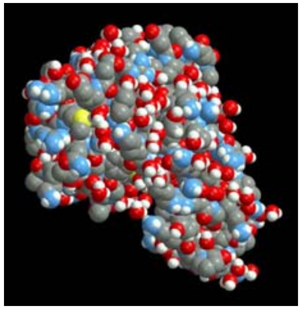
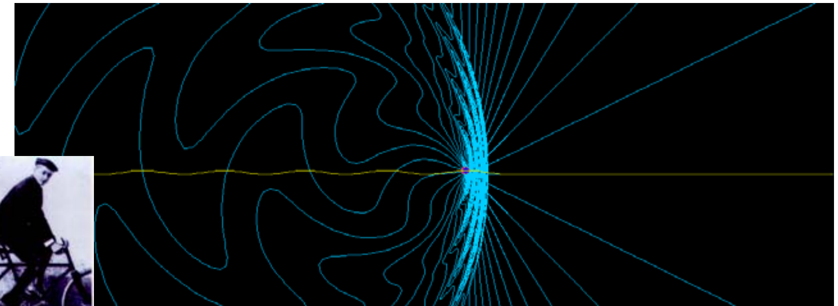
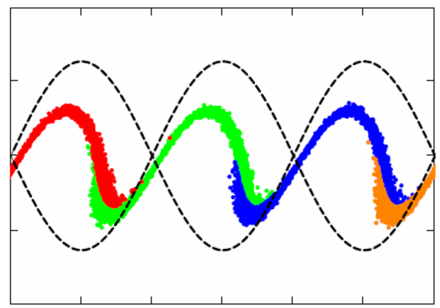
Free-Electron Laser

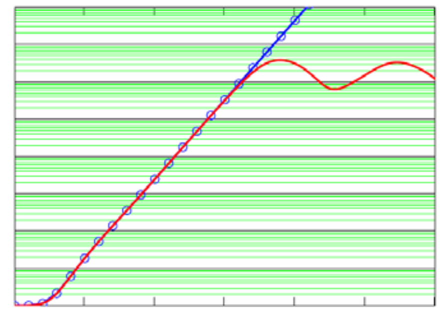
A) Motivation and Introduction



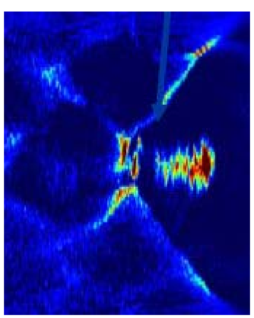


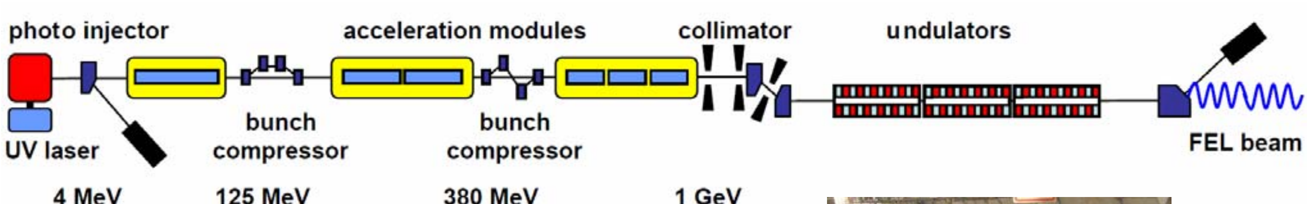
B) Theoretical Approach





C) Experimental Realization / Challenges





Free-Electron Laser

A) Motivation and Introduction

Need for Short Wavelengths

Why FELs?

Free Electron $\leftrightarrow$ Wave Interaction

Amplifier and Oscillator

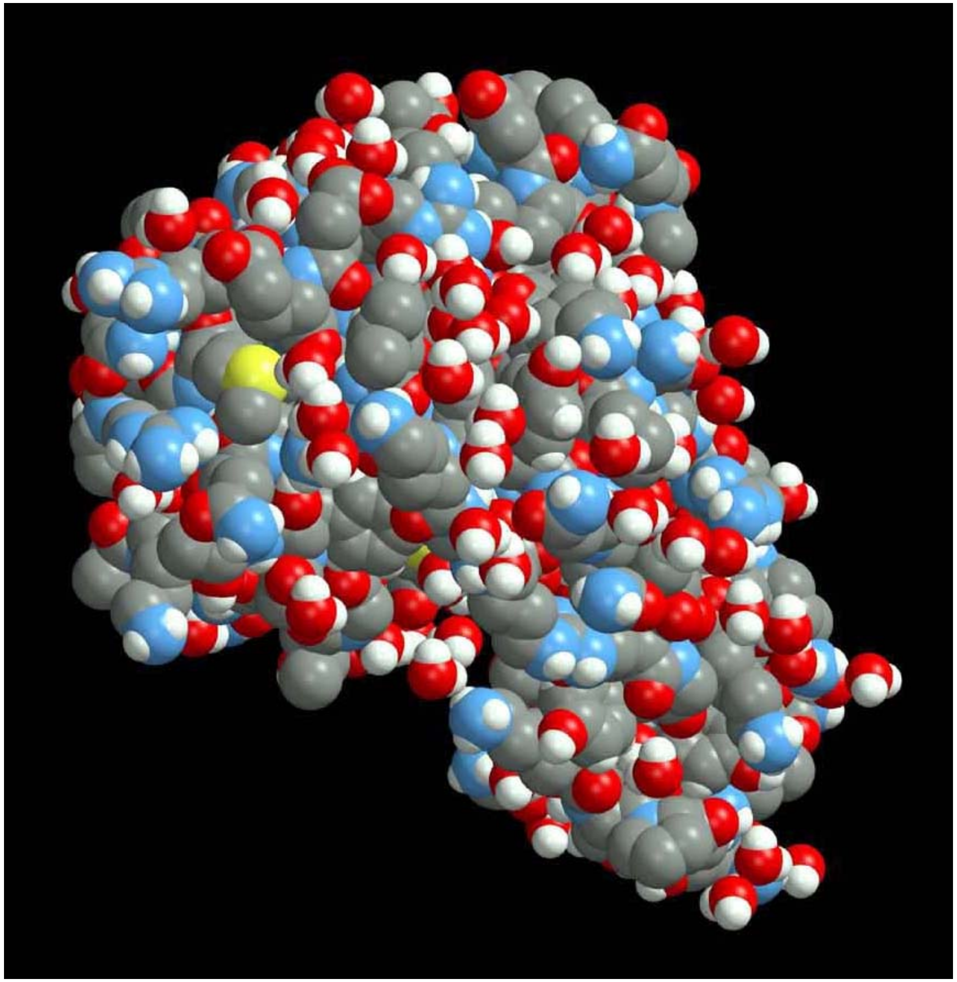
Self Amplifying Spontaneous Emission (SASE)

Why SASE?

Micro-Bunching

Coherent Radiation

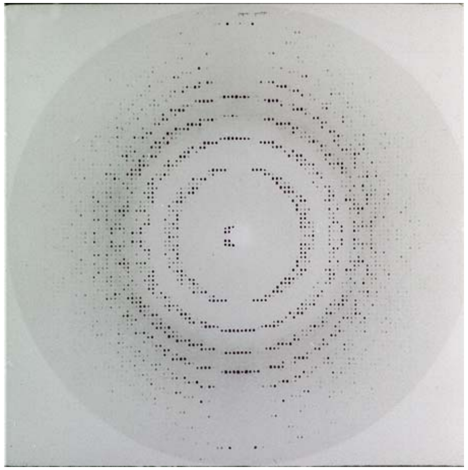
Need for Short Wavelengths



LYSOZYME   MW=19,806

state of the art:
structure of biological macromolecule

reconstructed from diffraction pattern of protein crystal:



needs $\approx 10^{15}$ samples

crystallized → not in life environment

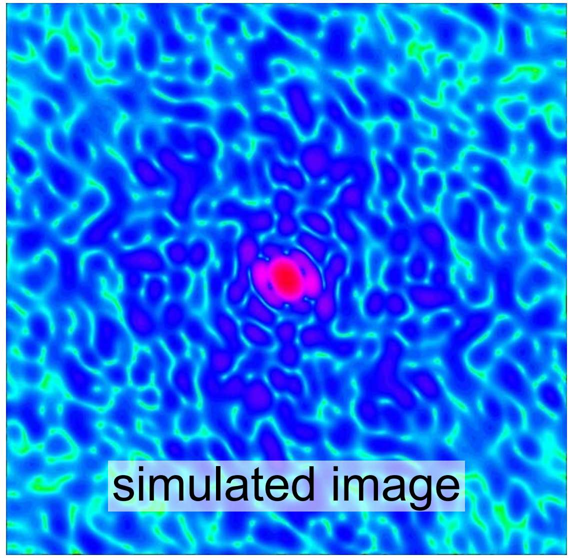
the crystal lattice imposes restrictions on molecular motion

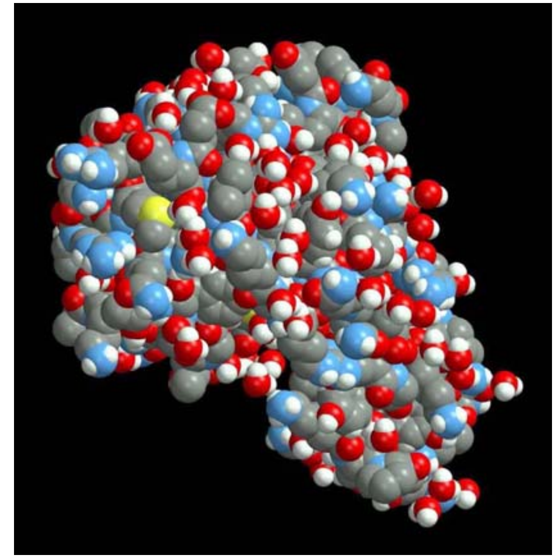
images courtesy Janos Hajdu, slide from Jörg Rossbach

Need for Short Wavelengths - 2

SINGLE MACROMOLECULE

courtesy Janos Hajdu





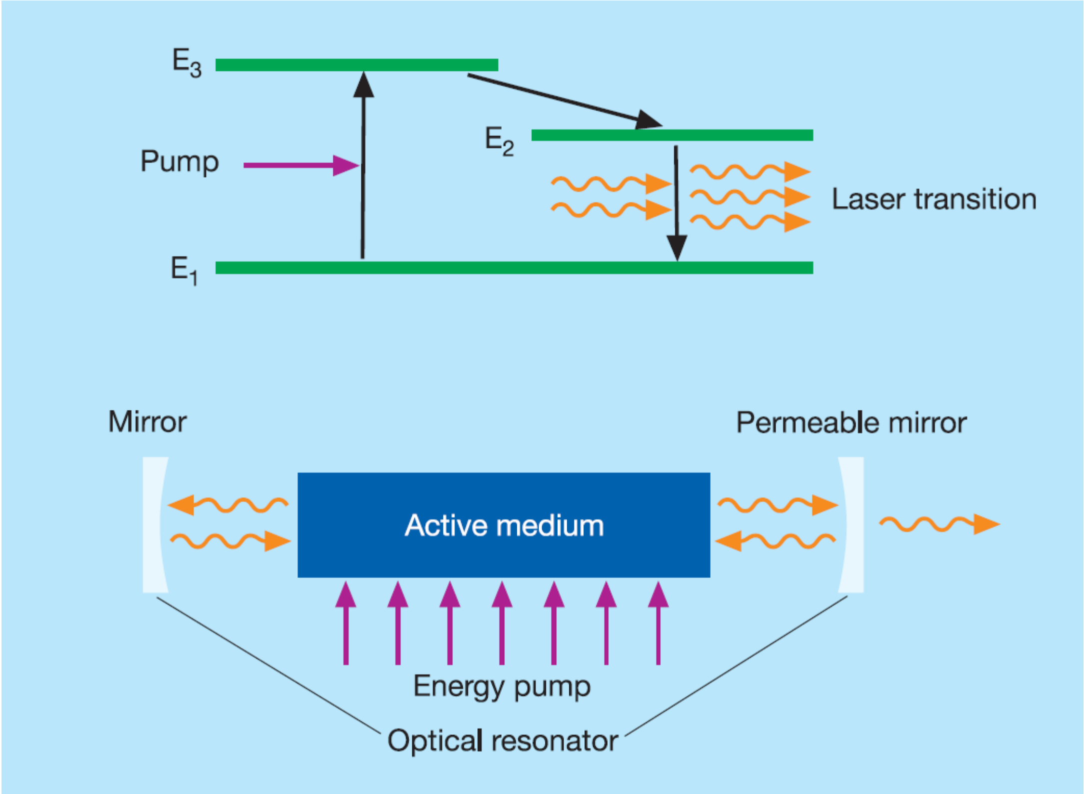
resolution does not depend on sample quality
needs very high radiation power @ $\lambda \approx 1$Å
can see dynamics if pulse length < 100 fs

we need a radiation source with

- very high peak and average power
- wavelengths down to atomic scale $\lambda \sim 1$Å
- spatially coherent
- monochromatic
- fast tunability in wavelength & timing
- sub-picosecond pulse length

Why FELs?

principle of a quantum laser



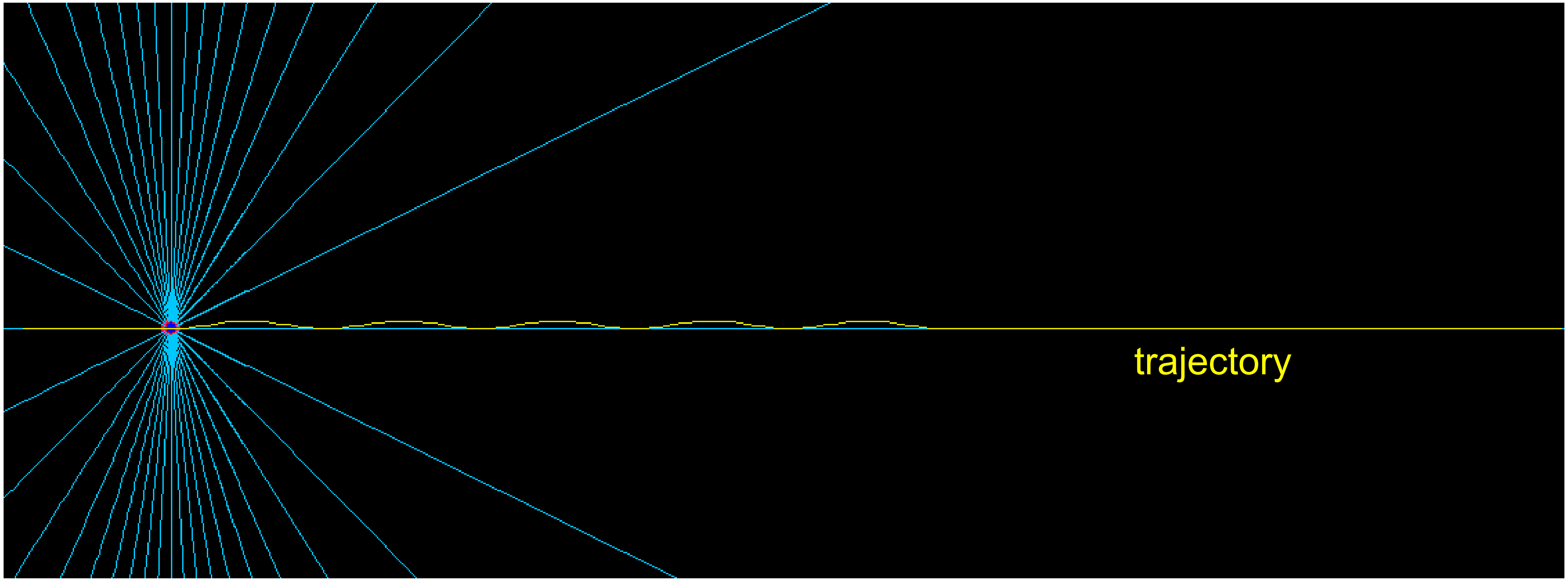
problem & solution:

active medium → free electron – EM wave interaction

Free Electron → Wave Interaction

free electron in uniform motion + electric field lines

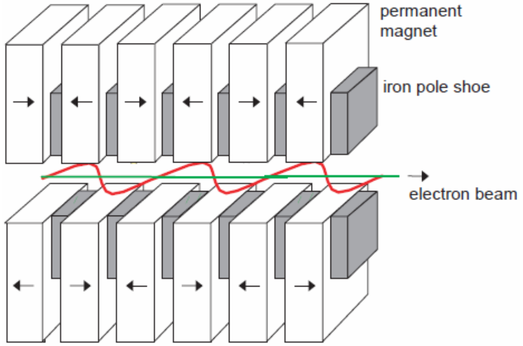
(before undulator)

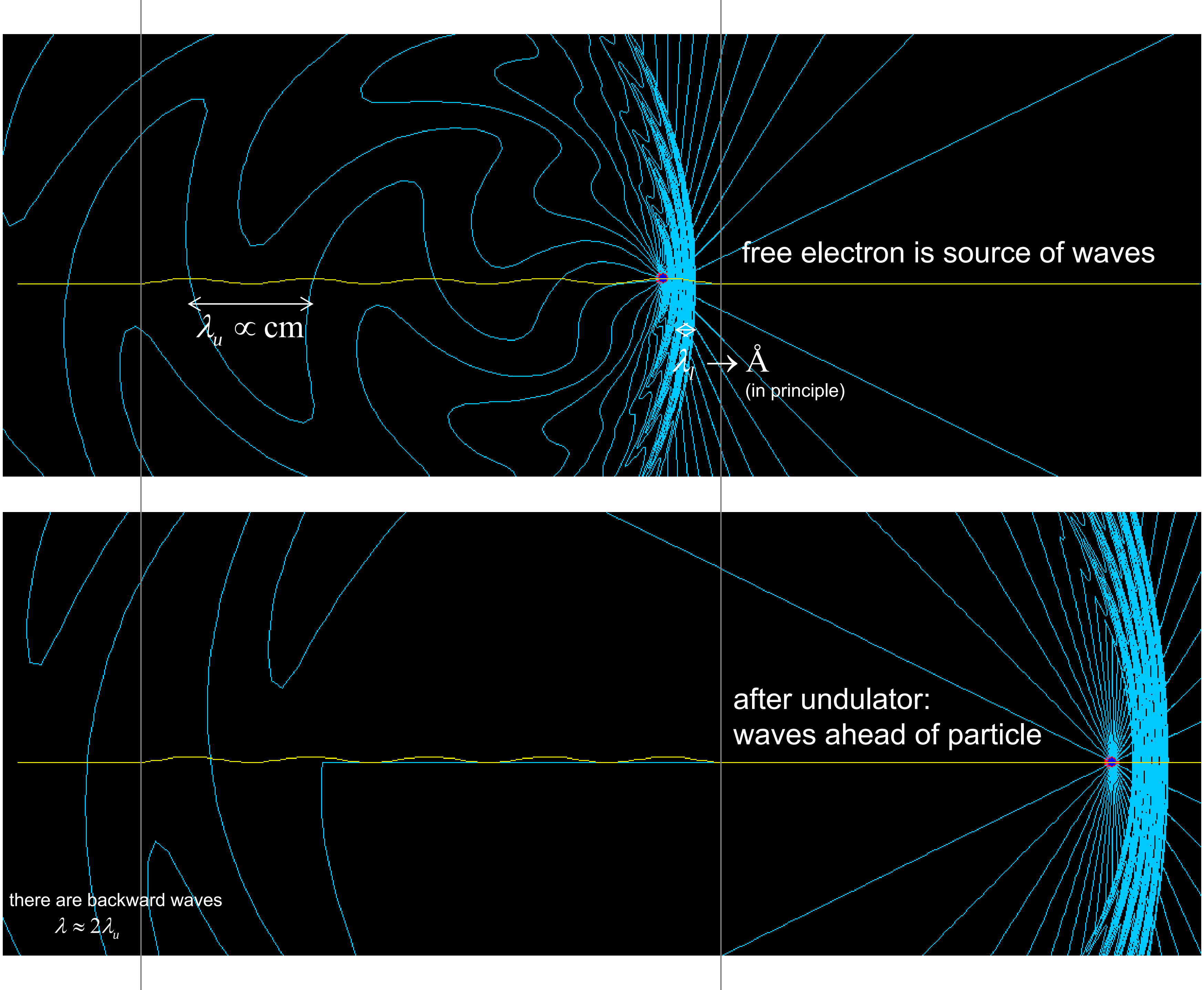


courtesy T. Shintake

http://www.shintakelab.com/en/enEducationalSoft.htm

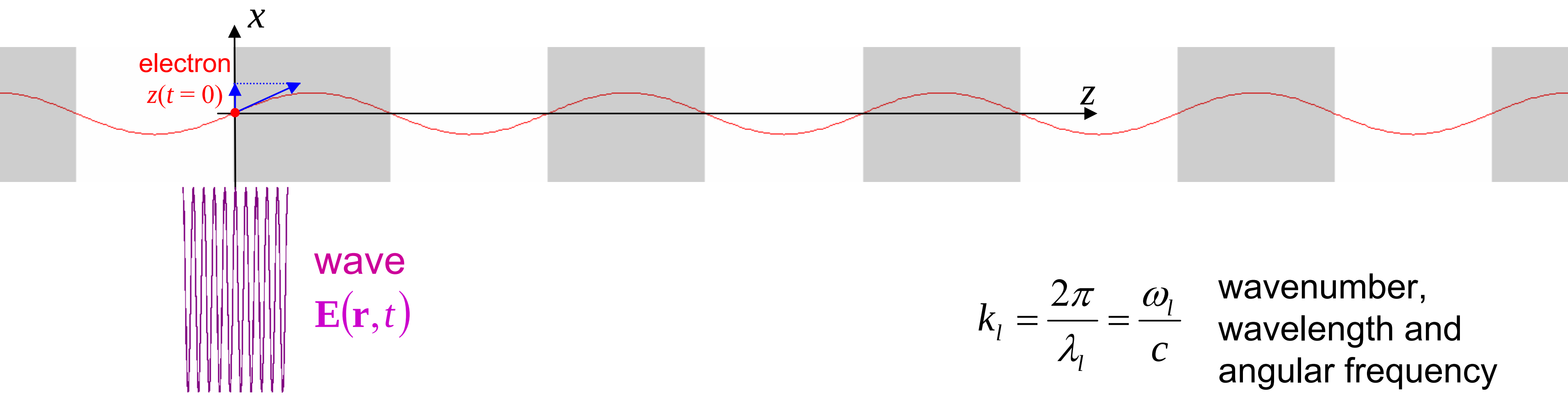
undulator





free electron is source of waves
$\lambda_u \propto$ cm
$\lambda_l \rightarrow$ Å
(in principle)
after undulator:
waves ahead of particle
there are backward waves
$\lambda \approx 2\lambda_u$

Free Electron ← Wave Interaction



$$k_l = \frac{2\pi}{\lambda_l} = \frac{\omega_l}{c}$$ wavenumber, wavelength and angular frequency

change of electron energy $$\frac{dW}{dt} = -e\boxed{\mathbf{v}(t)} \cdot \mathbf{E}\left(\boxed{\mathbf{r}(t)}, t\right)$$

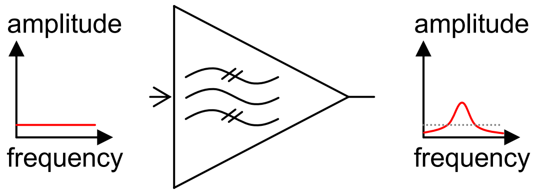
undulator trajectory

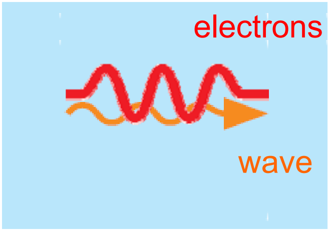
Amplifier and Oscillator

in principle

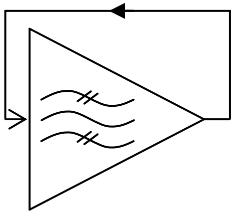
FEL

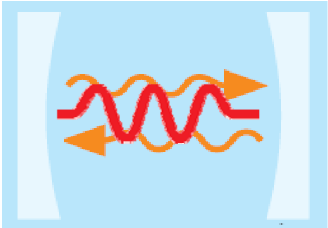
- amplifier:





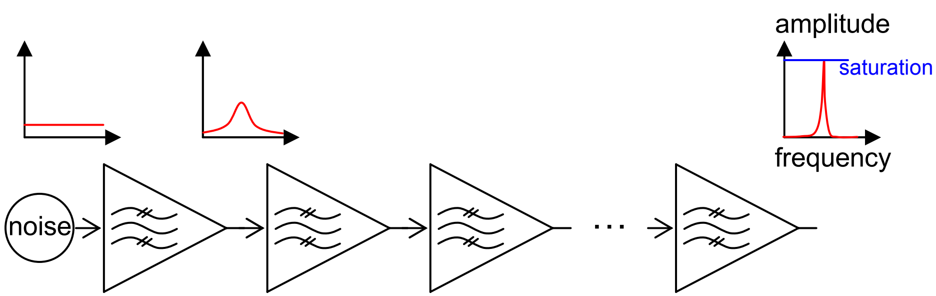
- oscillator:

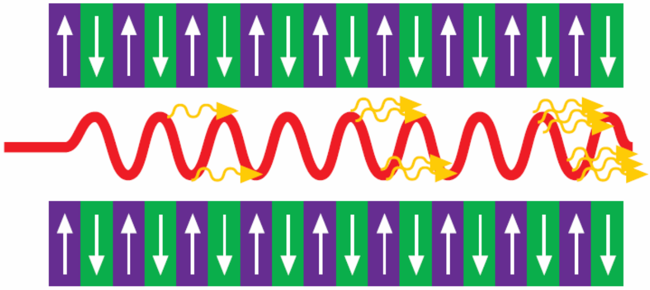




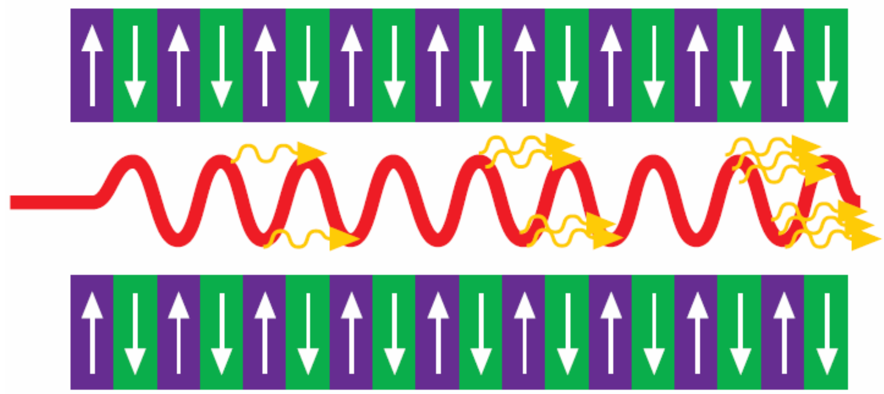
instability, driven by noise, growth until amplifier saturates

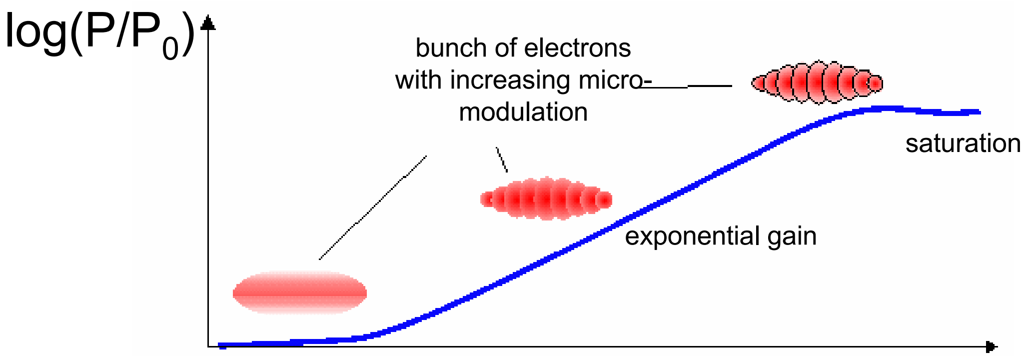
- amplified noise:





Self Amplifying Spontaneous Emission (SASE)



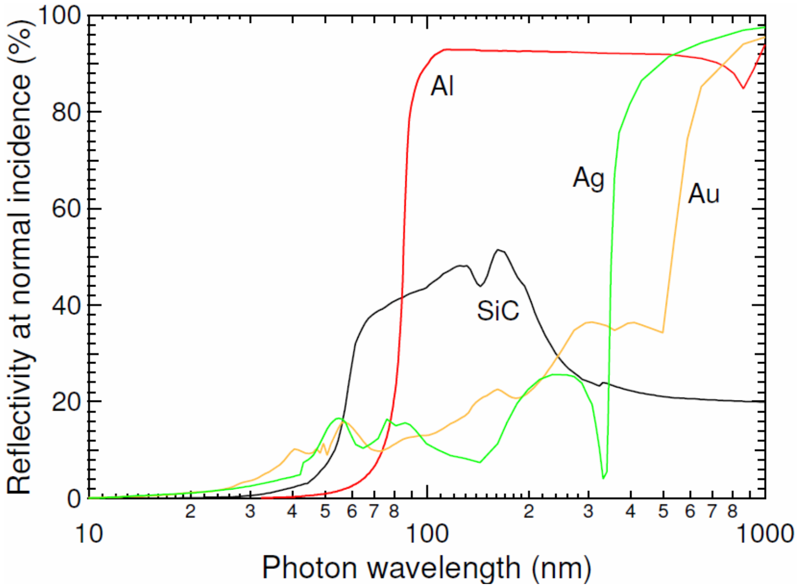


- uniform **random** distribution of particles at entrance
- **incoherent** emission of EM waves (noise, wide bandwidth)
- amplification ($\rightarrow$ resonant wavelength, micro-bunching)
- saturation, full **micro modulation**, **coherent** radiation

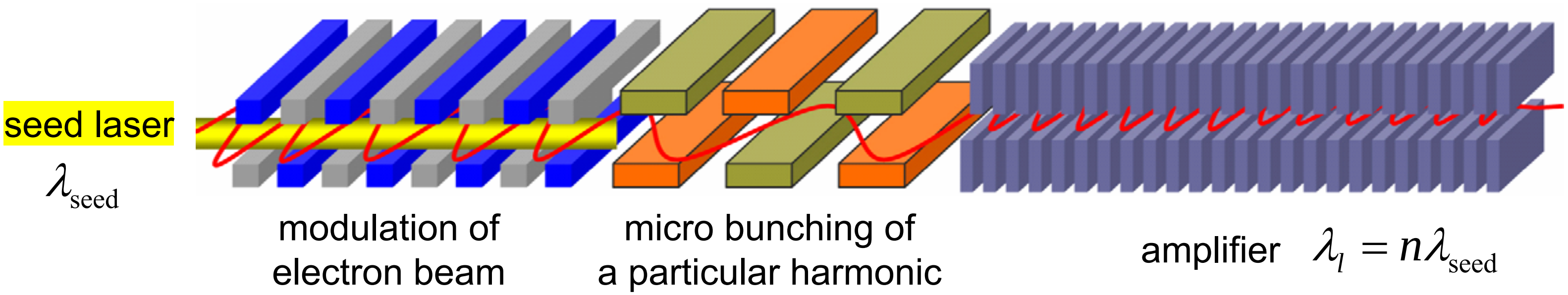
Why SASE?

- oscillator needs resonator

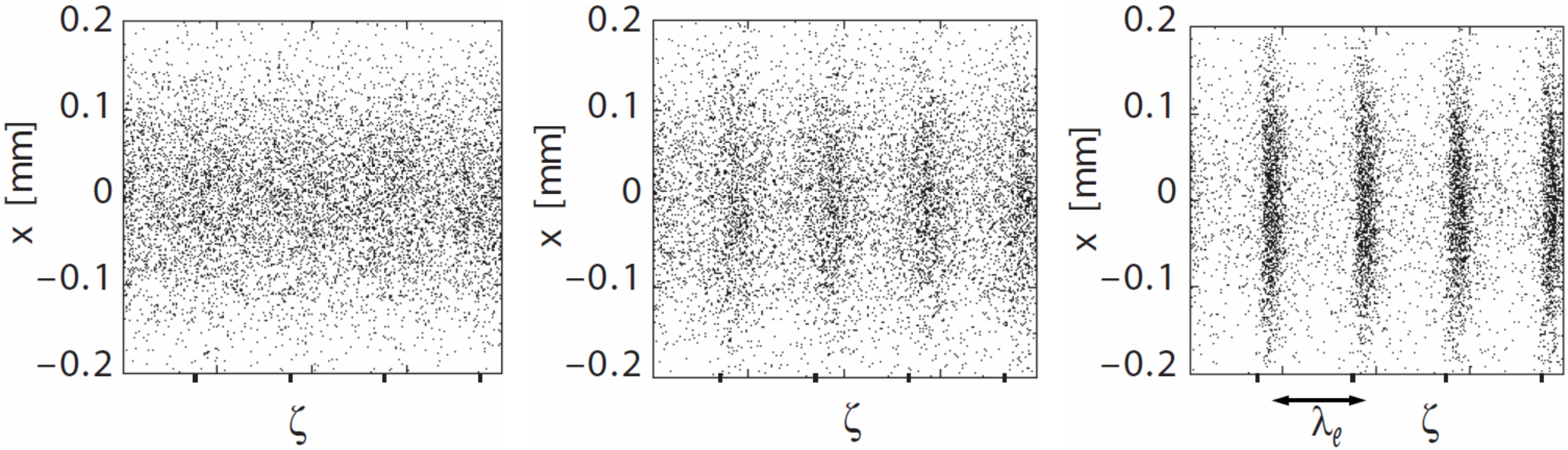
  but there are no mirrors for wavelengths < 100 nm



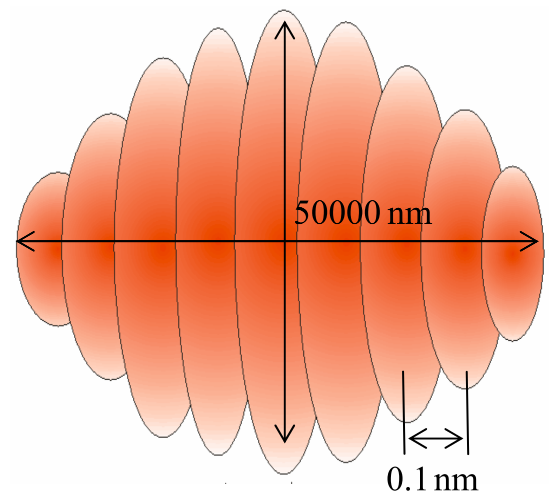
- alternative: seed laser + harmonic generation + amplifier



Micro-Bunching



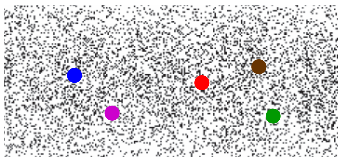
- longitudinal motion is trivial to 1st order, **but**
- micro-bunching is a 2nd order effect → theory
- transverse bunch structure is much larger than longitudinal sub-structure
  → 1d theory with plane waves



Coherent Radiation

electron in undulator $\rightarrow$ plane wave in far field

incoherent superposition of plane waves:

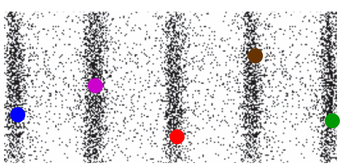


$$E = \sum_{\nu=1}^{N} \cos\left(k_l z - \omega_l t + \psi_\nu\right) = E_\Sigma \cos\left(k_l z - \omega_l t + \psi_\Sigma\right)$$

field amplitude: $E_\Sigma = \left| \sum_{\nu=1}^{N} \exp\left(i\varphi_\nu\right) \right| \propto \sqrt{N}$

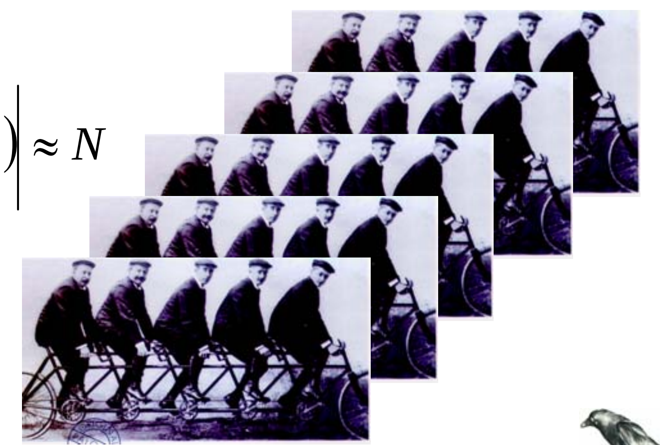
radiated power: $P_\Sigma \propto E_\Sigma^2 \propto N$

coherent superposition of plane waves:

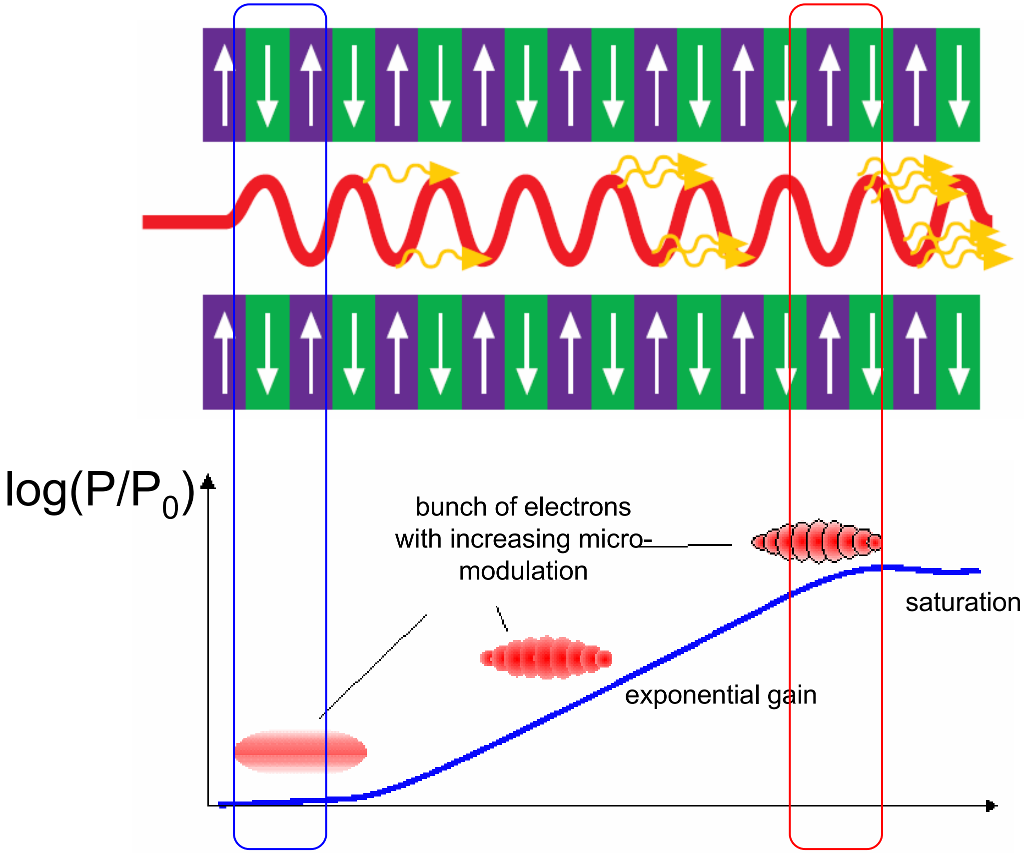


field amplitude: $E_\Sigma = \left| \sum_{\nu=1}^{N} \exp\left(i\varphi_\nu\right) \right| \approx N$

radiated power: $P_\Sigma \propto E_\Sigma^2 \approx N^2$



Coherent Radiation - 2



no micro-bunching
(only shot noise)
$P_0 \propto N$

saturation:
full micro-bunching
$P \propto N^2$

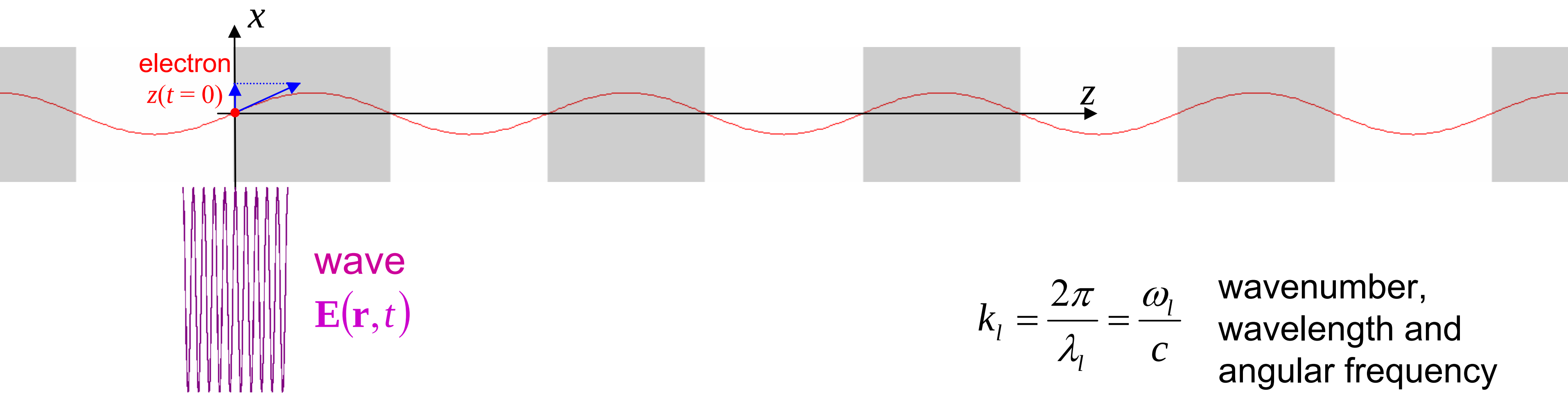
with $N =$ particles per λ_l

Free-Electron Laser

B) Theoretical Approach

- Resonance Condition
- Particle Energy and Ponderomotive Phase
- Longitudinal Equation of Motion
- Low Gain Theory
- FEL Gain
- Micro-Bunching
- High Gain Theory
- Continuous Phase Space
- Gain Length

Resonance Condition



$$k_l = \frac{2\pi}{\lambda_l} = \frac{\omega_l}{c}$$ wavenumber, wavelength and angular frequency

- EM wave is faster than electron, but
- periodic interaction **if** wave-electron slippage is one wavelength per undulator period

$$\frac{\lambda_u + \lambda_l}{c} = \frac{S_u}{v}$$

S_u trajectory length of one undulator period
(S_u and v depend on γ)

otherwise energy exchange cancels over many periods

- resonance condition

$$\boxed{\lambda_l = c\frac{S_u}{v} - \lambda_u \approx \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)}$$

γ Lorentz factor (~ energy)

K undulator parameter (~ 1)

Particle Energy and Ponderomotive Phase

- resonance condition: $\dfrac{\lambda_u + \lambda_l}{c} = \dfrac{S_u\left(\boxed{\gamma_{\mathrm{res}}}\right)}{v\left(\boxed{\gamma_{\mathrm{res}}}\right)}$ resonant energy

- if resonance condition fulfilled: $\left\langle \dfrac{dW}{dt} \right\rangle \propto -\hat{E}\cos\boxed{\psi}$ with ψ = const

  ponderomotive phase

  $\psi = 0 \rightarrow$ kinetic energy transfer EM $\rightarrow$ wave “laser”
  $\psi = \pi \rightarrow$ transfer EM wave $\rightarrow$ kinetic energy “accelerator”

- if resonance condition is not fullfilled: $\gamma \neq \gamma_{\mathrm{res}}$

$$\frac{\lambda_u + \boxed{\lambda_l - \zeta}}{c} = \frac{S_u(\gamma)}{v(\gamma)}$$

particle slips in one period by $\lambda_l - \zeta$

change of ponderomotive phase $\dfrac{\Delta\psi}{T_u} = k_l \zeta$ with $T_u = S_u / v$

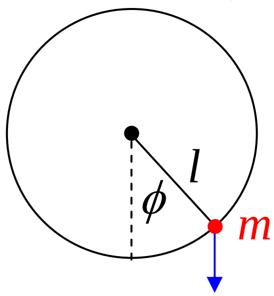
Longitudinal Equation of Motion
(in average)

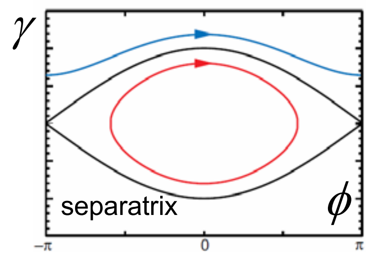
- longitudinal phase space

$$\frac{d\psi}{dt} \propto \gamma - \gamma_{\text{res}} \qquad\qquad \frac{d\gamma}{dt} \propto -\hat{E}\cos\psi$$

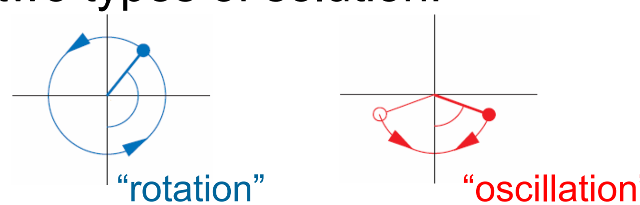
longitudinal position $\rightarrow \psi$
longitudinal momentum $\rightarrow \gamma$

- equations are analog to mathematical pendulum





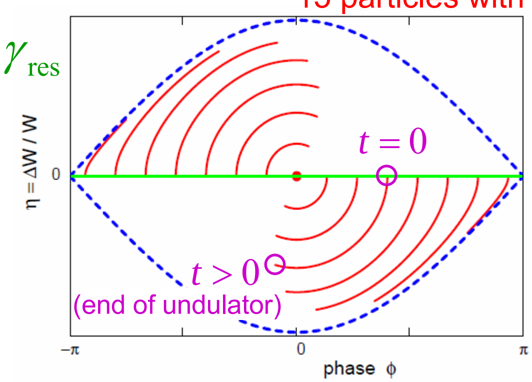
two types of solution:

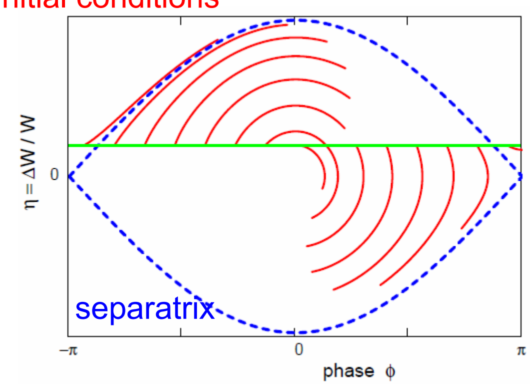


- trajectories in phase space

15 particles with different initial conditions

$\gamma_n(0) = \gamma_{\text{res}}$





$\gamma_n(0) > \gamma_{\text{res}}$

Low Gain Theory

- neglect change of field amplitude

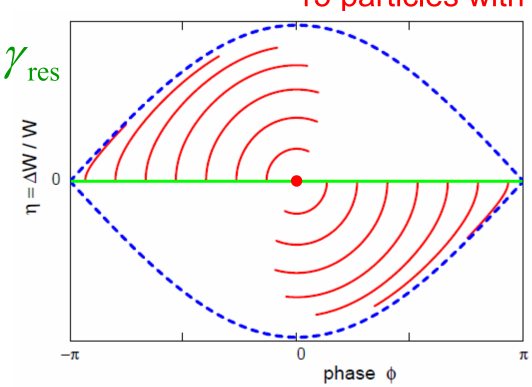
$$\frac{d\psi}{dt} \propto \gamma - \gamma_{\text{res}} \qquad\qquad \frac{d\gamma}{dt} \propto -\boxed{\hat{E}}\cos\psi$$

- indirect gain calculation

$$G = \frac{\text{gain of field energy}}{\text{initial field energy}} = \frac{\text{loss of particle energy}}{\text{initial field energy}} = \frac{W_\Sigma(\text{in}) - W_\Sigma(\text{out})}{\text{initial field energy}}$$

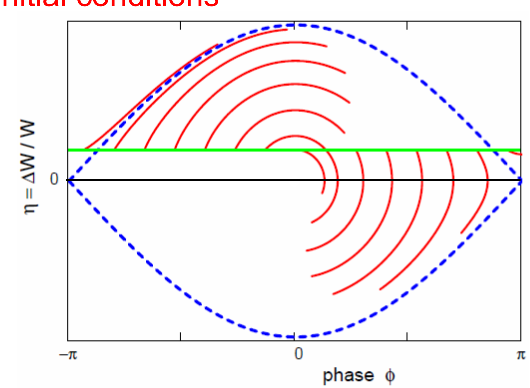
15 particles with different initial conditions

$\gamma_n(0) = \gamma_{\text{res}}$



$W_\Sigma(\text{in}) = W_\Sigma(\text{out})$

$G = 0$



$\gamma_n(0) > \gamma_{\text{res}}$

$W_\Sigma(\text{in}) < W_\Sigma(\text{out})$

$G > 0$

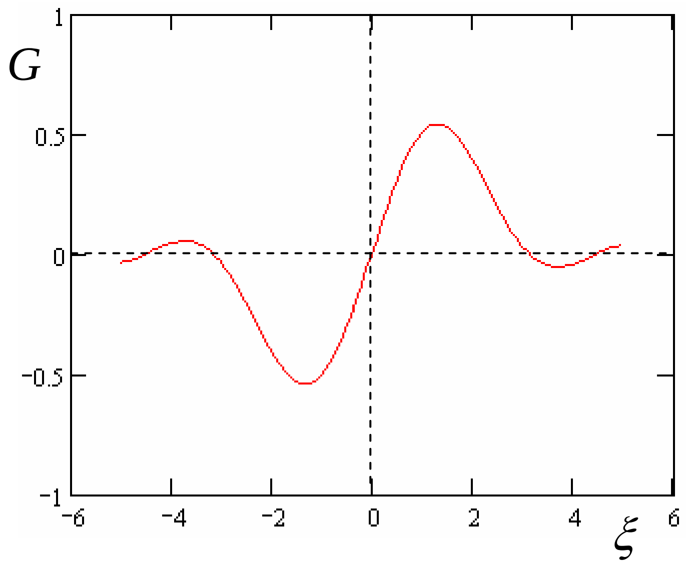
FEL Gain

- analytical analysis $\rightarrow$

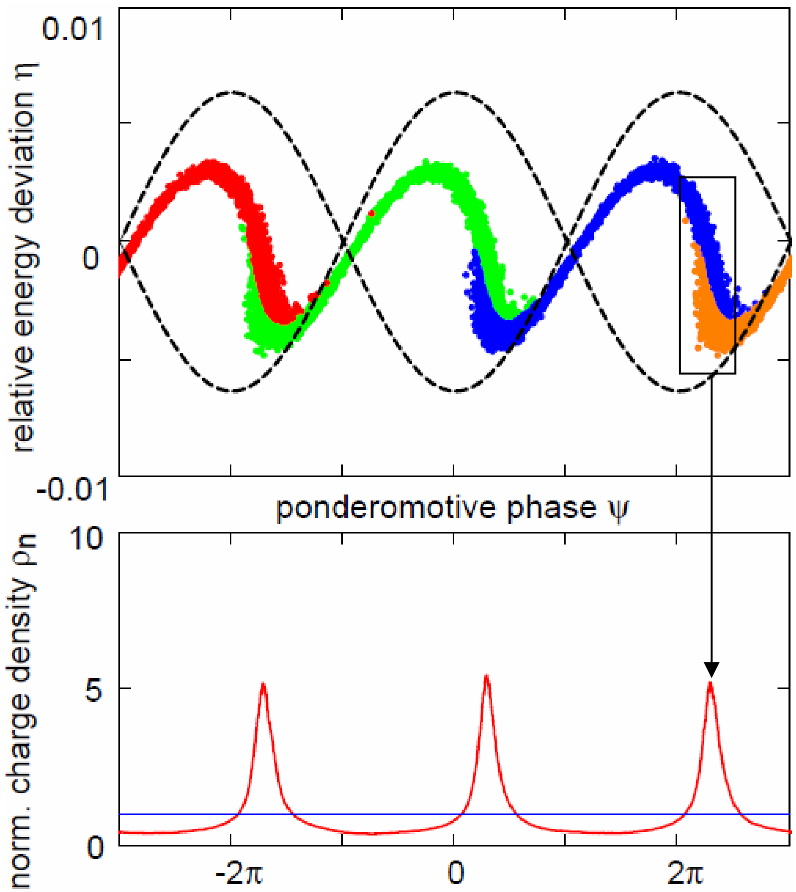
$$G \propto -\frac{d}{d\xi}\left(\frac{\sin^2 \xi}{\xi^2}\right)$$

$$\xi \propto \boxed{M} \frac{\gamma - \gamma_{res}}{\gamma_{res}}$$

periods of undulator



Micro-Bunching



- Fourier analysis of longitudinal particles positions
  $\rightarrow$ amplitude of micro modulation

$$\boxed{\hat{I} \propto \sum \exp(-i\,\psi_\nu)}$$

(fundamental mode)

High Gain Theory

- logitudinal position in undulator $z = \bar{v}t \approx ct$
- set of equations:

particles $\boxed{\frac{d\psi}{dz} \propto \gamma - \gamma_{\mathrm{res}} \qquad \frac{d\gamma}{dz} \propto -\mathrm{Re}\left\{\boxed{\hat{E}(z)}\exp^{i\psi}\right\}}$

amplitude (and phase) of EM wave

bunching $\boxed{\hat{I} \propto \sum \exp(-i\psi_\nu)}$

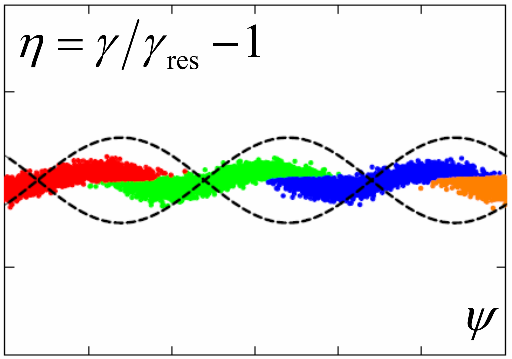
amplitude $\boxed{\frac{d}{dz}\hat{E} \propto K\hat{I}}$ (from Maxwell equations)

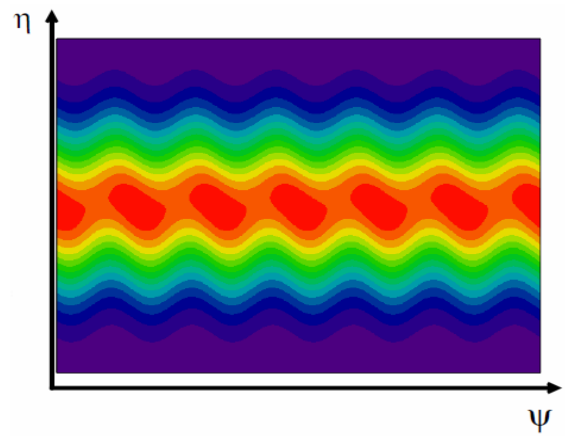
this set of equations + field equations can be solved numerically

- FEL codes include transverse motion and 3D EM field calculation

Continuous Phase Space

- phase space distribution
  many point particles ψ_n, γ_n $\rightarrow$ continuous density distribution $F(z,\psi,\eta)$





charge density $\lambda(z,\psi) = \int d\eta \times F(z,\psi,\eta)$

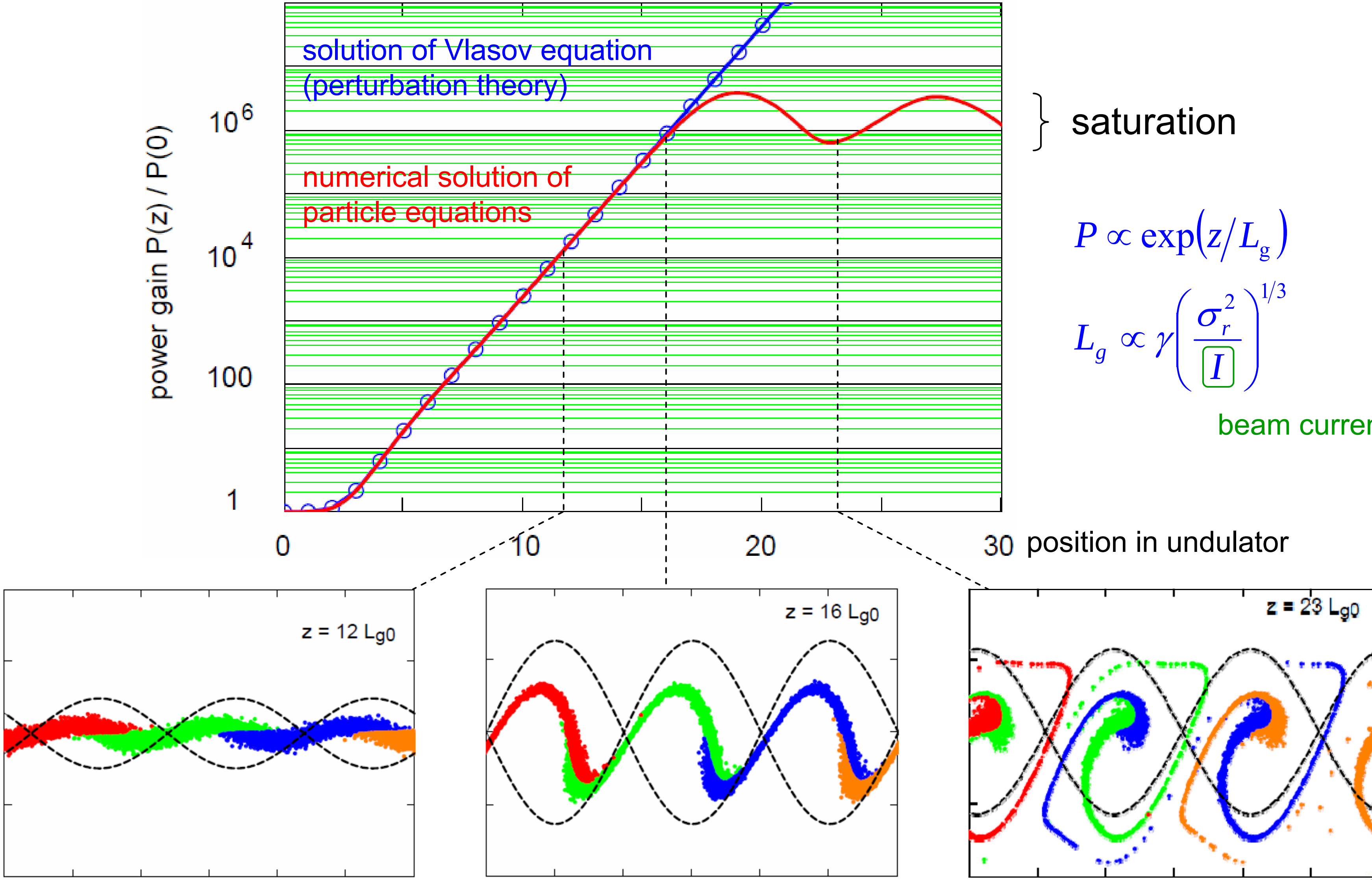
bunching $\hat{I} \propto \int d\psi \times \lambda(z,\psi) e^{-i\psi}$

- Vlasov equation

$$\boxed{\frac{dF}{dz} = \frac{\partial F}{\partial z} + \frac{\partial F}{\partial \psi}\frac{d\psi}{dz} + \frac{\partial F}{\partial \eta}\frac{d\eta}{dz} = 0}$$

$$\boxed{\frac{d\psi}{dz}} \propto \gamma - \gamma_{\text{res}} \qquad \boxed{\frac{d\eta}{dz}} \propto -\text{Re}\left\{\hat{E}(z)\exp^{i\psi}\right\}$$

Gain Length



$$P \propto \exp(z/L_g)$$

$$L_g \propto \gamma \left(\frac{\sigma_r^2}{\boxed{I}} \right)^{1/3}$$

beam current

Free-Electron Laser

C) Experimental Realization / Challenges

Linac Coherent Light Source - LCLS

Scales

Challenges

RF Gun

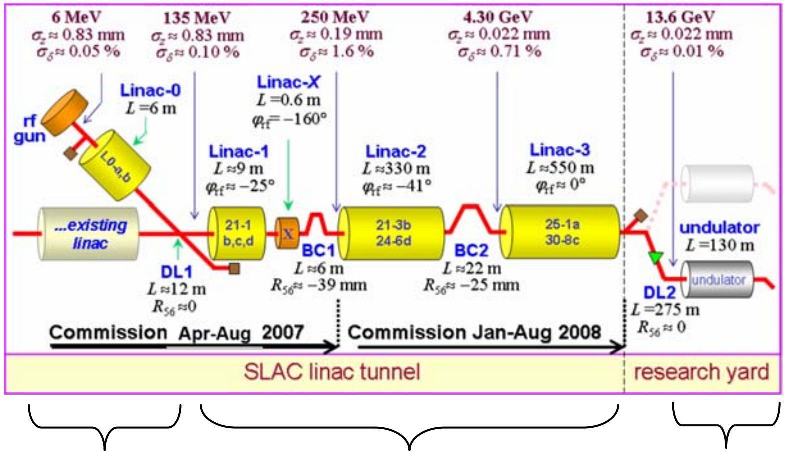
Bunch Compression

European X-FEL

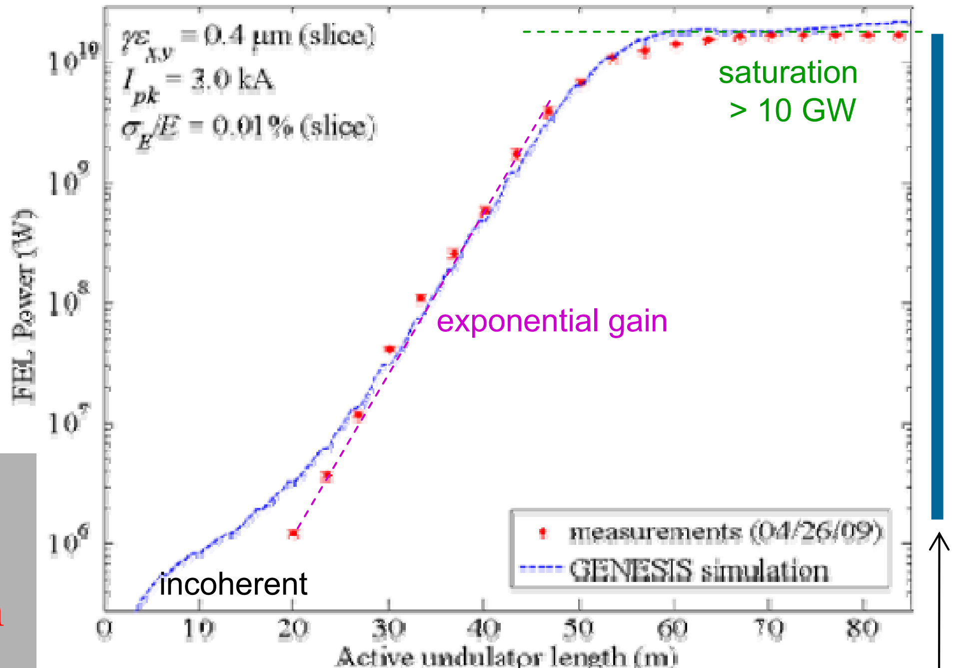
Table Top FEL

Linac Coherent Light Source- LCLS

SLAC mid-April 2009 – first lasing at 1.5 Å



injector	linac & bunch compression	undulator
	$L \propto 10^3\,\mathrm{m}$	$L \propto 100\,\mathrm{m}$
	$E = 13.6\,\mathrm{GeV}$	$\lambda_u \approx 3\,\mathrm{cm}$
$q \approx 0.25\,\mathrm{nC}$	$I = 3\,\mathrm{kA}$	$L_g \approx 3.3\,\mathrm{m}$
		$\boxed{\lambda_l = 1.5\,\text{Å}}$



particles per λ_l

$$N = \frac{I\lambda_l}{ec} \approx \boxed{10^4}$$

Scales

photon wavelength	$\lambda_l \propto 10^{-10}\,\mathrm{m} \quad \propto \lambda_u/\gamma^2$	
cooperation length	$L_l = N_c \lambda_l \propto 10^{-8}\,\mathrm{m}$	about N_c micro-bunches in cooperation
bunch length	$L_b = cq/I \propto 10^{-5}\,\mathrm{m}$	
bunch width	$\sigma_w \propto \sqrt{\lambda_l L_g} \propto 10^{-5}\,\mathrm{m}$	(overlap electron-beam EM wave)
undulator period	$\lambda_u \propto 10^{-2}\,\mathrm{m}$	
power gain length	$L_g = N_c \lambda_u \approx 1..10\,\mathrm{m} \qquad \propto \sqrt[3]{\gamma^3 \lambda_u \sigma_w^2 / I}$	
Rayleigh length	$L_R \approx L_g$	describes widening of EM wave
saturation length	$L_s \approx 10 L_g .. 20 L_g < L_u$	
undulator length	$L \propto 100\,\mathrm{m}$	
total length	$L \propto 10^3\,\mathrm{m}$	

Challenges

$$\lambda_l = \frac{\lambda_u}{2\gamma^2}\left(1+\frac{K^2}{2}\right)$$

$$L_g = \frac{1}{\sqrt{3}}\left(\frac{4mc}{\mu e}\frac{\gamma^3\lambda_u}{K^2}\boxed{\frac{\sigma_r^2}{I}}\right)^{1/3}$$

$$\sigma_r^2 \propto \lambda_l L_g$$

(undulator parameter K $\propto$ 1)

- $\lambda_l \rightarrow$ Å
- Energy $\rightarrow$ 10 .. 20 GeV
- gain length L_g <~ 10 m
- high peak current >~ kA
- transverse beam size $\sigma_r \propto 10\ \mu\mathrm{m}$
- energy spread
- overlap electron-photon beam

---

transverse: generate low emittance beam
preservation of emittance

longitudinal: compression

acceleration

diagnostic and steering

undulator alignment

$$\boxed{\frac{\sigma_r^2}{I}} = \frac{\boxed{\sigma_r^2 L_b}}{\boxed{q}c}$$

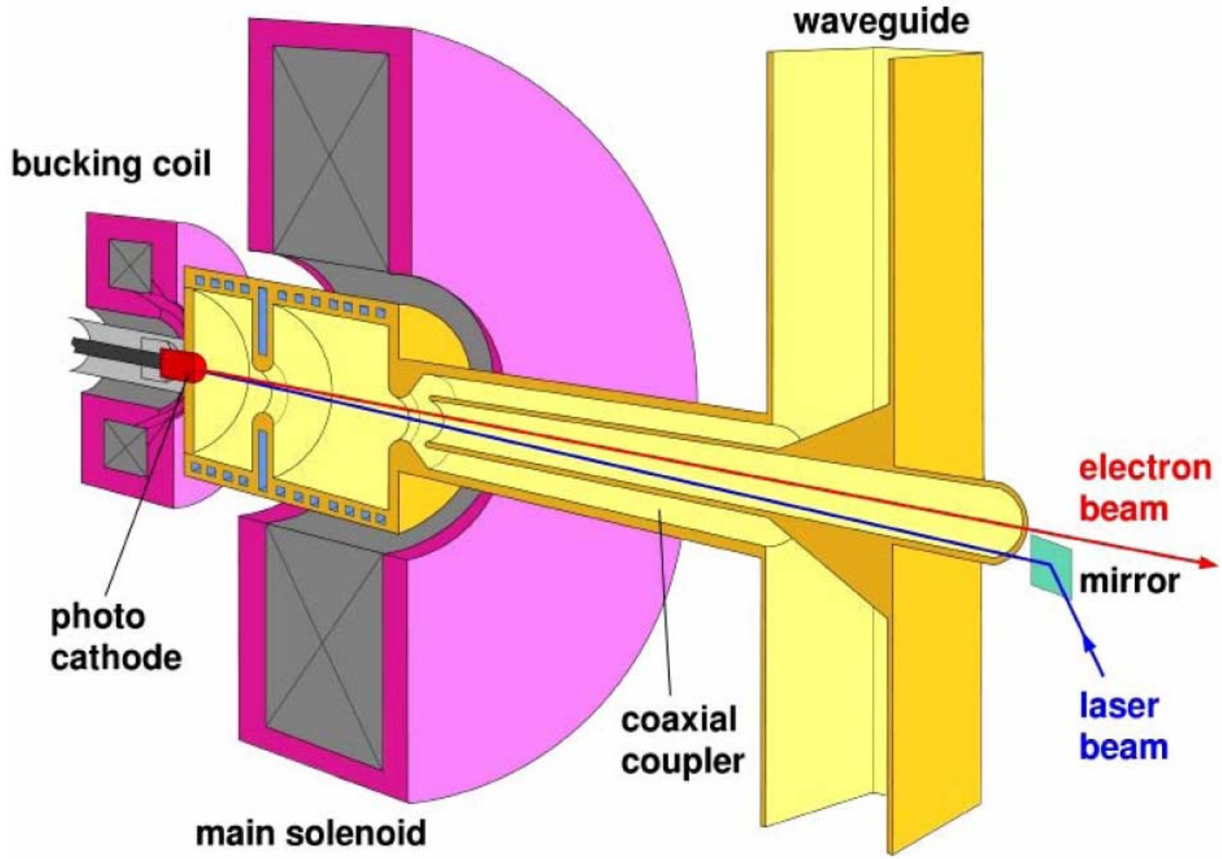
volume

bunch charge

space charge forces:

$$E_{sq} \propto \frac{1}{\gamma^2}\frac{q}{\sigma_r^2}$$

RF Gun



typical parameters of FLASH & European XFEL:

$$q \propto 1\,\mathrm{nC} \qquad E \propto 5\,\mathrm{MeV} \qquad I \propto 50\,\mathrm{A}$$

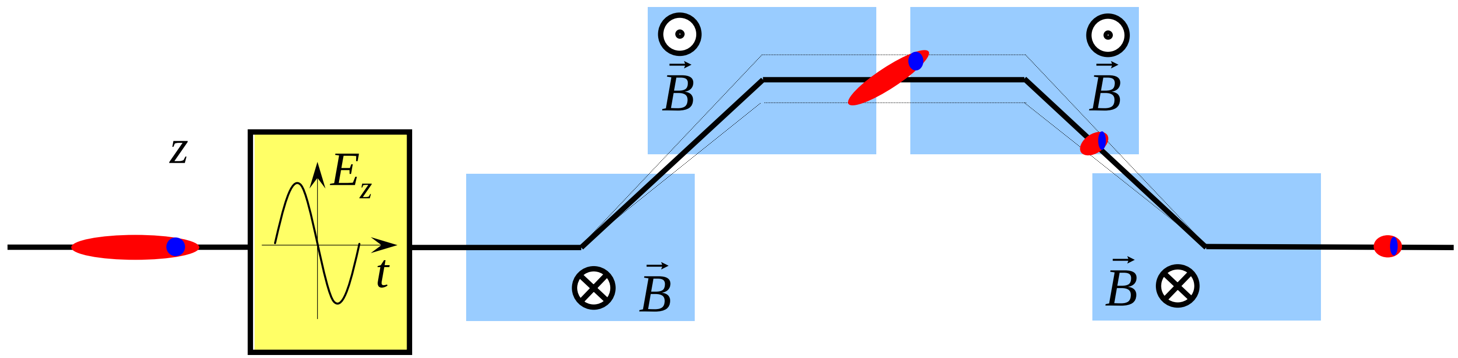
$$\gamma \propto 10$$

longitudinal compression $1 \to 0.01$ needed !

Magnetic Bunch Compression

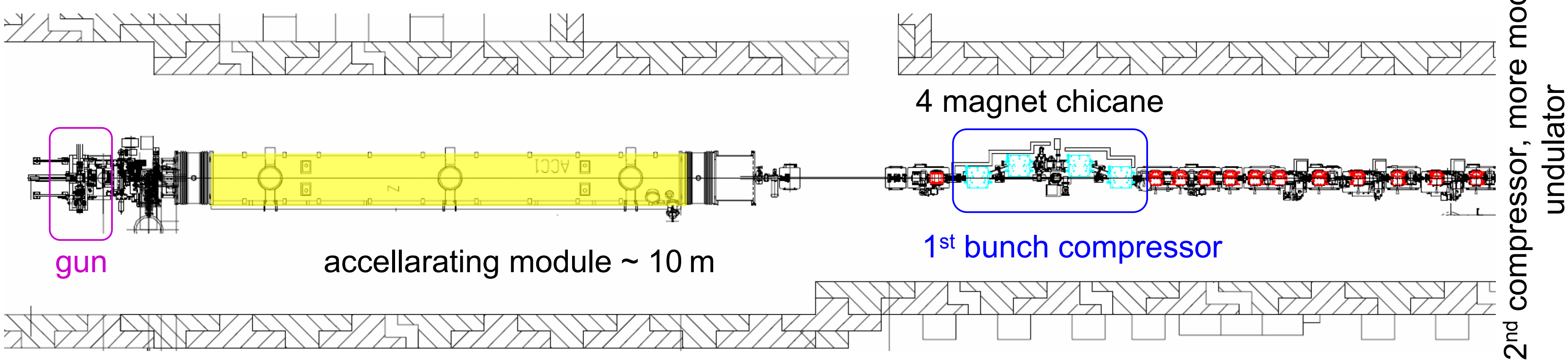
$\gamma >> 1 \rightarrow$ velocity differences are too small for effective compression

magnetic compression: path length depends on energy



accelleration “off crest” $\rightarrow$
head particle with less energy than tail

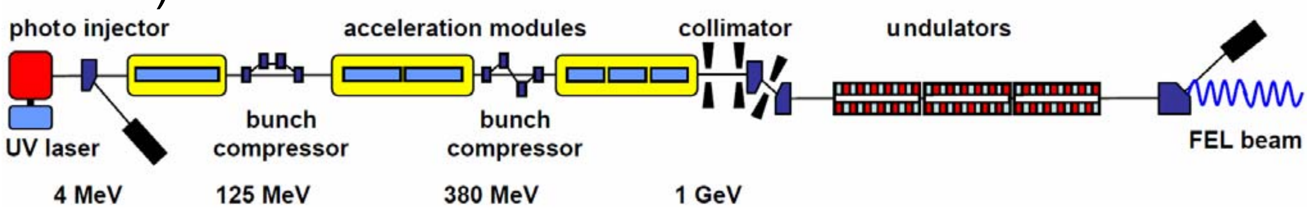
FLASH:



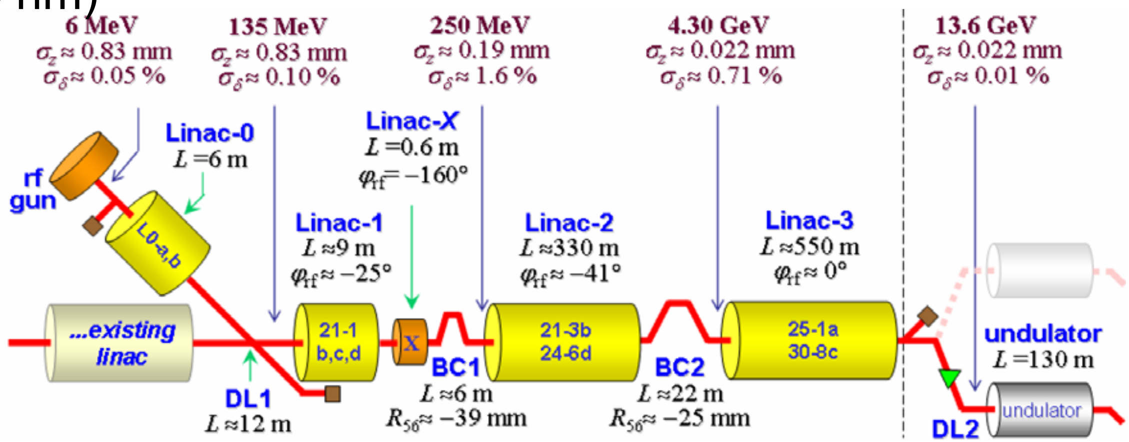
beam dynamics with space charge and CSR effects

Magnetic Bunch Compression - 2

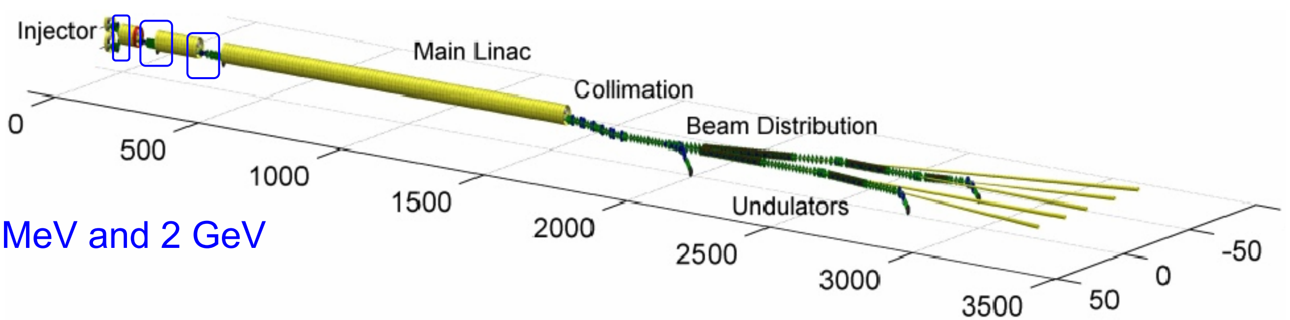
FLASH (1 GeV, 6.5 nm)



LCLS (14 GeV, 0.15 nm)



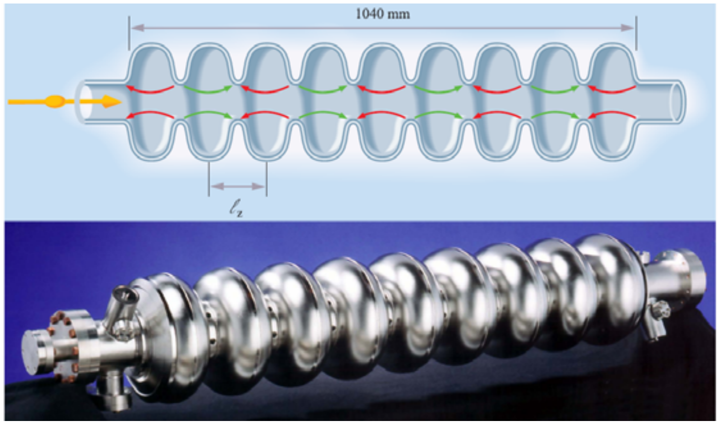
European XFEL (17.5 GeV, 0.1 nm)

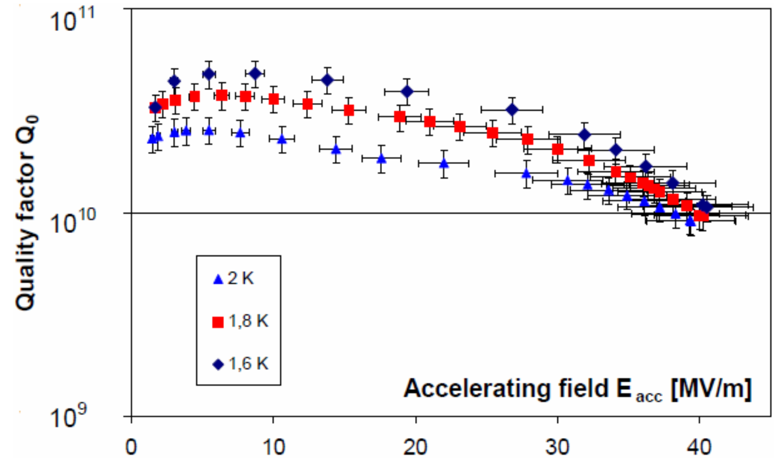


BCs at 130 MeV, 500 MeV and 2 GeV

European X-FEL

superconducting cavity, 1.3 GHz $E_{\mathrm{acc}} \rightarrow$ 40 MeV/m
22.5 MeV/m are needed for 17.5 GeV & 1Å





FLASH tunnel: cryo module

undulator

European X-FEL -2

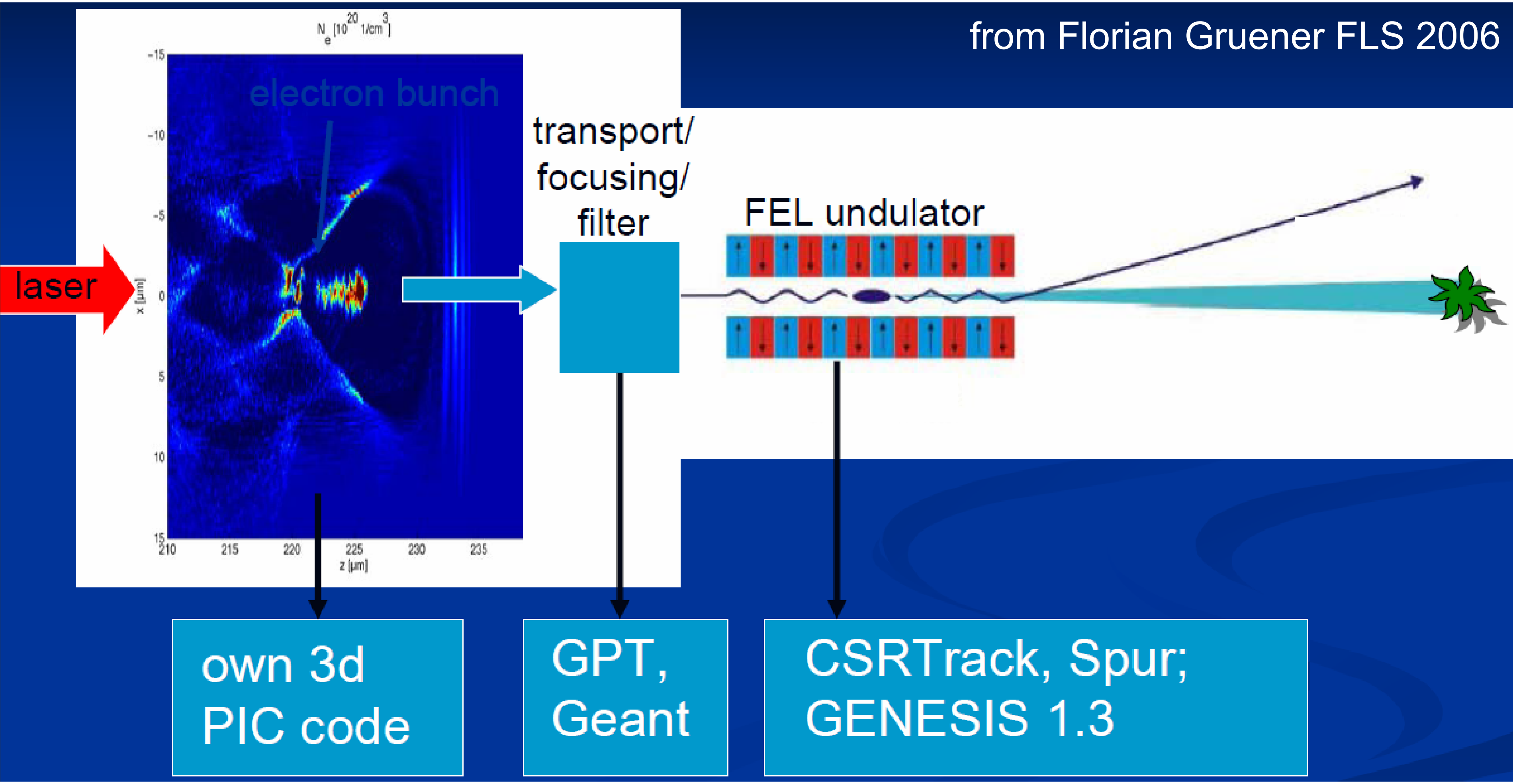
	LCLS	SCSS	European XFEL
Abbreviation for	Linac Coherent Light Source	Spring-8 Compact SASE Source	European X-Ray Free-Electron Laser
Location	California, USA	Japan	Germany
Start of commissioning	2009	2010	2014
Accelerator technology	normal conducting	normal conducting	superconducting
Number of light flashes per second	120	60	30 000 multi bunch operation
Minimum wavelength of the laser light	0.15 nanometres	0.1 nanometres	0.1 nanometres
Maximum electron energy	14.3 billion electron volts (14.3 GeV)	6-8 billion electron volts (6-8 GeV)	17.5 billion electron volts (17.5 GeV)
Length of the facility	3 Kilometer	750 Meter	3.4 Kilometer
Number of undulators (magnet structures for light generation)	1	3	5
Number of experiment stations	3-5	4	10
Peak brilliance [photons / s / mm^2 / mrad^2/ 0.1% bandwidth]	$8.5 \cdot 10^{32}$	$5 \cdot 10^{33}$	$5 \cdot 10^{33}$

beamlines

	SASE 1	SASE 2	SASE 3	U1	U2
wavelengths	0.1 nm	0.1-0.4 nm	0.4-1.6 nm	0.01-0.06 nm	0.01-0.06 nm
laserlike/coherent	yes	yes	yes	no	no
undulator length	201.3 m	256.2 m	128.1 m	61 m	61 m
number of segments	33	42	21	10	10

Table Top FEL

Laser-Pasma accelerators: “bubble acceleration”



$q \approx 1$ nC, $\sigma_r \approx 1$ μm
$I \sim 100$ kA, $\gamma < 10^4$

all types of effects; usual approximations fail

A challenge in experiment and theory !