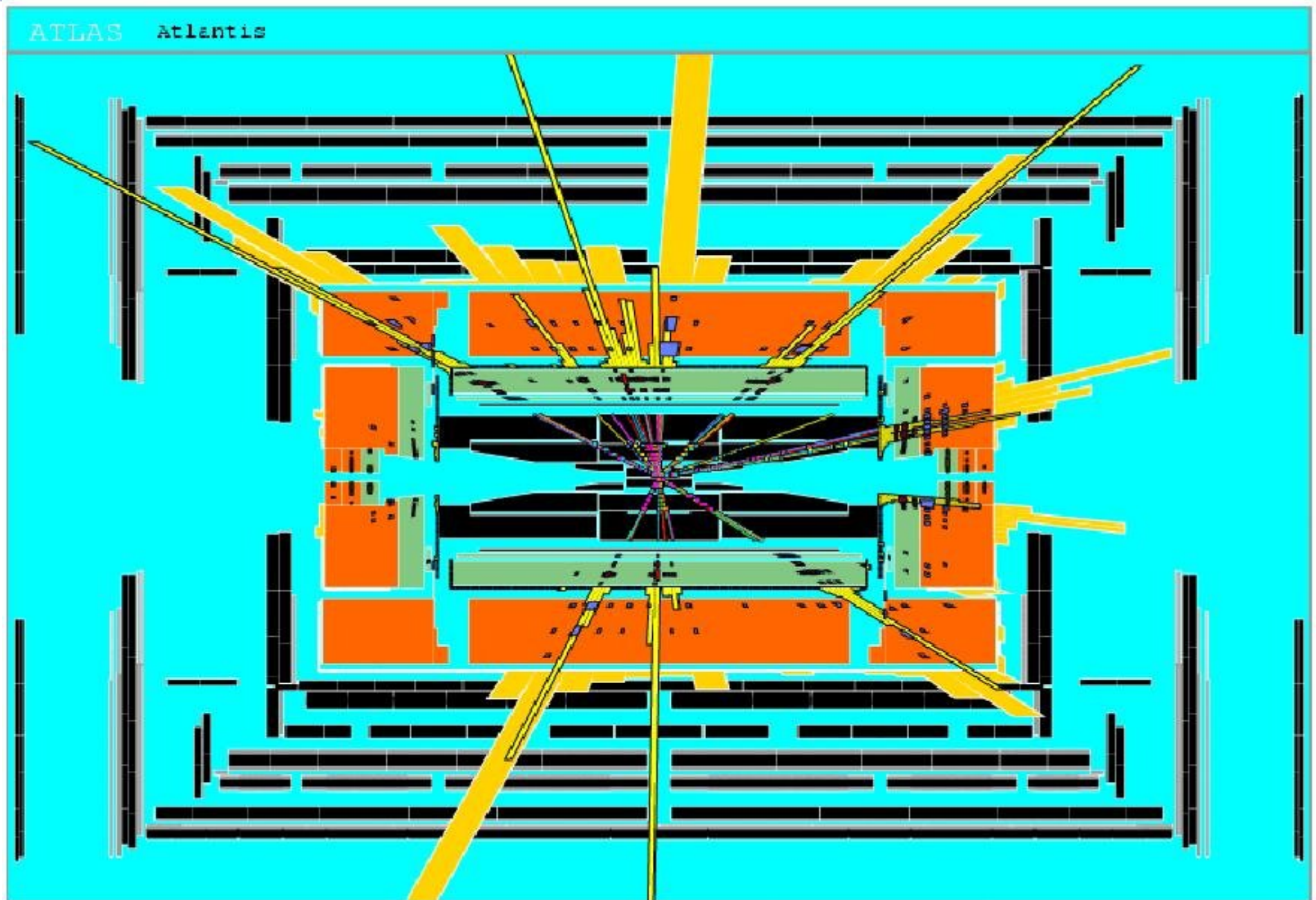


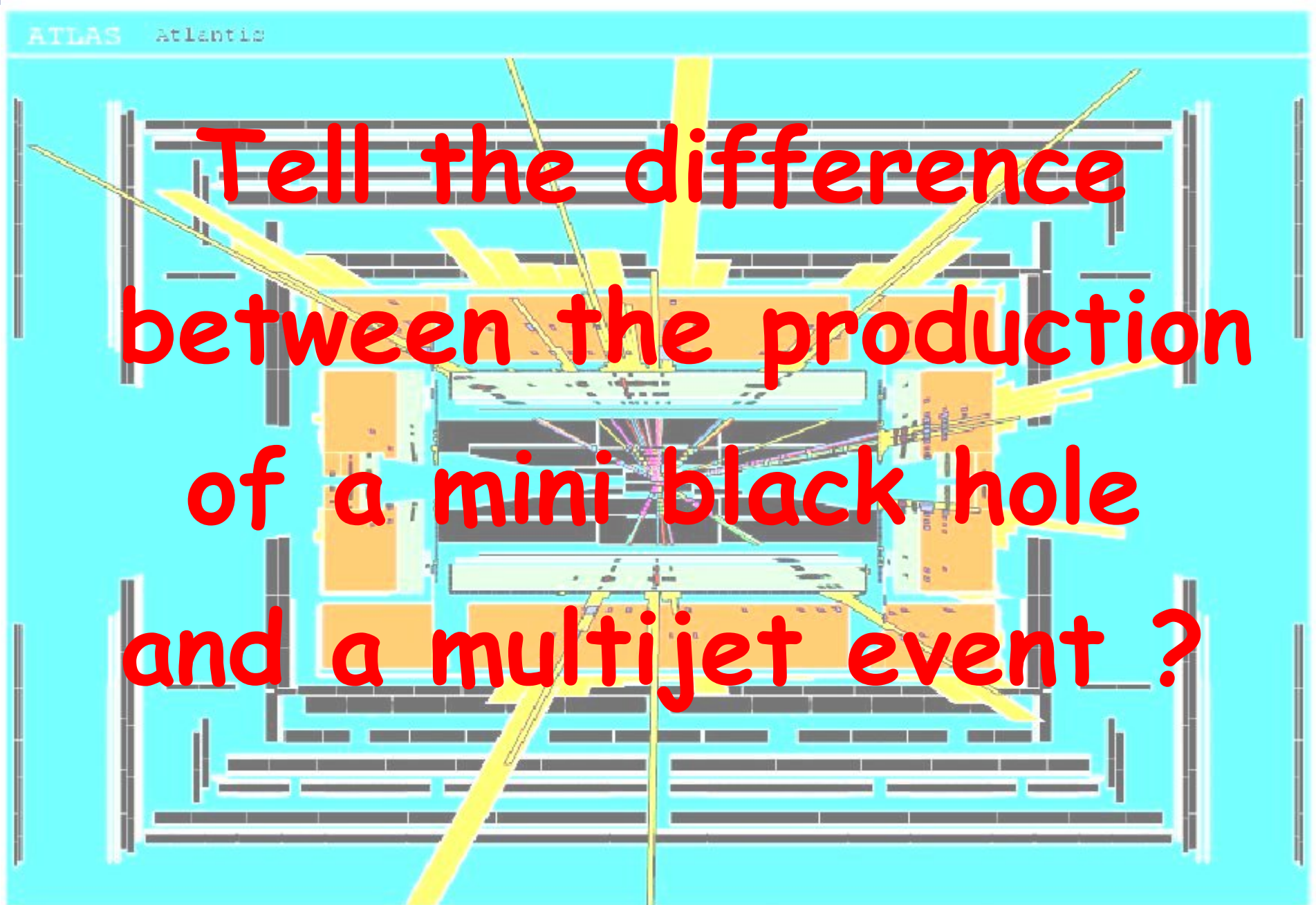
Simulations in High Energy Physics

H. Jung (DESY)

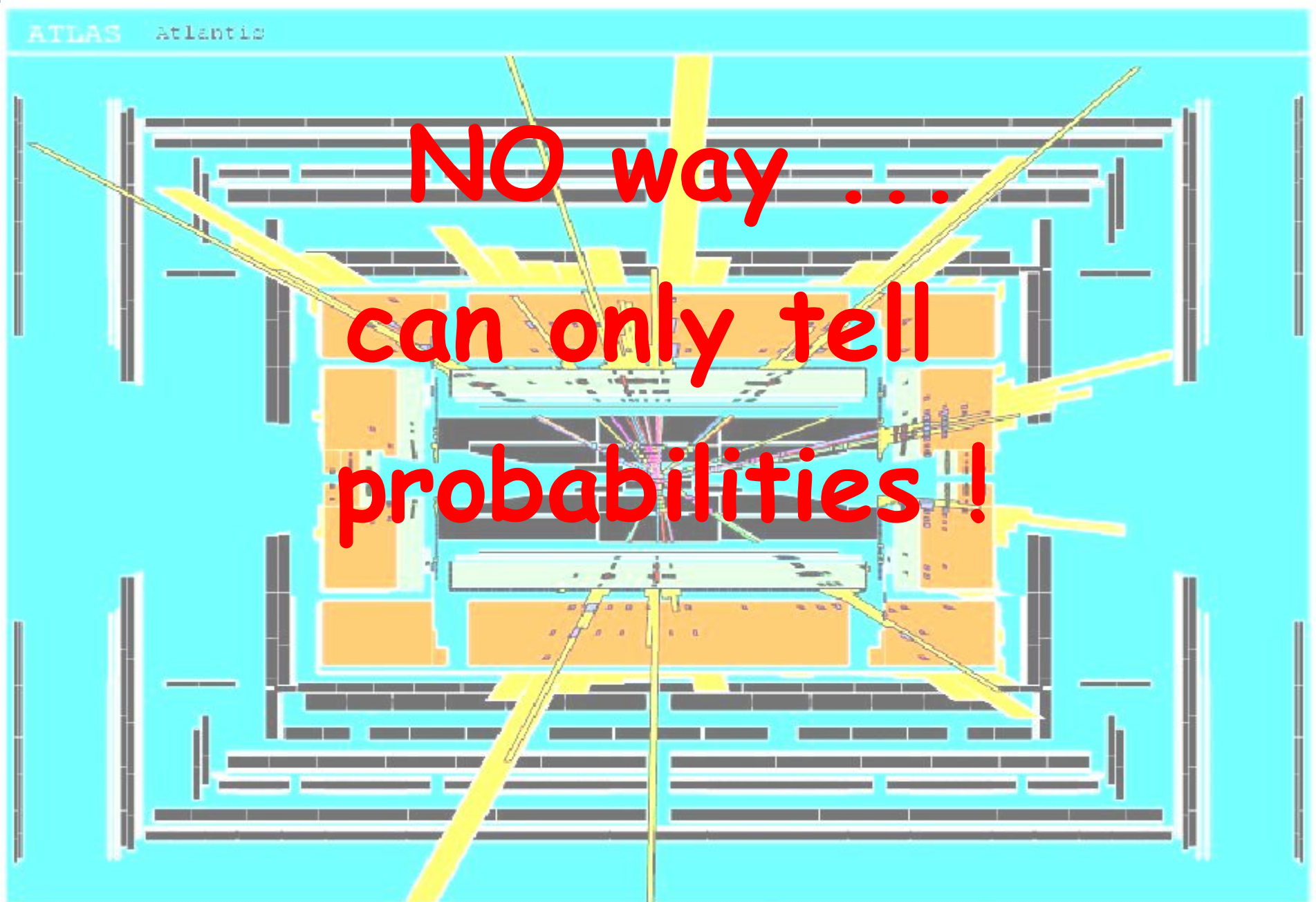
What is this ?



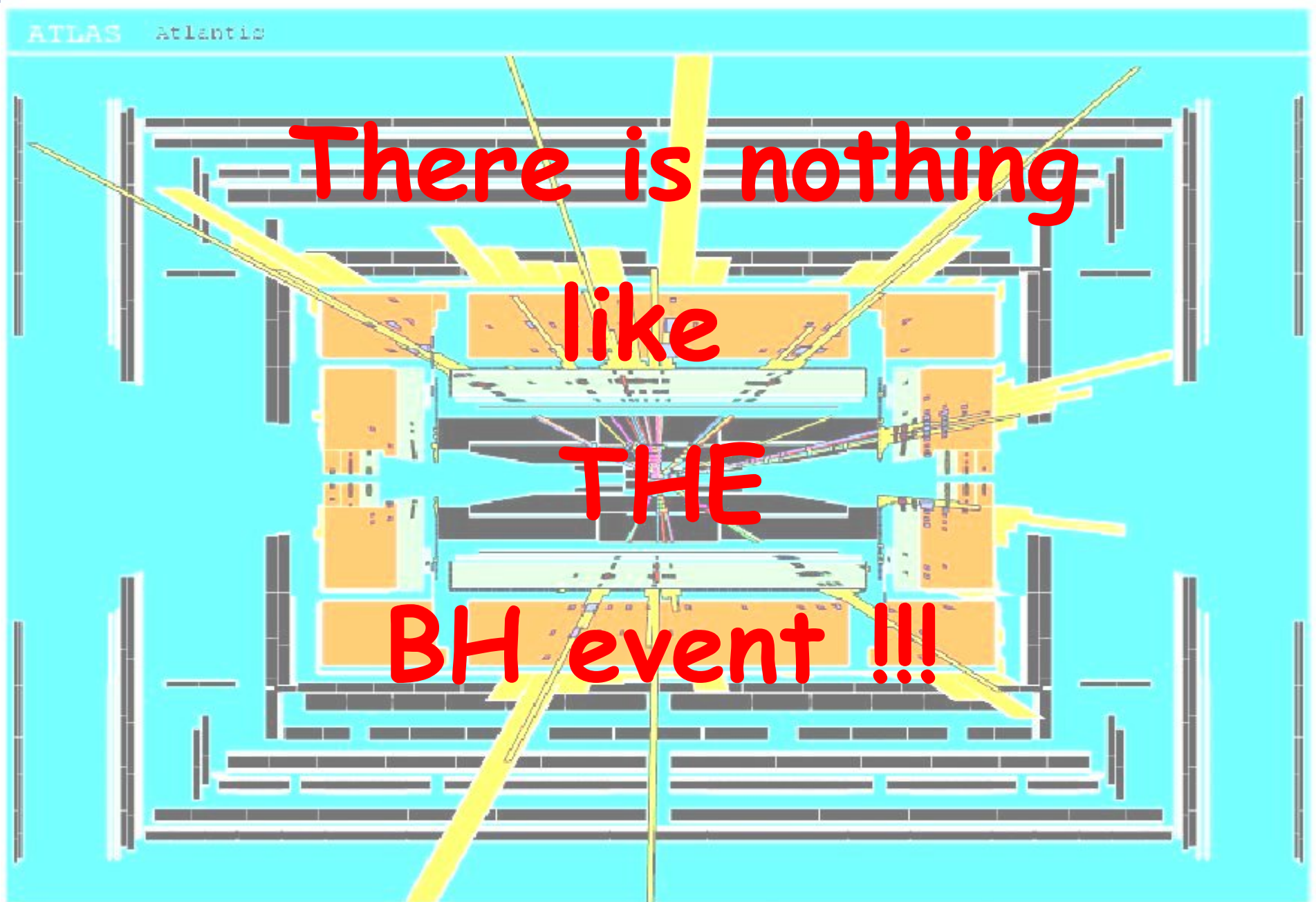
What is this ?



What is this ?



What is this ?



What this is about ...

The long, long way from

$$e^+e^- \rightarrow e^+e^-$$

via

$$ep \rightarrow e'X$$

to

$$pp \rightarrow h + X$$

.... or

from gambling



.... to

Econophysics Colloquium 2008

C | A | U

Christian-Albrechts-Universität zu Kiel

Econophysics Colloquium, August 28-30, 2008 in Kiel, Germany

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[Accepted abstracts](#)

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Useful links:

[Conference "Income Distribution and the Family"](#)

Econophysics Colloquium 2008

August 28-30, 2008 in Kiel, Germany

Aims

The annual colloquium, now in its fourth consecutive year, provides a platform for the presentation of interdisciplinary ideas from different communities, for instance economics and finance, physics, mathematics, biology, computer science, engineering, etc.

The conference's main aim is to foster an open-minded, cross-fertilizing, and regular exchange of ideas among scholars and practitioners of the different fields in a friendly environment.

Conference Topics

Conference topics traditionally include the application of methods and modeling paradigms from statistical physics and complexity science to socio-economic questions and problems, for instance in the following fields:


- Statistical and probabilistic methods in economics and finance
- Multi-scaling analysis and modeling
- Complex socio-economic networks
- Agent-based models in economics and finance
- Evolutionary economics
- Information, bounded rationality, and learning
- Markets as complex adaptive systems
- Non-linear dynamics and econometrics



... via applications in economy

What is monte carlo simulation? montecarlo analysis?

<http://www.decisioneering.com/monte-carlo-simulation.html>



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RISK ANALYSIS OVERVIEW

WHAT IS MONTE CARLO SIMULATION?

What do we mean by "simulation?"

When we use the word **simulation**, we refer to any analytical method meant to imitate a real-life system, especially when other analyses are too mathematically complex or too difficult to reproduce.

Without the aid of simulation, a spreadsheet model will only reveal a single outcome, generally the most likely or average scenario. Spreadsheet risk analysis uses both a spreadsheet model and simulation to automatically analyze the effect of varying inputs on outputs of the modeled system.

One type of spreadsheet simulation is **Monte Carlo simulation**, which randomly generates values for uncertain variables over and over to simulate a model.

How did Monte Carlo simulation get its name?

Monte Carlo simulation was named for Monte Carlo, Monaco, where the primary attractions are casinos containing games of chance. Games of chance such as roulette wheels, dice, and slot machines, exhibit random behavior.

The random behavior in games of chance is similar to how Monte Carlo simulation selects variable values at random to simulate a model. When you roll a die, you know that either a 1, 2, 3, 4, 5, or 6 will come up, but you don't know which for any particular roll. It's the same with the variables that have a known range of values but an uncertain value for any particular time or event (e.g. interest rates, staffing needs, stock prices, inventory, phone calls per minute).

Overview Start
What is Risk?
What is a Model?
Traditional Risk Analysis
Spreadsheet Risk Analysis
Monte Carlo Simulation
Analysis of Results
Benefits of Risk Analysis
Optimization
Time-series

... applications in risk management


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World's most widely used risk analysis tool.

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- @RISK in Italiano
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KNOW YOUR OPTIONS with The DecisionTools Suite

Make better decisions with
Risk and Decision Analysis Solutions
 Any commercial venture is affected by risk. Calculate risk with Palisade software and training. And start making better decisions today.

News



Pension, Insurance Researchers Develop @RISK Model to Project Key Indices
Three leading actuarial authorities teamed up to develop an @RISK model for pension and insurance risk planning. The resulting model provides a free, and publicly accessible integrated framework for sampling future financial scenarios.



2006 Palisade User Conference: Americas, November 13-14 Miami
Join us in Miami for America's most innovative risk and decision analysis forum. From hands-on software training to real-world case studies and best practices, the 2006 Palisade User Conference has something for everyone.



@RISK Models EU Blood Screening
The only significant disease for which blood transfusions continue to pose a health risk is hepatitis B. The Hospital Clinic in Barcelona, Spain used @RISK to model the best ways to manage this risk.



Palisade Featured in Quality Digest
Highlighting the growing popularity of Palisade tools in the quality control community, trade leader Quality Digest recently published the article "Neural Networks Software Crunches the Big Numbers," highlighting NeuralTools.



Introducing NeuralTools
NeuralTools, Palisade's new Neural Networks add-in for Excel, is now available. Over 2000 people participated in the NeuralTools Beta. Featuring Live Prediction that works with Evolver and Solver, NeuralTools

Risk Seminars

- NEW! London: 15 August**
1 Day @RISK for beginners course
- Frankfurt: 14 - 15 September**
Risk Assessment
- London: 21 - 22 September**
Risk Assessment

Palisade Customers



Merck Uses @RISK for Value-at-Risk

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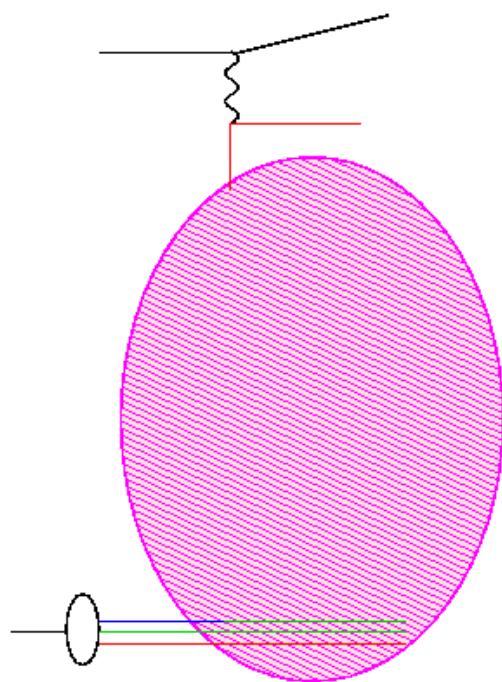
Why do we need all that ?

- because physics and life is more complicated than a simple equation, which can be solved analytically

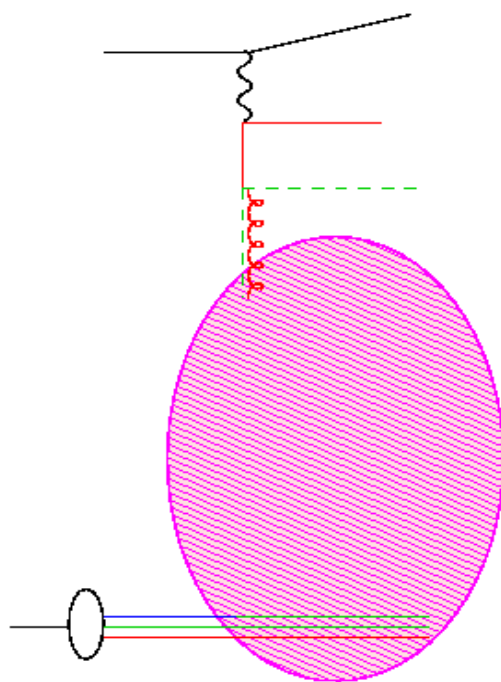
BUT

- Monte Carlo techniques are
 - widely used
 - are of enormous advantages
 - can be used to simulate any complicated process
 - are now **EVEN** used in **particle physics theory** !!!!!

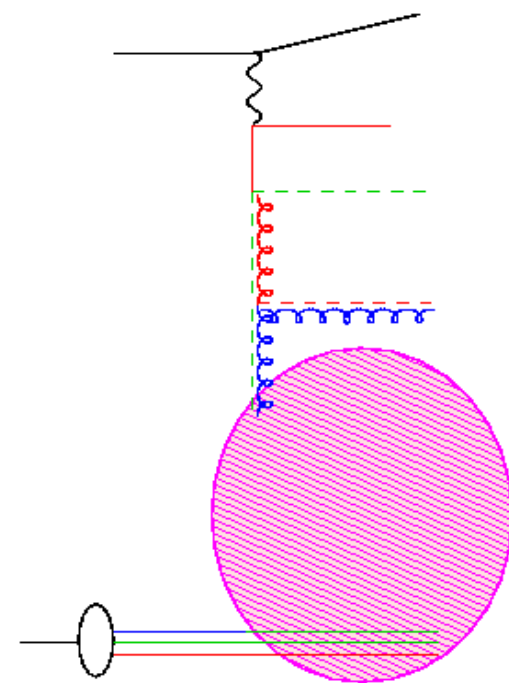
Where is the problem ?



QPM process
total x-section



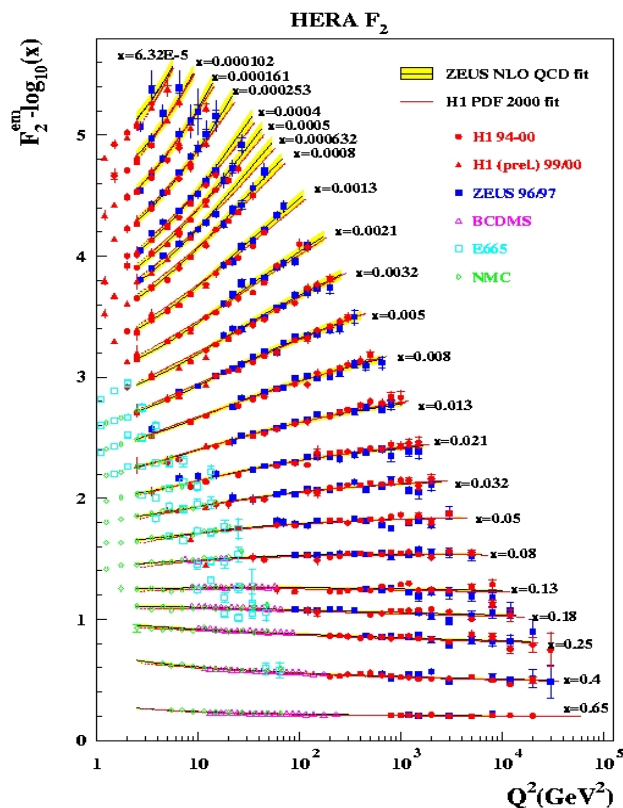
BGF
heavy quarks (charm & bottom)
2-jet
 $\mathcal{O}(\alpha_s)$



process
3-jet
 $\mathcal{O}(\alpha_s^2)$

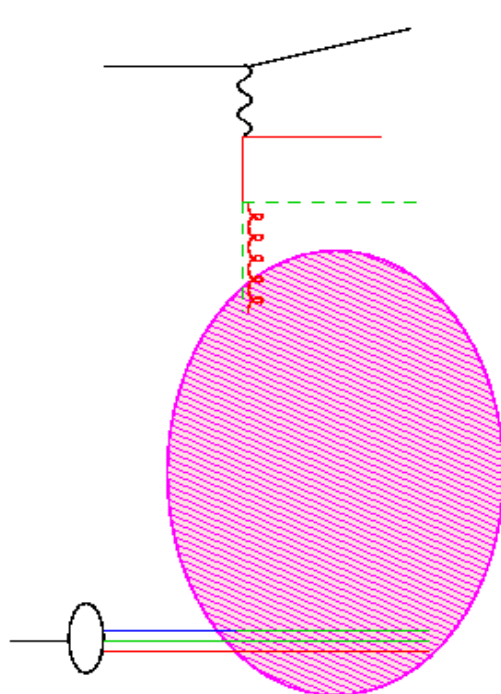
Where is the problem ?

$$F_2 \sim \sigma(\gamma^* p)$$



QPM process

total x-section

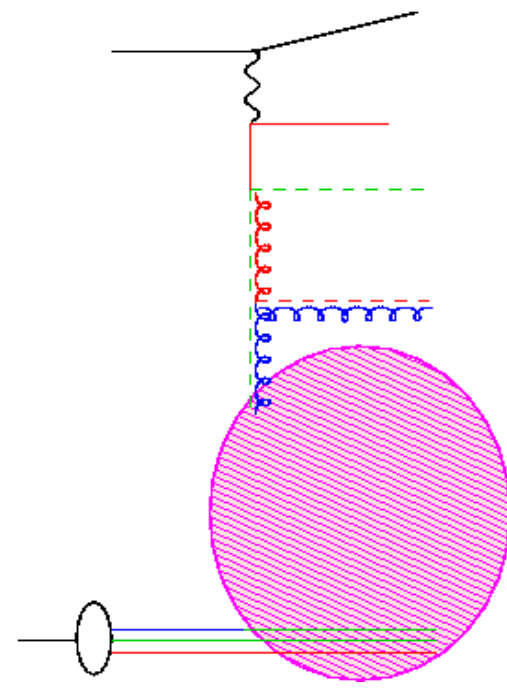


BGF

heavy quarks (charm & bottom)

2-jet

$\mathcal{O}(\alpha_s)$



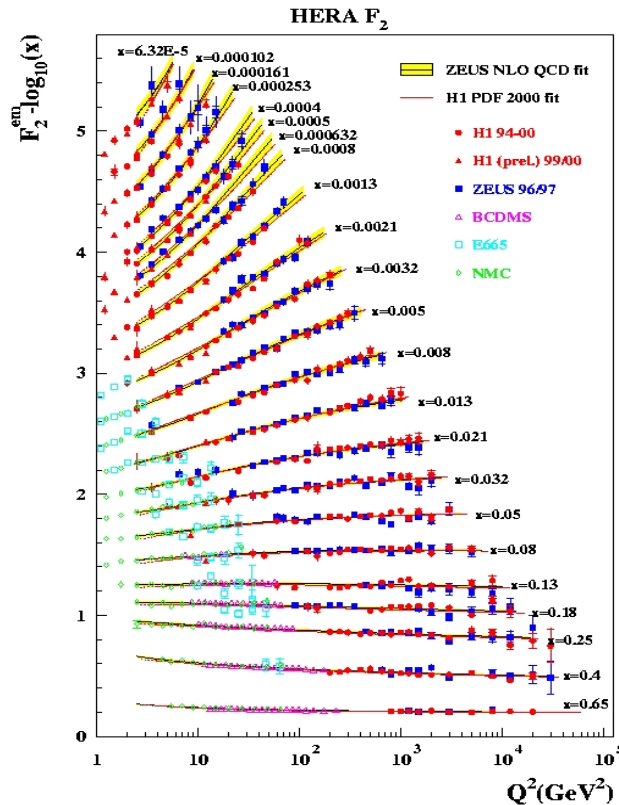
process

3-jet

$\mathcal{O}(\alpha_s^2)$

Where is the problem: hadronic final state

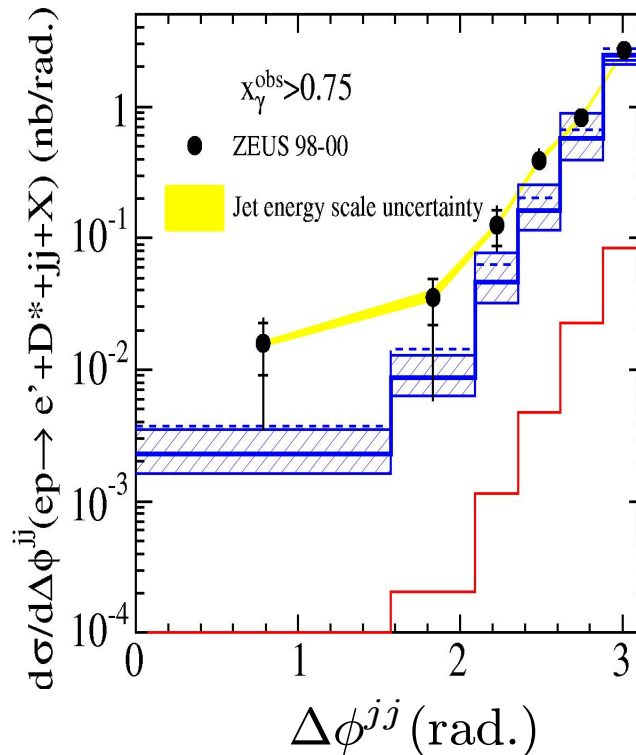
$$F_2 \sim \sigma(\gamma^* p)$$



QPM process

total x-section

ZEUS

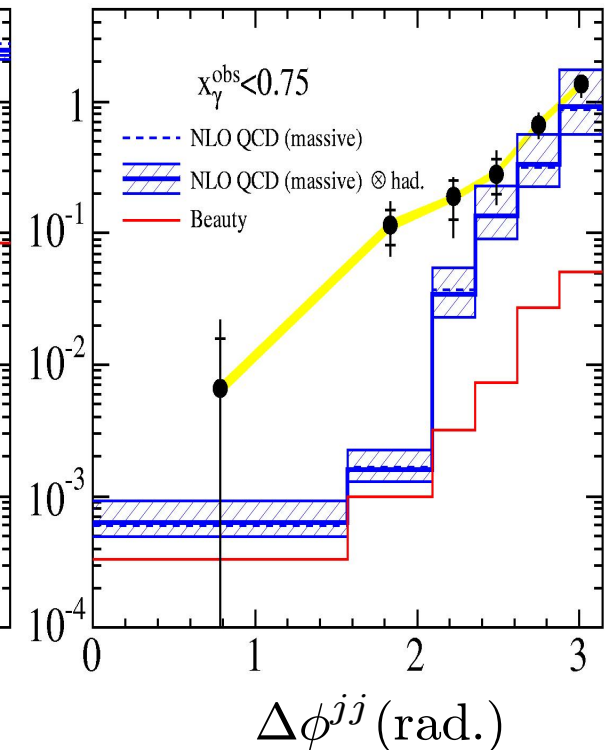


BGF

heavy quarks (charm & bottom)

2-jet

$\mathcal{O}(\alpha_s)$

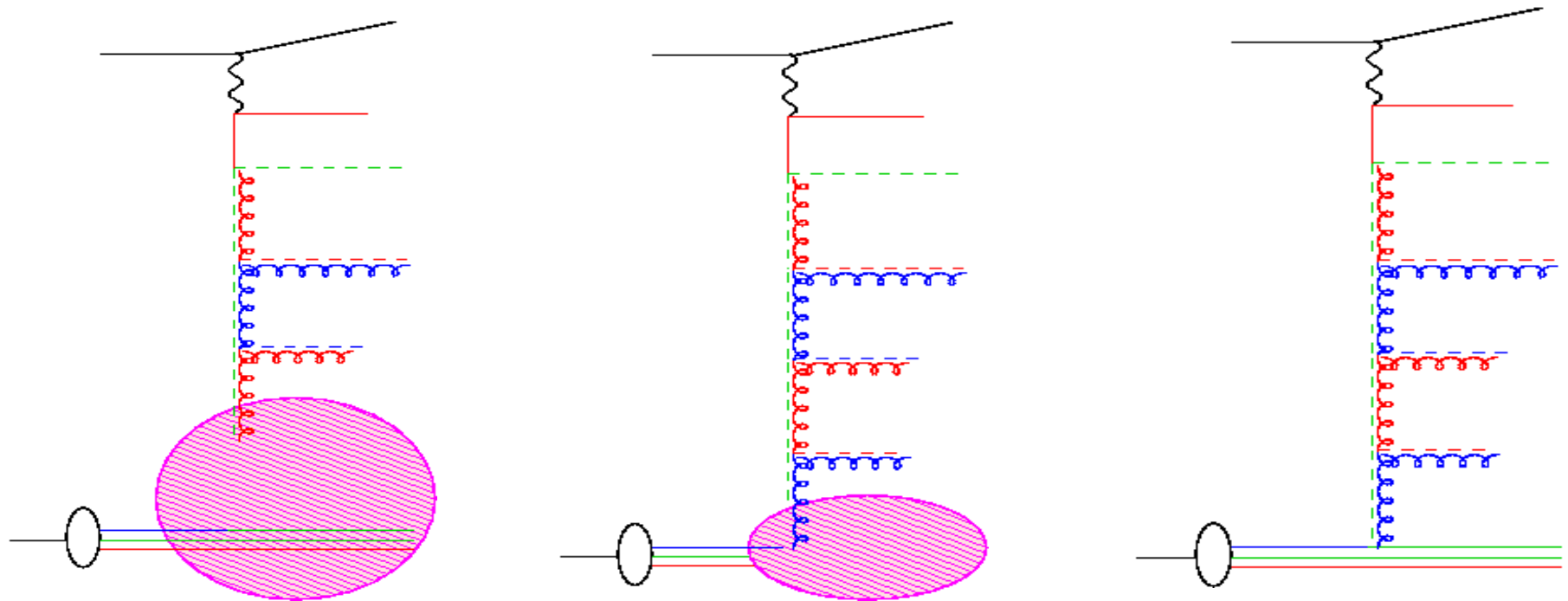


process

3-jet

$\mathcal{O}(\alpha_s^2)$

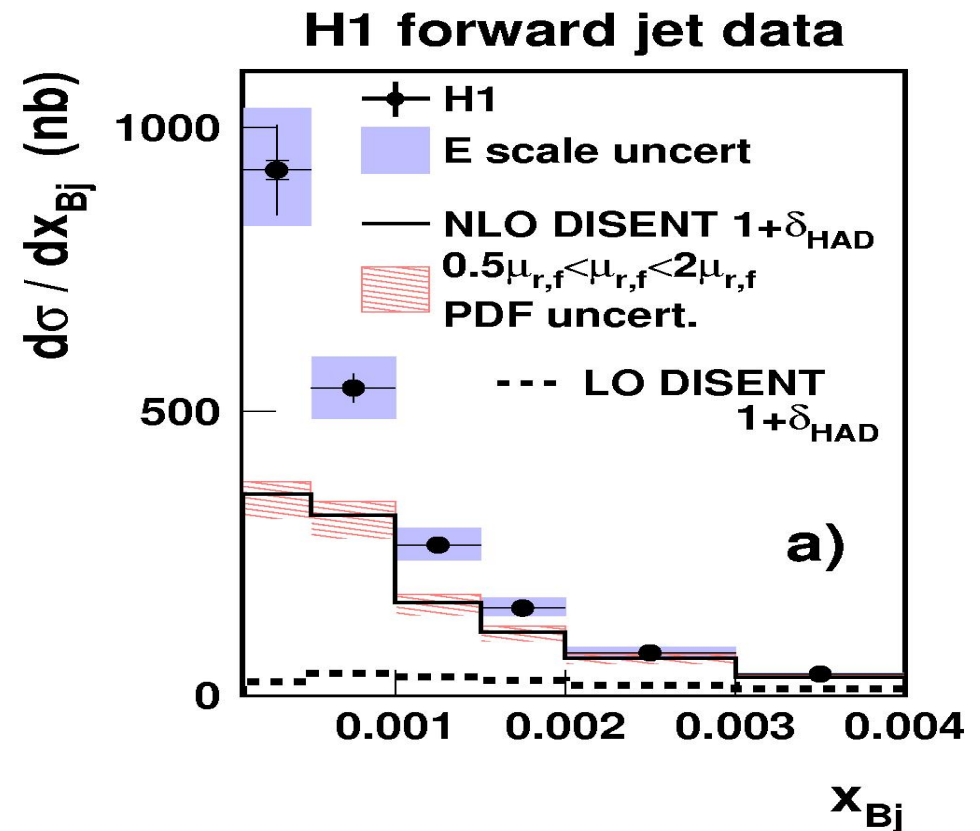
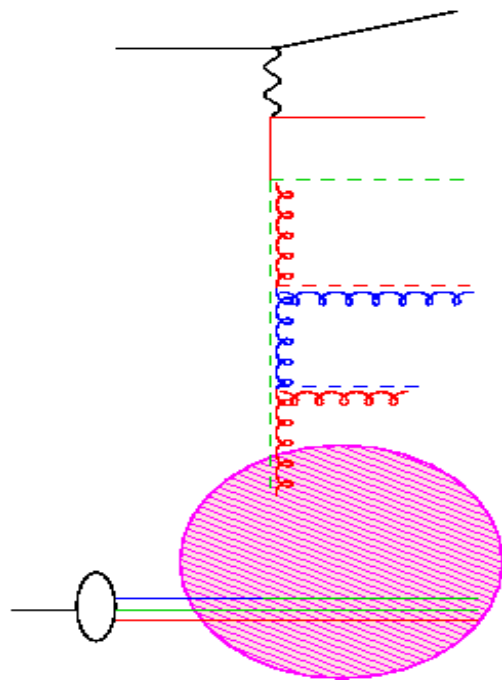
Where is the problem: hadronic final state



processes of $\mathcal{O} > \alpha_s^3$ have not yet been calculated ...

interesting to go closer to outgoing proton remnant
forward jets !!!

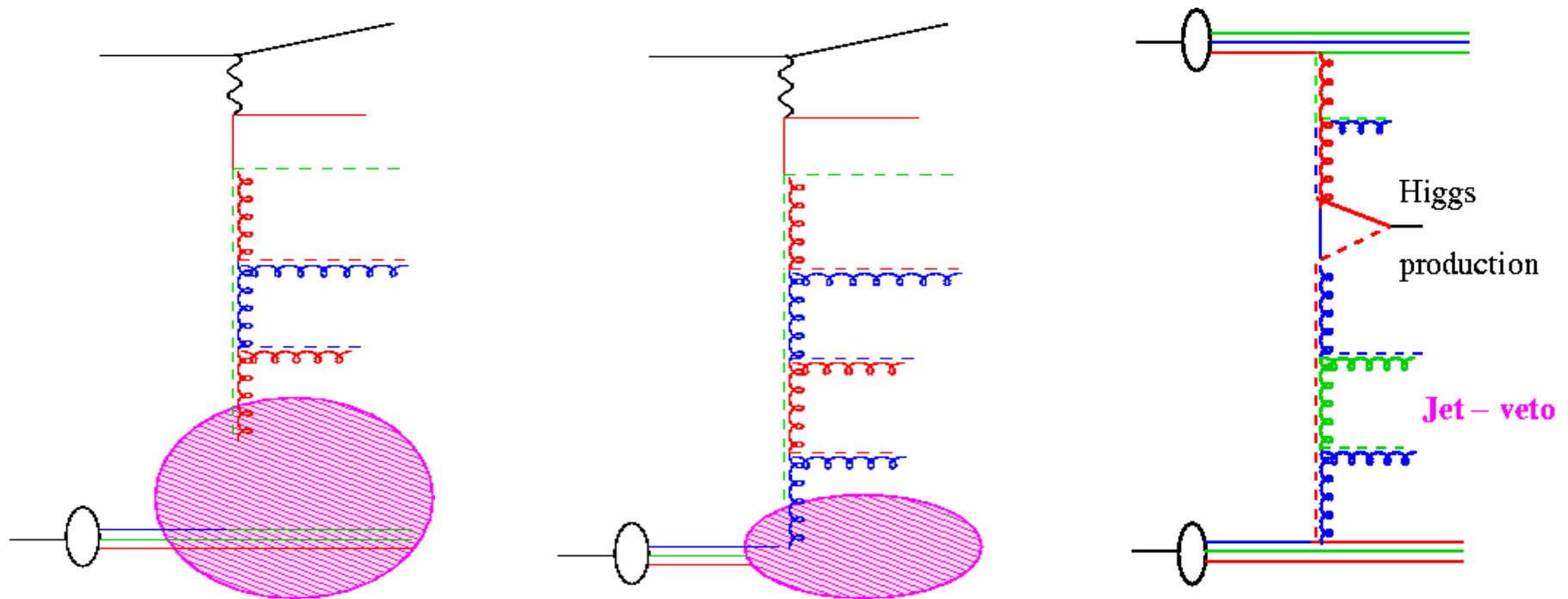
Where is the problem: hadronic final state



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Where is the problem: hadronic final state



- processes of $\mathcal{O} > \alpha_s^3$ have not yet been calculated ...
- interesting to go closer to outgoing proton remnant
- jet veto in Higgs production at LHC

How to simulate
these
processes ?

Monte Carlo method

- Monte Carlo method
 - **refers** to any procedure that makes use of random numbers
 - **uses** probability statistics to solve the problem
- Monte Carlo methods are used in:
 - Simulation of natural phenomena
 - Simulation of experimental apparatus
 - Numerical analysis
- Random number:

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 - No such thing as a single random number**

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 - Simulation of natural phenomena
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 - Numerical analysis
- Random number:
 - one of them is **3**
 - No such thing as a single random number**
 - A sequence of random numbers is a set of numbers that have nothing to do with the other numbers in a sequence

Going out to Monte Carlo



- Obtain true Random Numbers from Casino in Monte Carlo
- Puhhh... Going out every night
- ...



Random Numbers

- In a uniform distribution of random numbers in $[0,1]$ every number has the same chance of showing up
- Note that 0.000000001 is just as likely as 0.5

To obtain random numbers:

- Use some chaotic system like roulette, lotto, 6-49, ...
- Use a process, inherently random, like radioactive decay
- Tables of a few million truly random numbers exist

(.....until a few years ago.....)

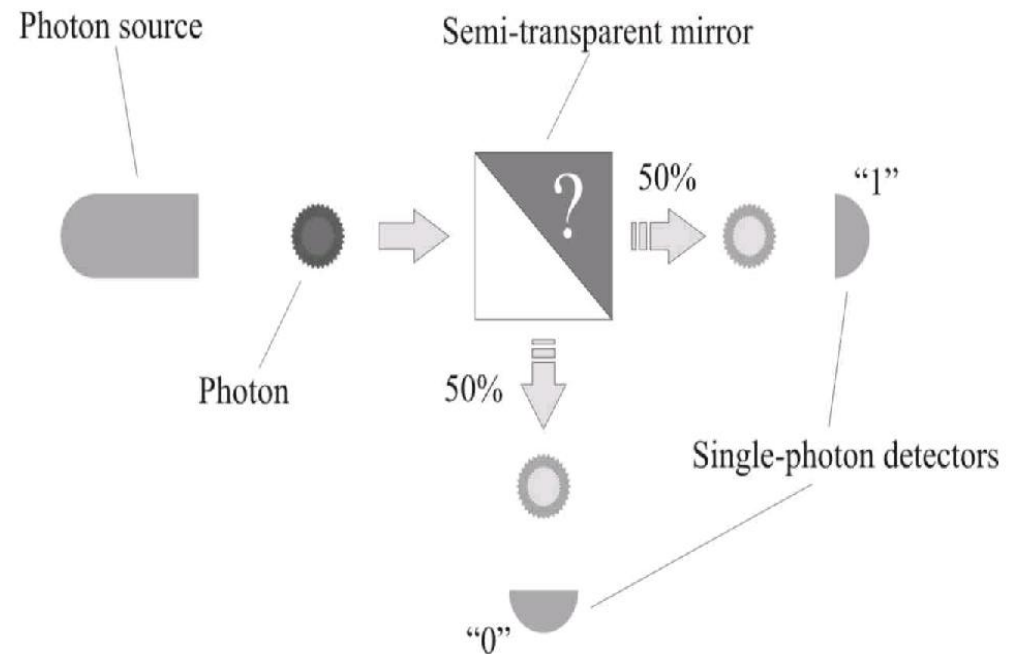
BUT not enough for most applications

→ we have true random number generators ...

True Random Numbers

- Random numbers from **classical physics: coin tossing**
evolution of such a system can be predicted, once the initial condition is known... however it is a chaotic process ... extremely sensitive to initial conditions.
- Truly Random numbers used for
 - Cryptography
 - Confidentiality
 - Authentication
 - Scientific Calculation
 - Lotteries and gambling
 - do not allow to increase chance of winning by having a bias too bad

- Random numbers from **quantum physics: intrinsic random**
photons on a semi-transparent mirror



- Available and tested in MC generator by a summer student
- Generator is however very slow...

Random Numbers

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BUT not enough for most applications

- Hooking up a random machine to a computer is NOT **toooooo good**, as it leads to irreproducible results, making debugging difficult....

→ **Develop Pseudo Random Number generators !!!!**

Pseudo Random Numbers

Pseudo Random Numbers

- are a sequence of numbers generated by a computer algorithm, usually uniform in the range $[0,1]$
- **more precisely:** algo's generate integers between 0 and M , and then $r_n = I_n / M$
- A very early example: **Middle Square (John van Neumann, 1946):**
generate a sequence, start with a number of 10 digits, square it, then take the middle 10 digits from the answer, as the next number etc.:
 $5772156649^2 = 33317792380594909291$
Hmmmm, sequence is not random, since each number is determined from the previous, but it **appears** to be random
- this algorithm has problems
- **BUT** a more complex algo does not necessarily lead to better random sequences

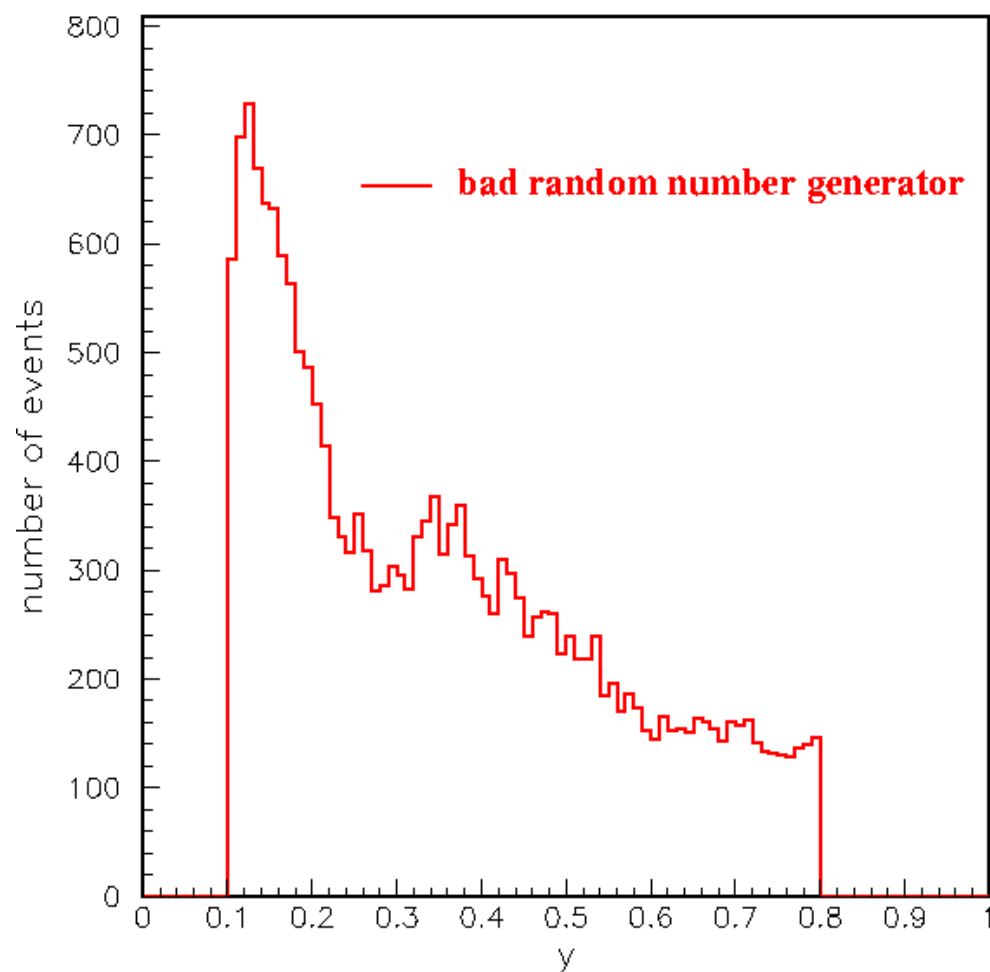
Better us an algo that is well understood

Random Number generators

Compare random number generators with physics process

- γ spectrum of electron
 - observe peaks
 - coming from physics ?

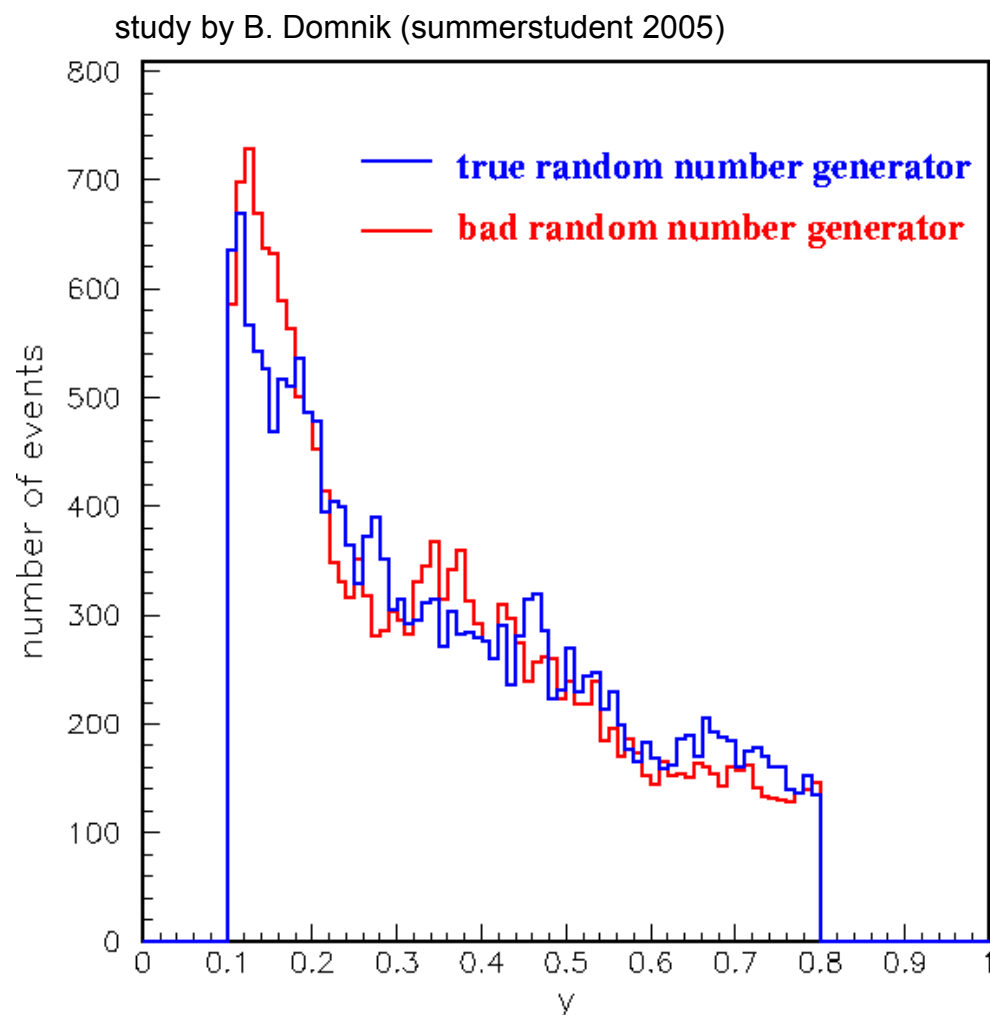
study by B. Domnik (summerstudent 2005)



Random Number generators

Compare random number generators with physics process

- γ spectrum of electron
 - observe peaks
 - coming from physics ?
- BUT coming from bad random number generator



From now on assume:
we have good random number generator

Simulating Radioactive Decay

- radioactive decay is a truly random process
- $dN = -N \alpha dt$ i.e. $N = N_0 e^{-\alpha t}$
- probability of decay is **constant** ... independent of the age of the nuclei:
probability that nucleus undergoes radioactive decay in time Δt is p :
 $p = \alpha \Delta t$ (for $\alpha \Delta t \ll 1$)
- Problem:**
consider a system initially having N_0 unstable nuclei.
How does the number of parent nuclei, N , change with time?
- Algorithm:**
LOOP from $t=0$ to t , step Δt
 LOOP over each remaining parent nucleus
 Decide if nucleus decays:
 IF (random # < $\alpha \Delta t$) then
 reduce number of parents by 1
 ENDIF
 END LOOP over nuclei
 Plot or record N vrs t
END LOOP over time
END

The first simulation: radioactive decay

- implement algo into a small program
- show results after 3000 sec for:

$$N_0 = 100, \alpha = 0.01 \text{ s}^{-1}$$

$$\Delta t = 1 \text{ s}$$

$$N_0 = 5000, \alpha = 0.03 \text{ s}^{-1}$$

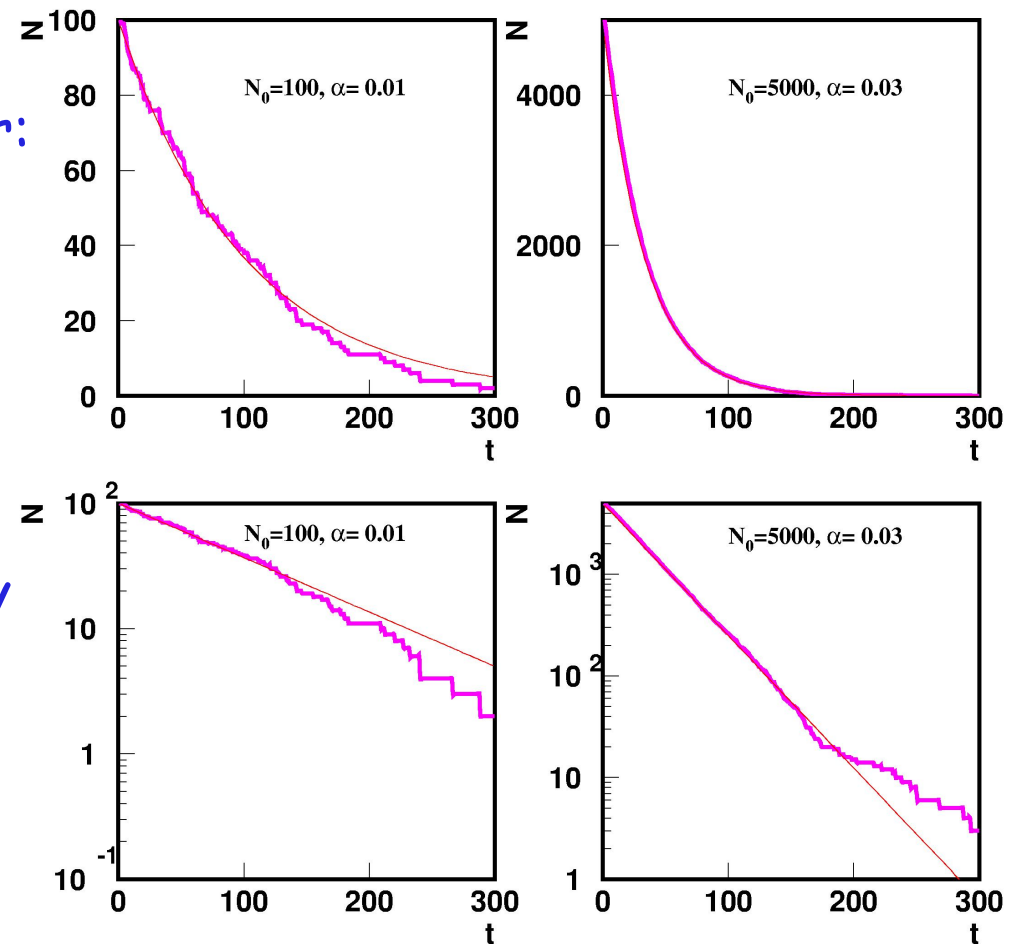
$$\Delta t = 1 \text{ s}$$

- algo:

```
alpha1 = 0.01
N01 = 100
deltat = 1
do I=1,300
  it = it + 1
  do j = 1, N01
    x = RN1
    fr = deltat*alpha1
    if(x.lt.fr) then
c  reduce number of parents N01
      N01 = N01 - 1
    endif
c  fill for each time it number N01
    call hfill(400,real(it+0.3),0,1.) !
  enddo
```

The first simulation: radioactive decay

- implement algo into a small program
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 $N_0 = 100, \alpha = 0.01 \text{ s}^{-1}$
 $\Delta t = 1 \text{ s}$
 $N_0 = 5000, \alpha = 0.03 \text{ s}^{-1}$
 $\Delta t = 1 \text{ s}$
- MC experiment does not exactly reproduce theory
- results from MC experiment show statistical fluctuations ...
-as expected



Monte Carlo technique: basics

- **Law of large numbers**

chose N numbers u_i randomly, with probability density uniform in $[a,b]$,
evaluate $f(u_i)$ for each u_i :

$$\frac{1}{N} \sum_{i=1}^N f(u_i) \rightarrow \frac{1}{b-a} \int_a^b f(u) du$$

for large enough N Monte Carlo estimate of integral converges to correct answer.

- **Convergence**

is given with a certain probability ...

**THIS is a mathematically serious and
precise statement !!!!**

Expectation values and variance

- Expectation value (defined as the average or mean value of function f):

$$E[f] = \int f(u) dG(u) = \left(\frac{1}{b-a} \int_a^b f(u) du \right) = \frac{1}{N} \sum_{i=1}^N f(u_i)$$

for uniformly distributed u in $[a,b]$ *then* $dG(u) = du/(b-a)$

- rules for expectation values:

$$E[cx + y] = cE[x] + E[y]$$

- Variance

$$V[f] = \int (f - E[f])^2 dG = \left(\frac{1}{b-a} \int_a^b (f(u) - E[f])^2 du \right)$$

- rules for variance:

if x, y uncorrelated: $V[cx + y] = c^2 V[x] + V[y]$

if x, y correlated $V[cx + y] = c^2 V[x] + V[y] + 2cE[(y - E[y])(x - E[x])]$

Central Limit Theorem

- Central Limit Theorem

for large N the sum of independent random variables is **always** normally (Gaussian) distributed:

$$f(x) = \frac{1}{s\sqrt{2\pi}} \exp \left[-\frac{(x - a)^2}{2s^2} \right]$$

- example: take sum of uniformly distributed random numbers:

$$R_n = \sum_{i=1}^n R_i$$

$$E[R_1] = \int u du = 1/2,$$

$$V[R_1] = \int (u - 1/2)^2 du = 1/12$$

$$E[R_n] = n/2$$

$$V[R_n] = n/12$$

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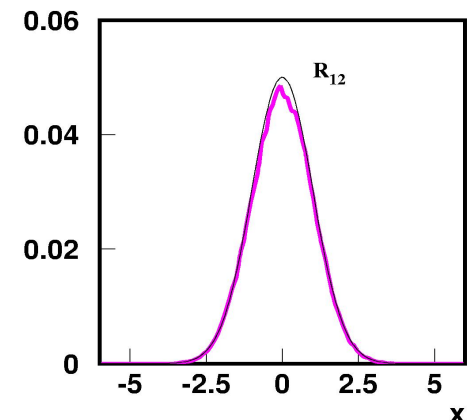
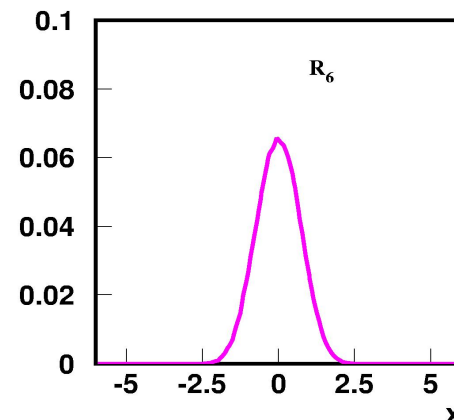
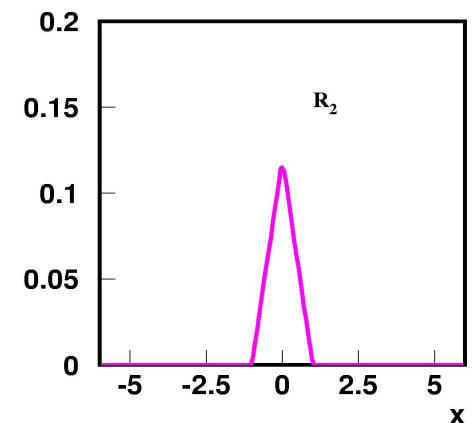
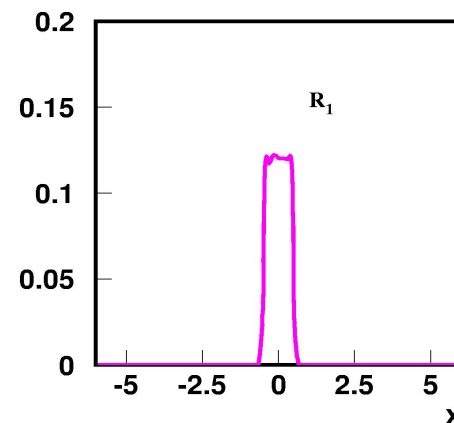
$$V[R_1] = \int (u - 1/2)^2 du = 1/12$$

$$E[R_n] = n/2$$

$$V[R_n] = n/12$$

- for Gaussian with mean=0 and variance=1, take for n=12:

$$N(0, 1) \rightarrow \frac{R_n - n/2}{n/12}$$



Resume: Monte Carlo technique

- Law of large numbers

$$\frac{1}{N} \sum_{i=1}^N f(u_i) \rightarrow \frac{1}{b-a} \int_a^b f(u) du$$

MC estimate converges to true integral

- Central limit theorem

MC estimate is asymptotically normally distributed
it approaches a Gaussian density

$$\sigma = \frac{\sqrt{V[f]}}{\sqrt{N}} \sim \frac{1}{\sqrt{N}}$$

with effective variance $V(f)$

→ to decrease σ , either reduce $V(f)$ or increase N

- advantages for n-dimensional integral ...

i.e. phase space integrals 2 → n processes

is where other approaches tend to fail

Monte Carlo: Buffons Needle - Hit & Miss

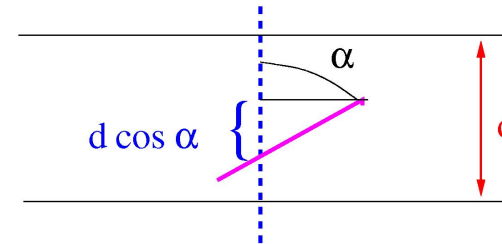
- **Buffons needle** (Buffon 1777)
pattern of parallel lines with distance d ,
randomly throw **needle** with **length d** onto stripes,
count hit, when needle crosses strip count miss, if not
- probability for hit is:

$$\frac{d \cos(\alpha)}{d} = \cos(\alpha)$$

all angles are equally likely:

$$\frac{\int_0^{\pi/2} \cos(\alpha) d\alpha}{\pi/2} = \frac{2}{\pi}$$

<http://www.angelfire.com/wa/hurben/buff.htm>



```
loop over ntrials
  x=RN(1) * d
  alpha = RN(2) * 3.1415 * 2
  y = d * abs(cos(alpha))
  if((x+y).gt. d) nhit = nhit
    + 1
endloop
write ' pi = ', 2*ntrial/nhit
```

trials	π	error
100	2.9850	0.2374
1000	3.2733	0.0749
10000	3.1645	0.0237
100000	3.1483	0.0075
1000000	3.1401	0.0024
10000000	3.1422	0.0008

Buffons Needle: Crude Monte Carlo

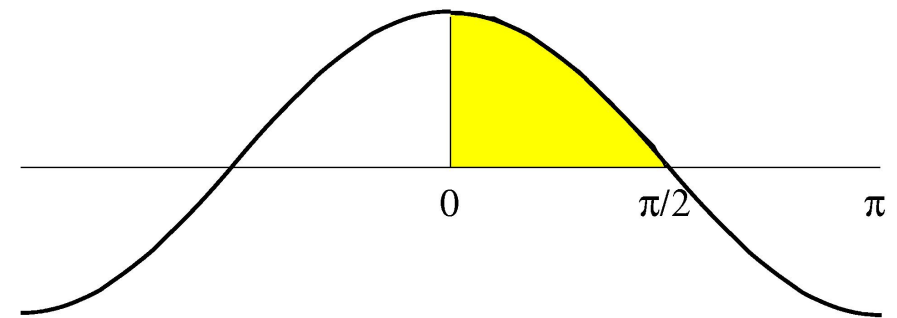
- Buffons needle (Buffon 1777) is essentially integration of

$$\int_0^{\pi/2} \cos(\alpha) d\alpha$$

- apply Law of large numbers:

$$\frac{1}{N} \sum_{i=1}^N f(u_i) \rightarrow \frac{1}{b-a} \int_a^b f(u) du$$

- compare Hit & Miss with Integration



trials	π (hit&miss)	π (integral)
100	3.27869	3.12265
1000	3.36700	3.11833
10000	3.14218	3.15129
100000	3.13087	3.13416
1000000	3.14127	3.14337
10000000	3.14154	3.14168
100000000	3.12174	3.14156

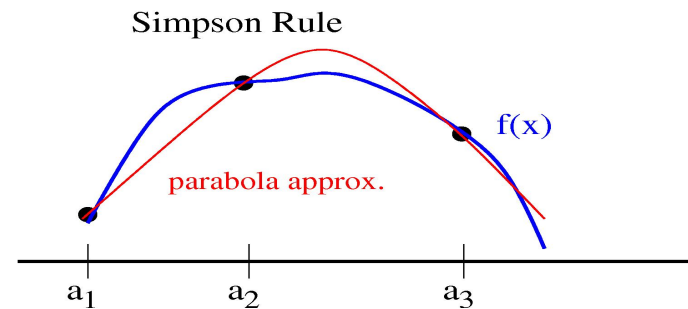
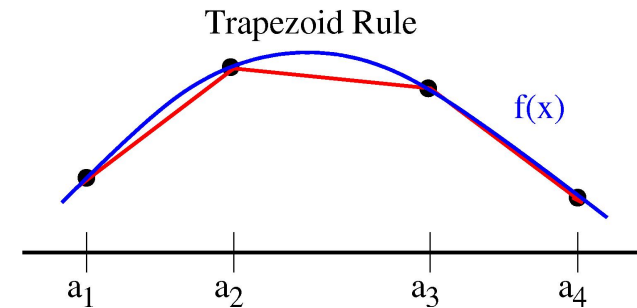
- 1st example of true Monte Carlo experiment
- equivalence of integration and MC event generation

Integration: Monte Carlo versus others

One dimensional quadrature

$$I = \int f(x) dx = \sum_{i=1}^n w_i f(x_i)$$

- Monte Carlo: Hit & Miss
 $w = 1$ and x_i chosen randomly
- Trapezoidal Rule:
 approximate integral in sub-interval by area of trapezoid below (above) curve
- Simpson quadrature
 approximate by parabola
- Gauss quadrature
 approximate by higher order polynomial



method	err (1d)	error
MC	$n^{-1/2}$	$n^{-1/2}$
Trapez	n^{-2}	$n^{-2/d}$
Simpson	n^{-4}	$n^{-4/d}$
Gauss	n^{-2m+1}	$n^{-(2m-1)/d}$

MC method: advantage of hit & miss

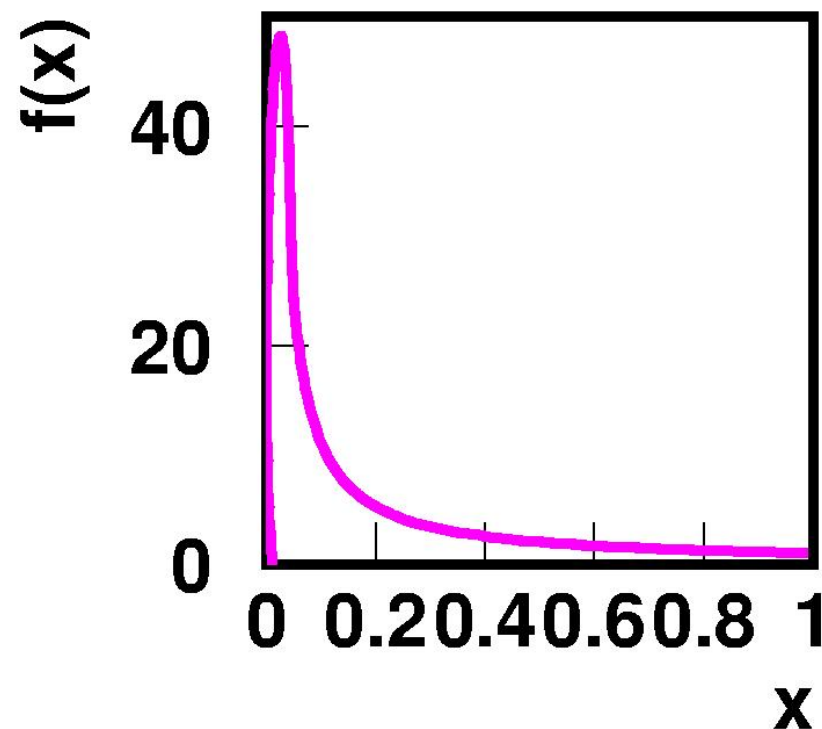
- integration • weighting events
 - large fluctuations from large weights
 - weights also to errors applied
 - difficult to apply further hadronization
- real events all have weight = 1 !!!
- Hit & Miss method:

MC for function $f(x)$:
get random number:
 $R1$ in $(0,1)$ and $R2$ in $(0,1)$
calculate $x = R1$
reject event if: $f_x < f_{\max} R2$

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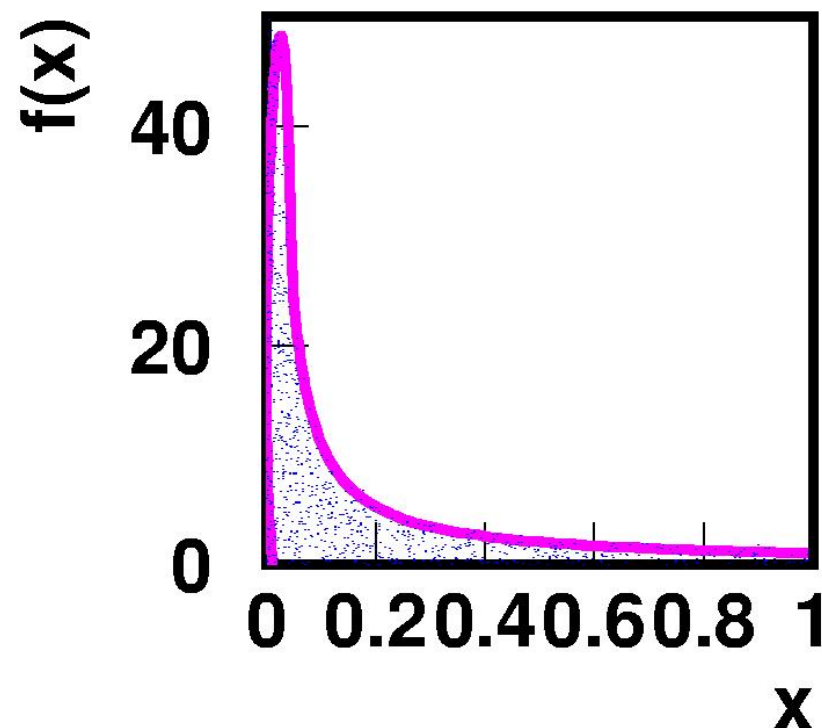
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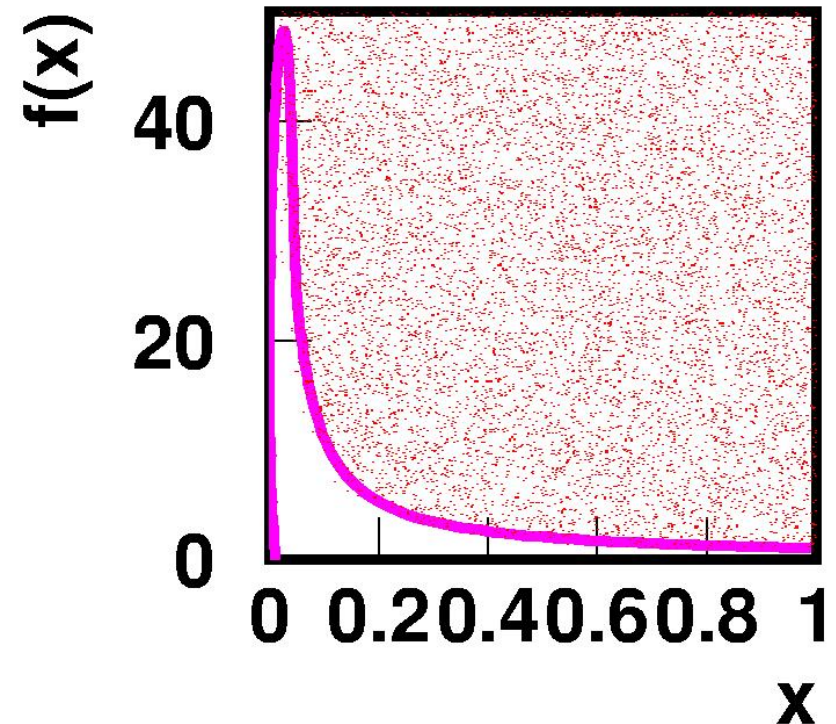
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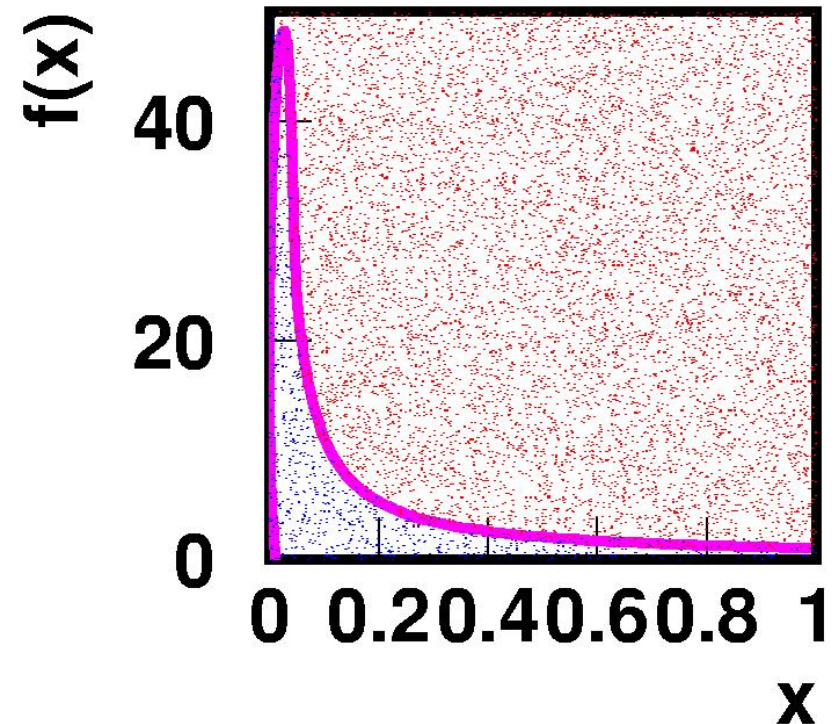
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MC method: advantage of hit & miss

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get random number:
 $R1$ in $(0,1)$ and $R2$ in $(0,1)$
calculate $x = R1$
reject event if: $f_x < f_{\max} R2$



- BUT: Hit & Miss method inefficient for peaked $f(x)$

MC method: do even better ...

- Importance sampling

MC for function $f(x)$

approximate $f(x) \sim g(x)$

with $g(x) > f(x)$ simple and integrable
generate x according to $g(x)$:

$$\int_{x_{min}}^x g(x') dx' = R1 \int_{x_{min}}^{x_{max}} g(x') dx'$$

example: $f(x) = 1/x^{0.7}$

$$g(x) = 1/x$$

$$x = x_{min} \cdot \left(\frac{x_{max}}{x_{min}} \right)^{R1}$$

reject event if: $f(x) < g(x) R2$

MC method: do even better ...

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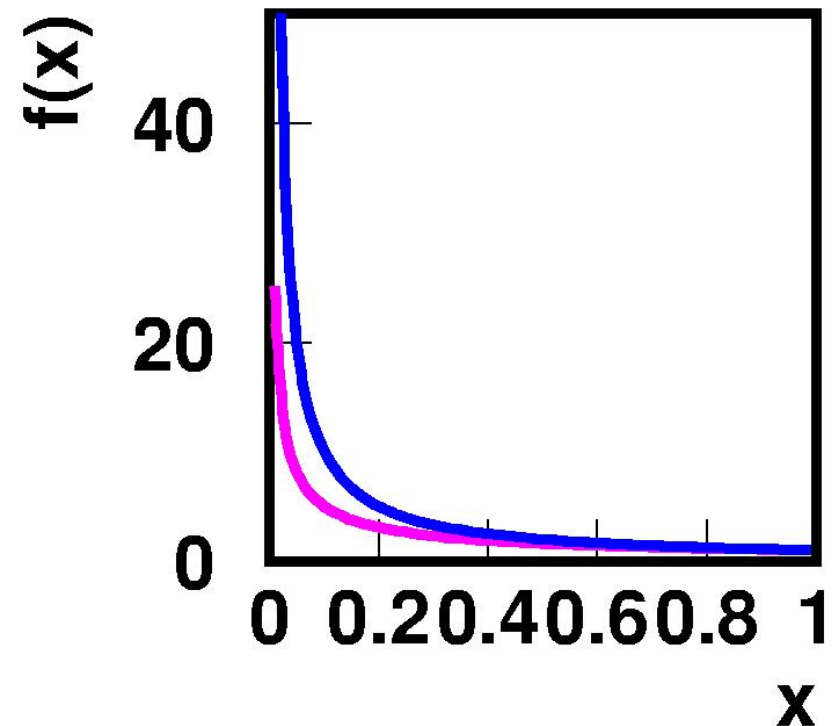
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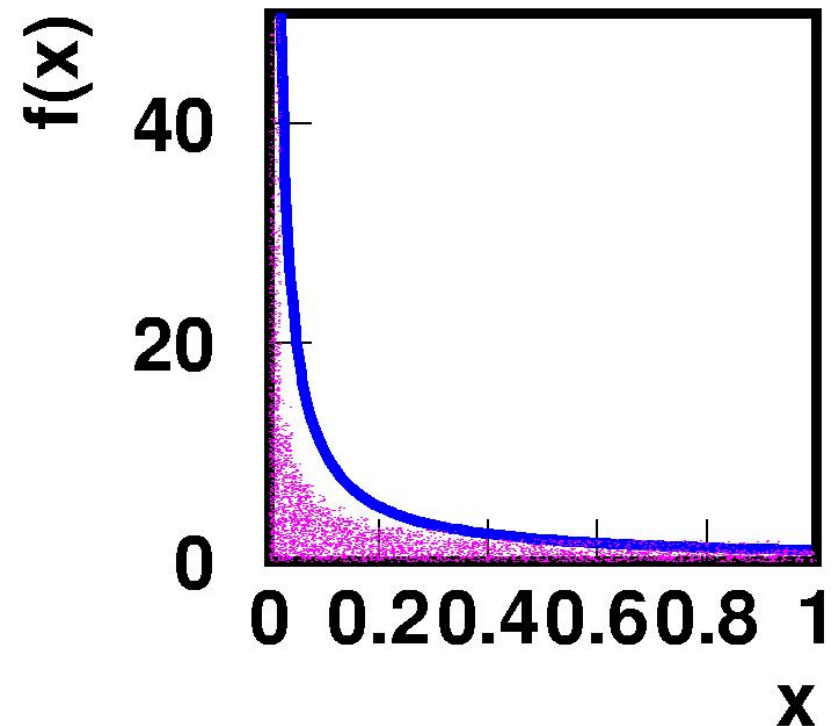
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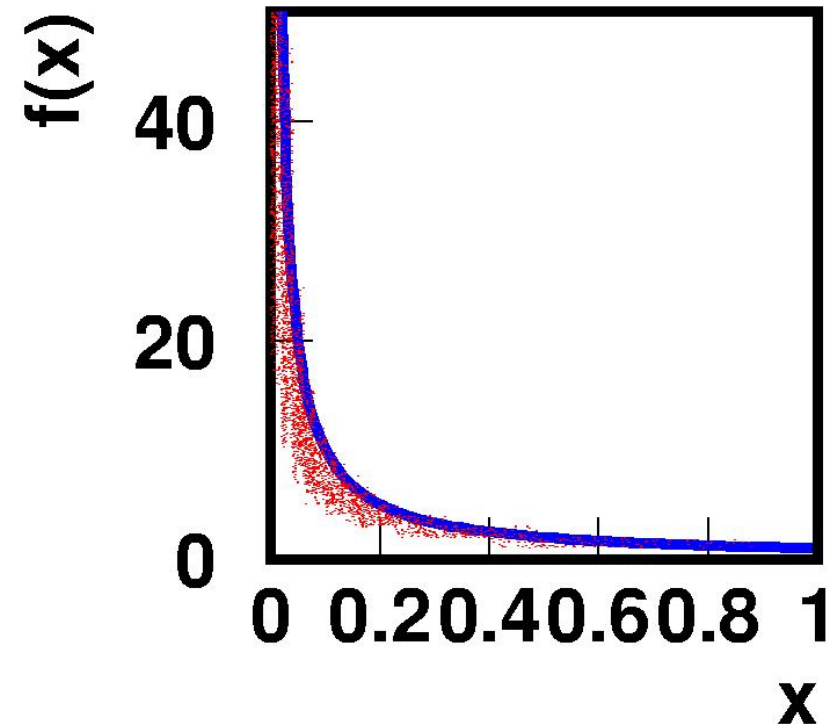
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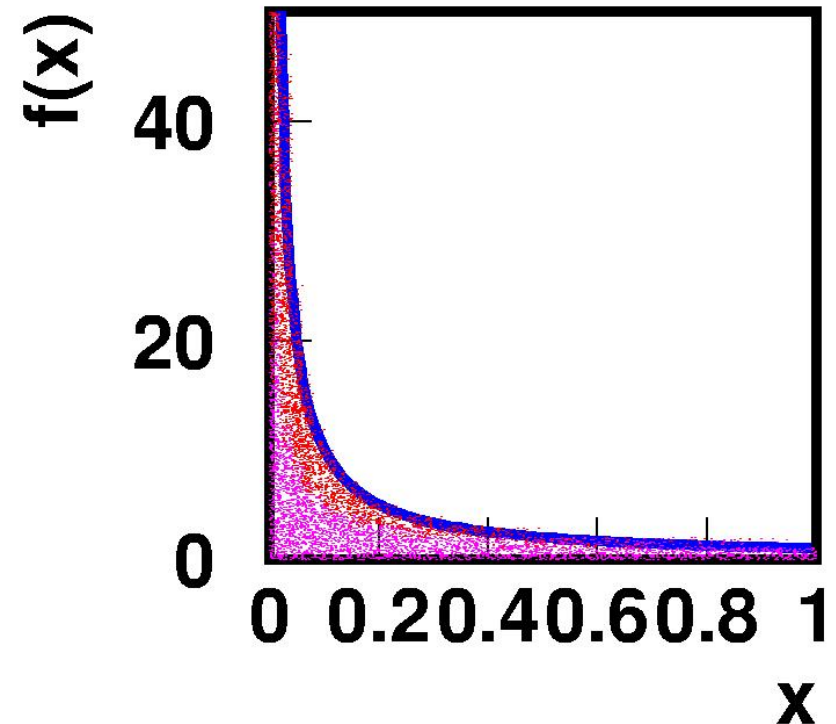
$$\int_{x_{min}}^x g(x') dx' = R1 \int_{x_{min}}^{x_{max}} g(x') dx'$$

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$$g(x) = 1/x$$

$$x = x_{min} \cdot \left(\frac{x_{max}}{x_{min}} \right)^{R1}$$

reject event if: $f(x) < g(x) R2$



- WOW !!! very efficient even for peaked $f(x)$

Importance Sampling

- MC calculations most efficient for small weight fluctuations:

$$f(x)dx \rightarrow f(x) dG(x)/g(x)$$

- chose point according to $g(x)$ instead of uniformly
- f is divided by $g(x) = dG(x)/dx$

- generate x according to:

$$R \int_a^b g(x') dx' = \int_a^x g(x') dx'$$

- relevant variance is now $V(f/g)$:

small if $g(x) \sim f(x)$

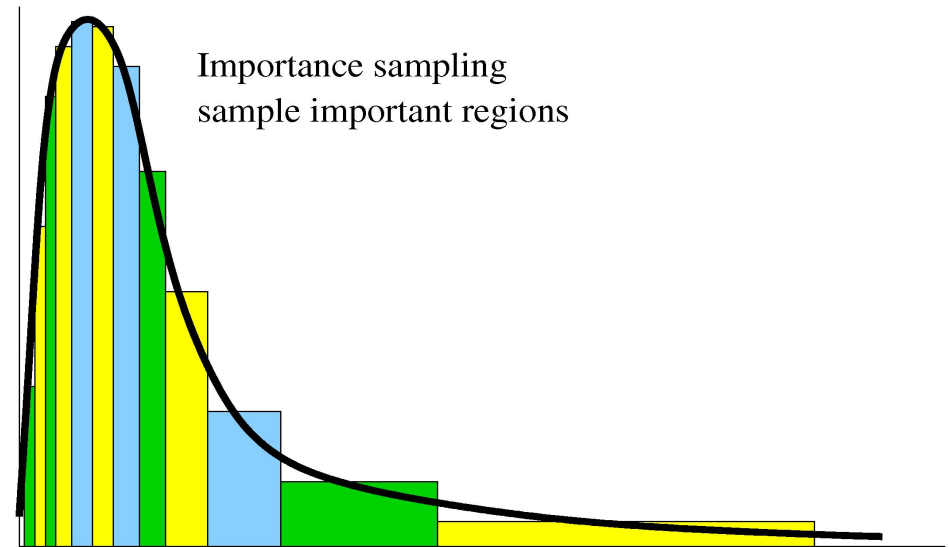
- how-to get $g(x)$

(1) $g(x)$ is probability: $g(x) > 0$ and $\int dG(x) = 1$

(2) integral $\int dG(x)$ is known analytically

(3) $G(x)$ can be inverted (solved for x)

(4) $f(x)/g(x)$ is nearly constant, so that $V(f/g)$ is small compared to $V(f)$



We have the method,....

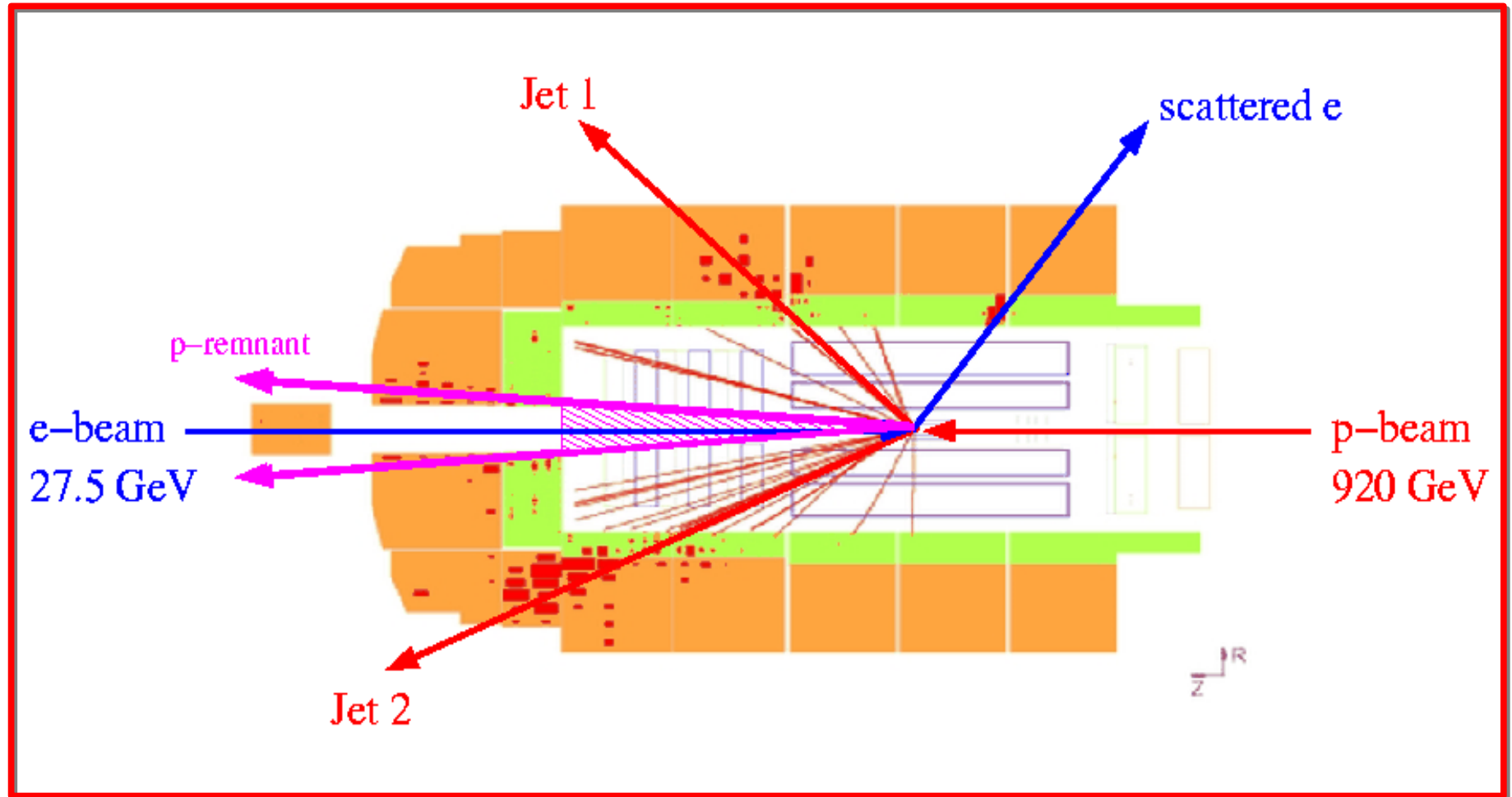
BUT

HOWTO

simulate the physics

?????

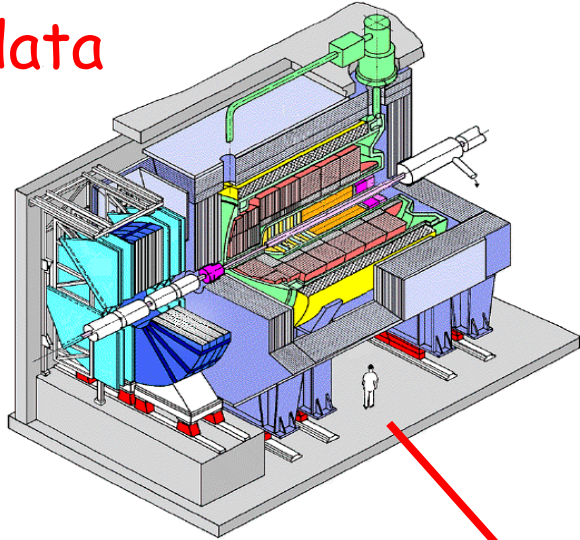
MC event: hadron and detector level



$$\sqrt{s} \sim 318 \text{ GeV}$$
$$x \sim 7 \cdot 10^{-5} \text{ at } Q^2 = 4 \text{ GeV}^2$$

From experiment to measurement

take data



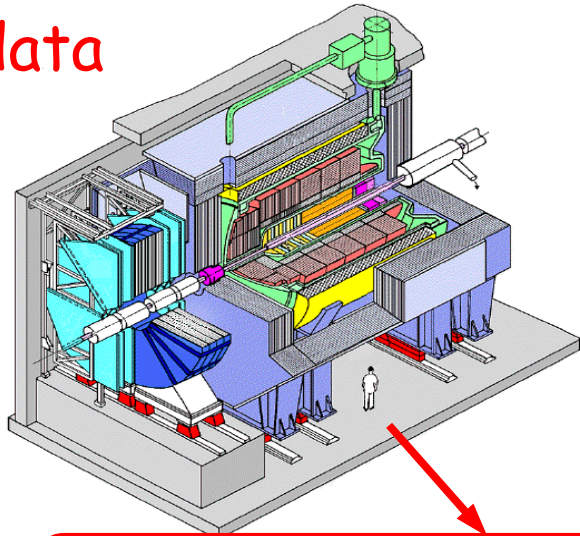
- run MC generator
- detector simulation



Upppps all measurements rely on proper
MC generators and MC simulation !!!!

From experiment to measurement

take data



- run MC generator
- detector simulation

define visible x - section in kinematic variables
calculate factor C_{corr} to correct from detector to hadron level

$$\frac{d\sigma_{had}^{data}}{dx} = \frac{d\sigma_{det}^{data}}{dx} C_{corr} \quad \text{with} \quad C_{corr} = \frac{\frac{d\sigma_{had}^{MC}}{dx}}{\frac{d\sigma_{det}^{MC}}{dx}}$$

visible x -section on hadron level

**Upppps all measurements rely on proper
MC generators and MC simulation !!!!**

Monte Carlo – different applications

- MC simulation of detector response
 - input: hadron level events - output: detector level events
 - Calorimeter ADC hits
 - Tracker hits
 - need knowledge of particle passage through matter, x-section ...
 - need knowledge of actual detector

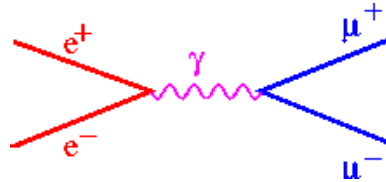
Monte Carlo – different applications

- MC simulation of detector response
 - input: hadron level events - output: detector level events
 - Calorimeter ADC hits
 - Tracker hits
 - need knowledge of particle passage through matter, x-section ...
 - need knowledge of actual detector
- multipurpose MC event generators:
 - x-section on parton level
 - including multi-parton (initial & final state) radiation
 - remnant treatment (proton remnant, electron remnant)
 - hadronization/fragmentation (more than simple fragmentation functions...)
- fixed order parton level theorists like it !!!!!!!!!!!!!!!!!!!!!
 - integration of multidimensional phase space

Not covered here

Constructing a MC for e^+e^- : the simple case

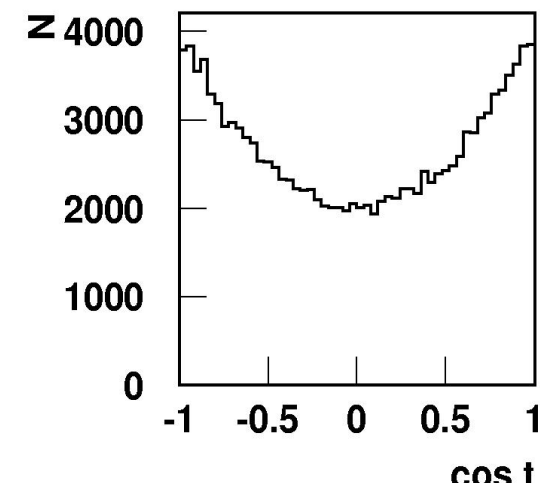
- process: $e^+e^- \rightarrow \mu^+ \mu^-$



- $$\frac{d\sigma}{d\cos\theta d\phi} = \frac{\alpha_{em}^2}{4s} (1 + \cos^2\theta)$$

- goal: generate 4-momenta of μ 's, need cm energy s , $\cos\theta$, ϕ

after 100000 events



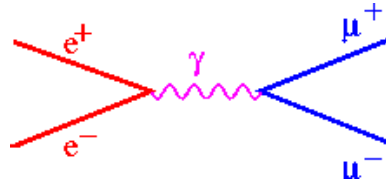
random number $R1(0,1)$ $\phi = 2\pi R1$
random number $R2(0,1)$ $\cos\theta = -1 + 2R2$

for every $R1, R2$ use weight with
repeat many times

$$\frac{d\sigma}{d\cos\theta d\phi}$$

Constructing a MC for e^+e^- : the simple case

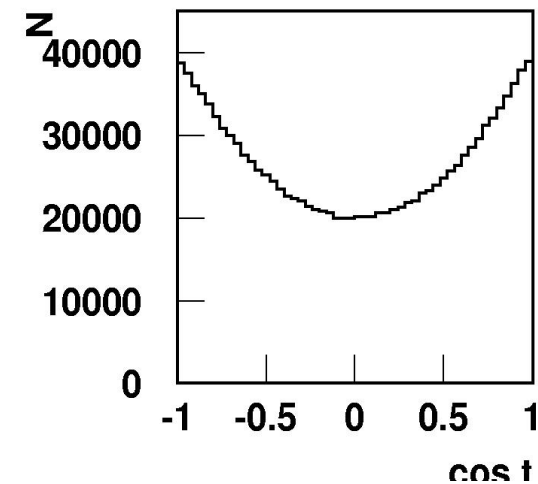
- process: $e^+e^- \rightarrow \mu^+ \mu^-$



- $$\frac{d\sigma}{d\cos\theta d\phi} = \frac{\alpha_{em}^2}{4s} (1 + \cos^2\theta)$$

- goal: generate 4-momenta of μ 's, need cm energy s , $\cos\theta$, ϕ

after 10^6 events



random number $R1(0,1)$ $\phi = 2\pi R1$
random number $R2(0,1)$ $\cos\theta = -1 + 2R2$

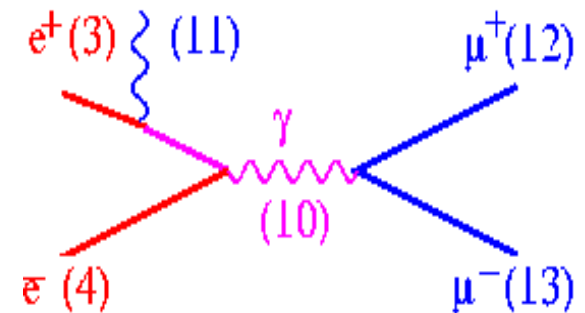
for every $R1, R2$ use weight with
repeat many times

$$\frac{d\sigma}{d\cos\theta d\phi}$$

Example event: $e^+e^- \rightarrow \mu^+ \mu^-$

- example from PYTHIA: Event listing

I	particle/jet	KS	KF	orig	p_x	p_y	p_z	E	m
1	!e+!	21	-11	0	0.000	0.000	30.000	30.000	0.001
2	!e-!	21	11	0	0.000	0.000	-30.000	30.000	0.001
=====									
3	!e+!	21	-11	1	0.000	0.000	30.000	30.000	0.000
4	!e-!	21	11	2	0.000	0.000	-30.000	30.000	0.000
5	!e+!	21	-11	3	0.143	0.040	26.460	26.460	0.000
6	!e-!	21	11	4	0.000	0.000	-29.998	29.998	0.000
7	!Z0!	21	23	0	0.143	0.040	-3.539	56.458	56.347
8	!mu-!	21	13	7	-9.510	1.741	24.722	26.546	0.106
9	!mu+!	21	-13	7	9.653	-1.700	-28.261	29.913	0.106
=====									
10	(Z0)	11	23	7	0.143	0.040	-3.539	56.458	56.347
11	gamma	1	22	3	-0.143	-0.040	3.539	3.542	0.000
12	mu-	1	13	8	-9.510	1.741	24.722	26.546	0.106
13	mu+	1	-13	9	9.653	-1.700	-28.261	29.913	0.106
=====									
sum:			0.00		0.000	0.000	0.000	60.000	60.000

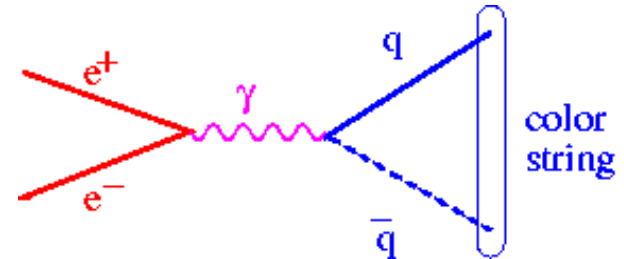


- technicalities/advantages
 - can work in any frame
 - Lorentz-boost 4-vectors back and forth
 - can calculate any kinematic variable
 - history of event process

Constructing a MC for $e^+e^- \rightarrow q\bar{q}$

- process $e^+e^- \rightarrow q\bar{q}$

- $$\frac{d\sigma}{d\cos\theta d\phi} = \frac{\alpha_{em}^2}{4s} (1 + \cos^2\theta)$$



- generate scattering as for $e^+e^- \rightarrow \mu^+\mu^-$
- **BUT** what about fragmentation/hadronization ???
- use concept of **local parton-hadron duality**

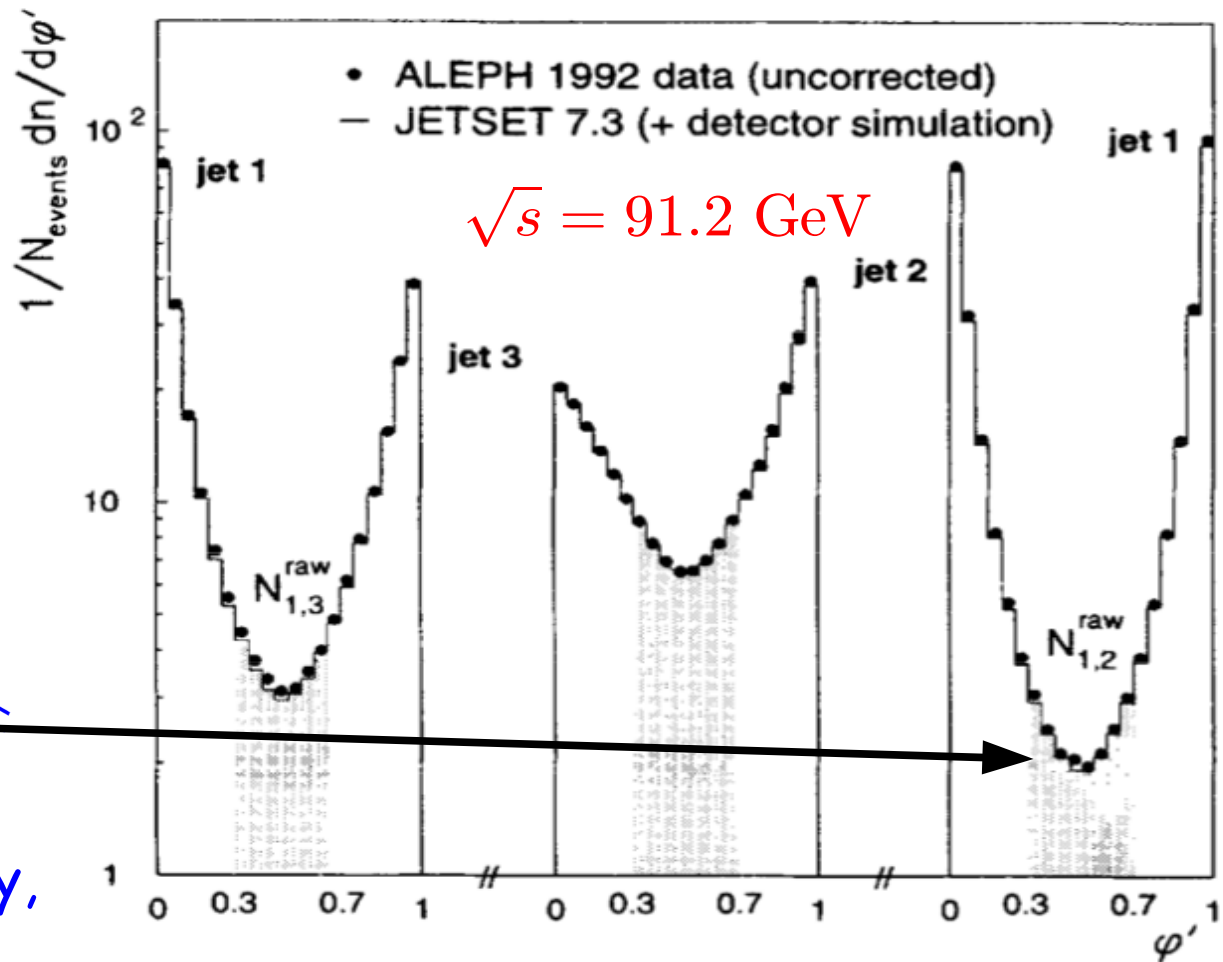
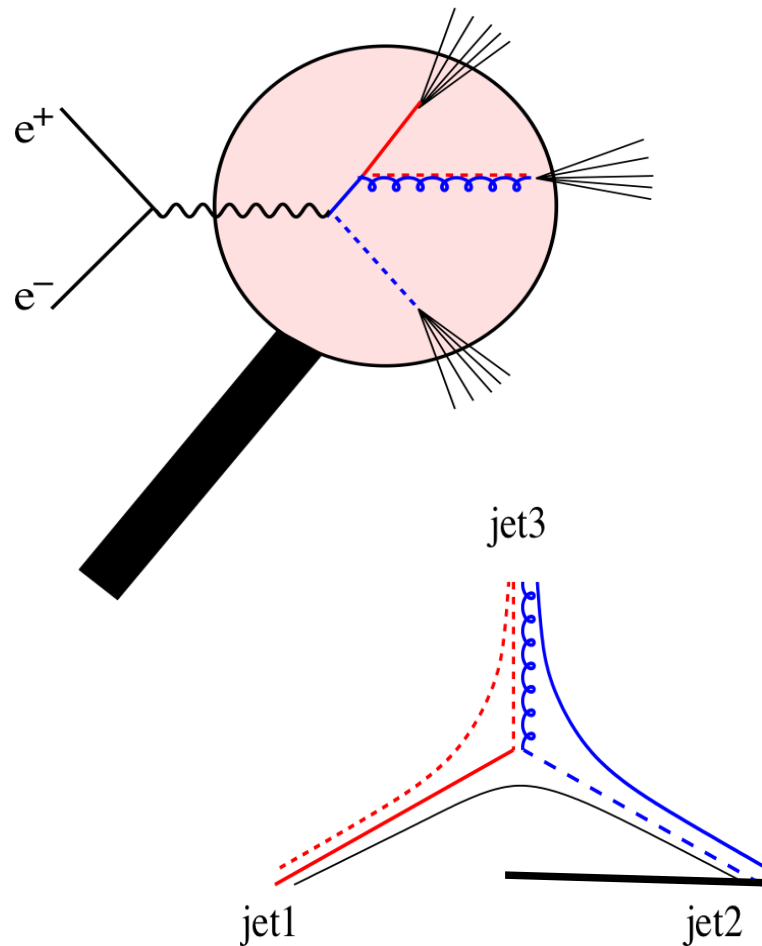
Different approaches to fragmentation/hadronization:

- ➔ independent fragmentation
- ➔ cluster fragmentation (HERWIG model)
- ➔ string fragmentation (Lund Model) NEXT PAGE

Where is QCD ?

How to find the gluon jets, Andersson, Gustafson, Sjostrand, PLB 94,211 (1980)

ALEPH Collaboration / Physics Reports 294 (1998) 1-165

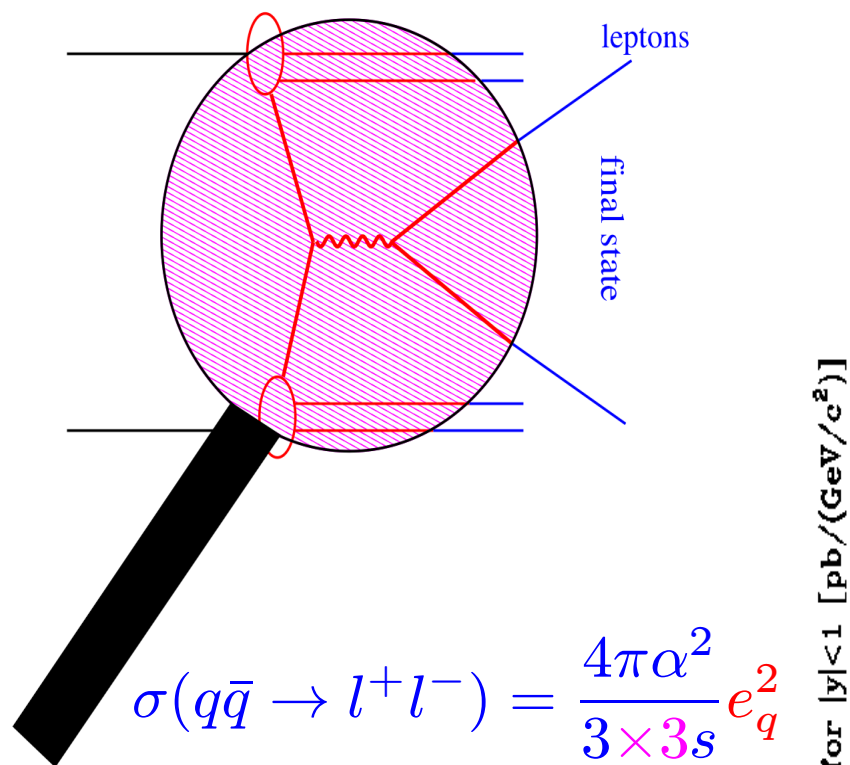


- jets ordered by energy, highest is quark (~94 %), lowest is gluon (~70%)

But e^+e^- is the simple
case...

What about pp ?

Rotating the diagrams

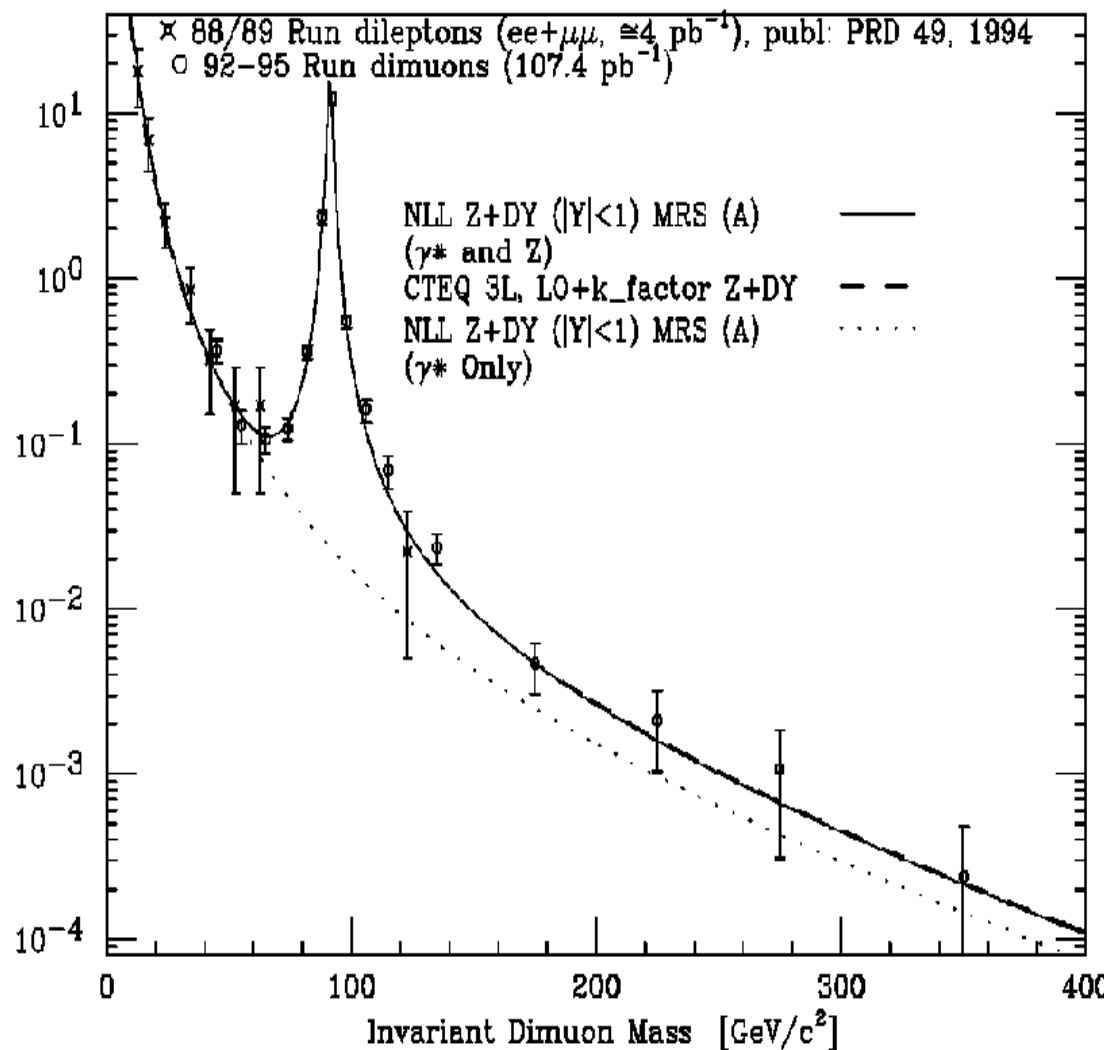


Factorisation:

- hard process: $q\bar{q} \rightarrow l^+l^-$
- parton densities: prob to find parton with x at scale Q^2 in proton: $q(x, Q^2)$, $g(x, Q^2)$...

Measurement of Z0 and Drell-Yan production cross-section using dimuons in anti-p p collisions at $S^{**}(1/2) = 1.8\text{-TeV}$.
CDF Collaboration F. Abe et al. Phys.Rev.D59:052002,1999.

Drell-Yan differential cross-section



W & Z cross sections

- Basic process: Drell - Yan

$$p + p \rightarrow l^+ + l^- + X$$

- Factorise process:

- $q + \bar{q} \rightarrow \gamma^* \rightarrow l^+ + l^-$

- and then

- $q + \bar{q} \rightarrow \gamma^*$

- $q + \bar{q} \rightarrow Z_0$

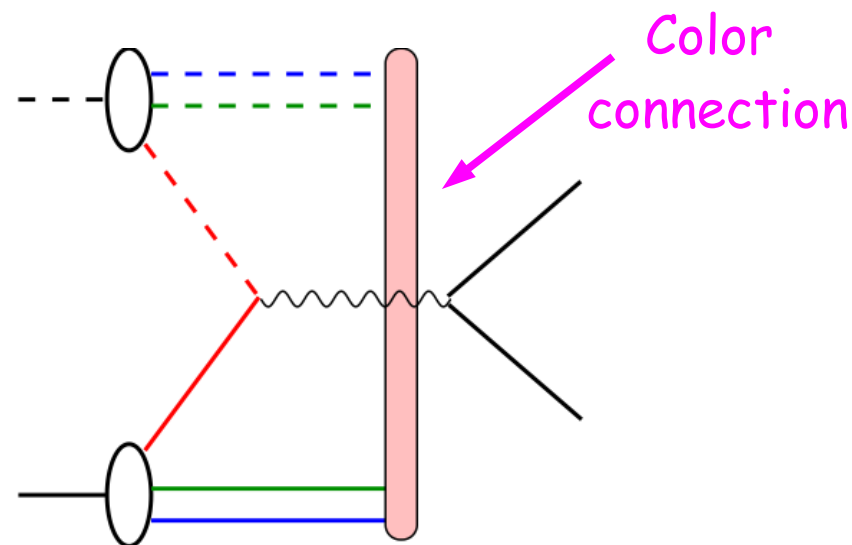
- $q + \bar{q}' \rightarrow W^\pm$

- and then

- $\gamma^* \rightarrow l^+ + l^-$

- $Z_0 \rightarrow l^+ + l^-$

- $W^\pm \rightarrow l + \nu$



- We need

- PDFs

- hard scattering

- Decays

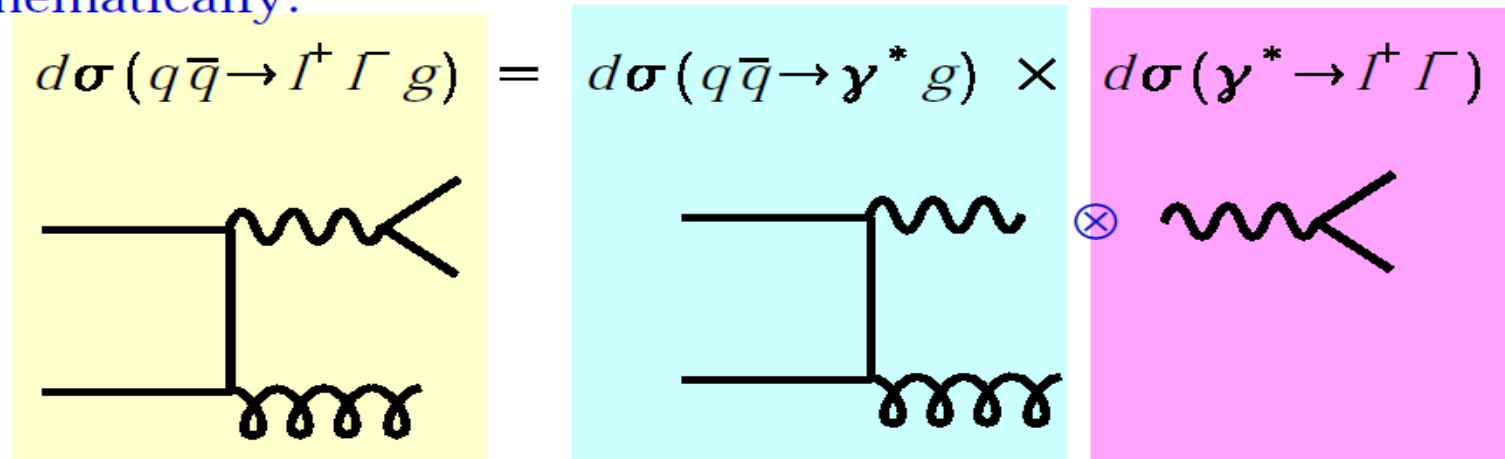
- remnant treatment

Factorizing x-section

Fred Olness, CTEQ
summerschool 2003

Side Note: From $pp \rightarrow \gamma/Z/W$, we can obtain $pp \rightarrow \gamma/Z/W \rightarrow l^+ l^-$

Schematically:

$$d\sigma(q\bar{q} \rightarrow l^+ \Gamma g) = d\sigma(q\bar{q} \rightarrow \gamma^* g) \times d\sigma(\gamma^* \rightarrow l^+ \Gamma)$$


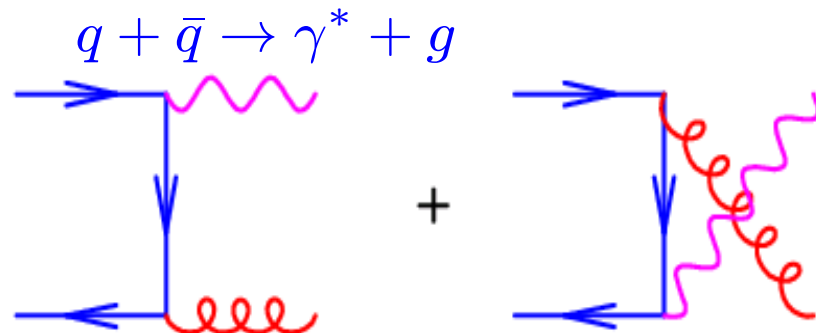
For example:

$$\frac{d\sigma}{dQ^2 d\hat{t}}(q\bar{q} \rightarrow l^+ \Gamma g) = \frac{d\sigma}{d\hat{t}}(q\bar{q} \rightarrow \gamma^* g) \times \frac{\alpha}{3\pi Q^2}$$

→ **it works also for pQCD processes**

QCD corrections for Drell-Yan

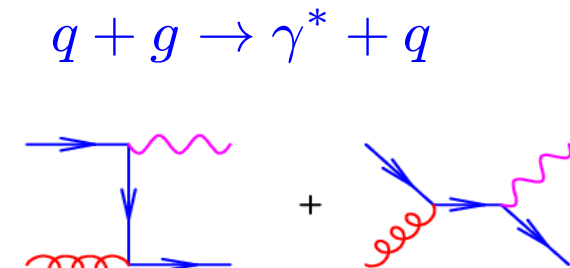
- Calculate annihilation process



$$\begin{aligned}
 |M|^2 &= 16\pi^2 \alpha_s \alpha \frac{8}{9} \left[\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} + \frac{2(M^2 \hat{s})}{\hat{u}\hat{t}} \right] \\
 &= 16\pi^2 \alpha_s \alpha \frac{8}{9} \left[\left(\frac{1+z^2}{1-z} \right) \right. \\
 &\quad \times \left. \left(\frac{-s}{t} + \frac{-s}{u} - 2 \right) \right] \\
 &= 16\pi^2 \alpha_s \alpha \frac{8}{9} [P_{qq}(z) \\
 &\quad \times \left(\frac{-s}{t} + \frac{-s}{u} - 2 \right)]
 \end{aligned}$$

- Calculate QCDC process

K. Ellis, LHC lecture,
<http://theory.fnal.gov/people/ellis/Talks>



Divergent !!!!!

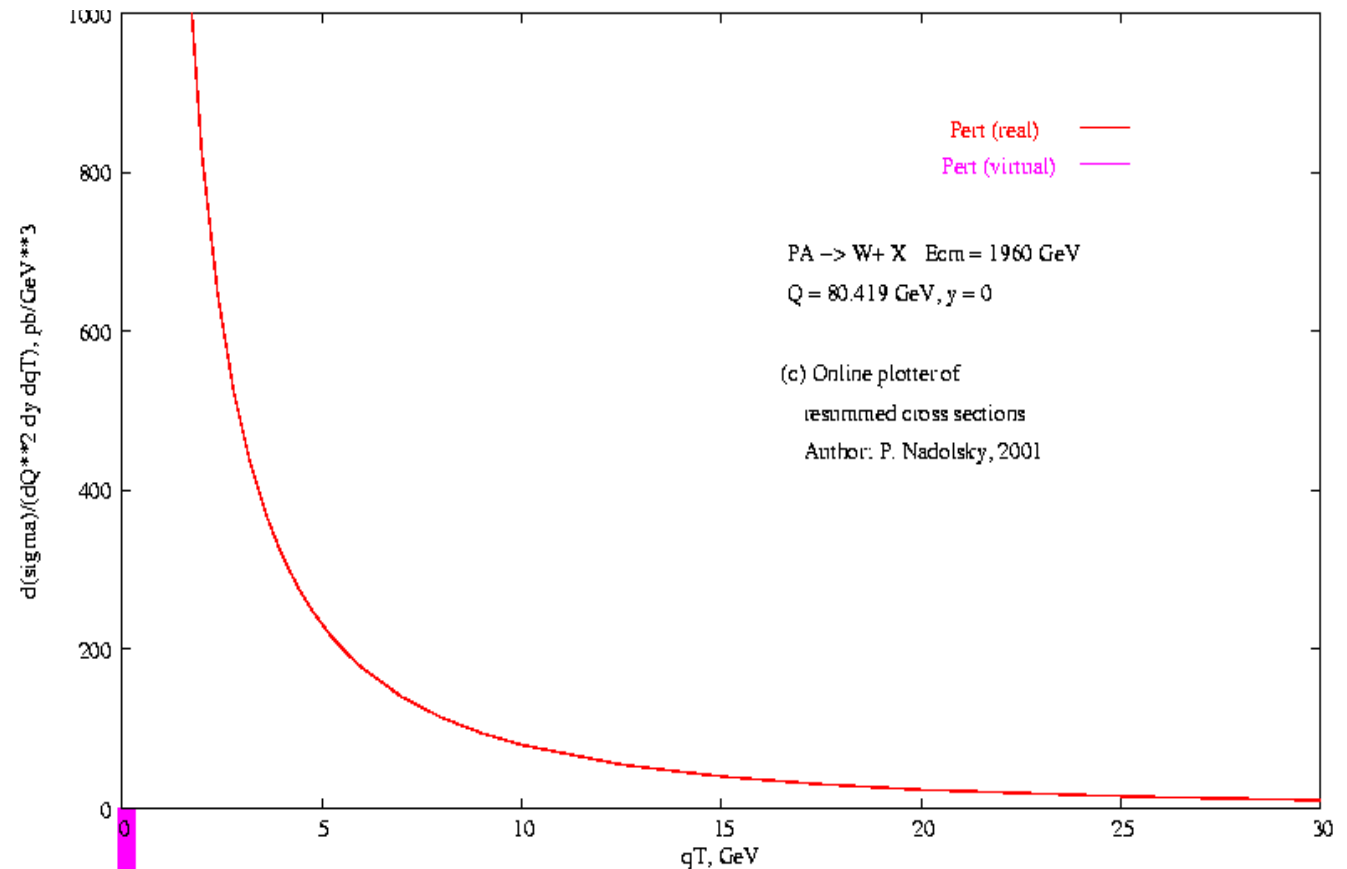
- How to generate this process ?
 - generate $1/t$ and $1/(1-z)$
 - use method for approximation described before....

Transverse Momentum of W/Z

Perturbative calculation

$$\mathcal{O}(\alpha_s), \mathcal{O}(\alpha_s^2)$$

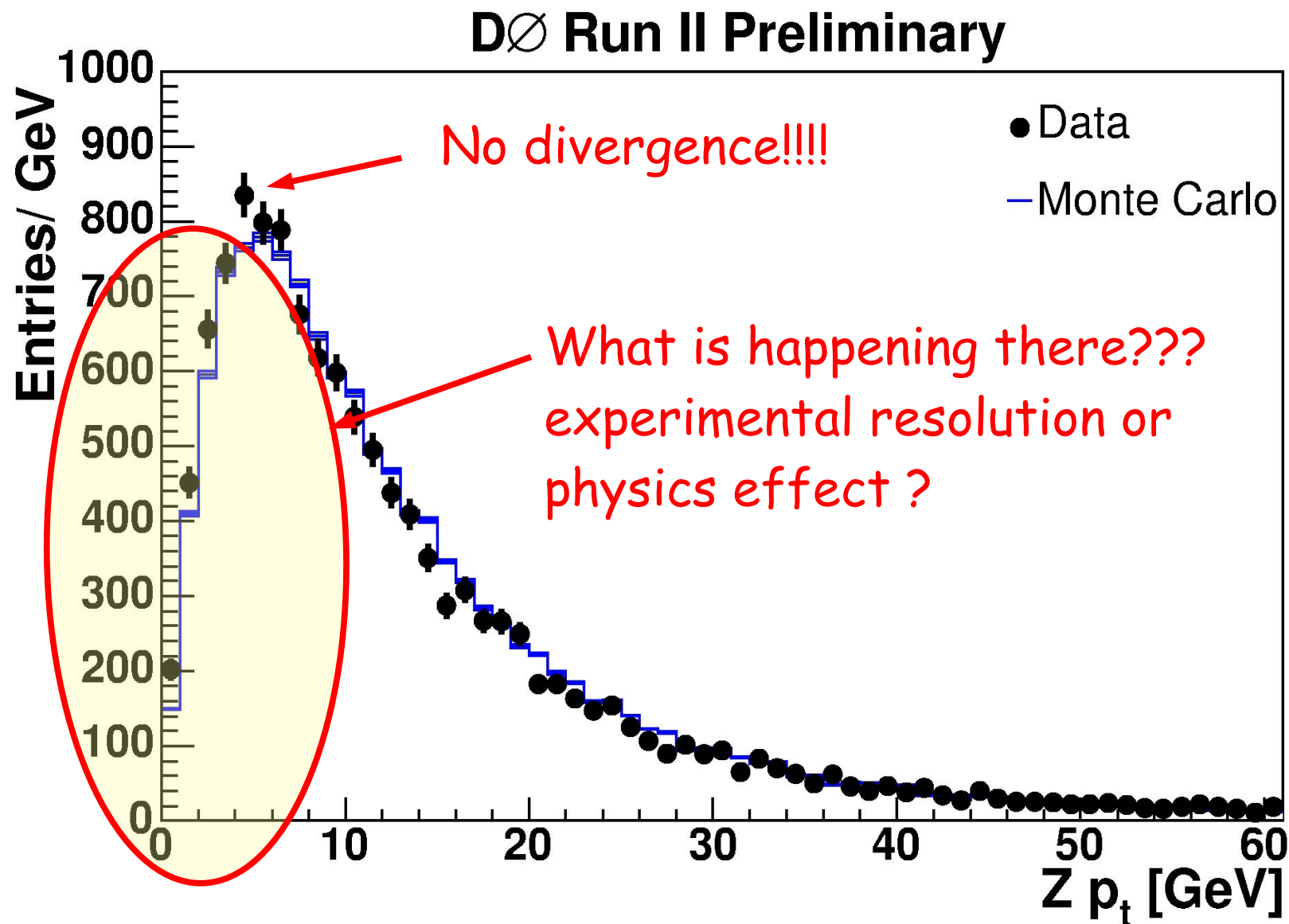
diverges for small p_\perp



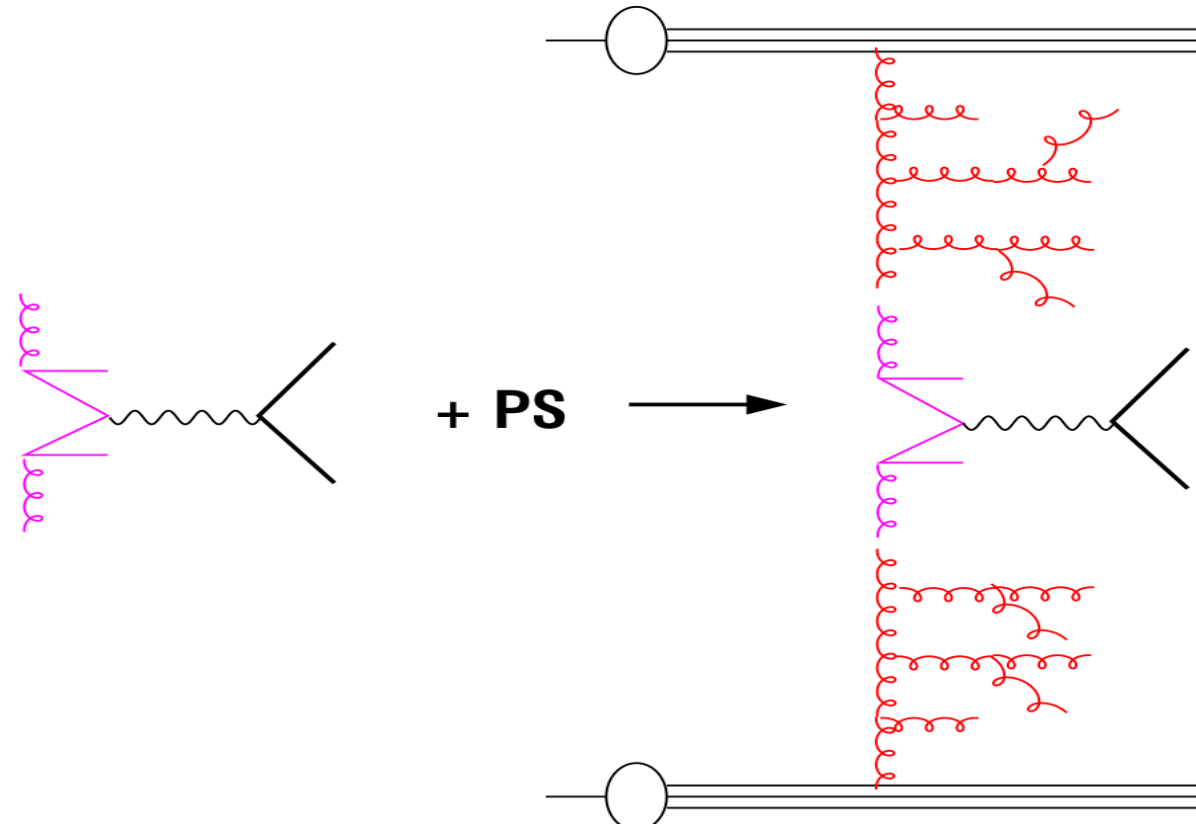
<http://hep.pa.msu.edu/wwwlegacy/>

- Need a p_\perp cut to avoid divergency
- or include virtual corrections and calculate at NLO
-

BUT: the data



All order approach: ME & parton evolution



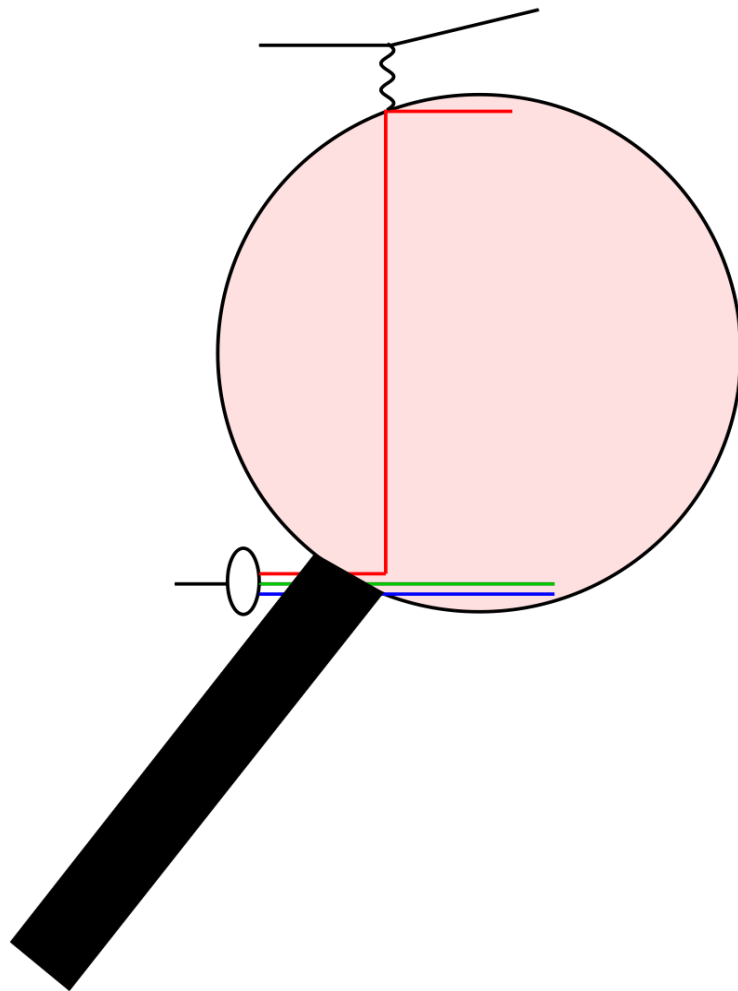
- use LO/NLO matrix element
- matching of cutoff in ME with parton showers
- apply hadronization
- obtain cross sections fully differential in any observable

Wait

Let's go one step back ...

Let's look at ep first !

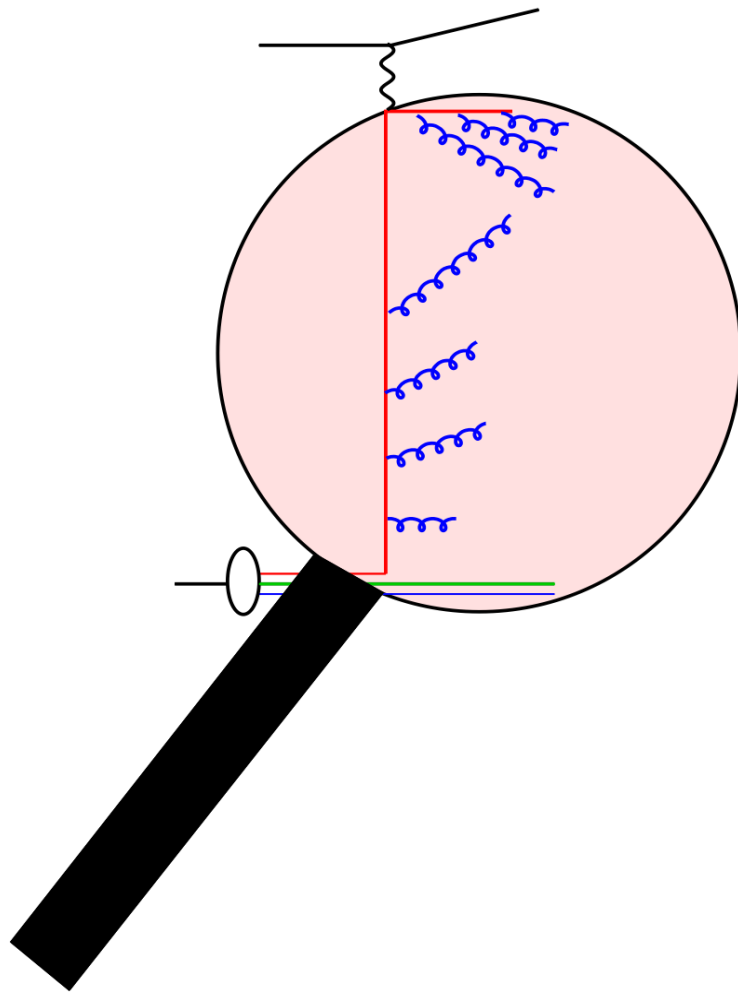
The fun with ep scattering



- Deep Inelastic Scattering is a incoherent sum of $e^+ q \rightarrow e + q$
- only 50 % of p momentum carried by quarks
- need a large gluon component
- partonic part convoluted with parton density function $f_i(x)$

$$\sigma(e^+ p \rightarrow e^+ X) = \sum_i f_i(x, \quad) \sigma(e^+ q_i \rightarrow e^+ q_i)$$

The fun with ep scattering

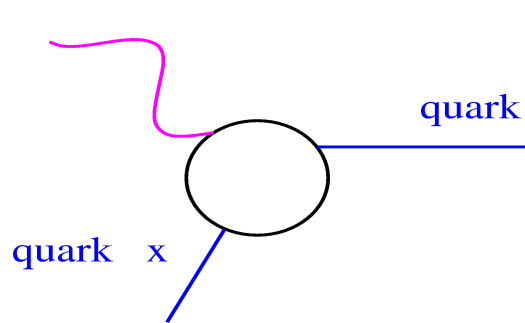


- Deep Inelastic Scattering is a incoherent sum of $e^+ q \rightarrow e + q$
- only 50 % of p momentum carried by quarks
- need a large gluon component
- partonic part convoluted with parton density function $f_i(x)$
- BUT we know, PDF depends on resolution scale Q^2

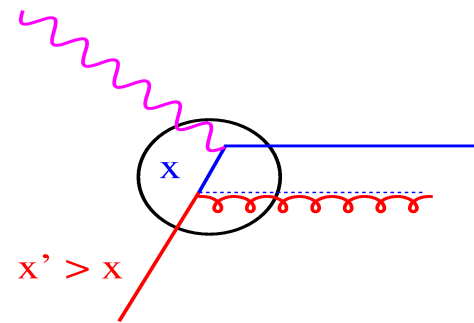
$$\sigma(e^+ p \rightarrow e^+ X) = \sum_i f_i(x, Q^2) \sigma(e^+ q_i \rightarrow e^+ q_i)$$

$F_2(x, Q^2)$: DGLAP evolution equation

- QPM: F_2 is independent of Q^2
- Q^2 dependence of structure function: **D**okshitzer **G**ribov **L**ipatov **A**ltarelli **P**arisi



Q^2 small
small resolution power



Q^2 small
better resolution power

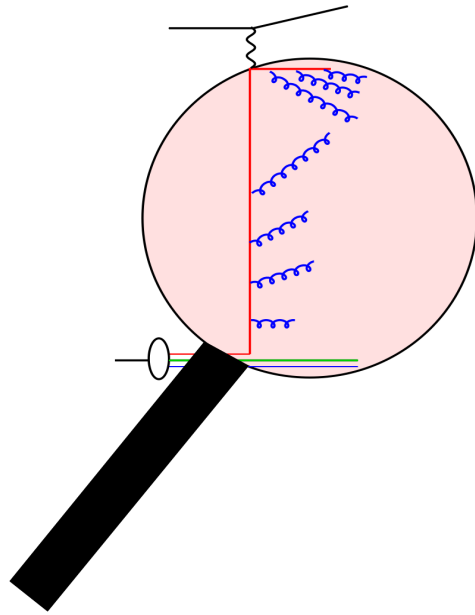
→ Probability to find
parton at small x
increases with Q^2

$$F_2 = \left| \begin{array}{c} \text{Diagram 1: Quark line with a wavy line and a loop} \\ \text{OPM} \end{array} \right|^2 + \left| \begin{array}{c} \text{Diagram 2: Quark line with a wavy line and a loop} \\ \text{QCDC} \end{array} \right|^2 + \left| \begin{array}{c} \text{Diagram 3: Quark line with a wavy line and a loop} \\ \text{BGF} \end{array} \right|^2$$

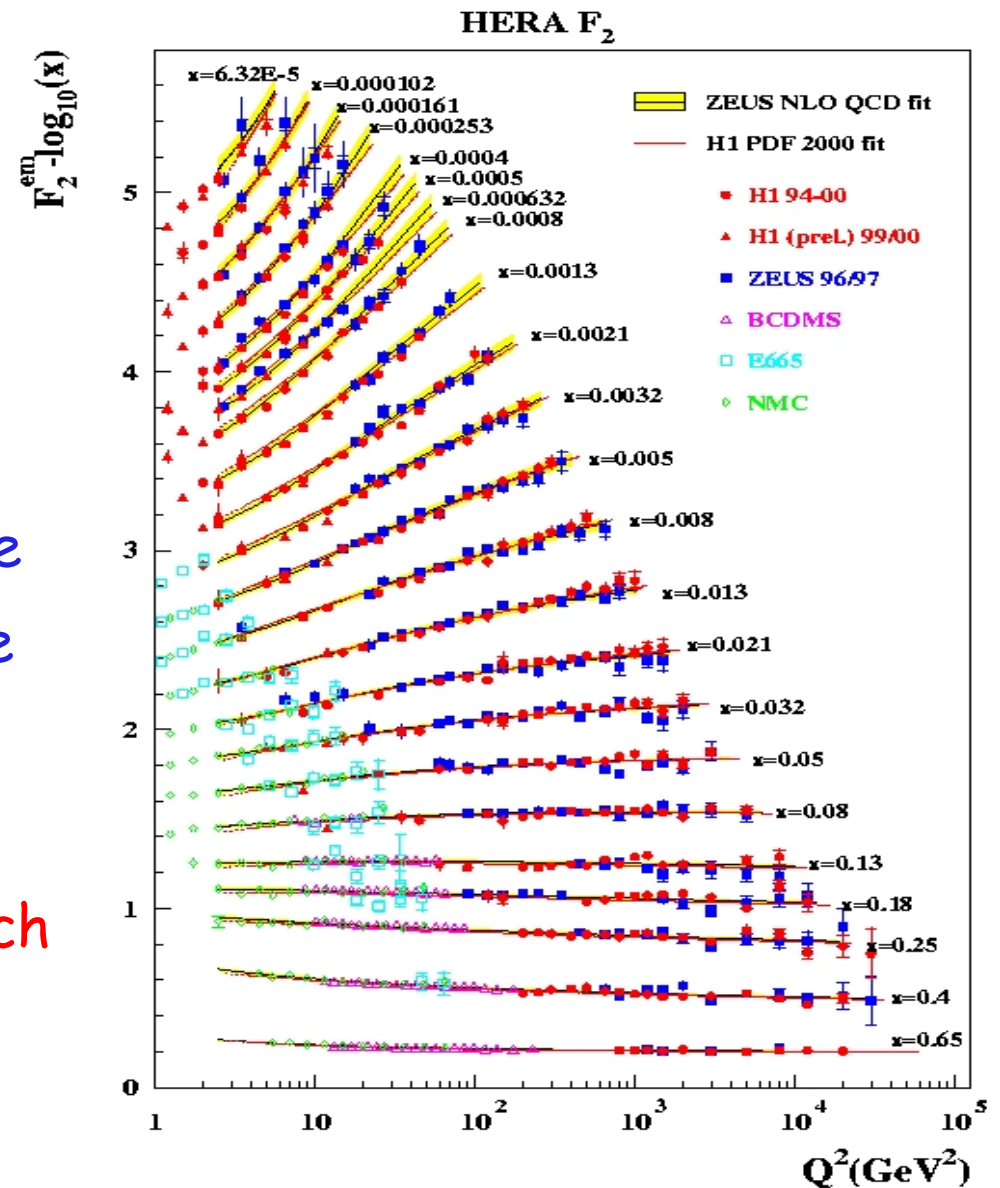
→ **Test of theory: Q^2 evolution of $F_2(x, Q^2)$!!!!!**

The fun with ep scattering

$$\sigma(e^+p \rightarrow e^+X) = \sum f_i(x, Q^2) \sigma(e^+q_i \rightarrow e^+q_i)$$



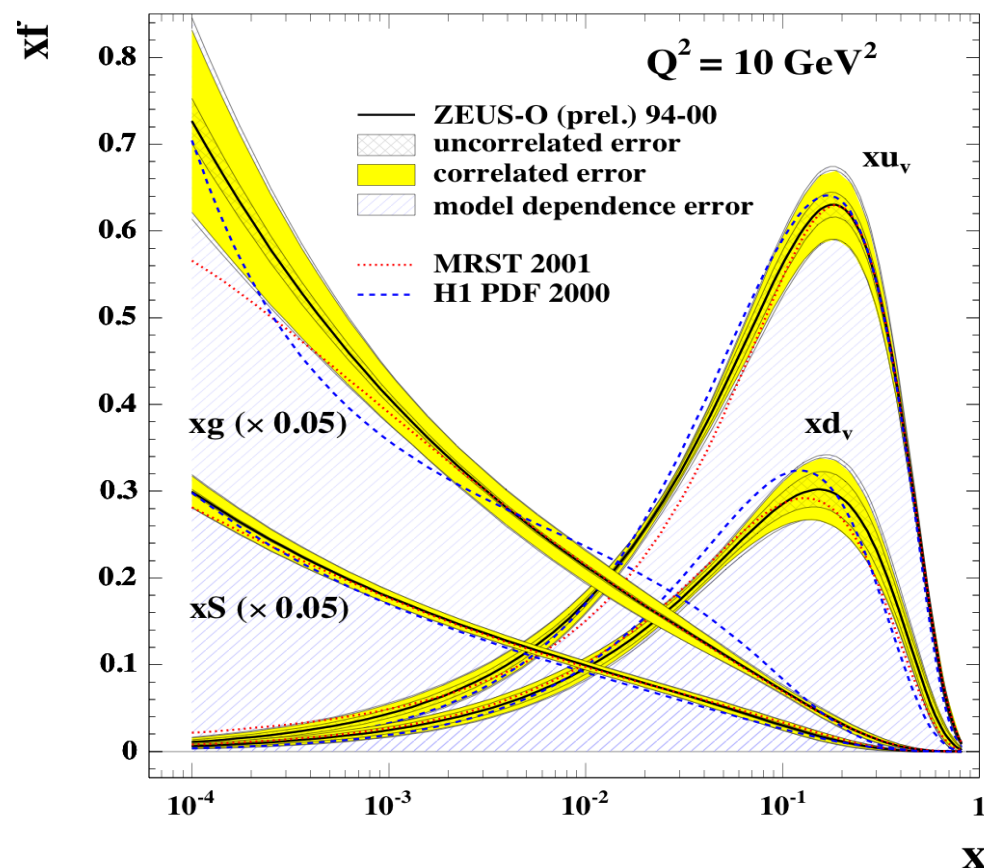
- perfect description of precise measurements of **HUGE** range in x and Q^2
- Theory works well.....
- ➔ extract parton densities, which are universal
- ➔ to be used at LHC.....



The proton PDFs ...

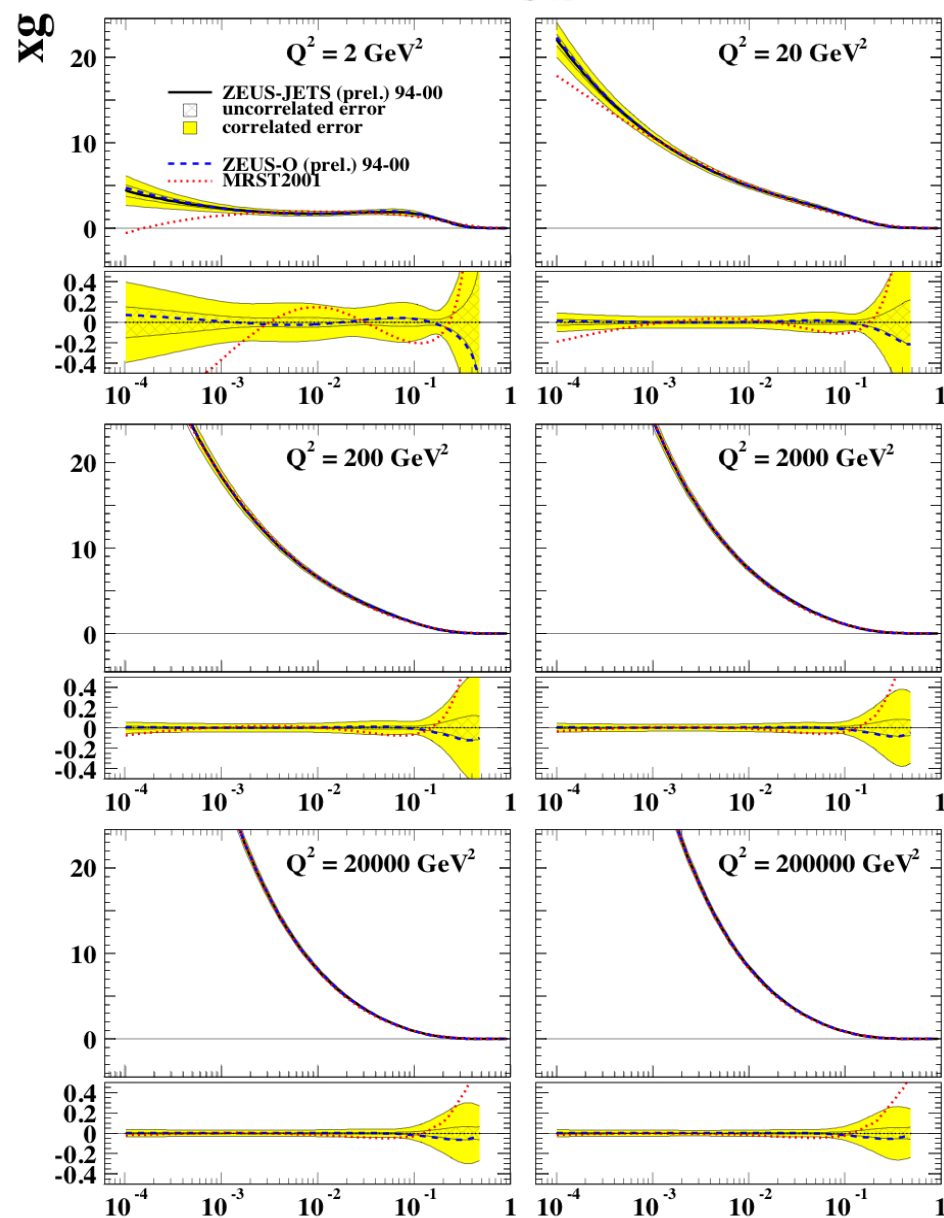
- quark and gluon PDFs

ZEUS



→ Very large gluon density, even at small resolution scales Q^2

ZEUS



The DIS process $ep \rightarrow epX$

- cross section $\frac{d\sigma(ep \rightarrow e' X)}{dy dQ^2} = \frac{4\pi\alpha^2}{yQ^4} \left(\left(1 - y + \frac{y^2}{2}\right) F_2^p(x, Q^2) - \frac{y^2}{2} F_L^p(x, Q^2) \right)$

with $F_2^p(x, Q^2) = \sum_f e_f^2 (xq_f(x, Q^2) + x\bar{q}_f(x, Q^2))$

using the PDFs from before.....

The DIS process $ep \rightarrow epX$

- cross section $\frac{d\sigma(ep \rightarrow e'X)}{dy dQ^2} = \frac{4\pi\alpha^2}{yQ^4} \left(\left(1 - y + \frac{y^2}{2}\right) F_2^p(x, Q^2) - \frac{y^2}{2} F_L^p(x, Q^2) \right)$

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using the PDFs from before.....

- generate y with $g(y)=1/y$, and Q^2 with $g(Q^2)=1/Q^2$ (why not $1/Q^4$?):

$$y = y_{min} \left(\frac{y_{max}}{y_{min}} \right)^{R_1} Q^2 = Q_{min}^2 \left(\frac{Q_{max}^2}{Q_{min}^2} \right)^{R_2}$$

$$\sigma(ep \rightarrow e'X) = \frac{1}{N} \sum_{i=1}^N \frac{\frac{d\sigma}{dy_i dQ_i^2}}{g(y_i)g(Q_i^2)} \int g(y)dy \int g(Q^2)dQ^2$$

The DIS process $ep \rightarrow epX$

- cross section $\frac{d\sigma(ep \rightarrow e'X)}{dy dQ^2} = \frac{4\pi\alpha^2}{yQ^4} \left(\left(1 - y + \frac{y^2}{2}\right) F_2^p(x, Q^2) - \frac{y^2}{2} F_L^p(x, Q^2) \right)$

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$$\sigma(ep \rightarrow e'X) = \frac{1}{N} \sum_{i=1}^N \frac{\frac{d\sigma}{dy_i dQ_i^2}}{g(y_i)g(Q_i^2)} \int g(y)dy \int g(Q^2)dQ^2$$

- calculate x-section with:

$$\sigma(ep \rightarrow e'X) = \frac{1}{N} \sum_{i=1}^N y_i Q_i^2 \frac{d\sigma}{dy_i dQ_i^2} \log \left(\frac{y_{max}}{y_{min}} \right) \log \left(\frac{Q_{max}^2}{Q_{min}^2} \right)$$

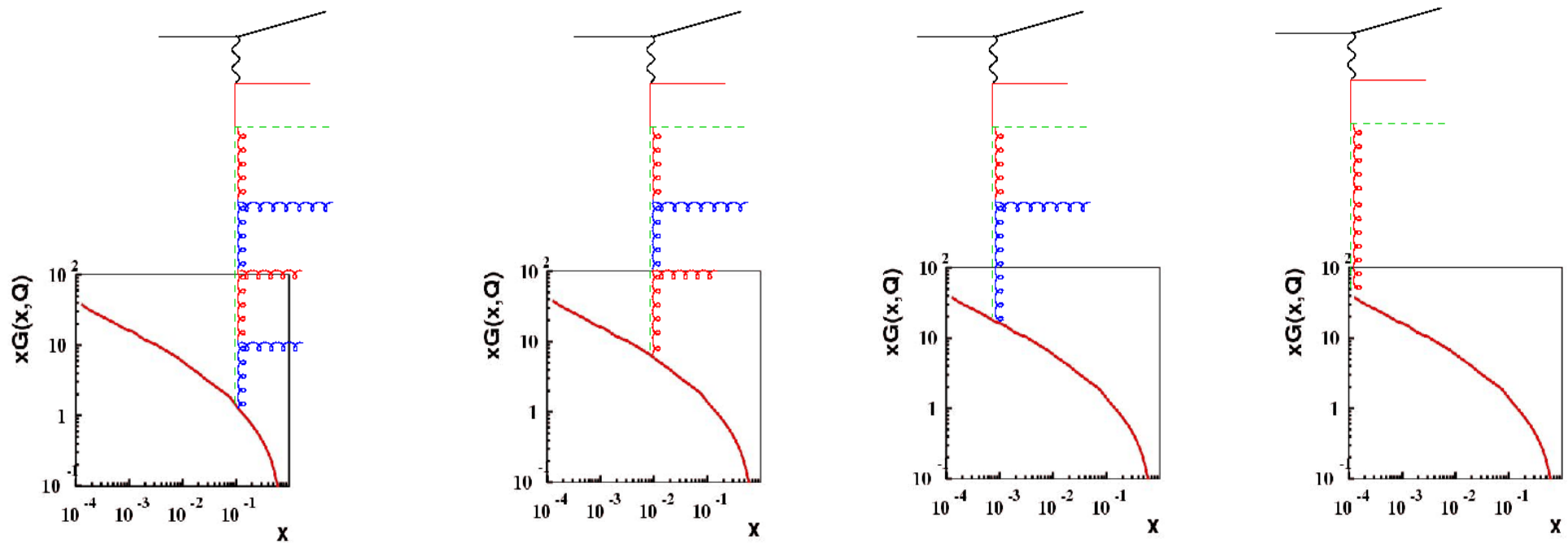
- calculate 4-momenta of scattered electron and virtual photon

We have now an event
generator for the
total cross section.

What about the final
states ?

DGLAP evolution equation... again...

- for fixed x and Q^2 chains with different branchings contribute
- iterative procedure, **spacelike** parton showering



$$f(x, t) = \sum_{k=1}^{\infty} f_k(x_k, t_k) + f_0(x, t_0) \Delta_s(t)$$

Parton Showers from evolution eq.

$$f(x, t) = f(x, t_0) \Delta_s(t) + \int \frac{dz}{z} \int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')} \tilde{P}(z) f\left(\frac{x}{z}, t'\right)$$

- solve integral equation via **explicit** iteration:

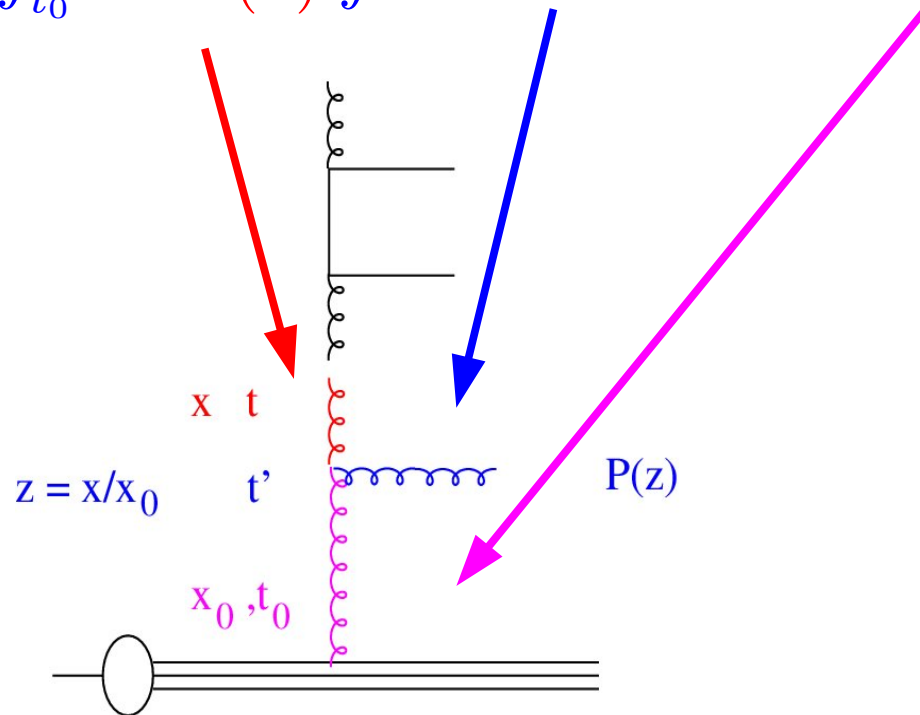
$$f_0(x, t) = f(x, t_0) \Delta(t)$$

from t' to t
w/o branching

branching at t'

from t_0 to t'
w/o branching

$$f_1(x, t) = f(x, t_0) \Delta(t) + \int_{t_0}^t \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} \tilde{P}(z) f(x/z, t_0) \Delta(t')$$



Sudakov form factor: all loop resum...

$g \rightarrow gg$ Splitting Fct $\tilde{P}(z) = \frac{\bar{\alpha}_s}{1-z} + \frac{\bar{\alpha}_s}{z} + \dots$

- **Sudakov form factor** all loop resummation

$$\Delta_S = \exp \left(- \int dz \int \frac{dq'}{q'} \frac{\alpha_s}{2\pi} \tilde{P}(z) \right)$$

$$\Delta_S = 1 + \left(- \int dz \int \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z) \right)^1 + \frac{1}{2!} \left(- \int dz \int \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z) \right)^2 \dots$$



$$\tilde{P}(z) \left[1 - \int \int dz \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z) + \frac{1}{2!} \left(- \int \int dz \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z) \right)^2 + \dots + \right]$$

All Order resummed evolution

$$f(x, t) = f(x, t_0) \Delta_s(t) + \int \frac{dz}{z} \int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')} \tilde{P}(z) f\left(\frac{x}{z}, t'\right)$$

- solve integral equation via iteration:

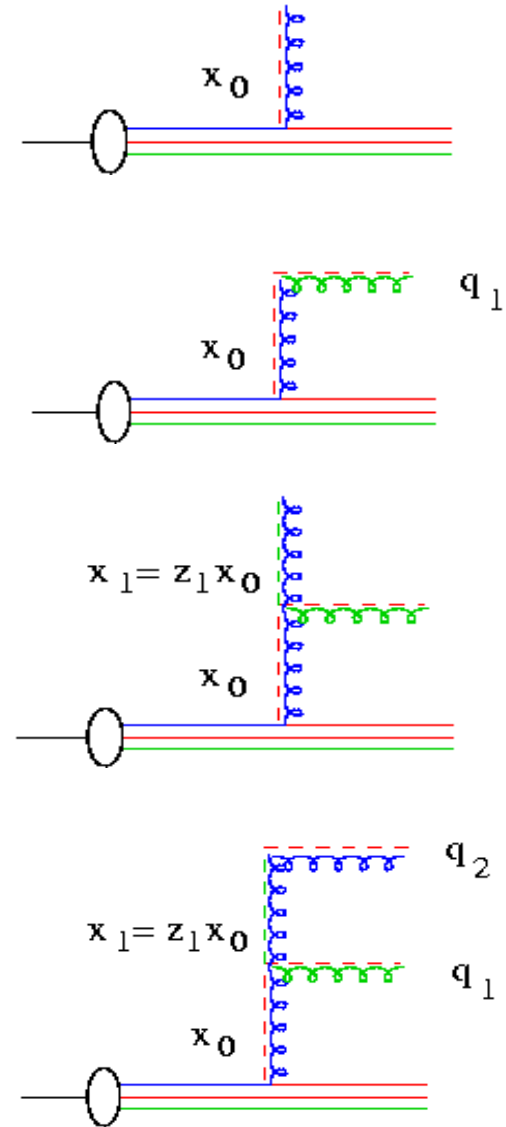
$$\begin{aligned}
 f_0(x, t) &= f(x, t_0) \Delta(t) && \boxed{\text{from } t' \text{ to } t \text{ w/o branching}} && \boxed{\text{branching at } t'} && \boxed{\text{from } t_0 \text{ to } t' \text{ w/o branching}} \\
 f_1(x, t) &= f(x, t_0) \Delta(t) + \int_{t_0}^t \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} \tilde{P}(z) f(x/z, t_0) \Delta(t') \\
 &= f(x, t_0) \Delta(t) + \log \frac{t}{t_0} A \otimes \Delta(t) f(x/z, t_0) \\
 f_2(x, t) &= f(x, t_0) \Delta(t) + \log \frac{t}{t_0} A \otimes \Delta(t) f(x/z, t_0) + \\
 &\quad \frac{1}{2} \log^2 \frac{t}{t_0} A \otimes A \otimes \Delta(t) f(x/z, t_0) \\
 f(x, t) &= \lim_{n \rightarrow \infty} f_n(x, t) = \lim_{n \rightarrow \infty} \sum_n \frac{1}{n!} \log^n \left(\frac{t}{t_0} \right) A^n \otimes \Delta(t) f(x/z, t_0)
 \end{aligned}$$

DGLAP re-sums $\log t$ to all orders !!!!!!!!!!!!!!!

Parton showers for the initial state

spacelike ($Q^2 < 0$) parton shower evolution

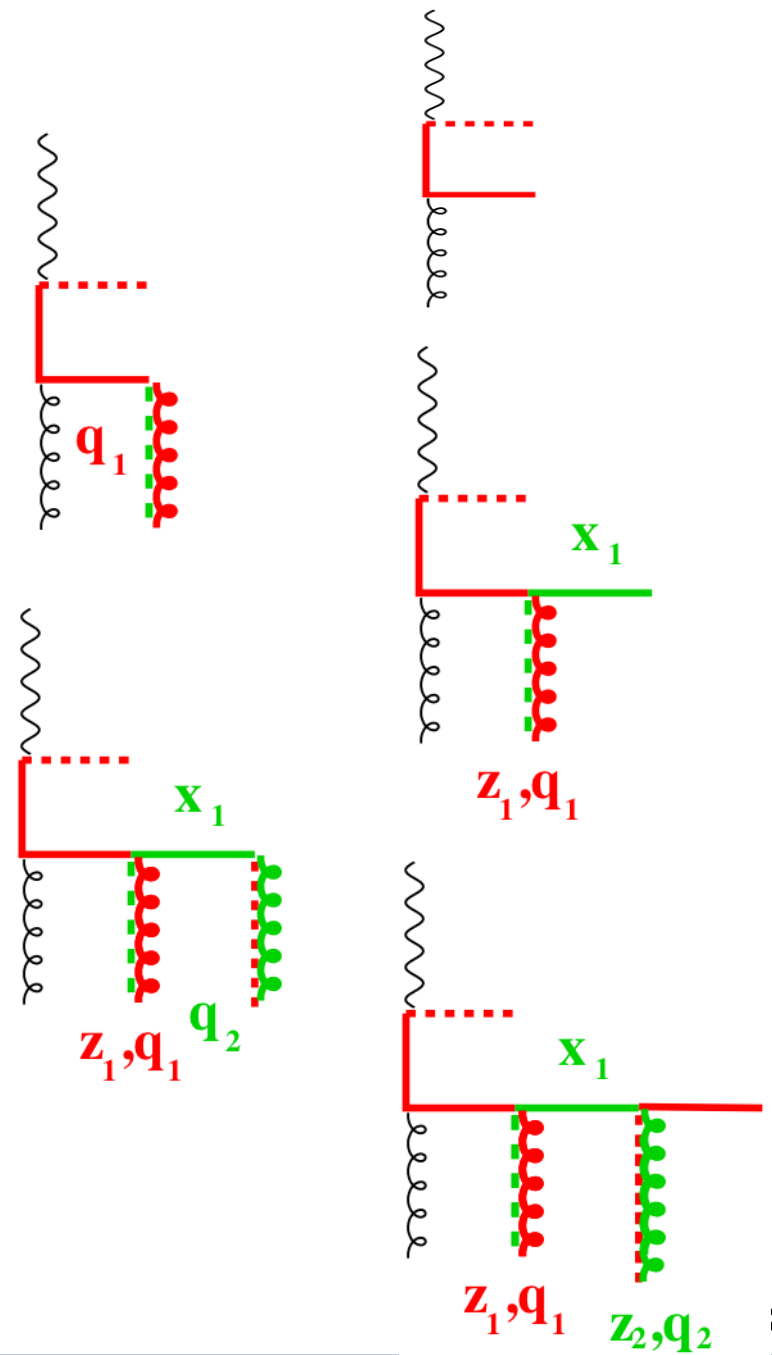
- starting from hadron (fwd evolution)
or from hard scattering (bwd evolution)
- select q_1 from Sudakov form factor
- select z_1 from splitting function
- select q_2 from Sudakov form factor
- select z_2 from splitting function
- stop evolution if $q_2 > Q_{\text{hard}}$



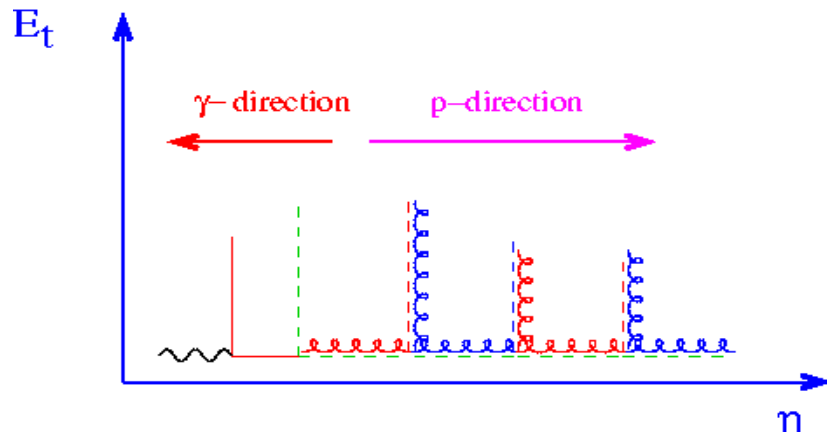
Parton Showers for the final state

timelike parton shower evolution

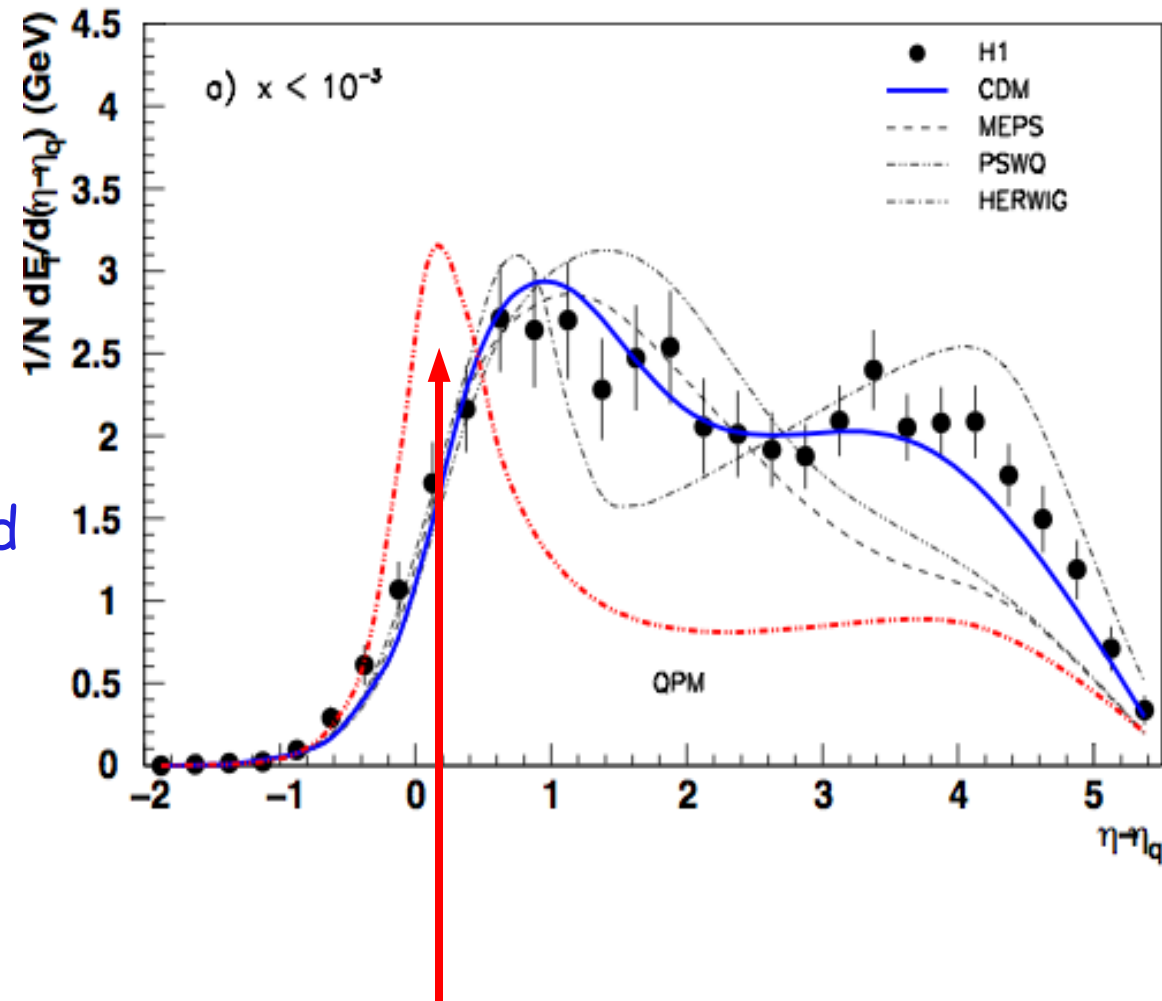
- starting with hard scattering
- select q_1 from Sudakov form factor
- select z_1 from splitting function
- select q_2 from Sudakov form factor
- select z_2 from splitting function
- stop evolution if $q_2 < q_0$



Hadronic final state: Energy flow



- E_t flow in DIS at small x and forward angle (p-direction):
- QPM is not enough
- clearly parton showers or higher order contributions needed



leading jet direction

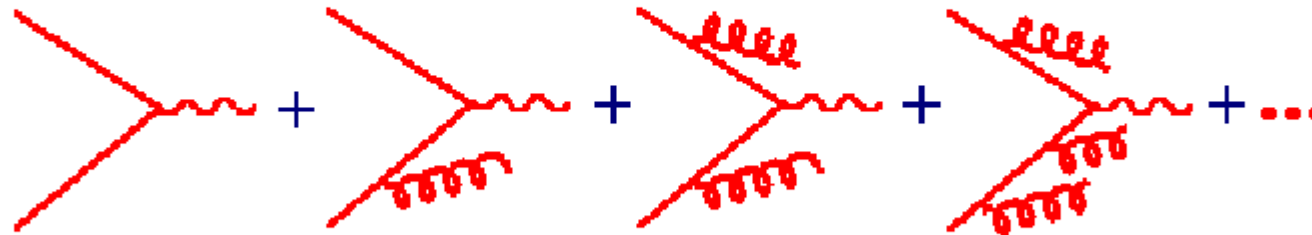
In ep higher order parton
radiation needed

What about pp ?

What about pt spectrum of Z_0 ?

C-P Yuan, CTEQ
summerschool 2002

Diagrammatically, **Resummation** is doing



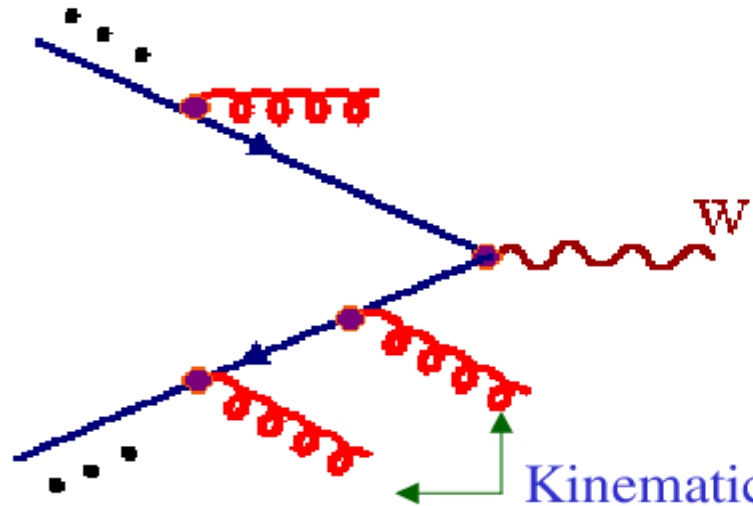
→ Resum large $\alpha_s^n \ln^m \left(\frac{Q^2}{q_T^2} \right)$ terms

$$\left. \frac{d\sigma}{dq_T^2 dy} \right|_{q_T \rightarrow 0} \sim \frac{1}{q_T^2} \sum_{n=1}^{\infty} \sum_{m=0}^{2n-1} \alpha_s^n \ln^m \left(\frac{Q^2}{q_T^2} \right) \cdot C_m^n$$

Monte-Carlo programs **ISAJET**, **PYTHIA**, **HERWIG** contain these physics.

pt spectrum in MC approach

C-P Yuan, CTEQ
summerschool 2002



Backward Radiation
(Initial State Rad.)

Kinematics of the radiated gluon, controlled by Sudakov form factor with some arbitrary cut-off.
(In contrast to perform integration in impact parameter space, i.e., **b space**.)



The shape of $q_T(w)$ is generated. But, the integrated rate remains the same as at Born level (finite virtual correction is not included).



Recently, there are efforts to include part of higher order effect in the event generator.

Transverse Momentum of W/Z

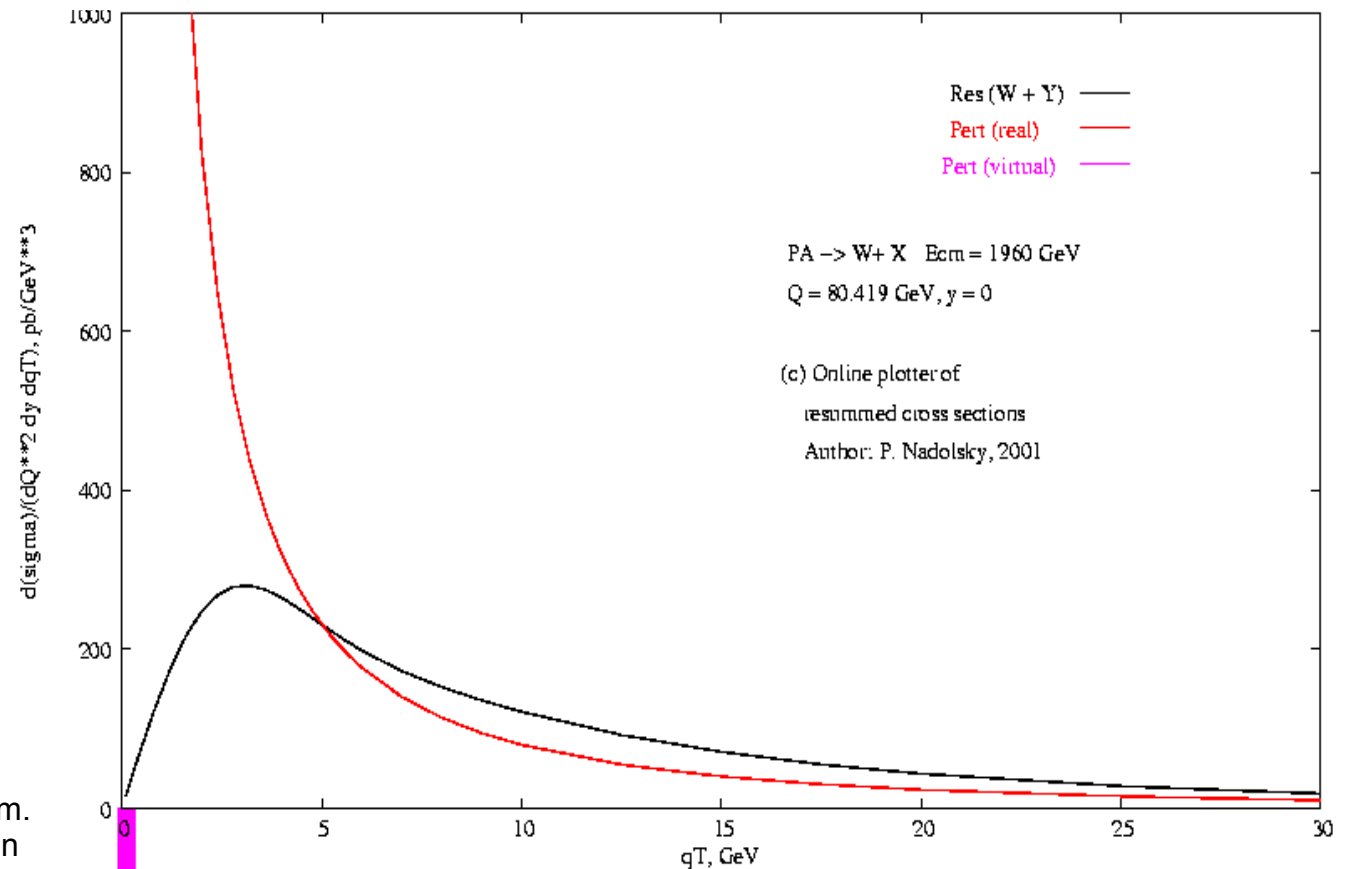
Perturbative calculation

$$\mathcal{O}(\alpha_s), \mathcal{O}(\alpha_s^2)$$

diverges for small p_\perp

Tevatron Run-1 Z boson data and
Collins-Soper-Sterman resummation formalism.
F. Landry, R. Brock, P.M. Nadolsky, C.P. Yuan
Phys.Rev.D67:073016,2003.
e-Print: hep-ph/0212159

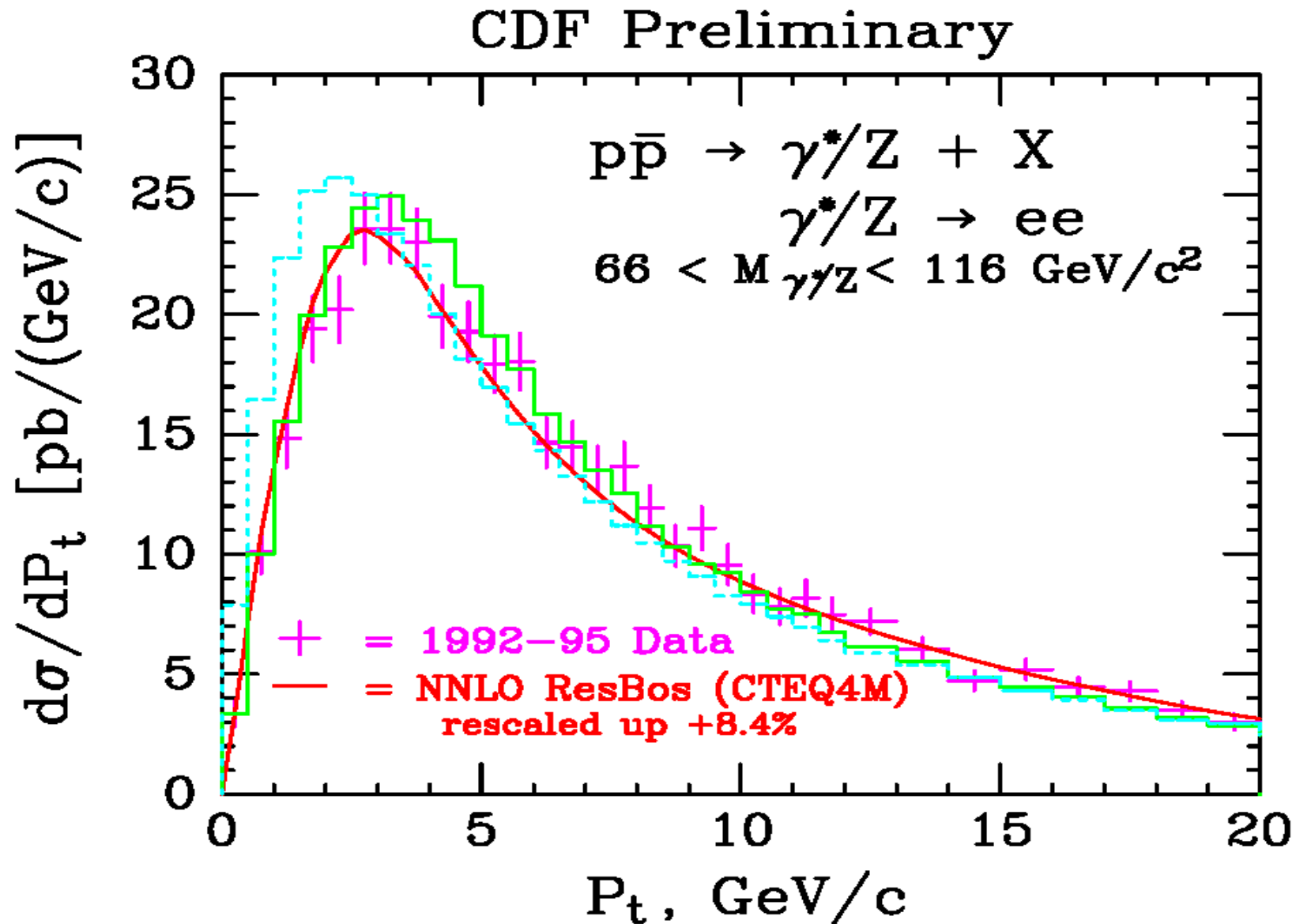
<http://hep.pa.msu.edu/wwwlegacy/>



Recall: Monte Carlo vrs ResBos

- Comparison of p_t spectrum from ResBos and PYTHIA

Campbell, Huston Stirling
Rep.Prog.Phys 70 (2007) 89



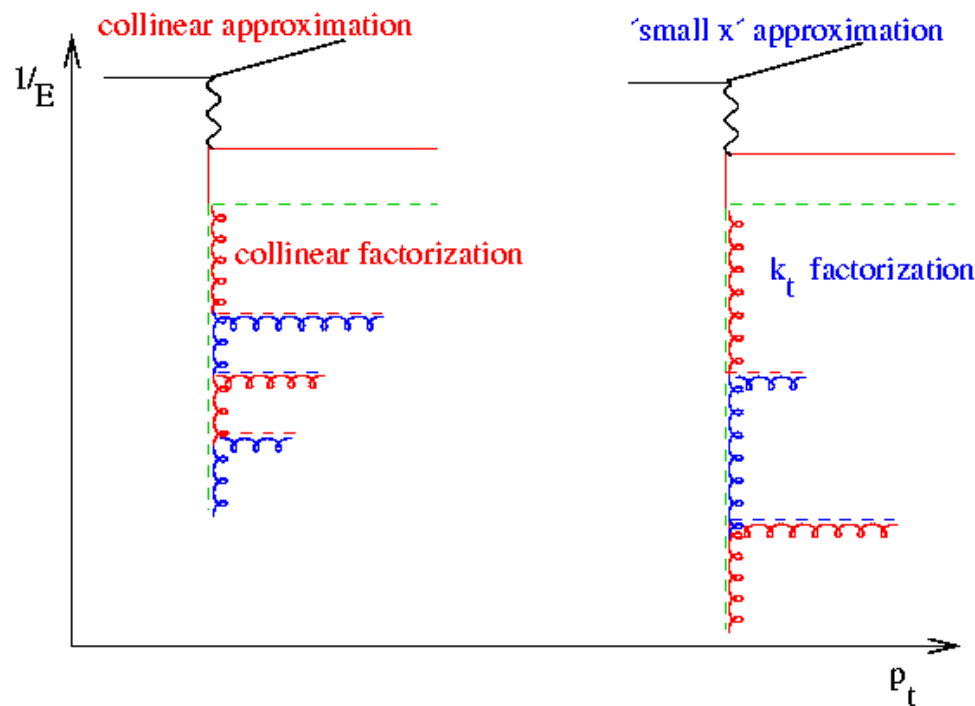
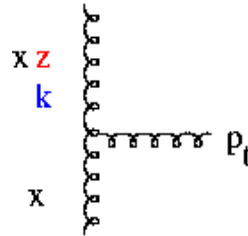
Parton Shower

- Sudakov form factor
 - gives probability for no-branching between q_0 and q
 - sums virtual contributions to all orders (via unitarity)
 - virtual (parton loop) and
 - real (non-resolvable) parton emissions
- Sudakov form factor particularly suited for Monte Carlo approach
 - need to specify scale of hard process (matrix element) $Q \sim p_T$
 - need to specify cutoff scale $Q_0 \sim 1 \text{ GeV}$
- Evolution equation with Sudakov form factor recovers exactly evolution equation (with $+$ prescription)

Theory recap: what are we doing ?

gluon bremsstrahlung

$$\sim \frac{1}{k^2} \left(\frac{1}{z} + \dots \right)$$



Dokshitzer Gribov Lipatov Altarelli Parisi

- collinear singularities factorized in pdf
- evolution in $Q^2 \sim k^2$, or k_t^2 or ?

$$\sigma = \sigma_0 \int \frac{dz}{z} C^a\left(\frac{x}{z}\right) f_a(z, Q^2)$$

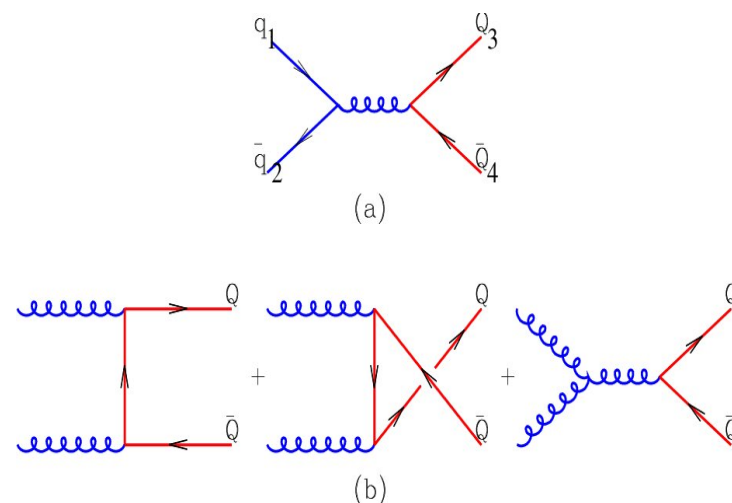
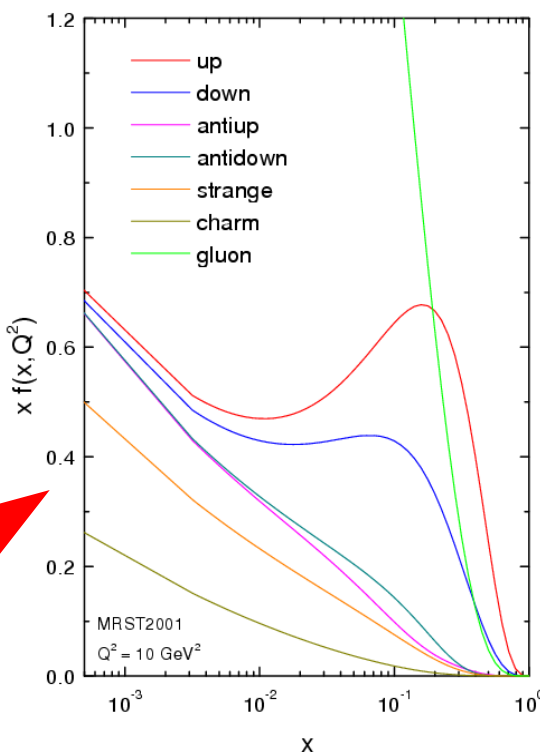
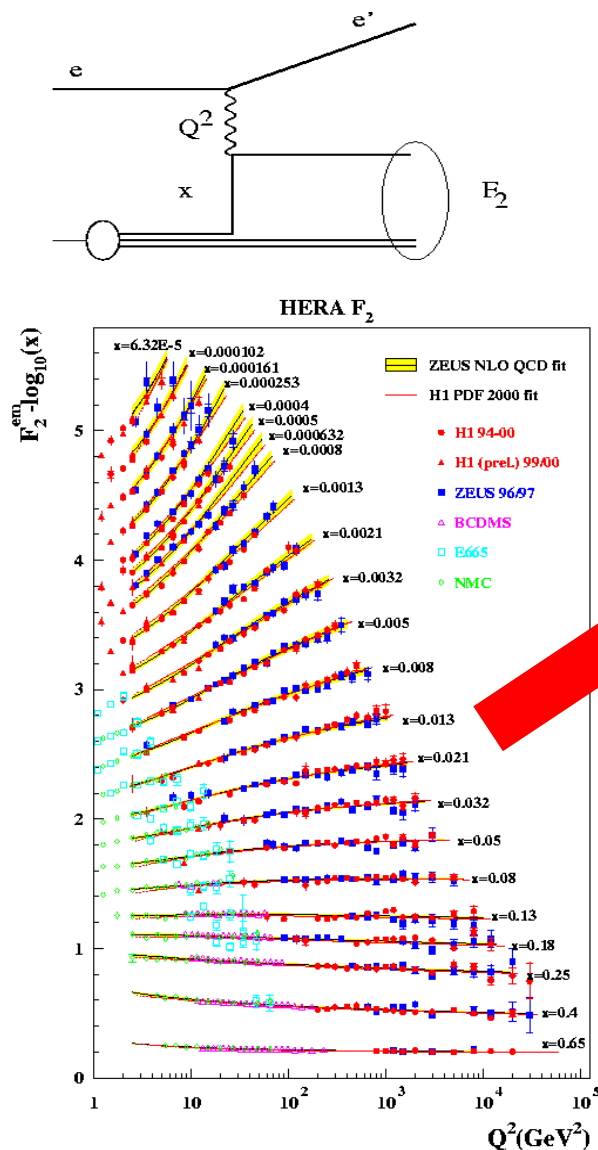
Balitski Fadin Kuraev Lipatov

- k_t dependent pdf \rightarrow unintegrated pdf

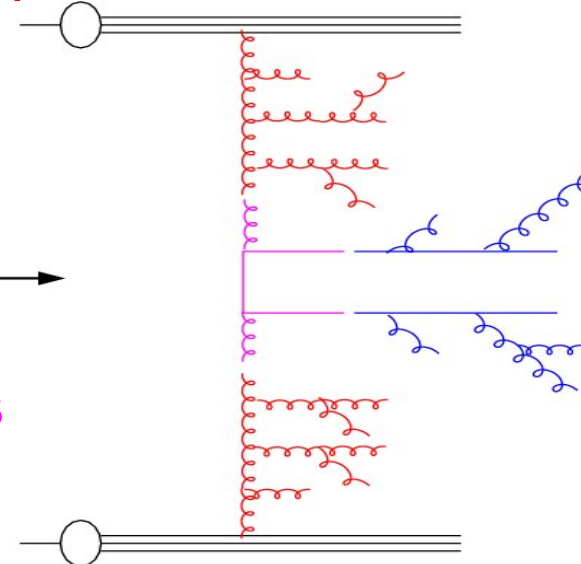
- evolution in x

$$\sigma = \int \frac{dz}{z} d^2 k_t \hat{\sigma}\left(\frac{x}{z}, k_t\right) \mathcal{F}(z, k_t)$$

From F_2 to Heavy Quarks in pp



calculate heavy quark

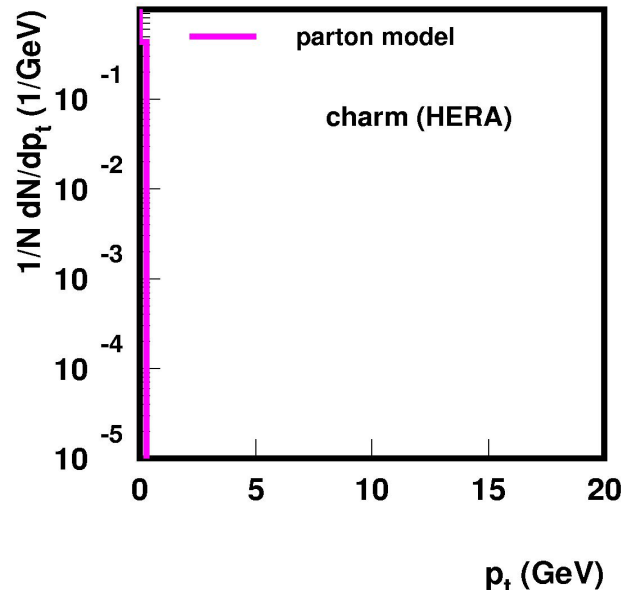
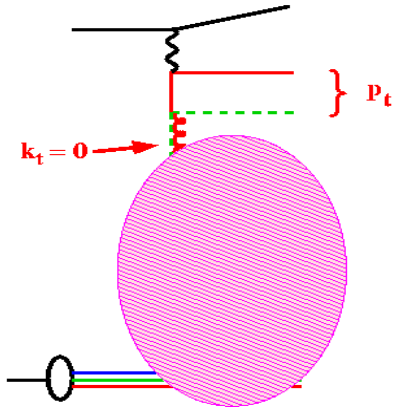


extract parton densities

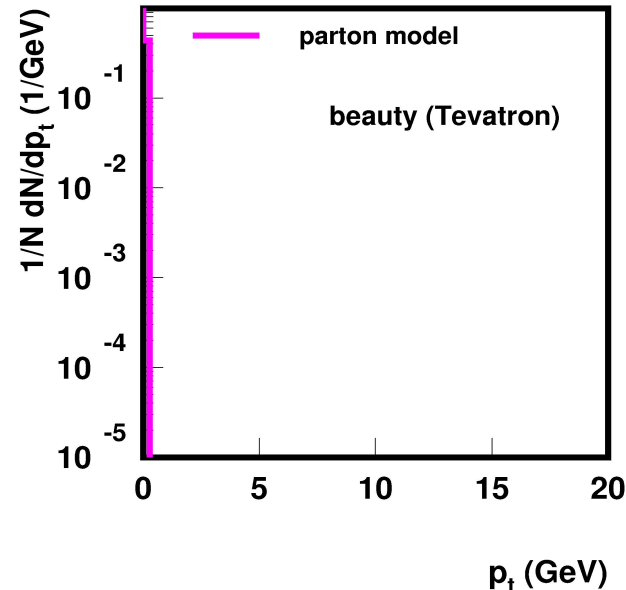
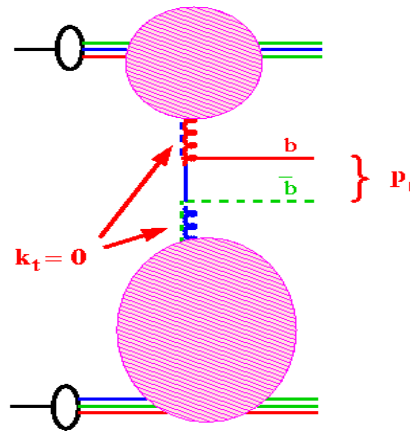
total x-section, F_2

Problems in Collinear Approximation

Jets/ heavy quarks at
HERA

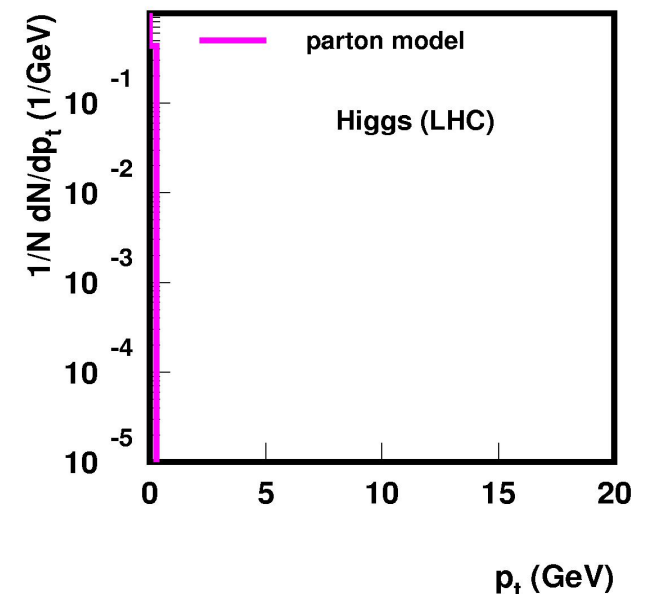
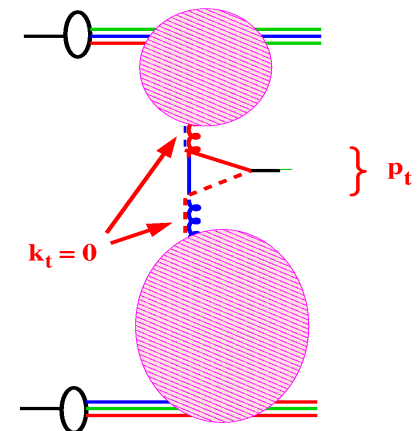


Jets/heavy quarks in pp



J. Collins, H. Jung hep-ph/0508280

Higgs in pp



➔ **NLO corrections will be very large for these LO processes**

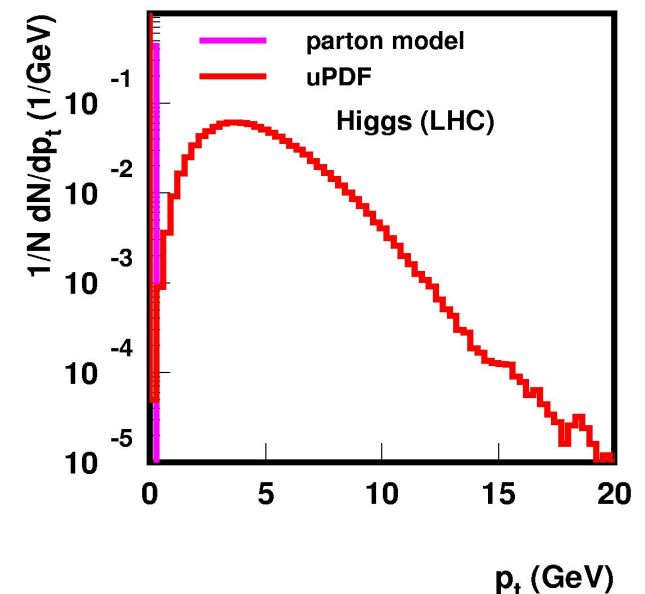
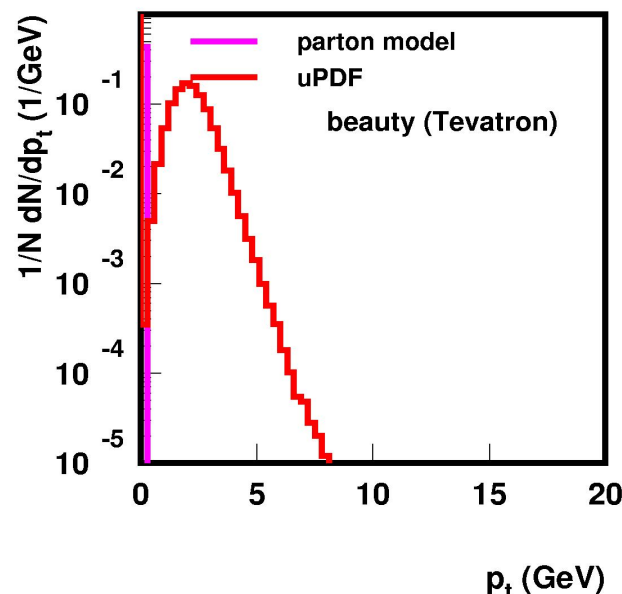
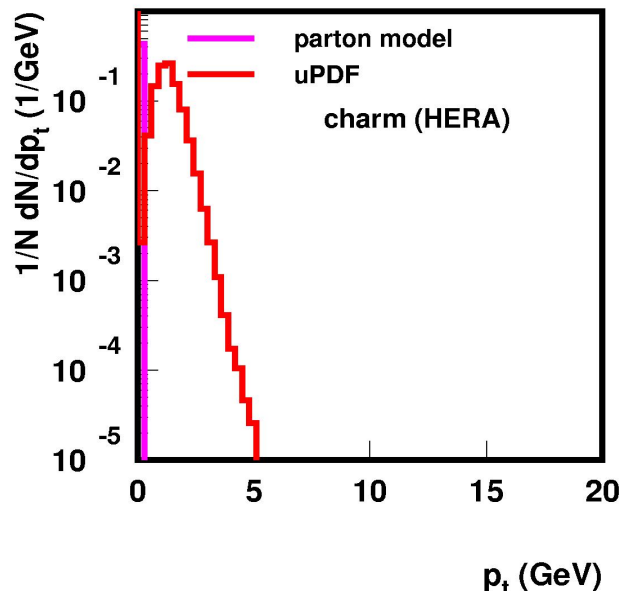
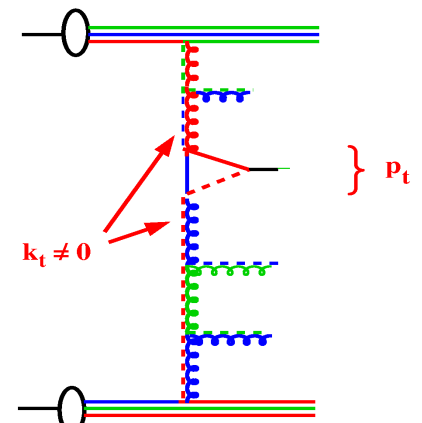
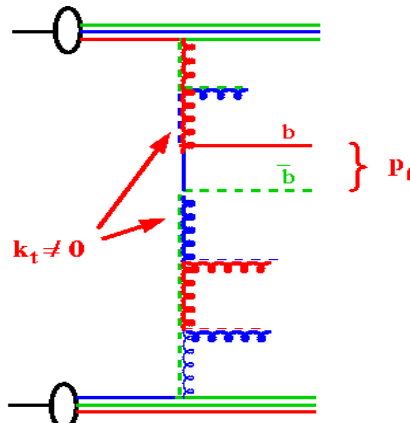
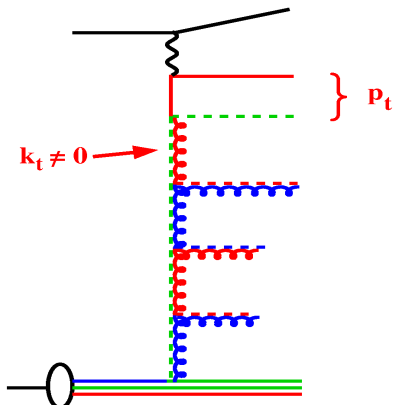
Doing much better with uPDFs ...

heavy quarks at HERA

Jets/heavy quarks in pp

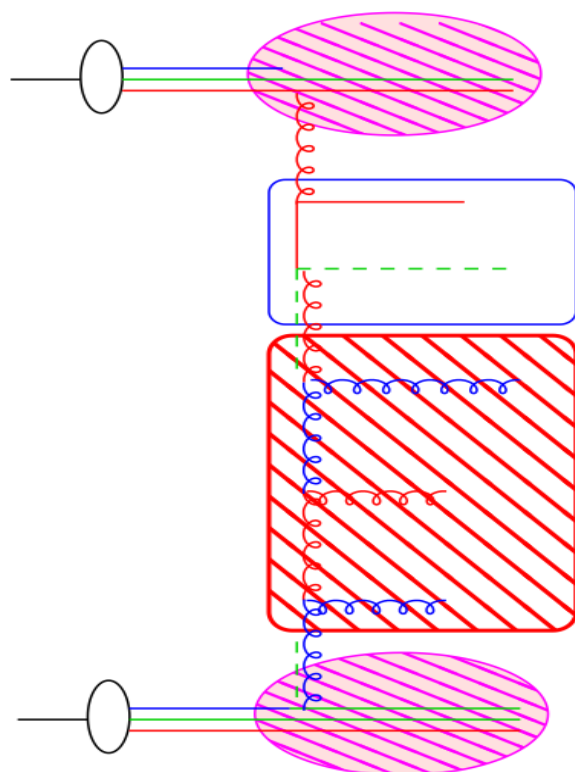
J. Collins, H. Jung hep-ph/0508280

Higgs in pp



→ doing kinematics correct at LO, reduces NLO corrs,.... NEED uPDFs !!!!

CASCADE - basic idea

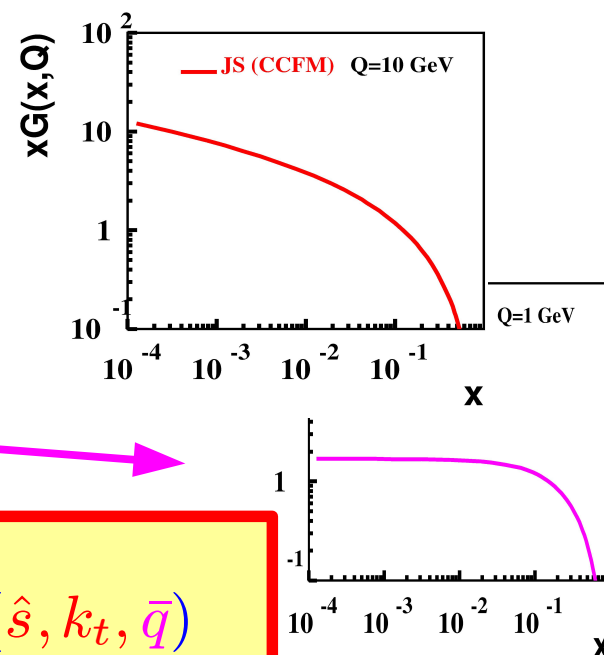


matrix element

evolution of parton cascade:

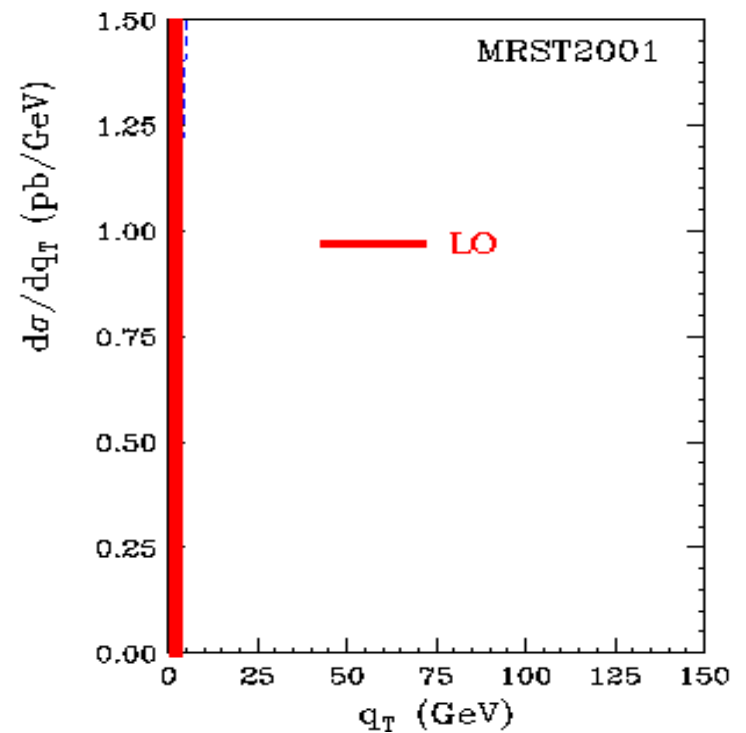
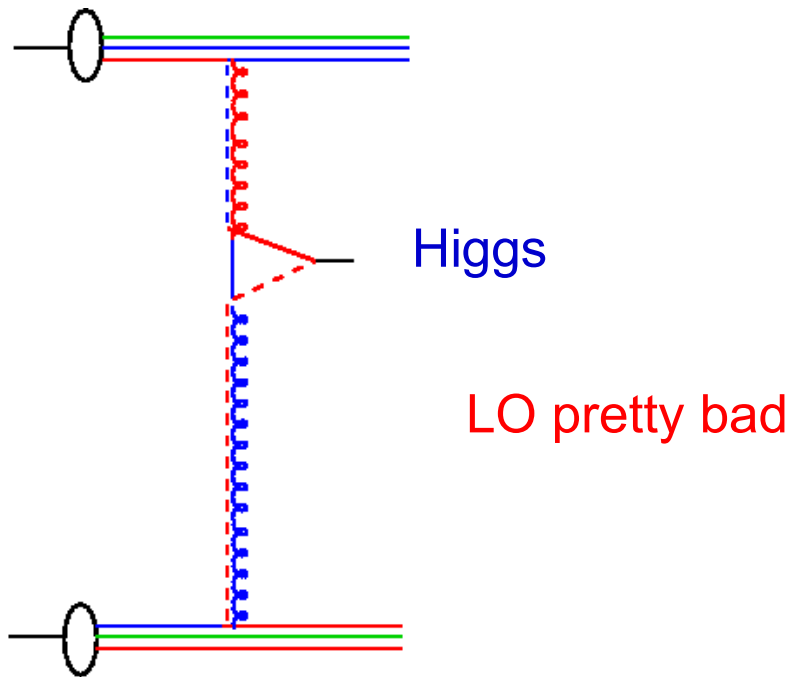
$$\tilde{P} = \bar{\alpha}_s \left(\frac{1}{1-z} + \frac{1}{z} + \dots \right)$$

initial distribution
~ flat

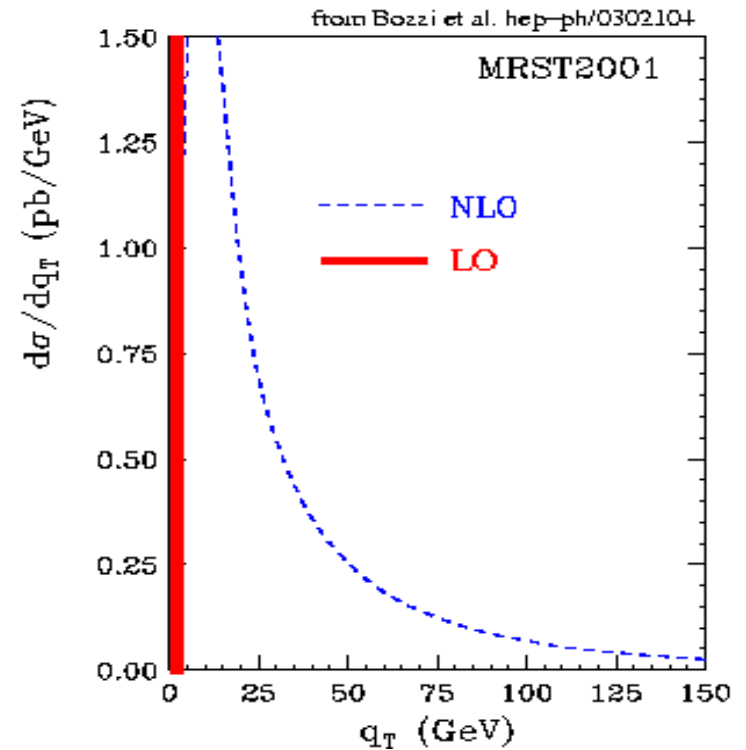
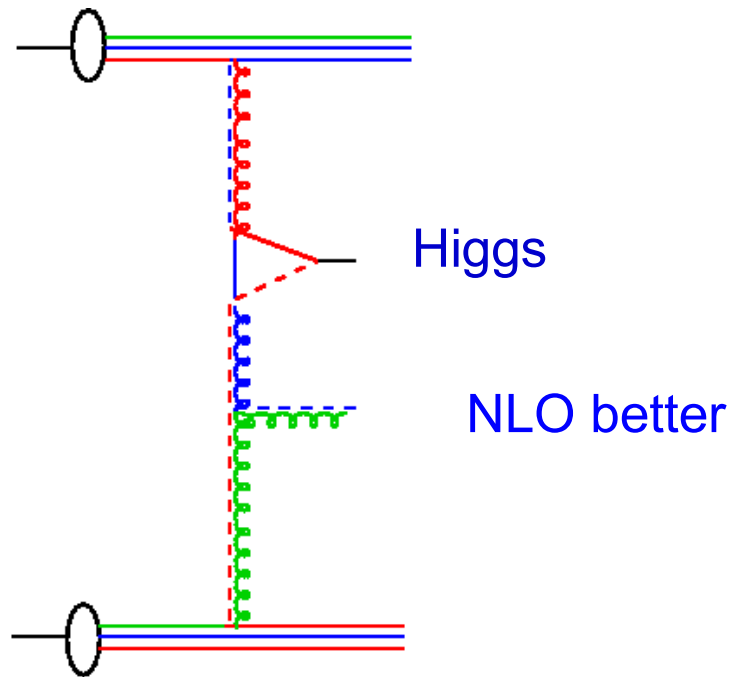


$$\begin{aligned} \sigma(pp \rightarrow q\bar{q} + X) &= \int \frac{dx_{g1}}{x_{g1}} \frac{dx_{g2}}{x_{g2}} \int d^2 k_{t1} d^2 k_{t2} \hat{\sigma}(\hat{s}, k_t, \bar{q}) \\ &\quad \times x_{g1} \mathcal{A}(x_{g1}, k_{t1}, \bar{q}) x_{g2} \mathcal{A}(x_{g2}, k_{t2}, \bar{q}) \\ \int d^2 k_t x_g \mathcal{A}(x_g, k_t, \bar{q}) &= x_g G(x_g, Q^2) \text{ if } \hat{\sigma} = \hat{\sigma}(\hat{s}, 0, \bar{q}) \end{aligned}$$

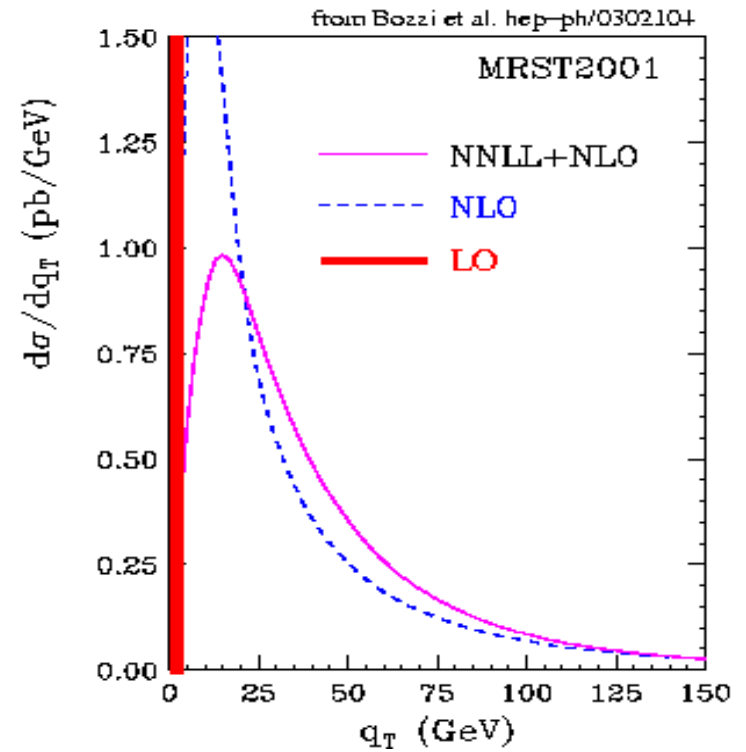
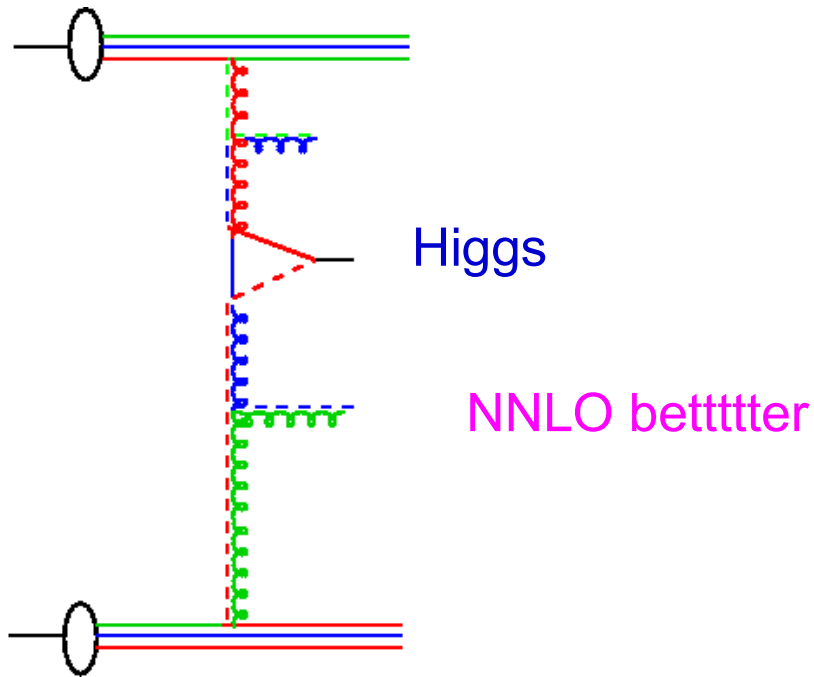
k_{\perp} effects LHC



k_t effects at LHC



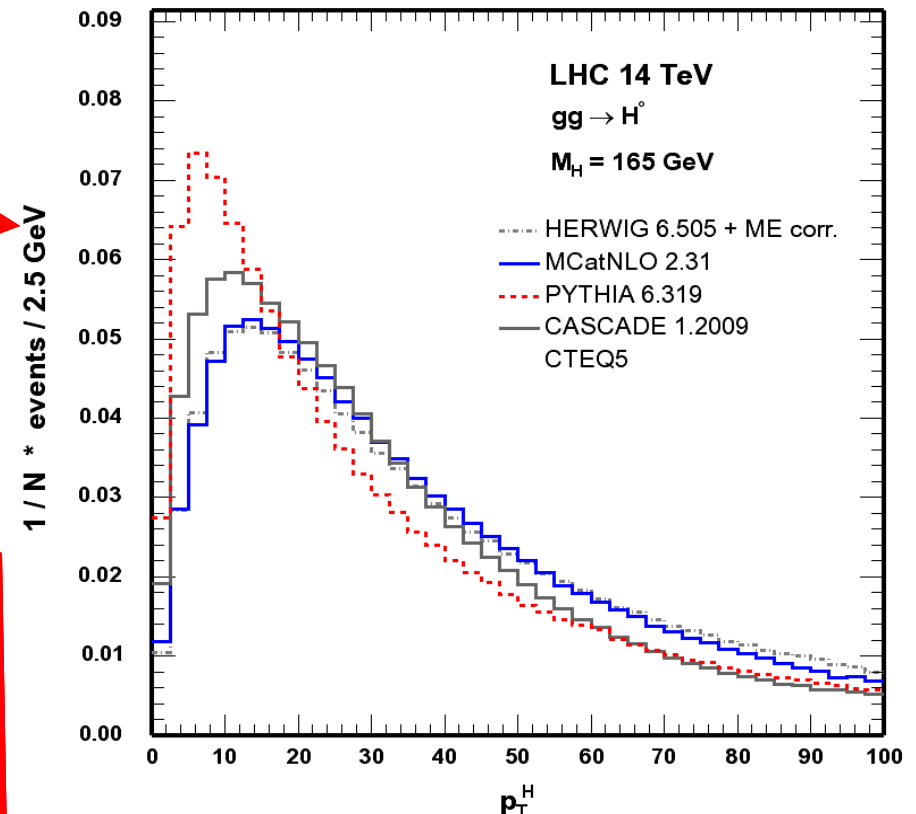
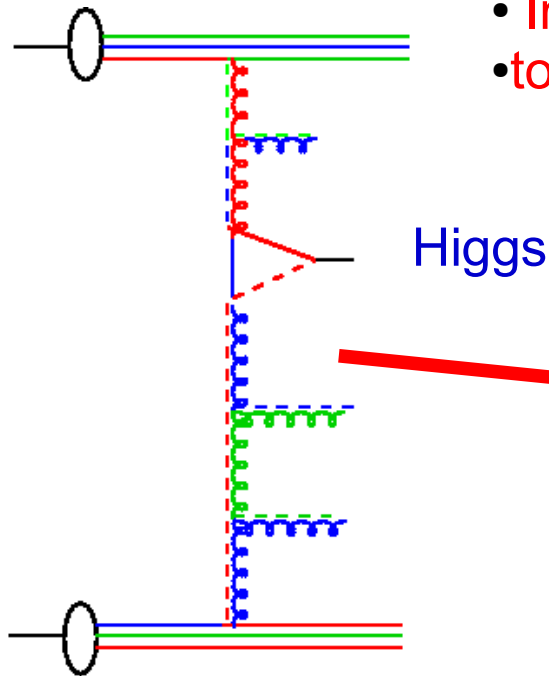
k_t effects at LHC



k_t effects at LHC

from G. Davatz

- Do we understand the p_t spectrum of Higgs at LHC?
- Important for the $gg \rightarrow \text{Higgs} \rightarrow W^+W^- \rightarrow l^+ \bar{\nu} l^- \nu$
- to understand the jet-veto for $t\bar{t}$ suppression...



$\langle k_t \rangle$ large

- unintegrated parton PDFs will be needed
- Need to be better constrained at HERA with final states

Heavy Quark production in pp

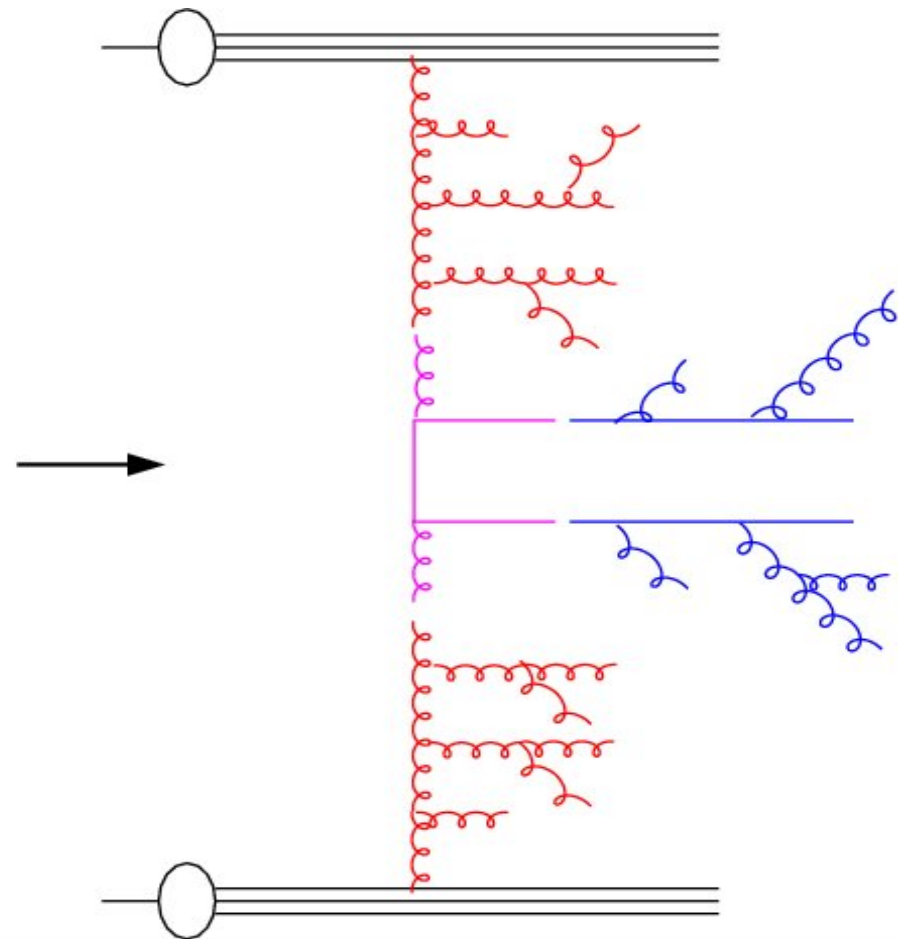
- x-section (i.e. for $t\bar{t}$ production need to know it for the Higgs..)

$$\sigma(pp \rightarrow Q\bar{Q}X) = \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} x_1 G(x_1, \bar{q}) x_2 G(x_2, \bar{q}) \times \hat{\sigma}(\hat{s}, \bar{q})$$

- with gluon densities $xG(x, \bar{q})$

- hard x-section:

$$\frac{d\sigma}{dt} = \frac{1}{64\hat{s}^2} |M_{ij}|^2$$



Heavy Quarks in pQCD

Ellis, Stirling, Webber
QCD & Collider physics p348

- Light Quarks

$$\hat{s} = (p_1 + p_2)^2$$

$$\hat{t} = (p_1 - p_3)^2$$

$$\hat{u} = (p_2 - p_3)^2$$

Process	$\bar{\sum} \mathcal{M} ^2 / g^4$
$q\bar{q} \rightarrow q'\bar{q}'$	$\frac{4}{9} \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}$
$gg \rightarrow q\bar{q}$	$\frac{1}{6} \frac{\hat{t}^2 + \hat{u}^2}{\hat{t}\hat{u}} - \frac{3}{8} \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}$

Divergent

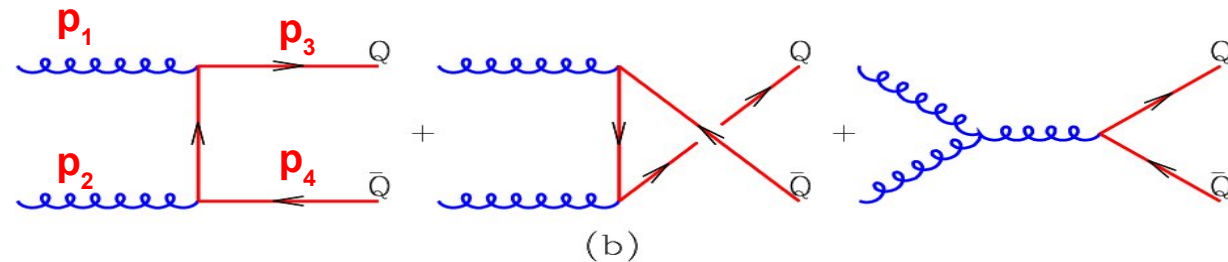
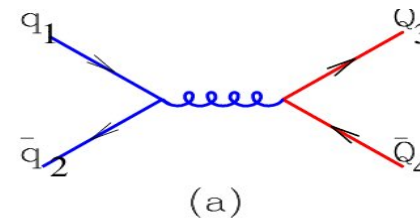
- Heavy Quarks

with

$$\tau_1 = -\frac{\hat{t} - m^2}{\hat{s}},$$

$$\tau_2 = -\frac{\hat{u} - m^2}{\hat{s}},$$

$$\rho = \frac{4m^2}{\hat{s}}$$

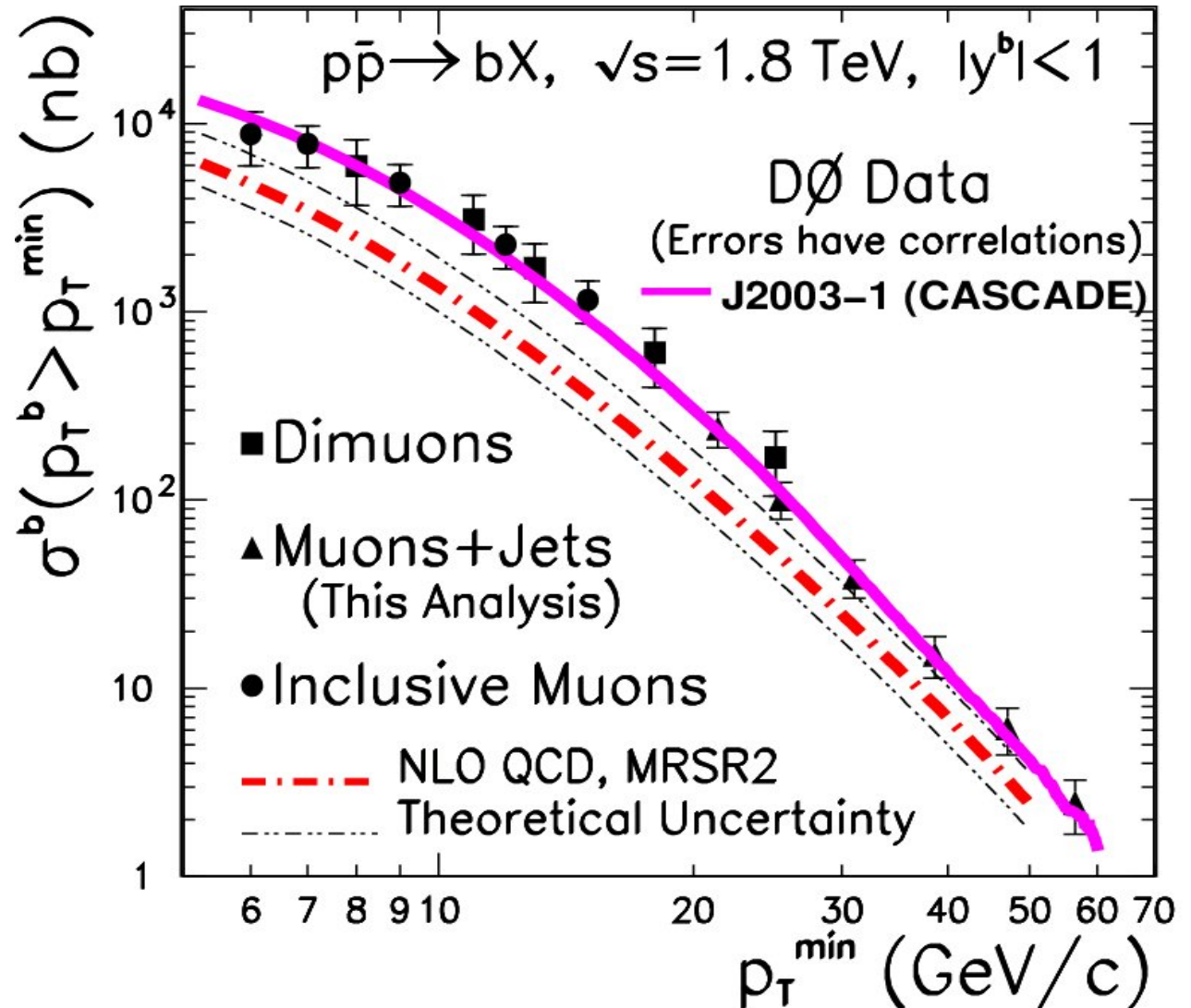


Process	$\bar{\sum} \mathcal{M} ^2 / g^4$
$q\bar{q} \rightarrow Q\bar{Q}$	$\frac{4}{9} (\tau_1^2 + \tau_2^2 + \frac{\rho}{2})$
$gg \rightarrow Q\bar{Q}$	$\left(\frac{1}{6\tau_1\tau_2} - \frac{3}{8} \right) \left(\tau_1^2 + \tau_2^2 + \rho - \frac{\rho^2}{4\tau_1\tau_2} \right)$

$b\bar{b}$ at TeVatron

H.Jung, PRD 65, 034015 (2002)
hep-ph/0110034

- comparison with measurements
- this is what you obtain....

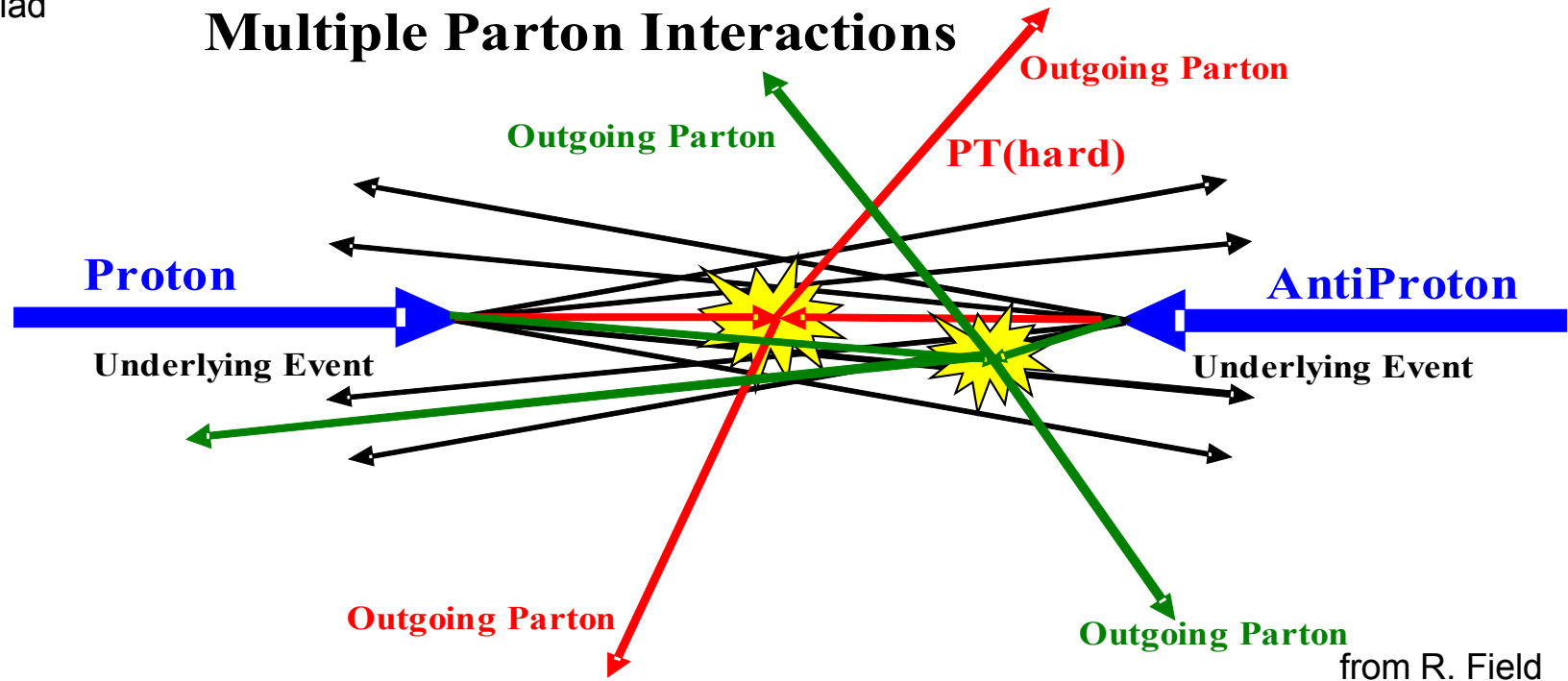


Is that now all ?

But with high parton
densities,
do we only have one
interaction ?

Multiparton Interactions

from L. Loennblad



What is the underlying event (UE)?

- Everything, except the **LO** process we're currently interested in
- parton showers
 - additional remnant - remnant interactions (multi-parton interactions, soft/hard)

X **NOT pile-up events** (luminosity dependent)

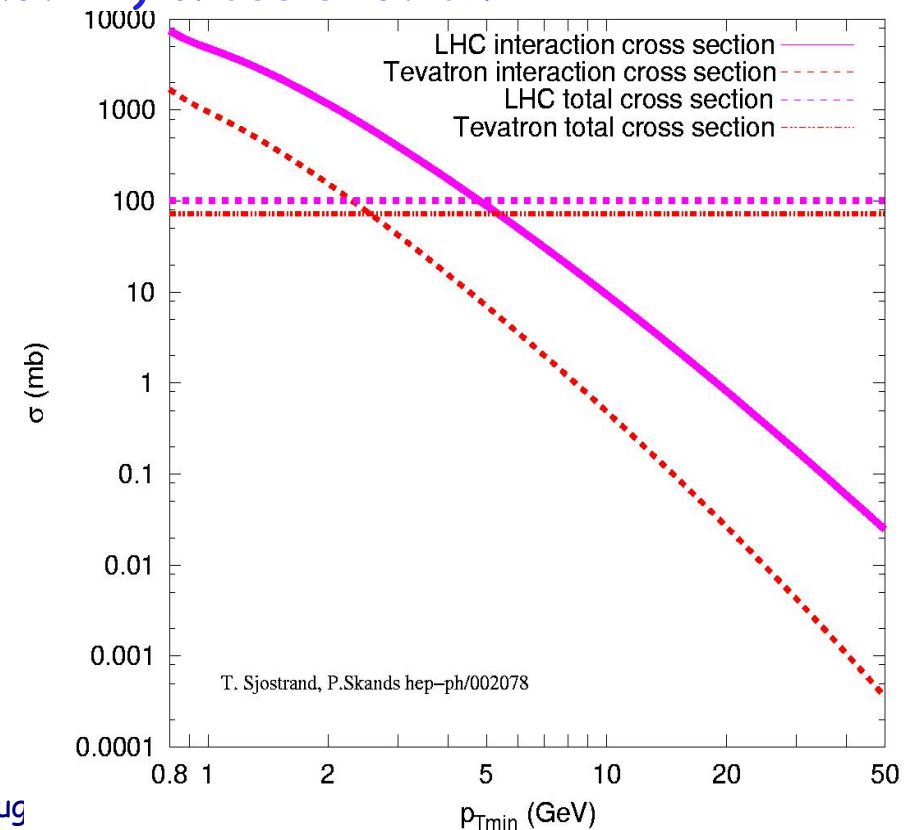
Underlying event – Multiple Interaction

- Basic partonic perturbative cross section

$$\sigma_{\text{hard}}(p_{\perp\text{min}}^2) = \int_{p_{\perp\text{min}}^2} \frac{d\sigma_{\text{hard}}(p_{\perp}^2)}{dp_{\perp}^2} dp_{\perp}^2$$

→ diverges faster than $1/p_{\perp\text{min}}^2$ as $p_{\perp\text{min}} \rightarrow 0$ and exceeds eventually total inelastic (non-diffractive) cross section

- Interaction x-section exceeds total xsection
- happens well above
- still in perturbative region



Underlying event – Multiple Interaction

- Basic partonic perturbative cross section

$$\sigma_{\text{hard}}(p_{\perp\text{min}}^2) = \int_{p_{\perp\text{min}}^2} \frac{d\sigma_{\text{hard}}(p_{\perp}^2)}{dp_{\perp}^2} dp_{\perp}^2$$

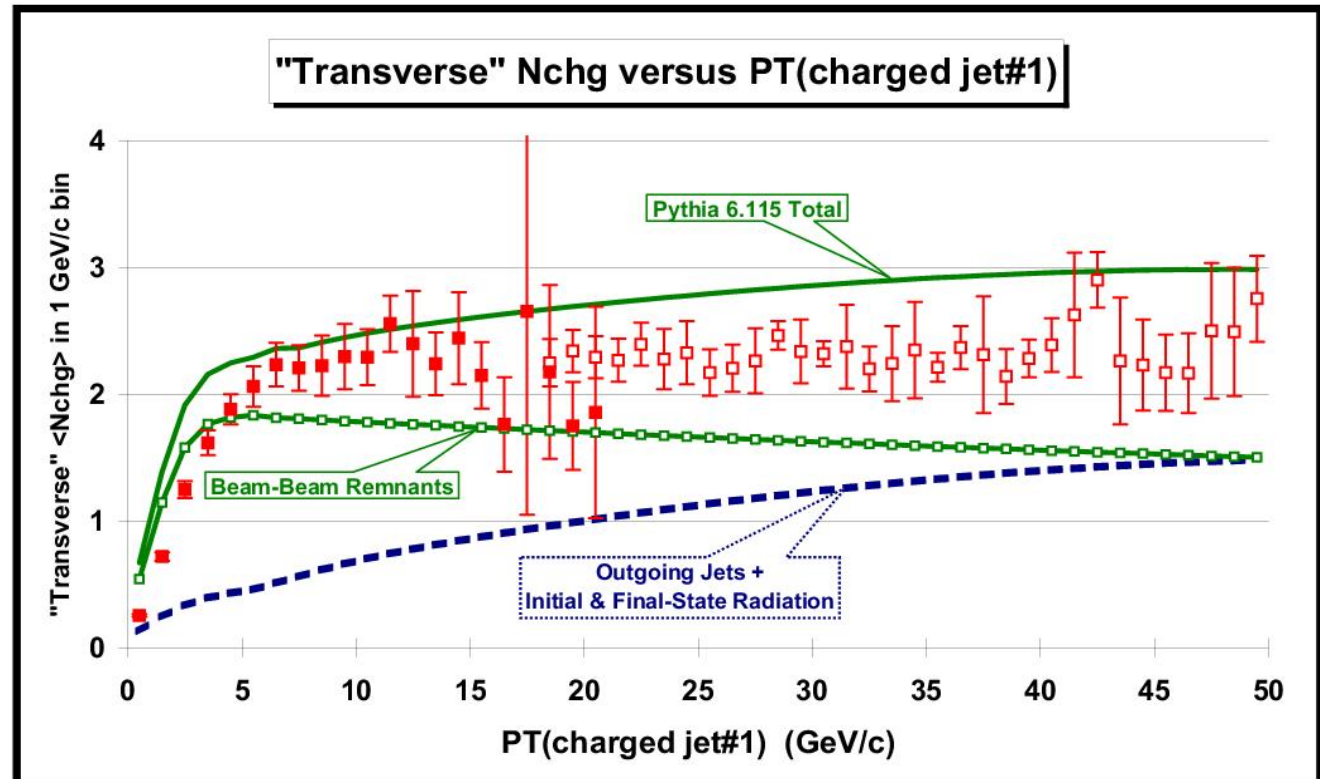
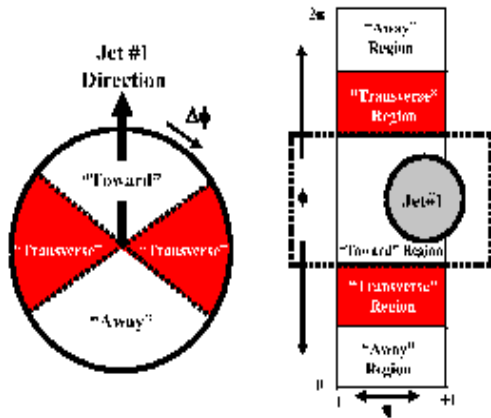
- diverges faster than $1/p_{\perp\text{min}}^2$ as $p_{\perp\text{min}} \rightarrow 0$ and exceeds eventually total inelastic (non-diffractive) cross section, resulting in more than 1 interaction per event (**multiparton interactions, MI**).
- Average number of interactions per event is given by:

$$\langle n \rangle = \frac{\sigma_{\text{hard}}(p_{\perp\text{min}})}{\sigma_{nd}}$$

- It depends on how soft interactions are treated, **BUT** also on the **parton densities** and **factorization scheme**, **parton evolution (DGLAP/BFKL) !!!!!!!**

Multiparton Interactions at TeVatron

CDF coll. PRD 65, 092002 (2002)



- Multiplicity distribution in region transverse to jet can only be described by adding multi-parton interactions (Remnant- Remnant Interactions)

Tuning to pp data... Color flow in MI

- possible scenarios for color string connection in multiparton events
- to describe underlying events.... need (CDF Tune A)

5 % quarks (default 33 %)

95 % gluons (default: 66%)

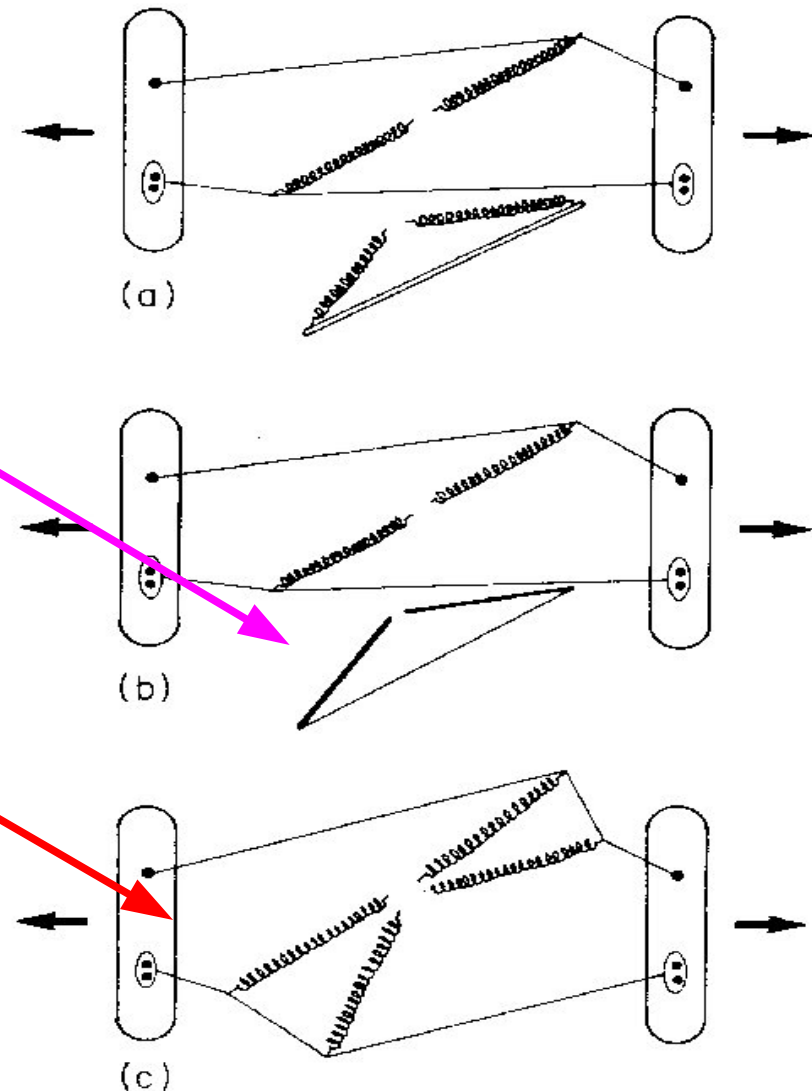
out of which 90 %
(default 33 %) are

→ smaller multiplicity
with large transverse energy

- Are there good physics reasons for this mix ???

- Highly nontrivial to describe multiplicity AND transverse energy distributions ...

T. Sjostrand, M. Ziji
PRD 36 (1987) 2019



Multiparton Interactions at LHC

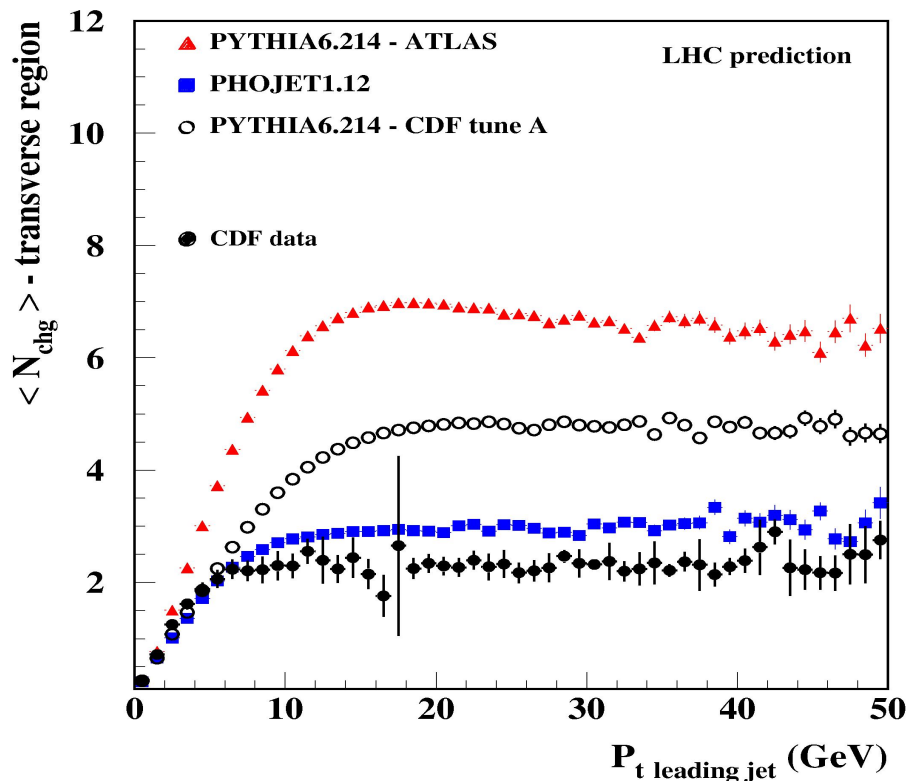
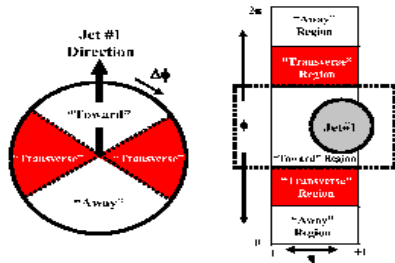
C. Buttar et al in HERA – LHC workshop proceedings hep-ph/0601012

Charged multiplicities in transverse

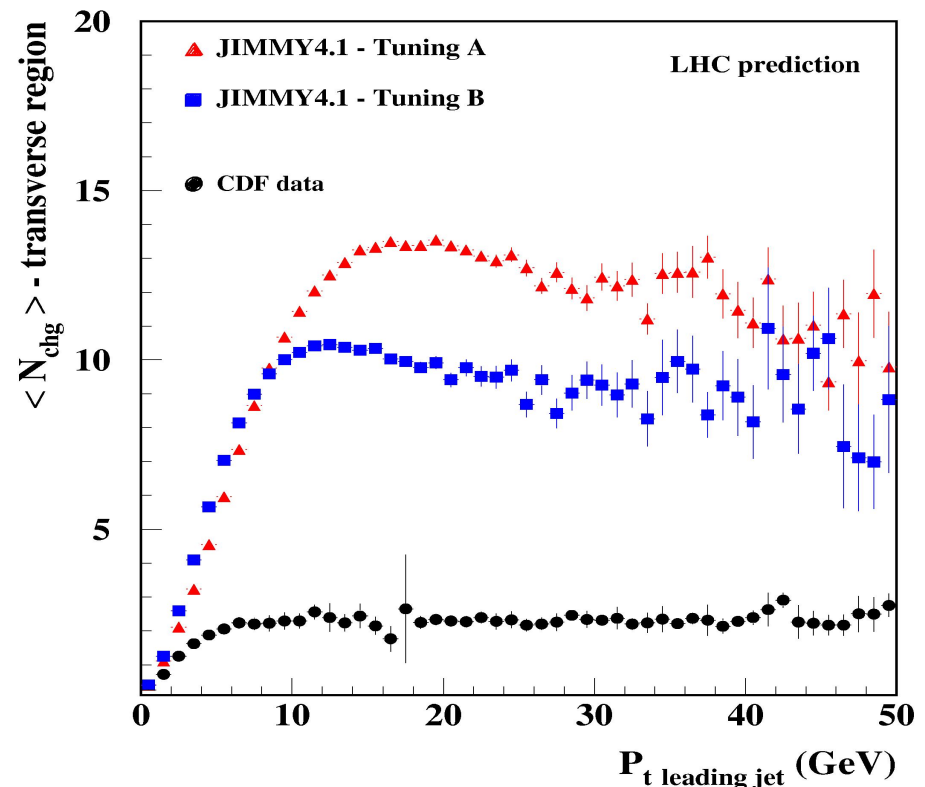
region Models tuned to TeVatron data

→ give **HUGE** differences at LHC ...

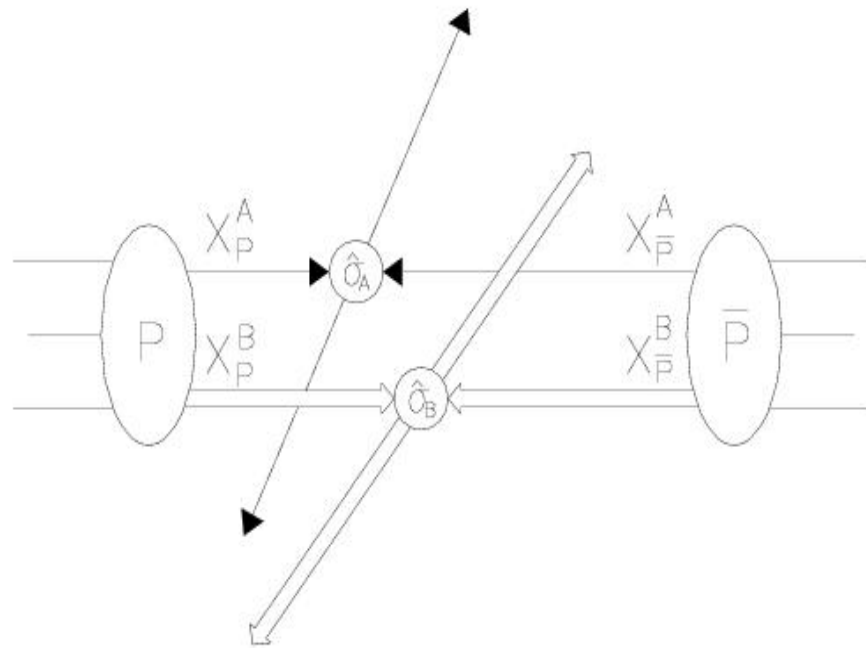
→ **better understand multiple**



ter
phc



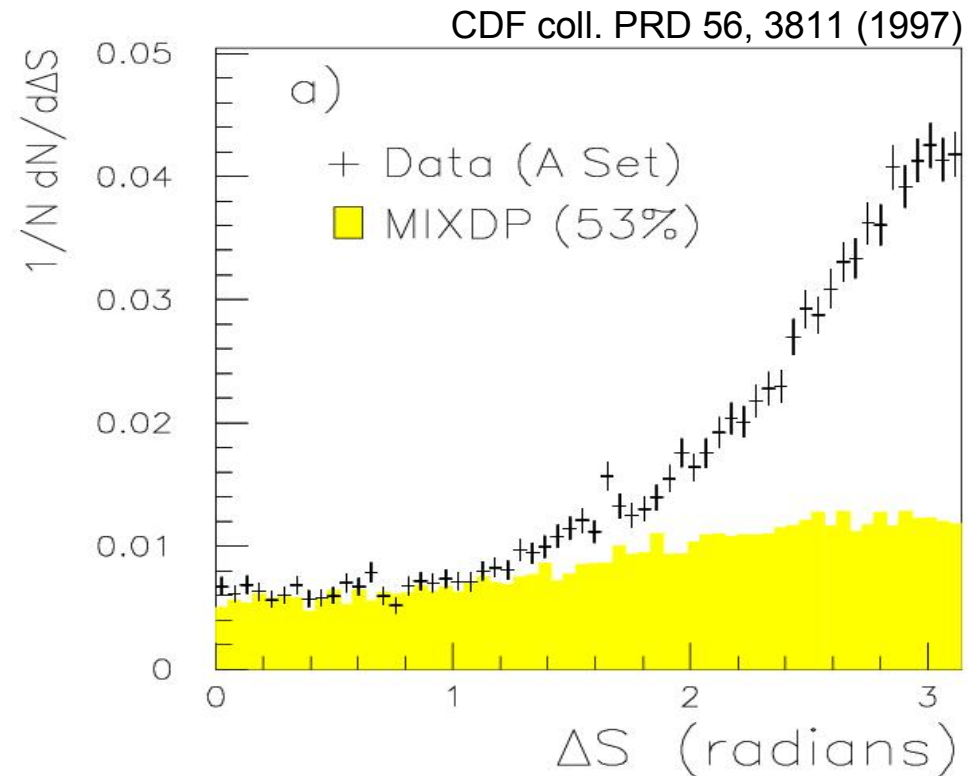
Evidence for Multi-Parton Interactions



- look at $\gamma + 3$ Jets

$$\text{with } \begin{aligned} E_T^\gamma &> 16\text{GeV} \\ E_T^{\text{Jets}} &> 5\text{GeV} \end{aligned}$$

- angular correlation of jet/photon pairs ΔS
- compare to $\gamma + 3$ Jets calculation
- **Need > 50 % double parton interaction to describe data**



Double-Parton Interactions at LHC

- xsection for $p + p \rightarrow b\bar{b}b\bar{b}$

single parton exchange (SP)

$$\sigma^{SP} \sim f^2 \hat{\sigma}(2 \rightarrow 4)$$

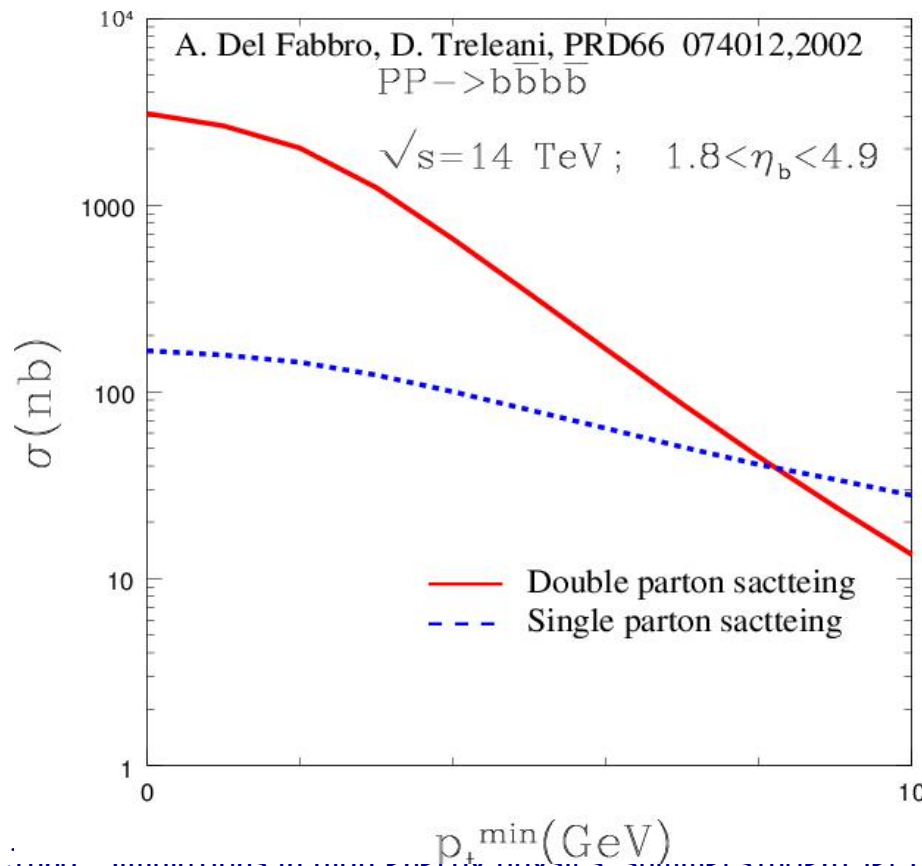
double parton exchange (DP)

$$\sigma^{DP} \sim f^4 \hat{\sigma}^2(2 \rightarrow 2)$$

- PYTHIA predictions:

$$\sigma^{DP} = 0.8 \cdots 11.1 \text{ } \mu\text{b}$$

→ Depending on model for underlying event/multi-parton interactions...



Multi-Parton Interactions at LHC

- Higgs: $p + p \rightarrow W + H + X$

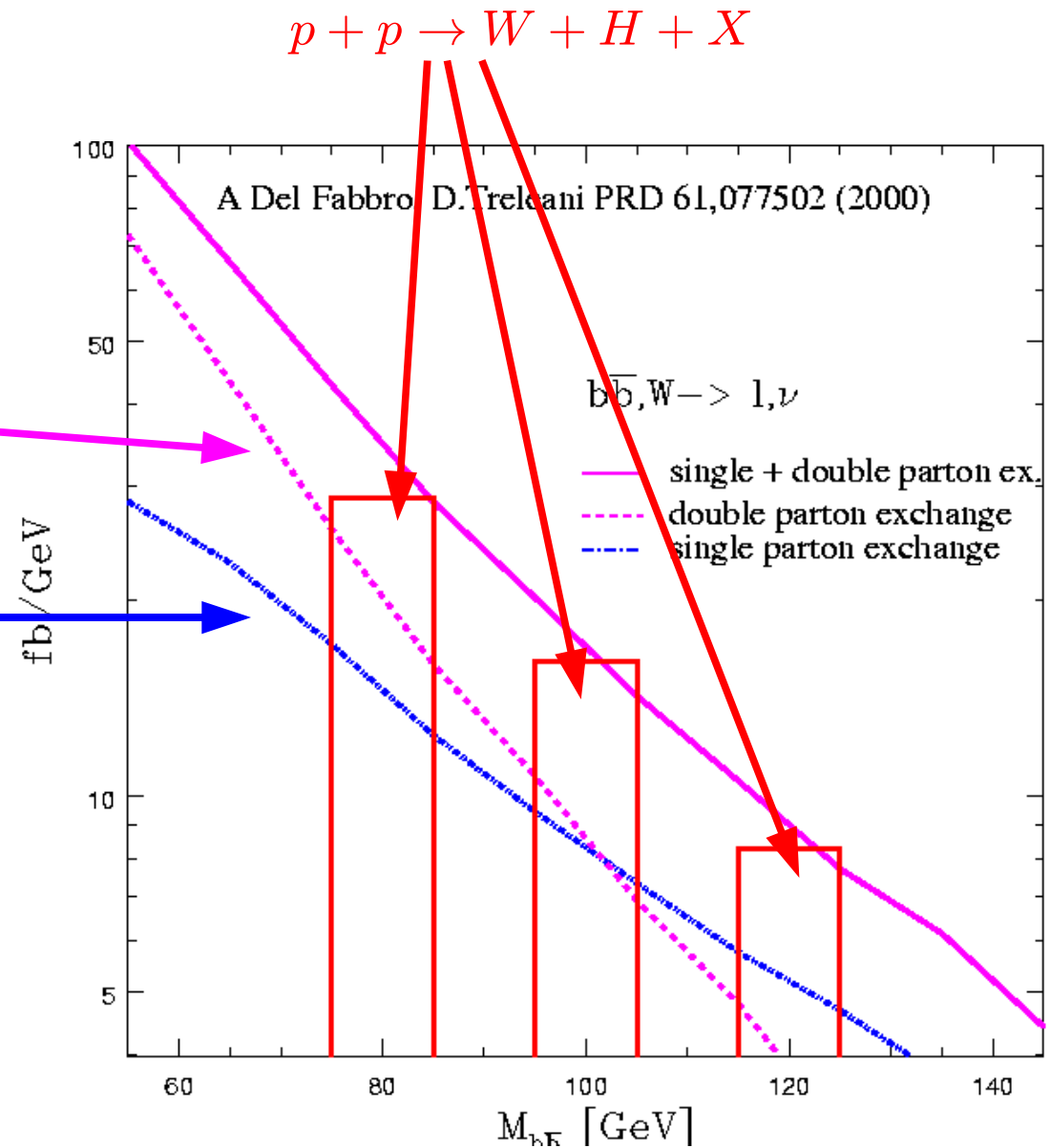
with $W \rightarrow l\nu$, $H \rightarrow b\bar{b}$

- Double parton scattering:

→ $p + p \rightarrow b\bar{b}X$

$p + p \rightarrow W + X$

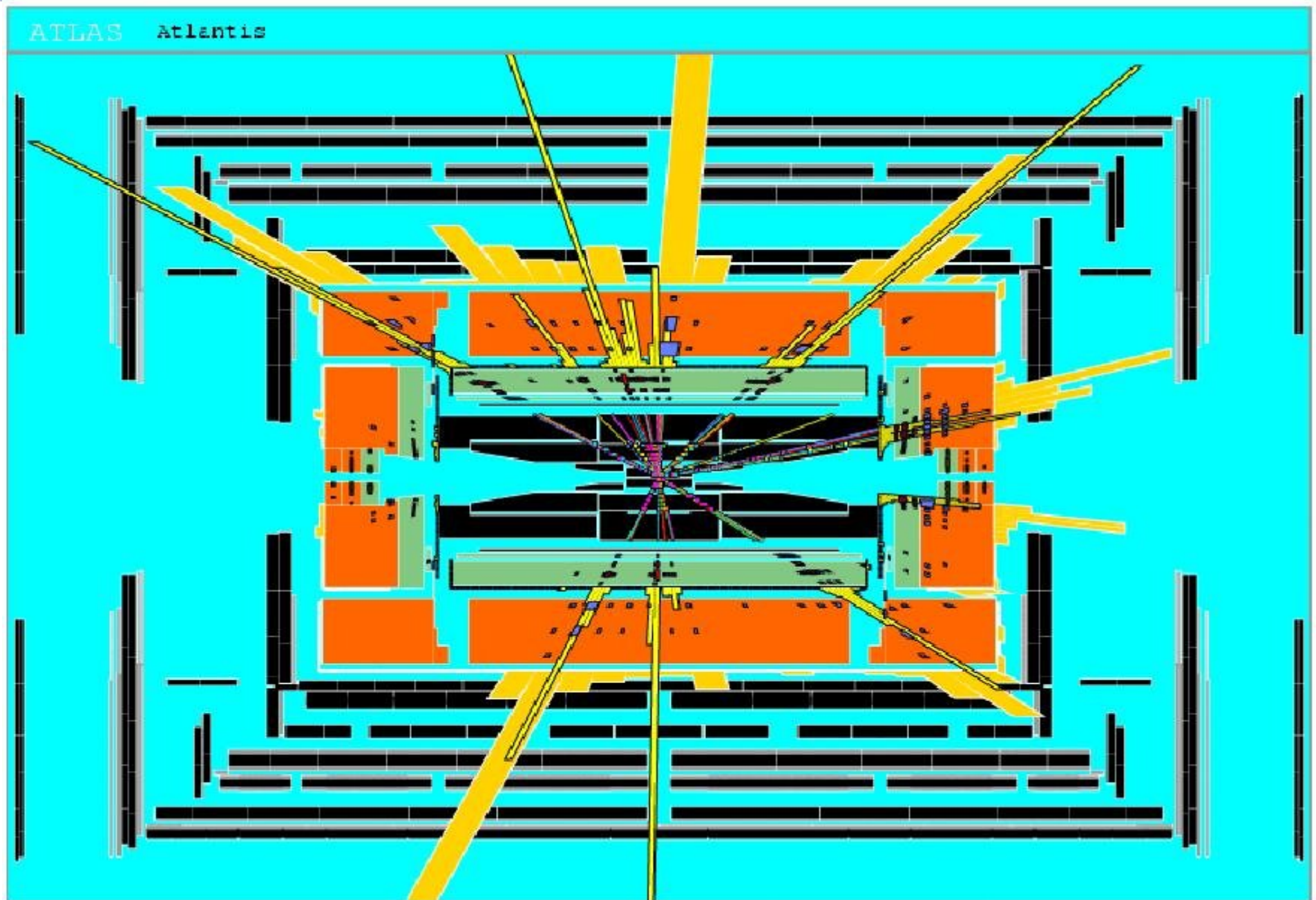
$p + p \rightarrow W + b\bar{b} + X$



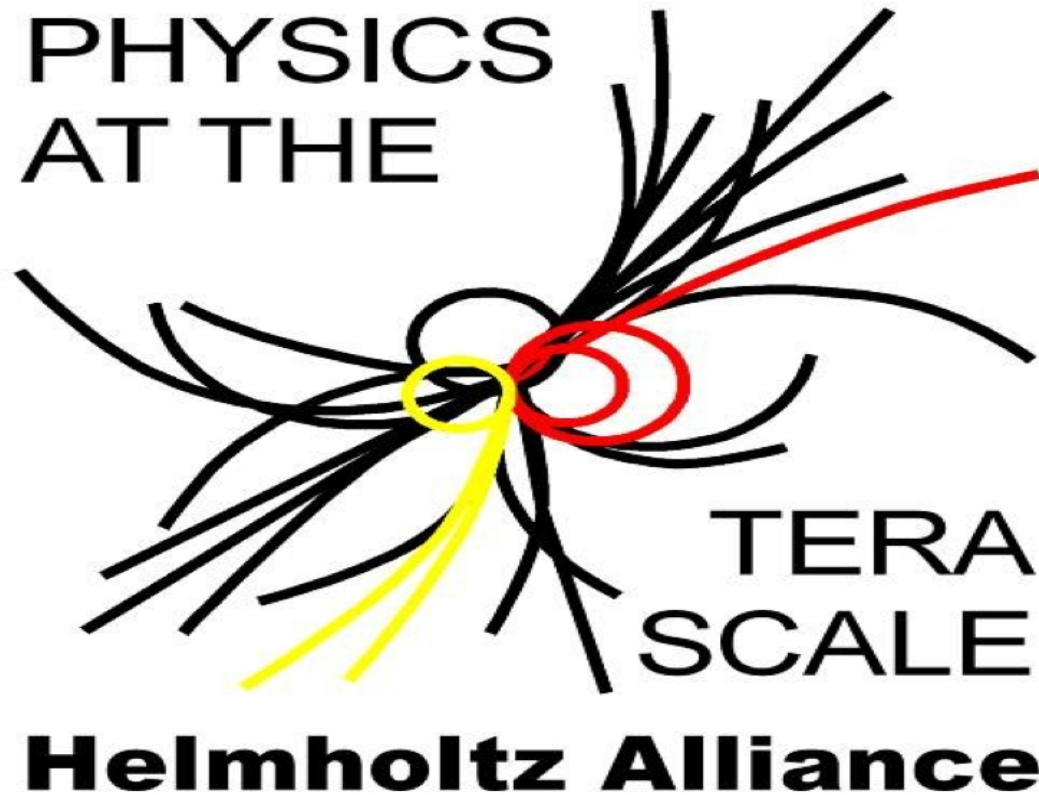


Can we now tell which type of event this is ?

What was this ?



The Analysis Centre in the



- Areas
 - Monte Carlo (user support, tuning, development ...)
 - Parton Distribution Functions
 - Statistics Tools
 - Collaborative tools (web based infos etc)

Monte Carlo group activities

- Development of Monte Carlo generators
 - Tuning of MC generators
 - PDF4MC
 - User support
- Training (schools, seminars)
 - MC school in April (in coll with MCnet)
 - > 100 participants
 - <http://www.terascale.de/mcs2008>
 - all lectures are video recorded:
<https://indico.desy.de/conferenceOtherViews.py?view=stc>
and in mp4 format for your ipod:
ftp://webcast.desy.de/Terascale_MonteCarloSchool2008

Monte Carlo School
PHYSICS AT THE TERASCALE
Strategic Helmholtz Alliance

PHYSICS AT THE TERA SCALE
Helmholtz Alliance

21-24 April 2008,
DESY Hamburg

Topics:

- Monte Carlo techniques and physics (L. Lönnblad)
- NLO Calculations (NN)
- NLO and parton showers (M. Dinsdale)
- Monte Carlo event generators
 - CASCADE (H. Jung)
 - HERWIG (S. Gieseke, P. Richardson)
 - PYTHIA (T. Sjöstrand)
 - SHERPA (F. Krauss)
- Exercises (L. Sonnenschein et al.)

The school covers Monte Carlo techniques and applications in NLO calculations as well as full hadron level Monte Carlo event generators. Predictions coming from different generators will be compared in practical exercises and first steps for comparison with measurements will be shown in tutorials.

Registration deadline: 15.03.2008
Please register via the school webpage.

Organising Committee: Hannes Jung, J. Katzy, A. Knutson, K. Kutak, Serguei Levonian
<http://www.terascale.de/mcs2008>

Deutsches Elektronen-Synchrotron DESY • Forschungszentrum Karlsruhe GmbH • Max-Planck-Institut für Physik München • Rheinisch-Westfälische Technische Hochschule Aachen
• Humboldt-Universität Berlin • Rheinische Friedrich-Wilhelms-Universität Bonn • Universität Dortmund • Technische Universität Dresden • Albert-Ludwigs-Universität
Freiburg • Julius-Liebig-Universität Gießen • Georg-August-Universität Göttingen • Universität Hamburg • Universität Heidelberg • Universität Karlsruhe • Johannes
Gutenberg Universität Mainz • Ludwig-Maximilians-Universität München • Universität Potsdam • Universität Regensburg • Julius-Maximilians-Universität Würzburg • Bergische
Universität Wuppertal

Monte Carlo group activities

If you are interested to do your

- diploma/masters thesis
- PhD
- postdoc

please get in contact with us...

There are plenty of possibilities and positions to do interesting physics with MC simulations and help to find extra dimensions or SUSY or new phenomena in QCD

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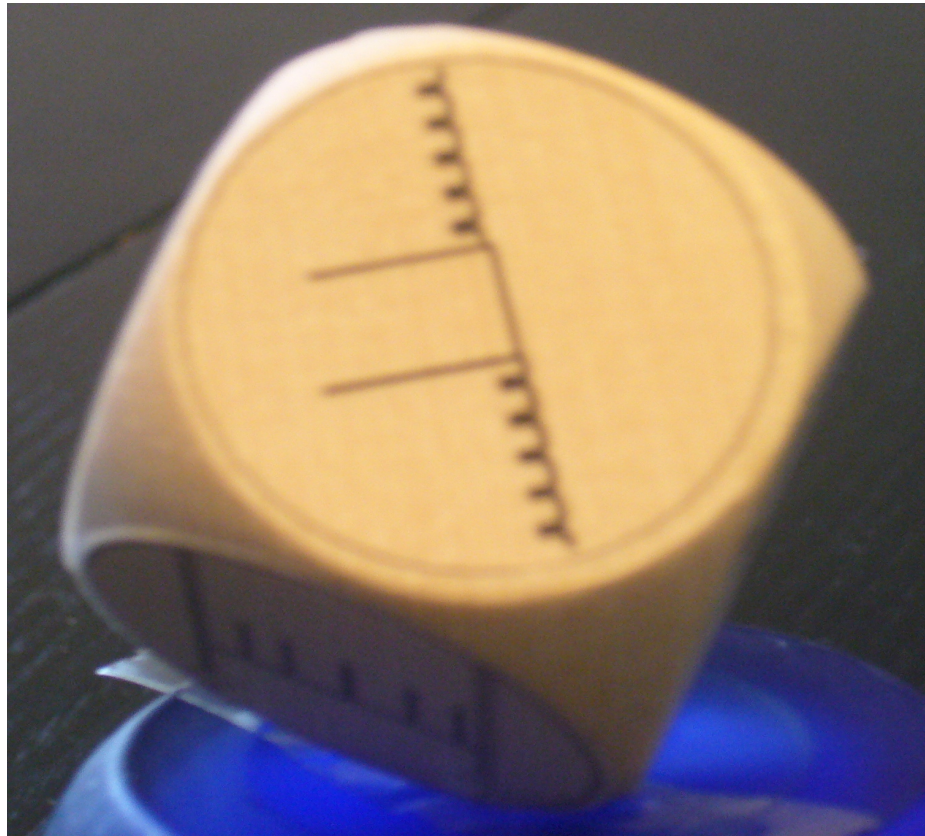
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Typical Monte Carlo toys ...

The necessary tool for a true Monte Carlo event generator:



Conclusion

- Monte Carlo event generators are needed to calculate multi-parton cross sections
- Monte Carlo method is a well defined procedure
 - parton shower are essential
 - hadronization is needed to compare with measurements
- MC approach extended from simple $e+e^-$ processes to
 - ep processes
 - pp processes and heavy Ion processes
- proper Monte Carlos are essential for any measurement

**Monte Carlo event generators
contain all our physics knowledge !!!!!**

List of available MC programs

- HERA Monte Carlo workshop: www.desy.de/~heramc
- **ARIADNE**
A program for simulation of QCD cascades implementing the color dipole model
- **CASCADE**
is a full hadron level Monte Carlo generator for ep and pp scattering at small x build according to the CCFM evolution equation, including the basic QCD processes as well as Higgs and associated W/Z production
- **HERWIG**
General purpose generator for Hadron Emission Reactions With Interfering Gluons; based on matrix elements, parton showers including color coherence within and between jets, and a cluster model for hadronization.
- **JETSET**
The Lund string model for hadronization of parton systems.

List of available MC programs

- LDCMC

A program which implements the Linked Dipole Chain (LDC) model for deeply inelastic scattering within the framework of ARIADNE. The LDC model is a reformulation of the CCFM model.

- PHOJET

Multi-particle production in high energy hadron-hadron, photon-hadron, and photon-photon interactions (hadron = proton, antiproton, neutron, or pion).

- PYTHIA

General purpose generator for e^+e^- pp and ep-interactions, based on LO matrix elements, parton showers and Lund hadronization.

- RAPGAP

A full Monte Carlo suited to describe Deep Inelastic Scattering, including diffractive DIS and LO direct and resolved processes. Also applicable for photo-production and partially for pp scattering.

Literature & References

- F. James Rep. Prog. Phys., Vol 43, 1145 (1980)
- Glen Cowan STATISTICAL DATA ANALYSIS. Clarendon, 1998.
- Particle Data Book S. Eidelman et al., Physics Letters B592, 1 (2004)
section on: Mathematical Tools (<http://pdg.lbl.gov/>)
- Michael J. Hurben Buffons Needle
(<http://www.angelfire.com/wa/hurben/buff.html>)
- J. Woller (Univ. of Nebraska-Lincoln) *Basics of Monte Carlo Simulations*
(<http://www.chem.unl.edu/zeng/joy/mclab/mcintro.html>)
- Hardware Random Number Generators:
A Fast and Compact Quantum Random Number Generator
(<http://arxiv.org/abs/quant-ph/9912118>)
Quantum Random Number Generator
(<http://www.idquantique.com/products/quantis.htm>)
Hardware random number generator (<http://en.wikipedia.org/wiki/>)
- Monte Carlo Tutorials
(<http://www.cooper.edu/engineering/chemechem/MMC/tutor.html>)
- History of Monte Carlo Method
(<http://www.geocities.com/CollegePark/Quad/2435/history.html>)

Literature & References (cont'd)

- T. Sjostrand et al
PYTHIA/JETSET manual - The Lund Monte Carlos
<http://www.thep.lu.se/tf2/staff/torbjorn/Pythia.html>
- H. Jung
[RAPGAP manual](http://www.desy.de/~jung/rapgap.html)
<http://www.desy.de/~jung/rapgap.html>
[CASCADE manual](http://www.desy.de/~jung/cascade.html)
<http://www.desy.de/~jung/cascade.html>
- V. Barger and R. J.N. Phillips
Collider Physics
Addison-Wesley Publishing Comp. (1987)
- R.K. Ellis, W.J. Stirling and B.R. Webber
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Cambridge University Press (1996)

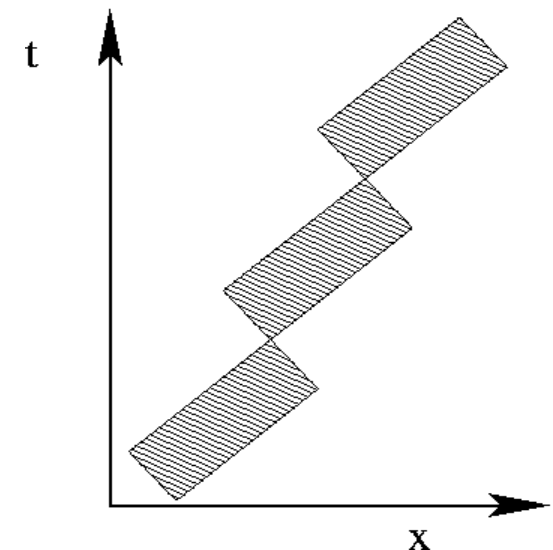
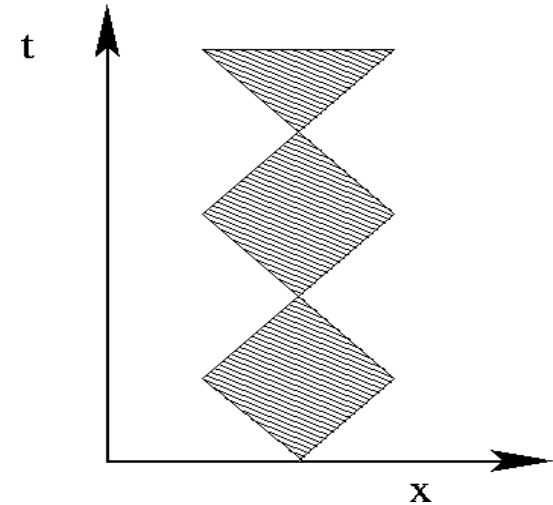
General literature

- Many new books are available in DESY library **NEW ... ask at the desk there ...**
- Statistische und numerische Methoden der Datenanalyse
V. Blobel & E. Lohrmann
- STATISTICAL DATA ANALYSIS. Glen Cowan.
- Particle Data Book S. Eidelman et al., Physics Letters B592, 1 (2004)
(<http://pdg.lbl.gov/>)
- Applications of pQCD R.D. Field Addison-Wesley 1989
- Collider Physics V.D. Barger & R.J.N. Phillips Addison-Wesley 1987
- Deep Inelastic Scattering. R. Devenish & A. Cooper-Sarkar, Oxford 2
- Handbook of pQCD G. Sterman et al
- Quarks and Leptons, F. Halzen & A.D. Martin, J.Wiley 1984
- QCD and collider physics R.K. Ellis & W.J. Stirling & B.R. Webber Cambridge 1996
- QCD: High energy experiments and theory G. Dissertori, I. Knowles, M. Schmelling Oxford 2003

Backup Slides

Lund string fragmentation

- in a color neutral qq-pair, a color force is created in between
- color lines of the force are concentrated in a narrow tube connecting q and \bar{q} , with a string tension of $\kappa \sim 1 \text{ GeV/fm} \sim 0.2 \text{ GeV}^2$
- as q and \bar{q} are moving apart in qq rest frame, they are de-accelerated by string tension, accelerated back etc ... (periodic oscillation)
- viewed in a moving system, the string is boosted



Fragmentation in the String Model

- hadronization: iterative process
- string breaks in $q\bar{q}$ pairs (still respecting color flow)
- select transverse motion with $m=m_{q\bar{q}}$ (and flavor)

$$P \sim \exp\left(-\frac{\pi m_t^2}{\chi}\right) = \exp\left(-\frac{\pi m^2}{\chi}\right) \exp\left(-\frac{\pi p_t^2}{\chi}\right)$$

- suppression of heavy quark production

$$u : d : s : c \sim 1 : 1 : 1 : 0.37 : 10^{-10}$$

actually leave it as a free parameter

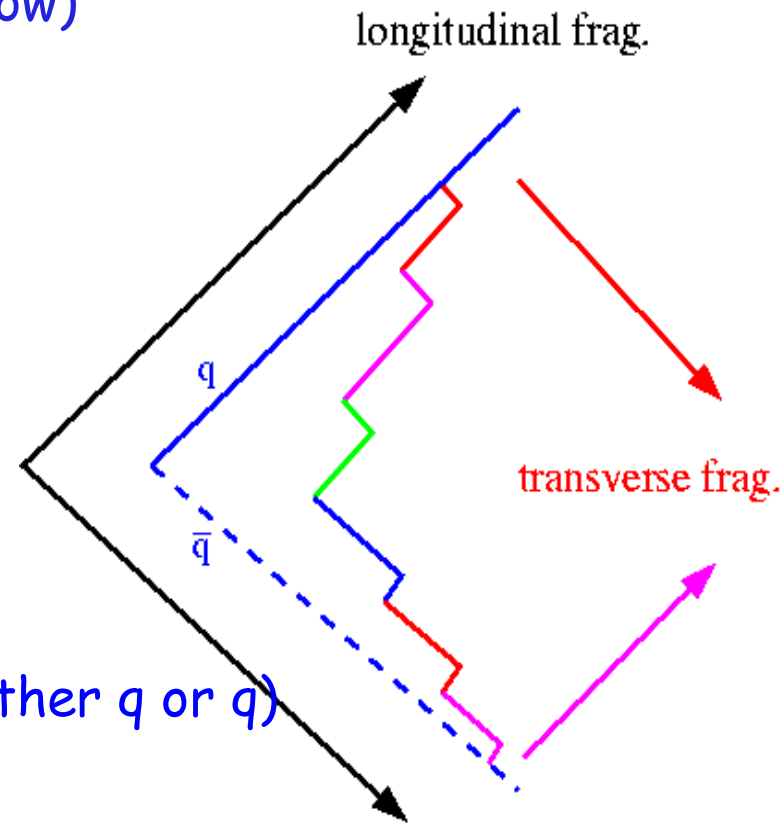
- longitudinal fragmentation

symmetric fragmentation function (from either q or \bar{q})

$$f(z) \sim z^{-1}(1-z)^a \exp(-b m_+^2/z)$$

harder spectrum for heavy quarks

- start from q or \bar{q}
- repeat until cutoff is reached
- heavy use of random numbers and importance sampling method



Hadronization: particle masses and decays

- particle masses

- taken from PDG, where known, otherwise from constituent masses

- particle widths

- in hard scattering production process short lived particles (ρ, Δ) have nominal mass, without mass broadening

- in hadronization use Breit-Wigner:

- lifetimes
$$\mathcal{P}(m)dm \propto \frac{1}{(m - m_0)^2 + \Gamma^2/4}$$
- related to widths ... but for practical purpose separated

- $P(\tau)d\tau \sim \exp(-\tau/\tau_0) d\tau$

- calculate new vertex position $v' = v + \tau p/m$

- decays

- taken from PDG, where known

- assume momentum distribution given by phase space only

- exceptions, like $\omega, \phi \rightarrow \pi^+ \pi^- \pi^0$, or $D \rightarrow K\pi$, $D^* \rightarrow K\pi\pi$ and some semileptonic decays use matrix elements