$pp \to b\bar{b} + X$



Can we now tell which type of event this is ?

What was this ?

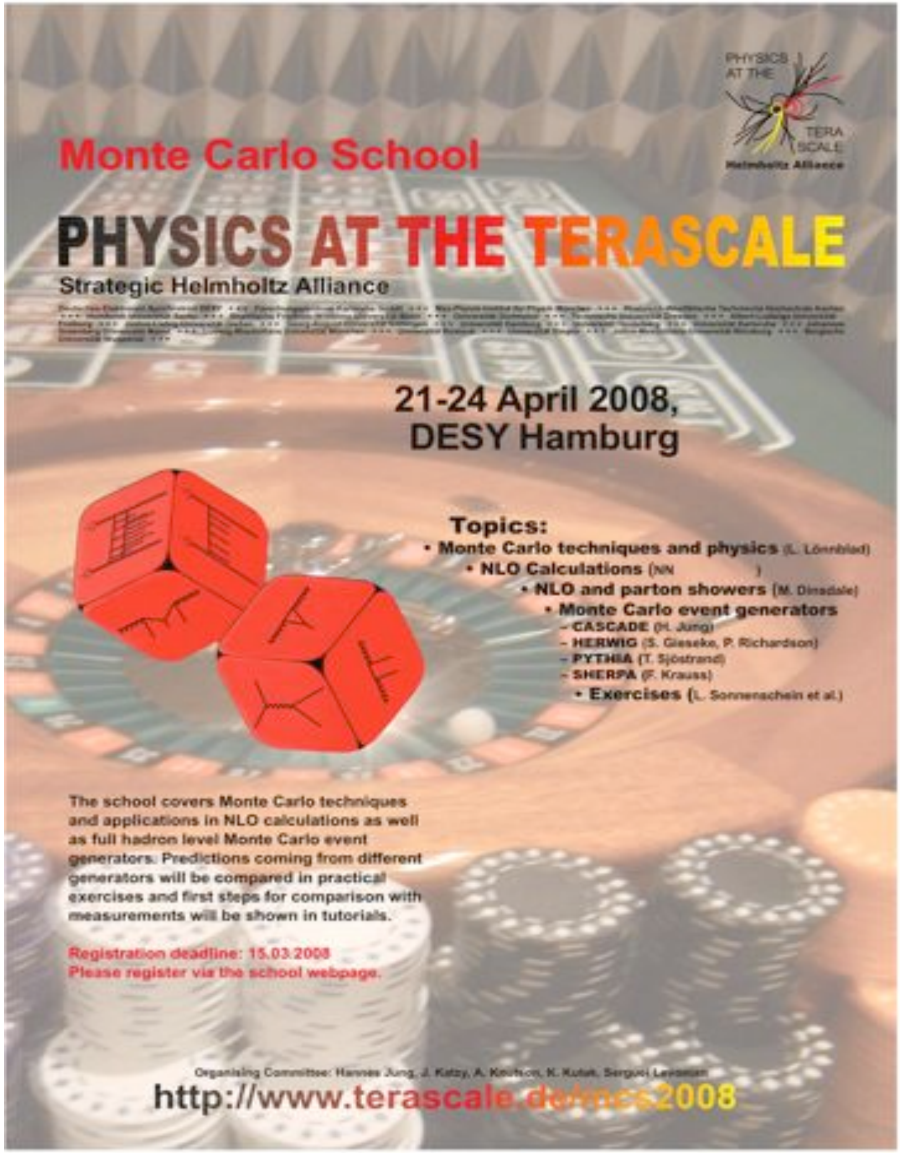


The Analysis Centre in the PHYSICS AT THE TERA SCALE Helmholtz Alliance

- Areas
  - Monte Carlo (user support, tuning, development ...)
  - Parton Distribution Functions
  - Statistics Tools
  - Collaborative tools (web based infos etc)

Monte Carlo group activities

- Development of Monte Carlo generators
  - Tuning of MC generators
  - PDF4MC
  - User support
- Training (schools, seminars)
  - MC schools in spring 2008, 2009
- link to MC group page



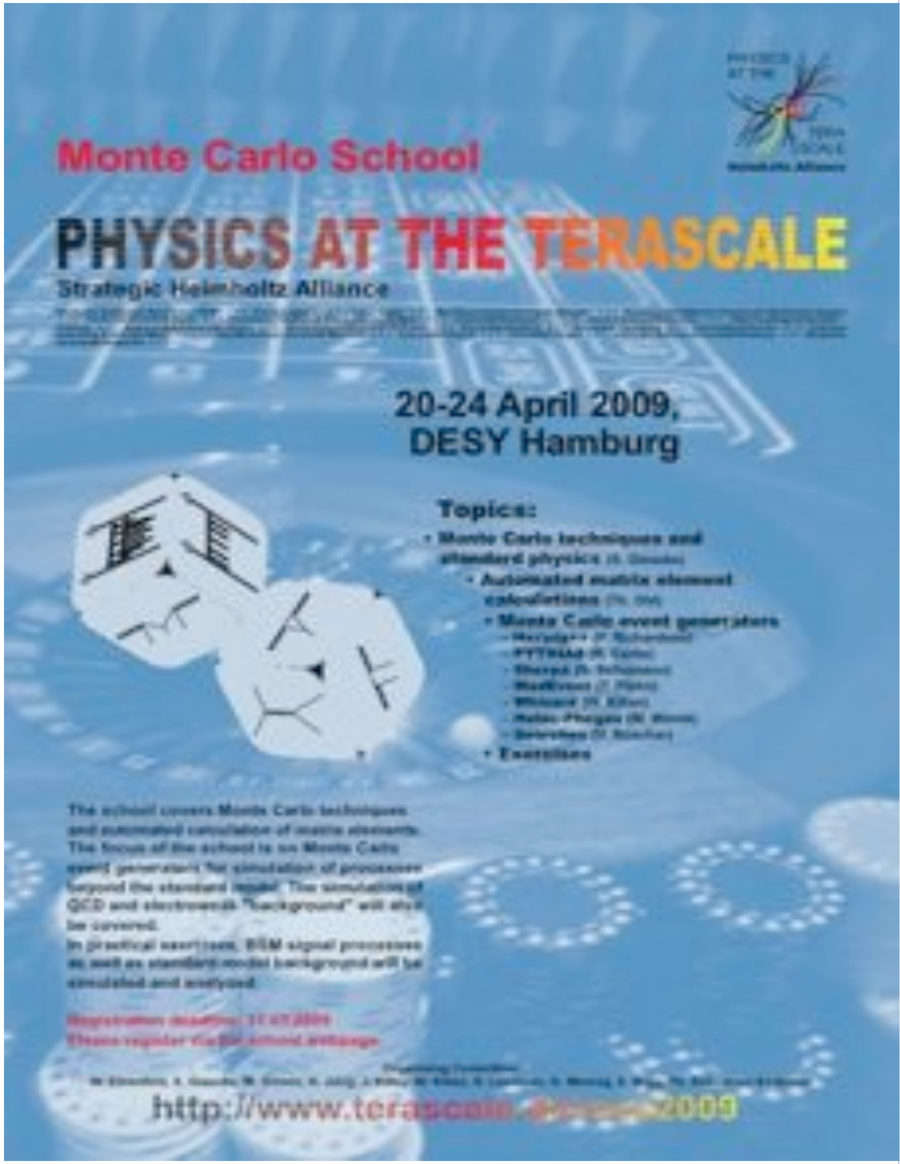
Monte Carlo group activities

If you are interested to do your

- diploma/masters thesis
- PhD
- postdoc

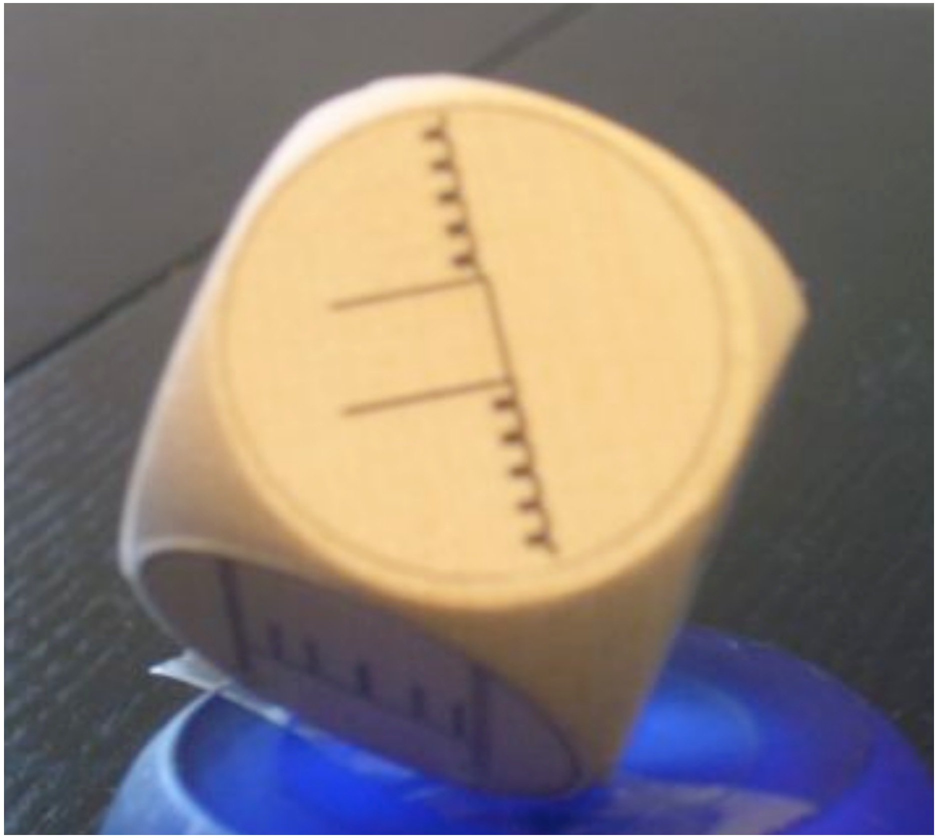
please get in contact with us...

There are plenty of possibilities and positions to do interesting physics with MC simulations ..... and help to find extra dimensions or SUSY or new phenomena in QCD



Typical Monte Carlo toys ...

The necessary tool for a true Monte Carlo event generator:



Conclusion

- Monte Carlo event generators are needed to calculate multi-parton cross sections
- Monte Carlo method is a well defined procedure
  - parton shower are essential
  - hadronization is needed to compare with measurements
- MC approach extended from simple e+e- processes to

  ep processes

  pp processes and heavy Ion processes
- proper Monte Carlos are essential for any measurement

Monte Carlo event generators contain all our physics knowledge !!!!!

List of available MC programs

- HERA Monte Carlo workshop: www.desy.de/~heramc
- ARIADNE
  - A program for simulation of QCD cascades implementing the color dipole model
- CASCADE
  - is a full hadron level Monte Carlo generator for ep and pp scattering at small x build according to the CCFM evolution equation, including the basic QCD processes as well as Higgs and associated W/Z production
- HERWIG
  - General purpose generator for Hadron Emission Reactions With Interfering Gluons; based on matrix elements, parton showers including color coherence within and between jets, and a cluster model for hadronization.
- JETSET
  - The Lund string model for hadronization of parton systems.

List of available MC programs

- LDCMC
  - A program which implements the Linked Dipole Chain (LDC) model for deeply inelastic scattering within the framework of ARIADNE. The LDC model is a reformulation of the CCFM model.
- PHOJET
  - Multi-particle production in high energy hadron-hadron, photon-hadron, and photon-photon interactions (hadron = proton, antiproton, neutron, or pion).
- PYTHIA
  - General purpose generator for e^+e^- pp and ep-interactions, based on LO matrix elements, parton showers and Lund hadronization.
- RAPGAP
  - A full Monte Carlo suited to describe Deep Inelastic Scattering, including diffractive DIS and LO direct and resolved processes. Also applicable for photo-production and partially for pp scattering.

Literature & References

- F. James Rep. Prog. Phys., Vol 43, 1145 (1980)
- Glen Cowan STATISTICAL DATA ANALYSIS. Clarendon, 1998.
- Particle Data Book S. Eidelman et al., Physics Letters B592, 1 (2004)
  section on: Mathematical Tools (http://pdg.lbl.gov/)
- Michael J. Hurben Buffons Needle
  (http://www.angelfire.com/wa/hurben/buff.html)
- J. Woller (Univ. of Nebraska-Lincoln) *Basics of Monte Carlo Simulations*
  (http://www.chem.unl.edu/zeng/joy/mclab/mcintro.html)
- Hardware Random Number Generators:
  A Fast and Compact Quantum Random Number Generator
  (http://arxiv.org/abs/quant-ph/9912118)
  Quantum Random Number Generator
  (http://www.idquantique.com/products/quantis.htm)
  Hardware random number generator (http://en.wikipedia.org/wiki/)
- Monte Carlo Tutorals
  (http://www.cooper.edu/engineering/chemechem/MMC/tutor.html)
- History of Monte Carlo Method
  (http://www.geocities.com/CollegePark/Quad/2435/history.html)

Literature & References (cont'd)

- T. Sjostrand et al
  PYTHIA/JETSET manual - The Lund Monte Carlos
  http://www.thep.lu.se/tf2/staff/torbjorn/Pythia.html
- H. Jung
  RAPGAP manual
  http://www.desy.de/~jung/rapgap.html
  CASCADE manual
  http://www.desy.de/~jung/cascade.html
- V. Barger and R. J.N. Phillips
  Collider Physics
  Addison-Wesley Publishing Comp. (1987)
- R.K. Ellis, W.J. Stirling and B.R. Webber
  QCD and collider physics
  Cambridge University Press (1996)

General literature

- Many new books are available in DESY library NEW ... ask at the desk there ...
- Statistische und numerische Methoden der Datenanalyse
  V. Blobel & E. Lohrmann
- STATISTICAL DATA ANALYSIS. Glen Cowan.
- Particle Data Book S. Eidelman et al., Physics Letters B592, 1 (2004)
  (http://pdg.lbl.gov/)
- Applications of pQCD R.D. Field Addison-Wesley 1989
- Collider Physics V.D. Barger & R.J.N. Phillips Addison-Wesley 1987
- Deep Inelastic Scattering. R. Devenish & A. Cooper-Sarkar, Oxford 2
- Handbook of pQCD G. Sterman et al
- Quarks and Leptons, F. Halzen & A.D. Martin, J.Wiley 1984
- QCD and collider physics R.K. Ellis & W.J. Stirling & B.R. Webber Cambridge 1996
- QCD: High energy experiments and theory G. Dissertori,I. Knowles, M. Schmelling Oxford 2003

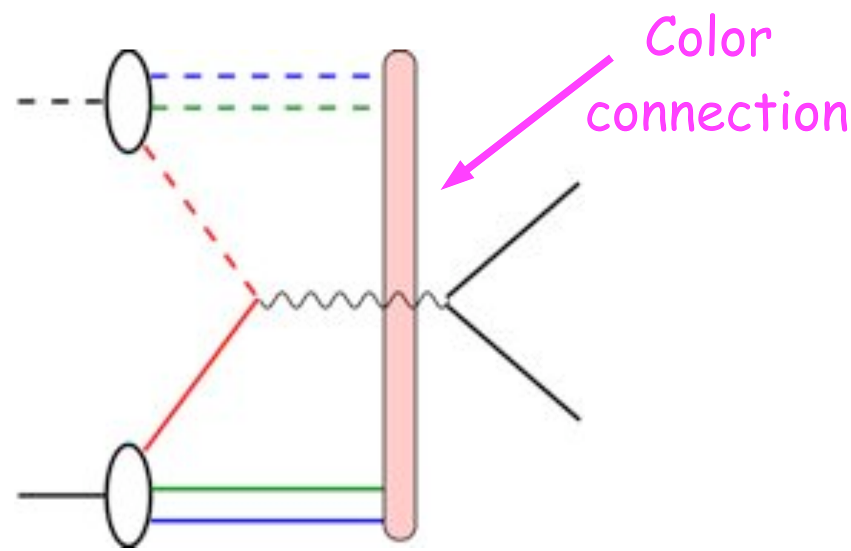
Backup Slides

W & Z cross sections

- Basic process: Drell - Yan

  $p + p \to l^+ + l^- + X$

- Factorize process:
  - $q + \bar{q} \to \gamma^* \to l^+ + l^-$
  - and then
    - $q + \bar{q} \to \gamma^*$
    - $q + \bar{q} \to Z_0$
    - $q + \bar{q}' \to W^\pm$
  - and then
    - $\gamma^* \to l^+ + l^-$
    - $Z_0 \to l^+ + l^-$
    - $W^\pm \to l + \nu$



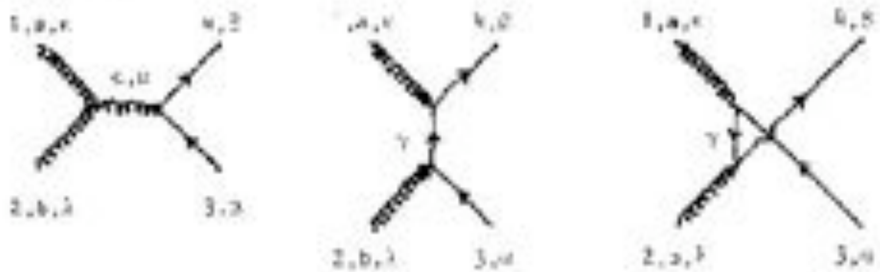
- We need
  - PDFs
  - hard scattering
  - Decays
  - remnant treatment

Color Flow in pp

The Lund Monte Carlo For High P(T) Physics H.U. Bengtsson
Comput.Phys.Commun.31:323,1984.

Process: $gg \to q_i\bar{q}_i$

Diagrams:



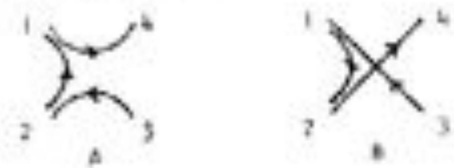
Amplitudes:

s: $g^2 f^{abc} T^c_{\alpha\beta} \bar{u}^\beta_i(q_4) \frac{\epsilon_1^\mu \epsilon_2^\lambda \gamma^\nu}{\hat{s}} C_{\nu\lambda\mu}(q_1, q_2, -q_1 - q_2) v^\alpha_i(q_3)$

t: $-ig^2 T^b_{\alpha\gamma} T^a_{\gamma\beta} \bar{u}^\beta_i(q_4) \not\epsilon_1 \frac{\not q_1 - \not q_4}{\hat{t}} \not\epsilon_2 v^\alpha_i(q_3)$

u: $-ig^2 T^a_{\alpha\gamma} T^b_{\gamma\beta} \bar{u}^\beta_i(q_4) \not\epsilon_2 \frac{\not q_2 - \not q_4}{\hat{u}} \not\epsilon_1 v^\alpha_i(q_3)$

Colour flows:



String configurations:

Colour factors: A: $T^b_{\alpha\gamma} T^a_{\gamma\beta}$; B: $T^a_{\alpha\gamma} T^b_{\gamma\beta}$

Amplitudes:

A: $-ig^2 \bar{u}^\beta_i(q_4) \left[\not\epsilon_1 \frac{\not q_1 - \not q_4}{\hat{t}} \not\epsilon_2 - \frac{\epsilon_1^\mu \epsilon_2^\lambda \gamma^\nu}{\hat{s}} C_{\nu\lambda\mu}(q_1, q_2, -q_1 - q_2) \right] v^\alpha_i(q_3)$

B: $-ig^2 \bar{u}^\beta_i(q_4) \left[\not\epsilon_2 \frac{\not q_2 - \not q_4}{\hat{u}} \not\epsilon_1 + \frac{\epsilon_1^\mu \epsilon_2^\lambda \gamma^\nu}{\hat{s}} C_{\nu\lambda\mu}(q_1, q_2, -q_1 - q_2) \right] v^\alpha_i(q_3)$

Cross-sections:

A: $\frac{\pi\alpha_s^2}{\hat{s}^2} \frac{1}{6} \left(\frac{\hat{u}}{\hat{t}} - 2\frac{\hat{u}^2}{\hat{s}^2} \right)$, B: $\frac{\pi\alpha_s^2}{\hat{s}^2} \frac{1}{6} \left(\frac{\hat{t}}{\hat{u}} - 2\frac{\hat{t}^2}{\hat{s}^2} \right)$

Color Flow in pp

The Lund Monte Carlo For High P(T) Physics H.U. Bengtsson
Comput.Phys.Commun.31:323,1984.

Process: $gg \to gg$

Diagrams:

Amplitudes:

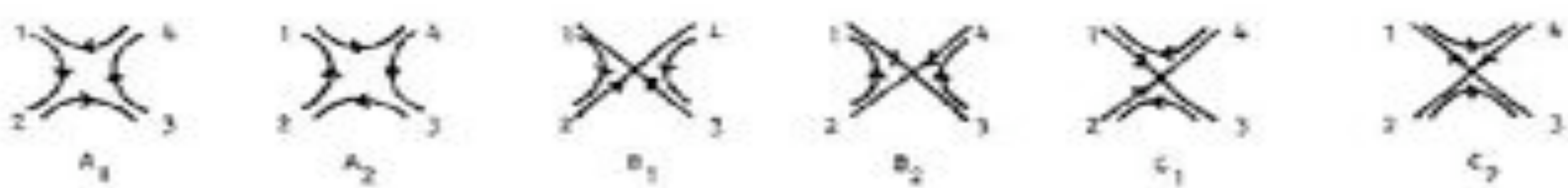
s: $-ig^2\frac{1}{s}f^{abe}f^{cde}\epsilon_1^{\kappa}\epsilon_2^{\lambda}\epsilon_3^{\mu}\epsilon_4^{\nu}C_{\kappa\lambda\sigma}(-q_1,-q_2,q_1+q_2)C_{\mu\nu}{}^{\sigma}(q_3,q_4,-q_3-q_4)$

t: $-ig^2\frac{1}{t}f^{ade}f^{bce}\epsilon_1^{\kappa}\epsilon_2^{\lambda}\epsilon_3^{\mu}\epsilon_4^{\nu}C_{\nu\kappa\sigma}(q_4,-q_1,q_1-q_4)C_{\lambda\mu}{}^{\sigma}(-q_2,q_3,q_2-q_3)$

u: $-ig^2\frac{1}{u}f^{ace}f^{bde}\epsilon_1^{\kappa}\epsilon_2^{\lambda}\epsilon_3^{\mu}\epsilon_4^{\nu}C_{\mu\kappa\sigma}(q_3,-q_1,q_1-q_3)C_{\lambda\nu}{}^{\sigma}(-q_2,q_4,q_2-q_4)$

4: $-ig^2f^{abe}f^{cde}(g_{\kappa\mu}g_{\lambda\nu}-g_{\kappa\nu}g_{\lambda\mu})\quad ig^2f^{dae}f^{bce}$
$\times(g_{\kappa\mu}g_{\lambda\nu}-g_{\kappa\lambda}g_{\nu\mu})+ig^2f^{ace}f^{bde}(g_{\kappa\lambda}g_{\mu\nu}-g_{\kappa\nu}g_{\lambda\mu})$

Colour flows:



Color Flow in $p\bar{p} \to b\bar{b} + X$

B physics at the Tevatron: Run II and beyond
E. Norrbin, hep-ph/0201071,p522

$p\bar{p} \to b\bar{b} + X$

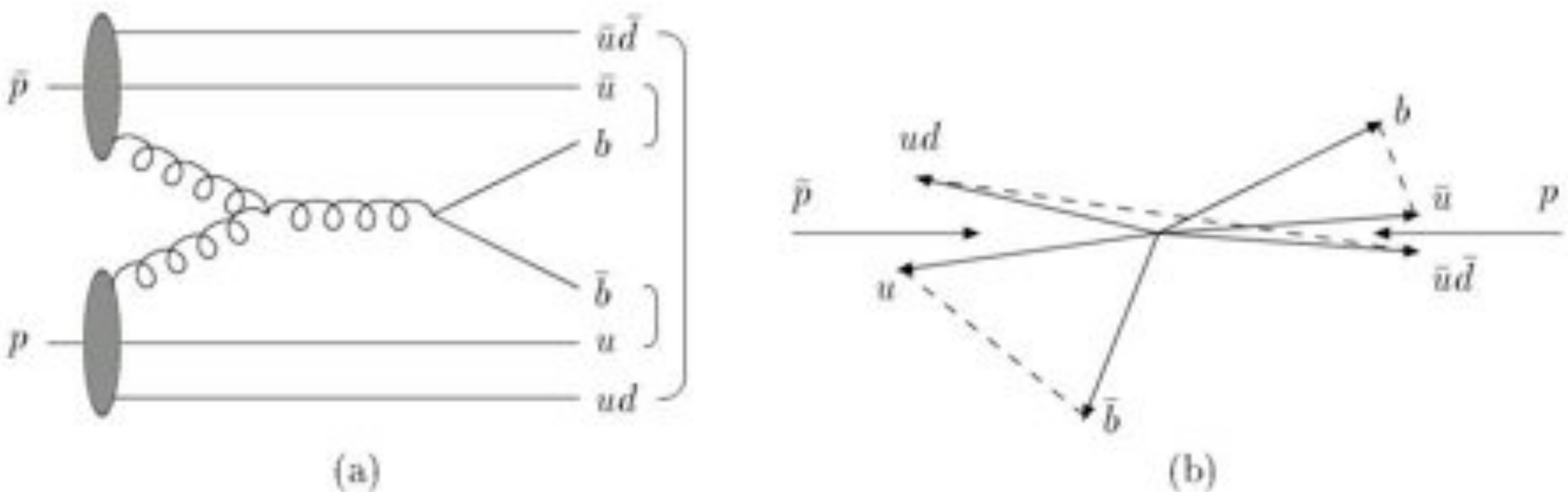


Figure 9.45: Example of a string configuration in a $p\bar{p}$ collision. (a) Graph of the process, with brackets denoting the final color singlet subsystems. (b) Corresponding momentum space picture, with dashed lines denoting the strings.

Beam - drag effect

- Due to color connection of produced b-quark with beam remnants, the rapidity distribution of b-quarks and B-hadrons is different.
- Asymmetry of $B^0 \bar{B}^0$

B physics at the Tevatron: Run II and beyond
E. Norrbin, hep-ph/0201071,p525

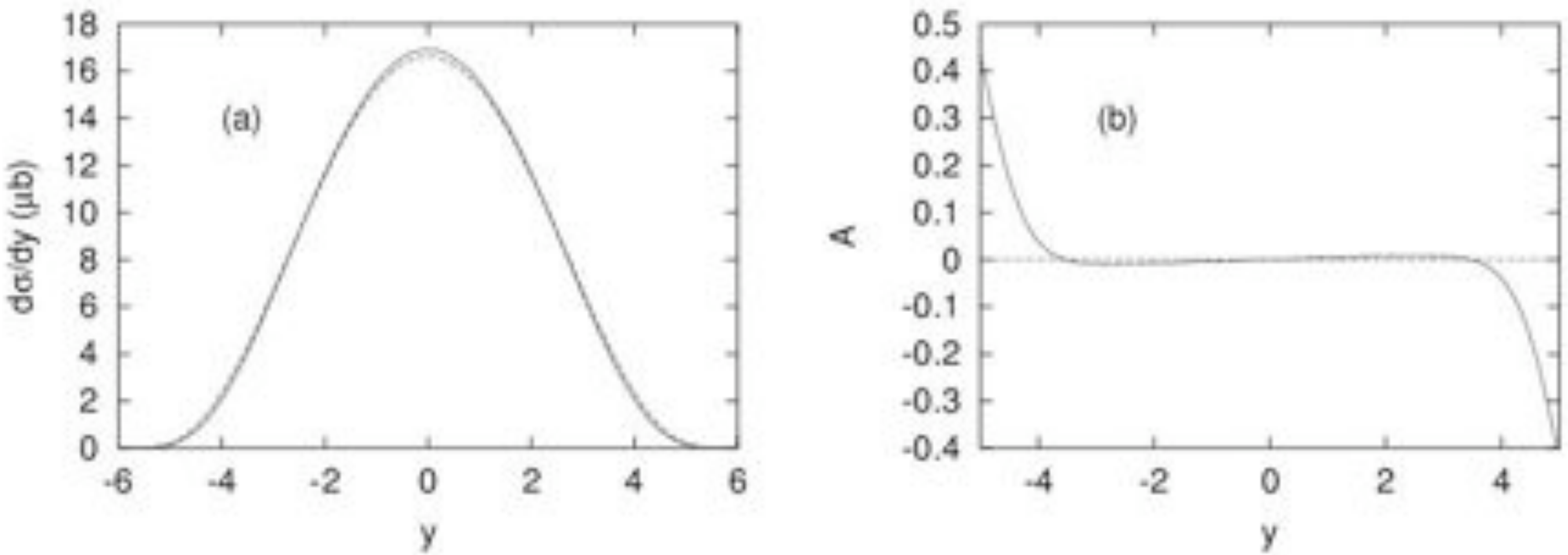


Figure 9.47: Bottom production at the Tevatron. (a) Rapidity distribution of bottom quarks (full) and the B hadrons produced from them (dashed). (b) The asymmetry $A = \frac{\sigma(B^0)-\sigma(\bar{B}^0)}{\sigma(B^0)+\sigma(\bar{B}^0)}$ as a function of rapidity. For simplicity, only pair production is included.

- **HowTo connect this to factorised fragmentation functions ?**

Is that now all ?

But with high parton densities,

do we only have one interaction ?

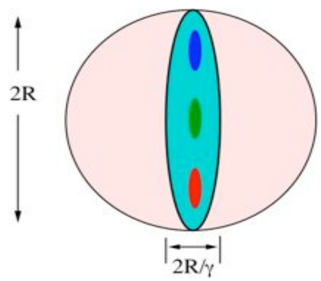
How many gluons are there ?

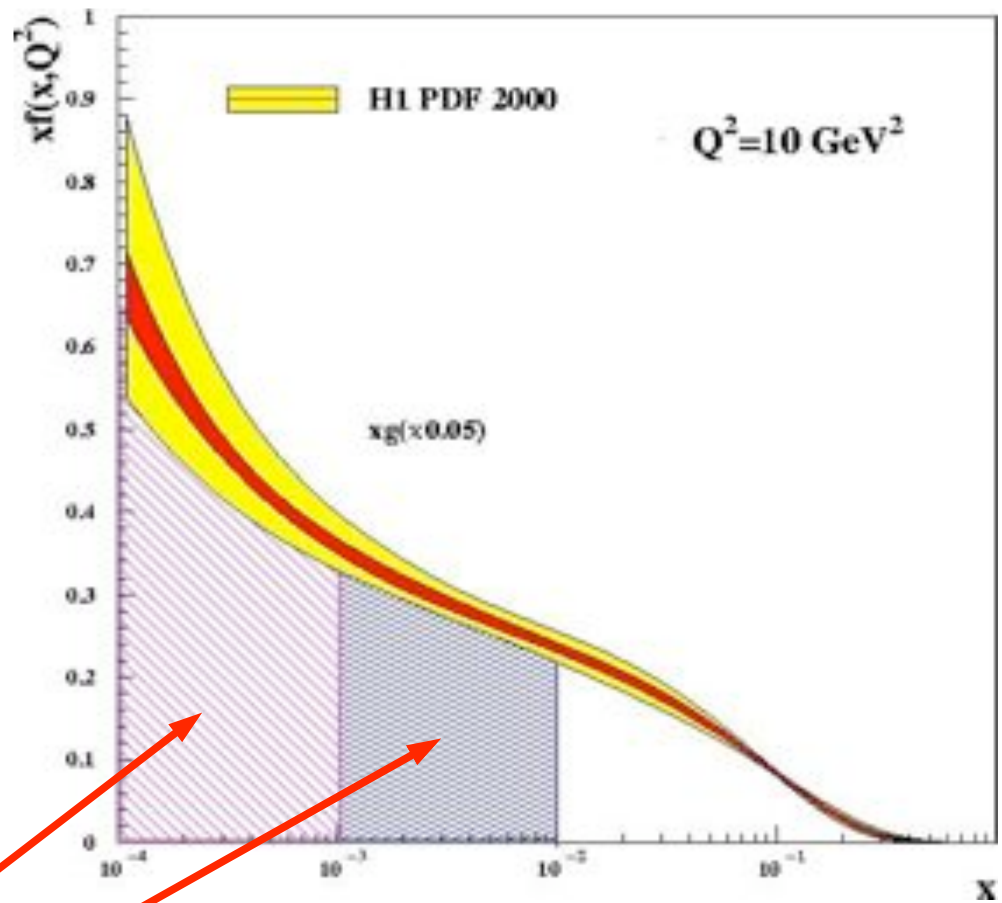
- number of gluons in long. phase space dx/x :

  $xg(x, \mu^2) dx/x$
- occupation area:

  nr of gluons x (trans size)^2

  $$g(x, \mu^2) \frac{1}{\mu^2}$$





- saturation starts when:

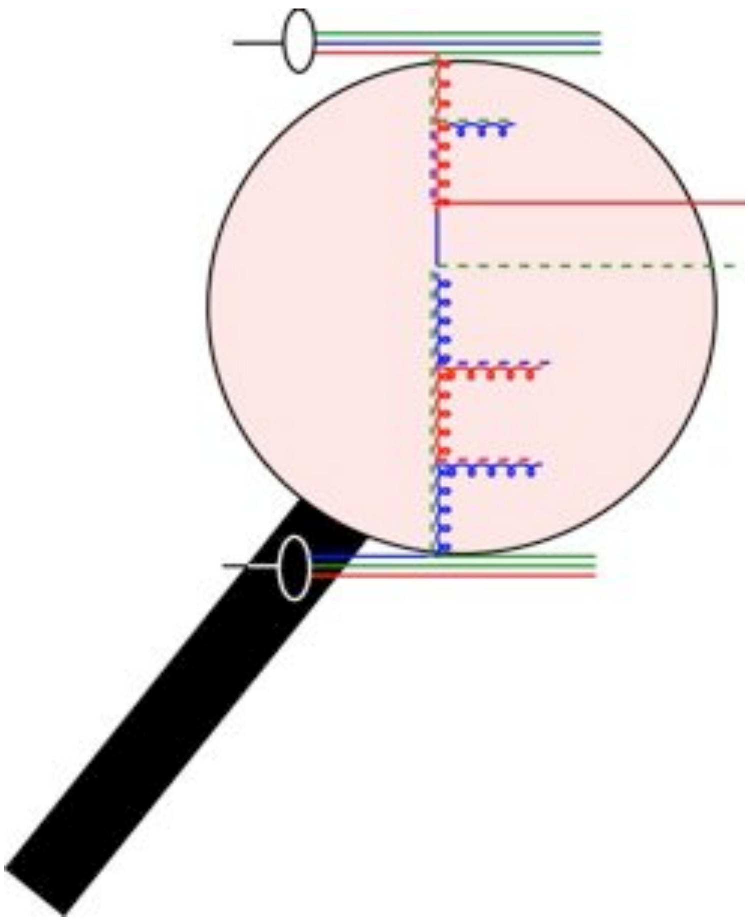
  $$\frac{\alpha_s(\mu^2)}{\mu^2} xg(x, \mu^2) \frac{dx}{x} \geq \pi R^2$$
- gluon density is very large:~ 90 or 45 Gluons !!!!!
- with R ~ 1 GeV^{-1} we obtain:

  $$\frac{0.2}{10GeV^2} 100 \sim \pi R^2 \sim \pi$$!!!!!

Partonic Cross sections

- Cross section

$$\sigma(p_1 + p_2 \to j_1 + j_2 + X) = f(x_1, \mu^2) \otimes \hat{\sigma}(x_1 p_1 + x_2 p_2 \to j_1 + j_2) \otimes f(x_2, \mu^2)$$



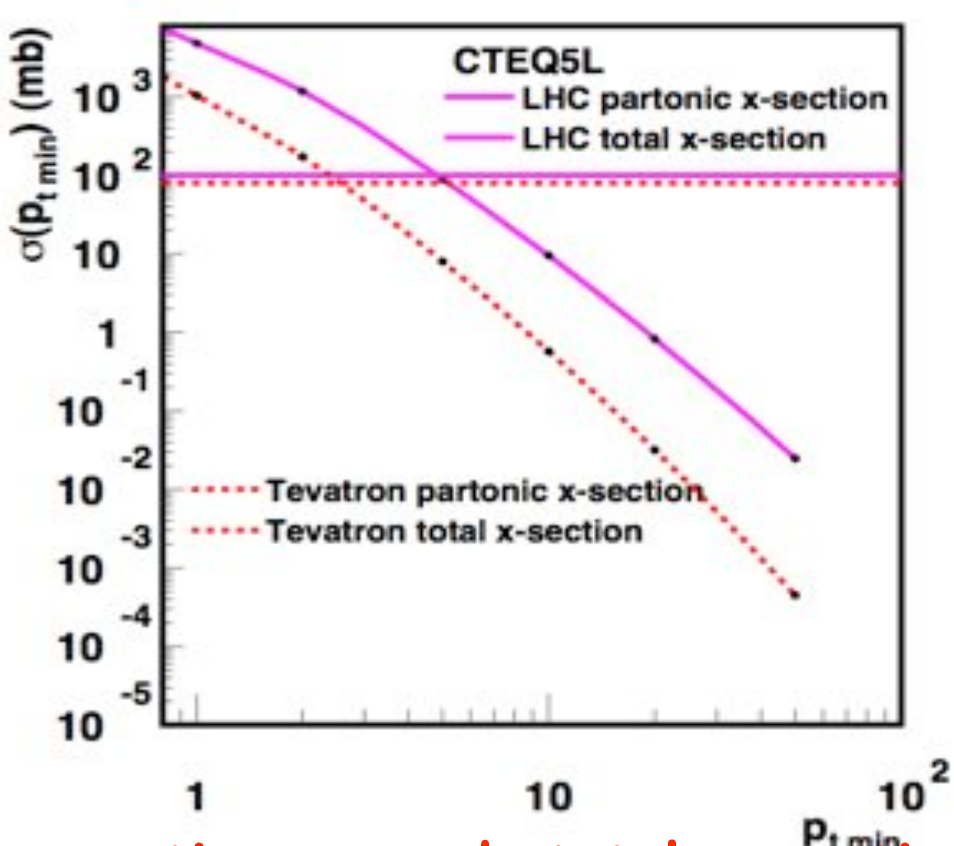
- partonic cross section diverges with p_t
- calculate x-section as function of p_{tmin}

$$\sigma_{\text{hard}}(p^2_{\perp \text{min}}) = \int_{p^2_{\perp \text{min}}} \frac{d\sigma_{\text{hard}}(p^2_\perp)}{dp^2_\perp} dp^2_\perp$$

Partonic Cross sections

$$\sigma_{\mathrm{hard}}(p^2_{\perp\min}) = \int_{p^2_{\perp\min}} \frac{d\sigma_{\mathrm{hard}}(p^2_{\perp})}{dp^2_{\perp}} dp^2_{\perp}$$

- Cross section at Tevatron/LHC



➔ Partonic x-section exceeds total x-section !!!

➔ with HERA PDFs .... at larger values of p_{tmin} !!!!!

Underlying event - Multiple Interaction

- Basic partonic perturbative cross section

$$\sigma_{\text{hard}}(p^2_{\perp \text{min}}) = \int_{p^2_{\perp \text{min}}} \frac{d\sigma_{\text{hard}}(p^2_\perp)}{dp^2_\perp} dp^2_\perp$$

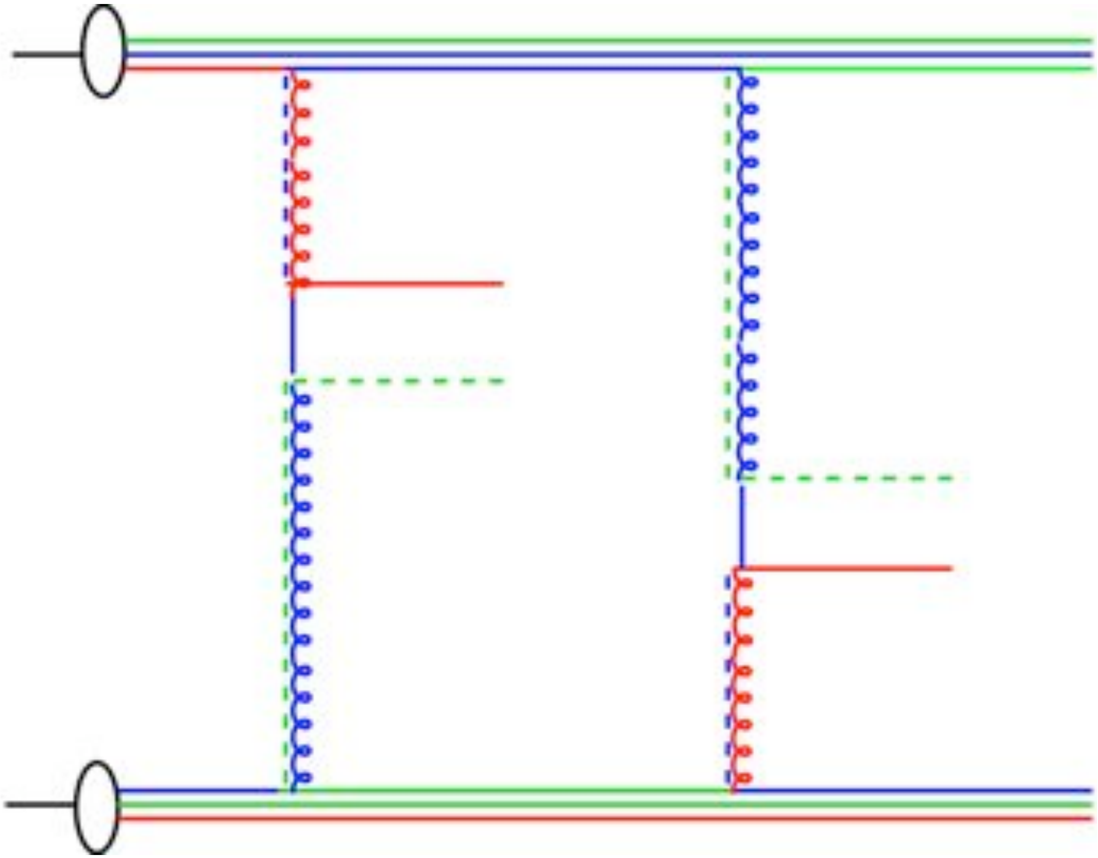
➔ diverges faster than $1/p^2_{\perp \text{min}}$ as $p_{\perp \text{min}} \to 0$ and exceeds eventually total inelastic (non-diffractive) cross section

HELP ....

HOW to solve this ?

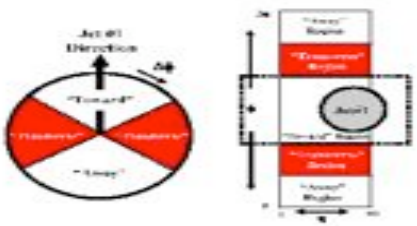
Models for Multi-Parton Interaction

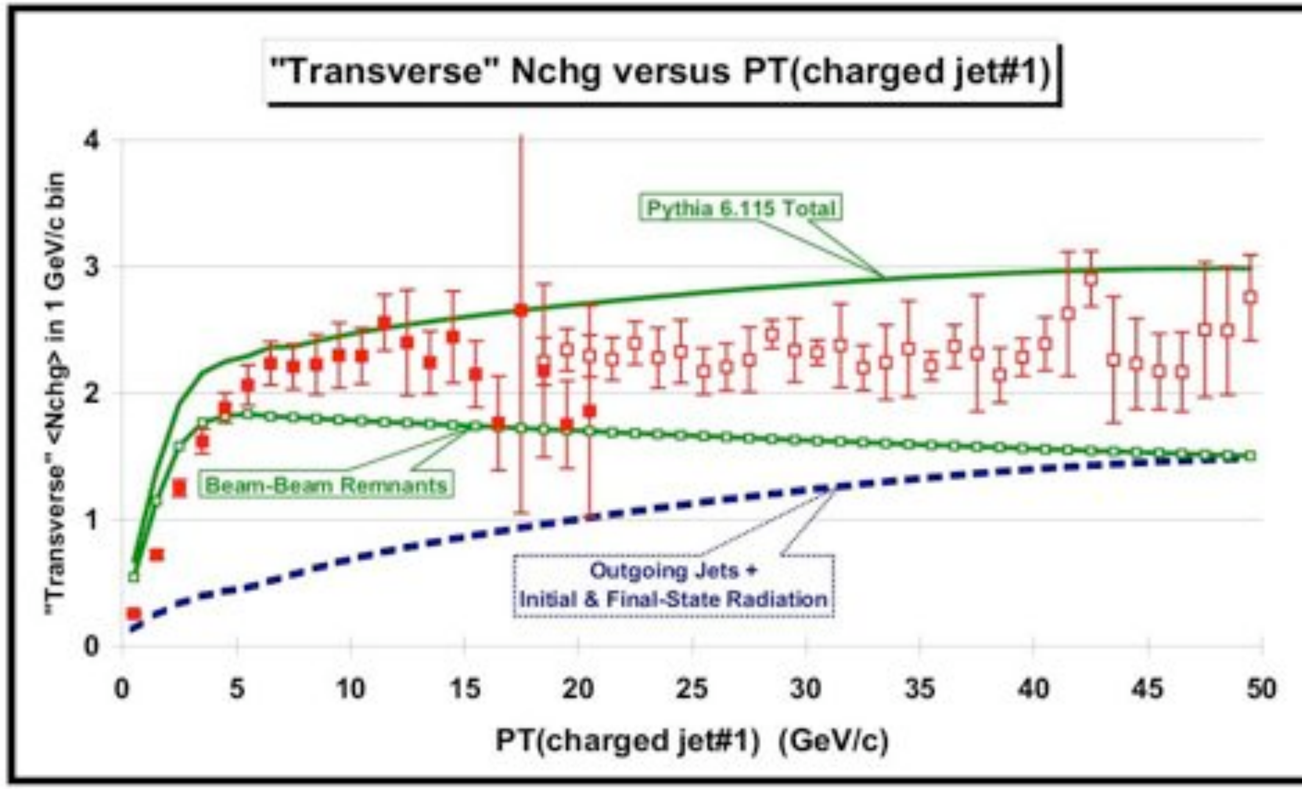
- The very simple model
  - ➔ add secondary interactions
  - ➔ first model by: T. Sjostrand, M. Zijl PRD 36 (1987) 2019



Multiparton Interactions at TeVatron

CDF coll. PRD 65, 092002 (2002)





- Multiplicity distribution in region transverse to jet can only be described by adding multi-parton interactions (Remnant- Remnant Interactions)

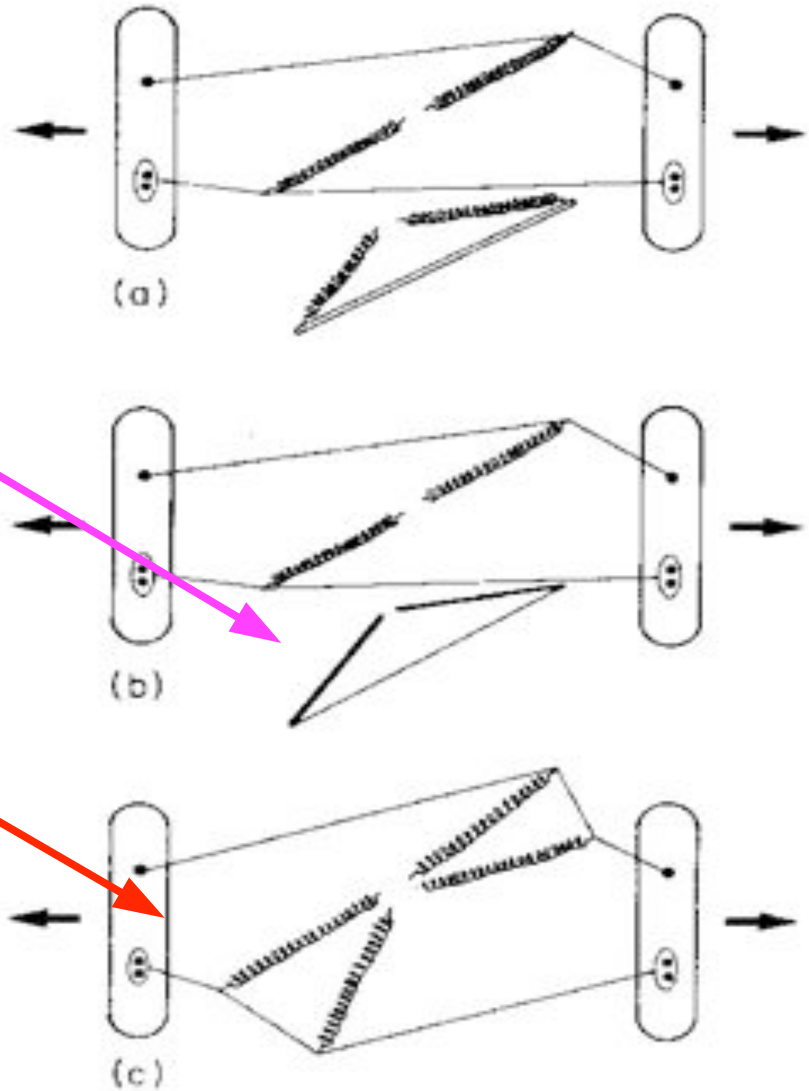
Tuning to pp data... Color flow in MI

- possible scenarios for color string connection in multiparton events
- to describe underlying events.... need (CDF Tune A)
  - 5 % quarks (default 33 %)
  - 95 % gluons (default: 66%) out of which 90 % (default 33 %) are

➔ smaller multiplicity with large transverse energy

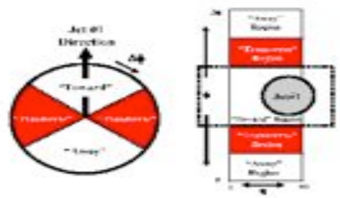
- Are there good phyiscs reasons for this mix ???
- **Highly nontrivial to describe multiplicity AND transverse energy distributions ...**

T. Sjostrand, M. Zijl
PRD 36 (1987) 2019



Multiparton Interactions at LHC

C. Buttar et al in HERA – LHC workshop proceedings hep-ph/0601012

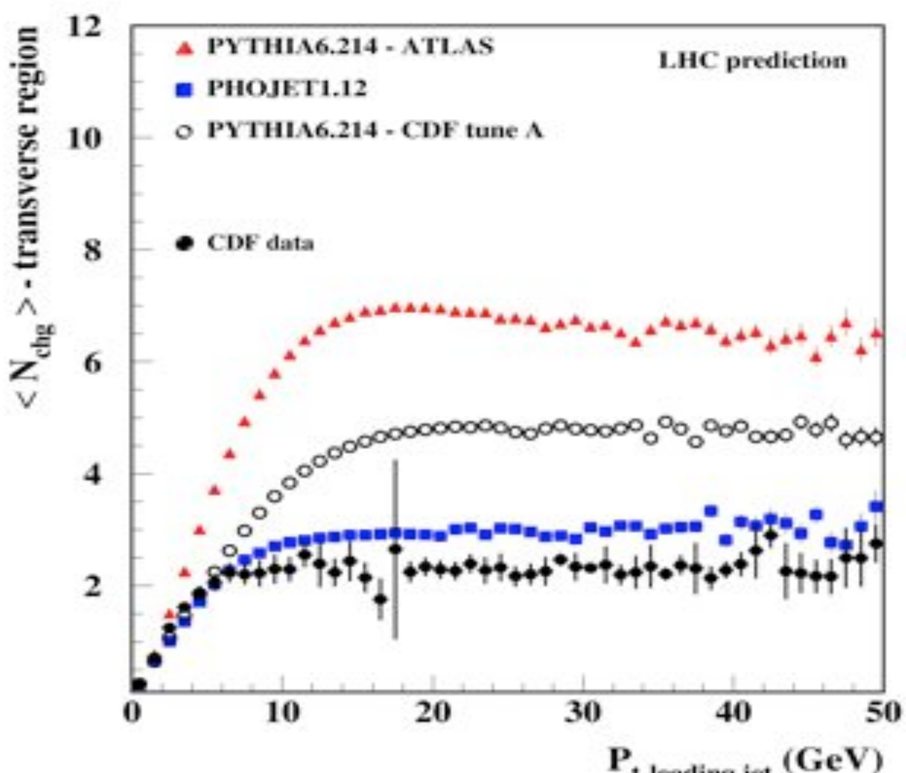


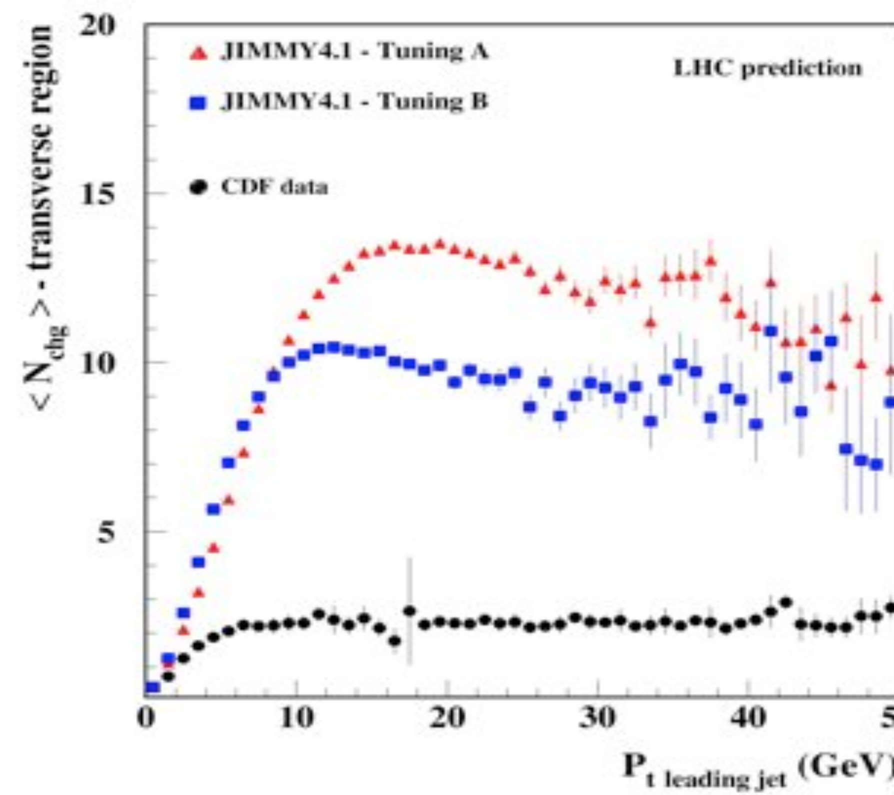
Charged multiplicities in transverse region

- Models tuned to TeVatron data

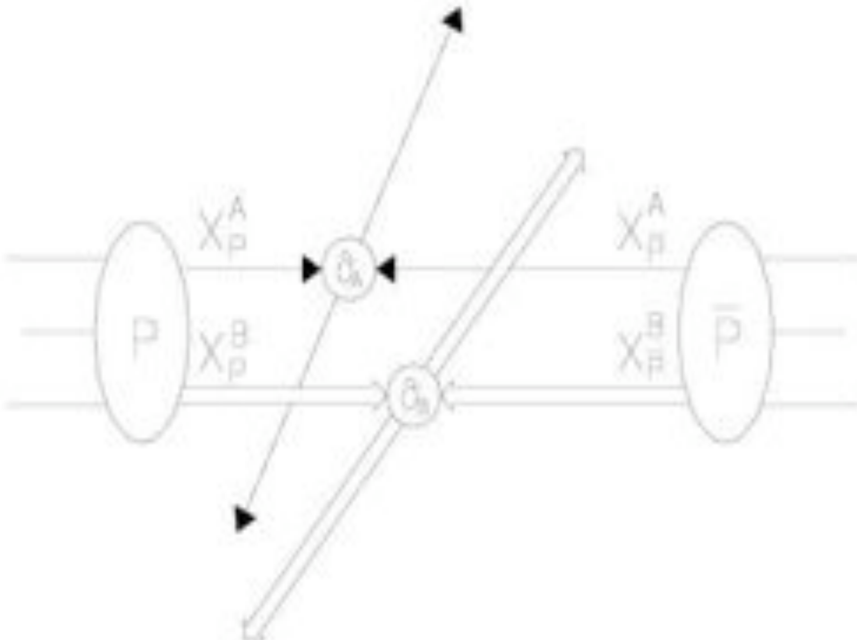
➔ give **HUGE** differences at LHC ...

➔ **better understand multiple interactions ...**

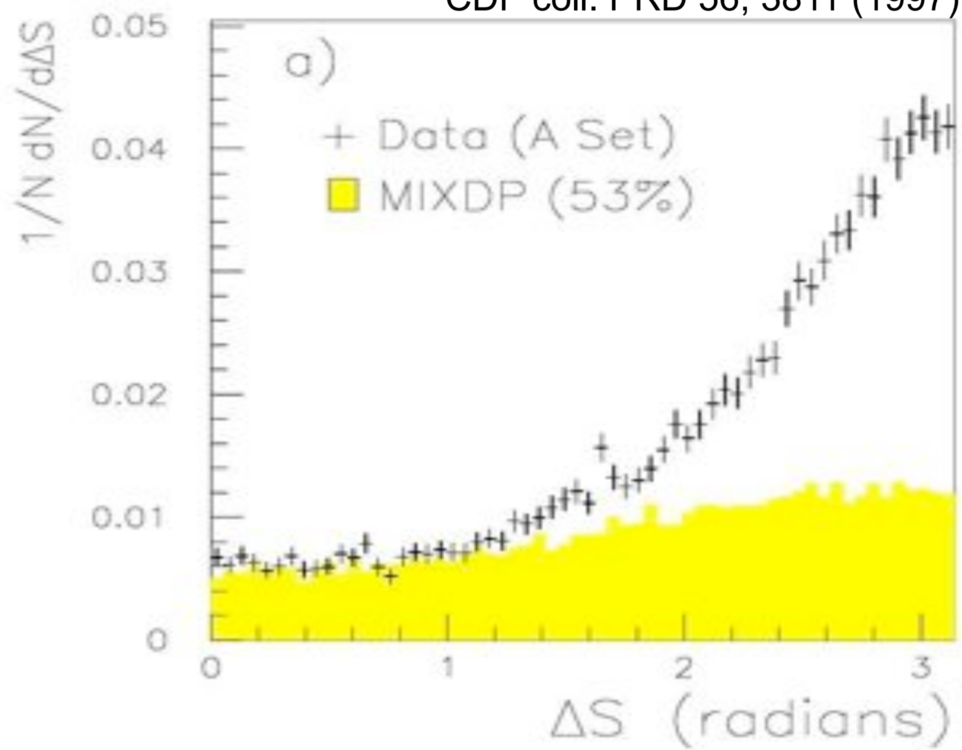




Evidence for Multi-Parton Interactions



CDF coll. PRD 56, 3811 (1997)



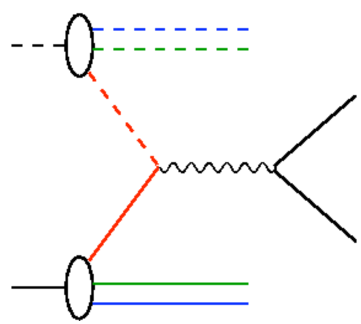
- look at $\gamma + 3$ Jets

  with $E_T^{\gamma} > 16 GeV$
  $E_T^{Jets} > 5 GeV$

- angular correlation of jet/photon pairs ΔS
- compare to $\gamma + 3$ Jets calculation
- **Need > 50 % double parton interaction to describe data**

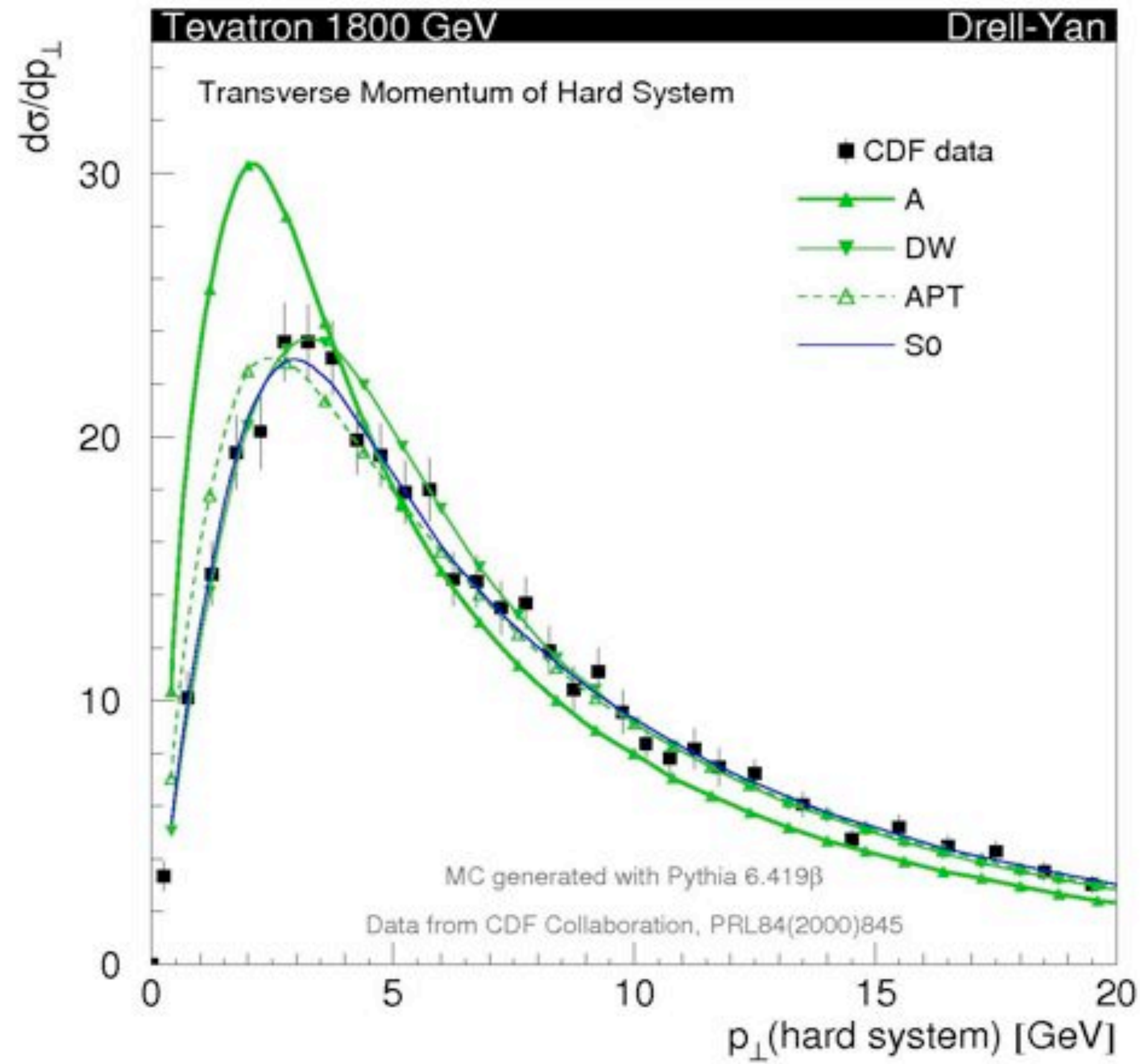
Does it matter for high pt physics ?

Drell Yan process is affected ...



- P_t of Drell Yan is affected by parton shower BUT also by the underlying events ....
- significant effects
- how to tune the truth ?

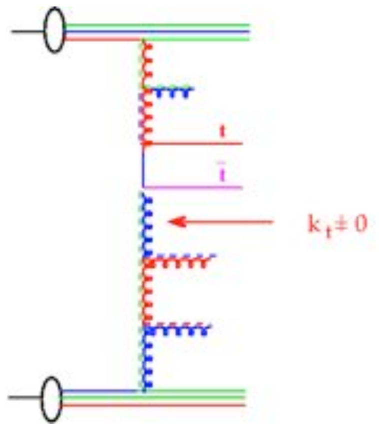
P. Skands, MPI@LHC, Perugia 2008

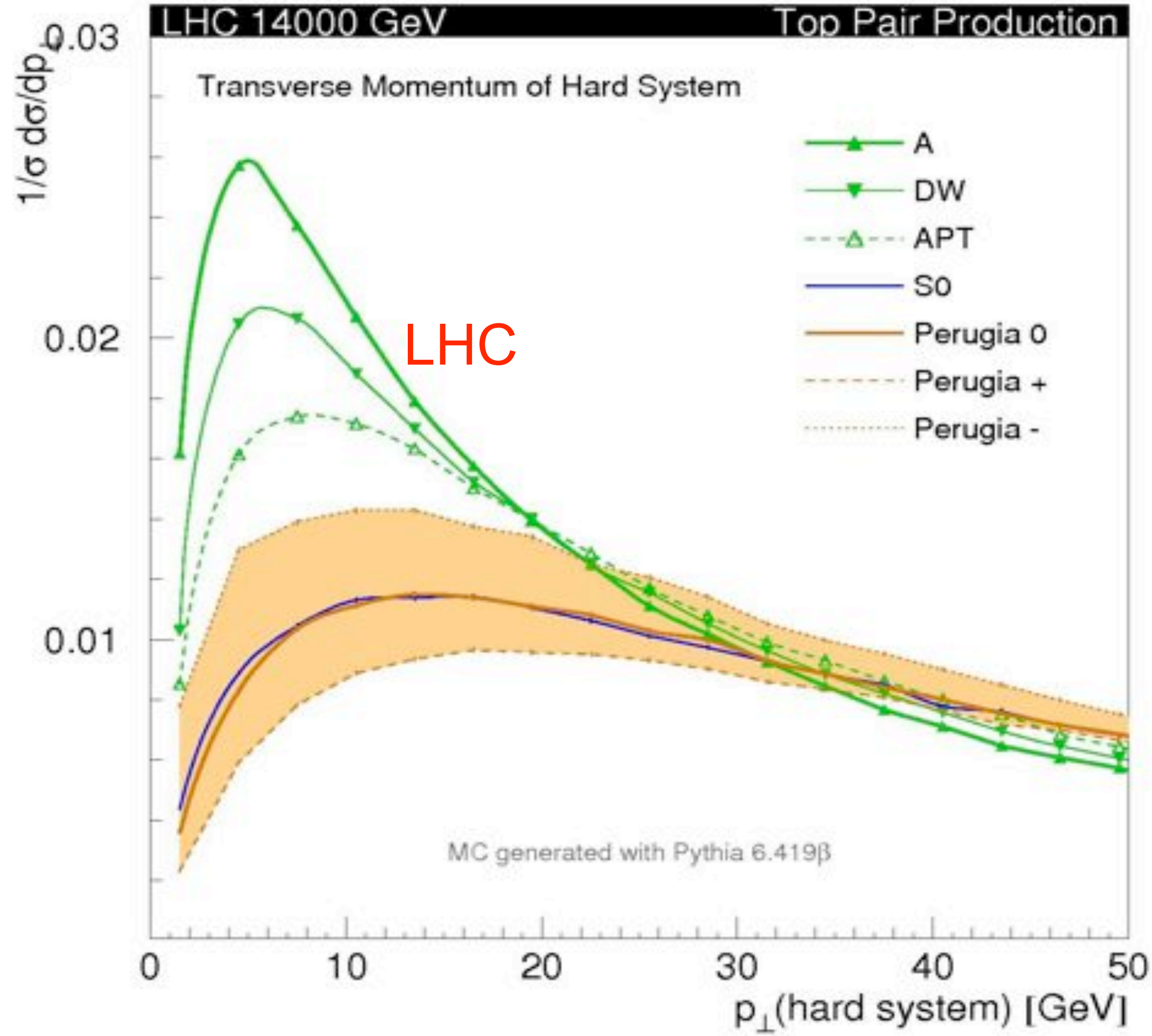


ttbar is also affected ...

P. Skands, MPI@LHC, Perugia 2008

- P_t of $t\bar{t}$ is affected by parton shower BUT also by the underlying events ....





- note: pt of the pair is plotted !!!
- **HUGE effects**

Double-Parton Interactions at LHC

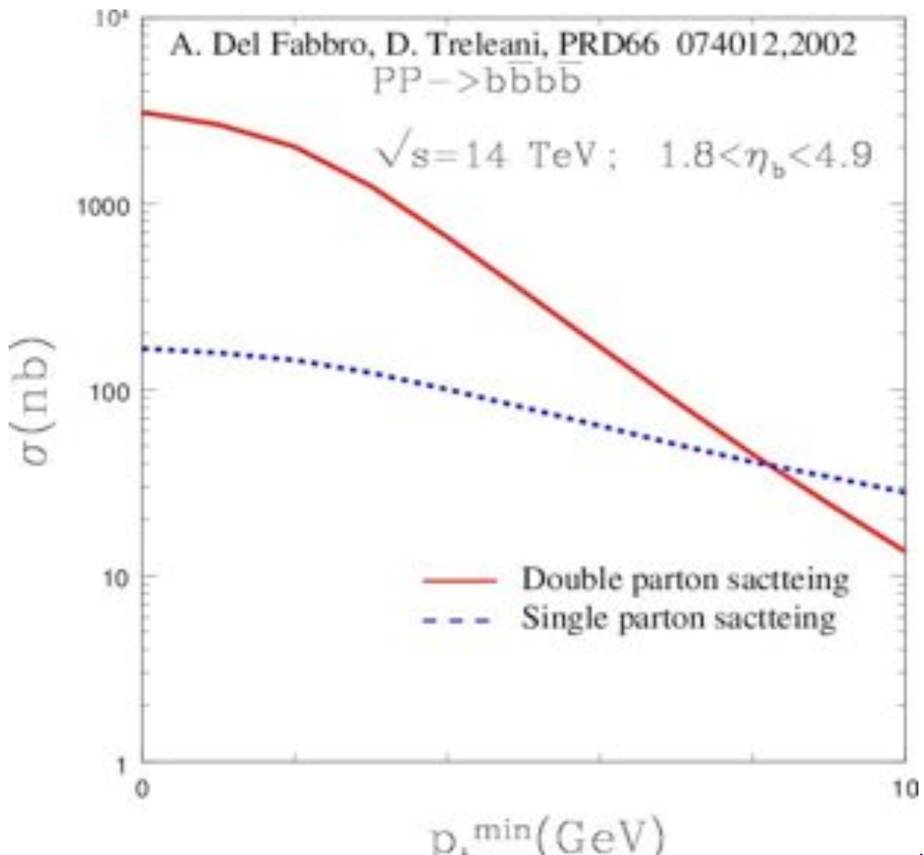
- xsection for $p + p \rightarrow b\bar{b}b\bar{b}$

  single parton exchange (SP)

  $$\sigma^{SP} \sim f^2 \hat{\sigma}(2 \rightarrow 4)$$

  double parton exchange (DP)

  $$\sigma^{DP} \sim f^4 \hat{\sigma}^2(2 \rightarrow 2)$$



- PYTHIA predictions:

  $$\sigma^{DP} = 0.8 \cdots 11.1 \ \ \mu b$$

➔ Depending on model for underlying event/muli-parton interactions...

Multi-Parton Interactions at LHC

- Higgs: $p + p \to W + H + X$

  with $W \to l\nu, \quad H \to b\bar{b}$
- Double parton scattering:

➔ $p + p \to b\bar{b}X$

$p + p \to W + X$

$p + p \to W + b\bar{b} + X$

-

$p + p \to W + H + X$