Introduction to Particle Accelerators

Bernhard Holzer, DESY

DESY Summer Student Lectures 2007

Introduction

historical development & first principles
components of a typical accelerator
„...the easy part of the story"

The state of the art in high energy machines:

The synchrotron: linear beam optics
colliding beams,
luminosity
„... how does it work ?"
„...does it ?"

Introduction to Particle Accelerators

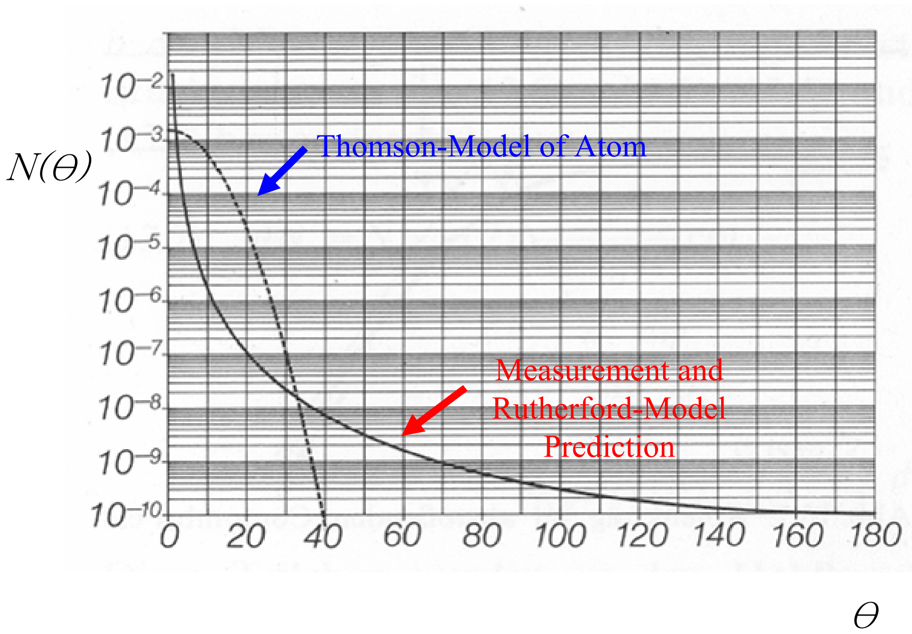
I.) Historical note:

... the first steps in particle physics

Rutherford Scattering, 1906...1913

Using radioactive particle sources:
α-particles of some MeV energy

*

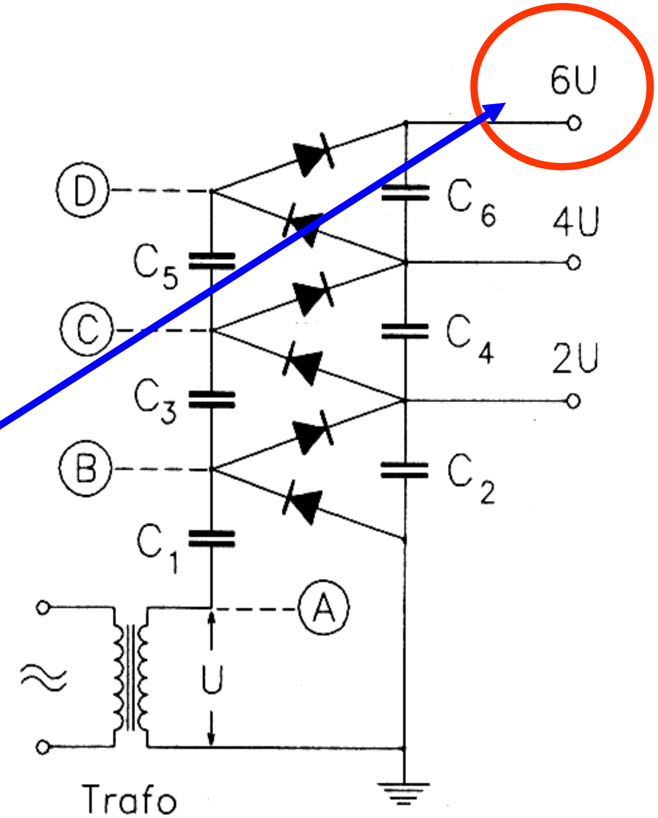


$$N(\theta) = \frac{N_i n t Z^2 e^4}{(8\pi\varepsilon_0)^2 r^2 K^2} * \frac{1}{\sin^4(\theta/2)}$$

Electrostatic Machines

II.) The Cockroft Walton Generator

create a high Voltage DC by rectifying an AC Voltage: „cascade generator“



Force: $$\left|\vec{F}\right| = q * \left|\vec{E}\right| = q * \frac{U}{d}$$

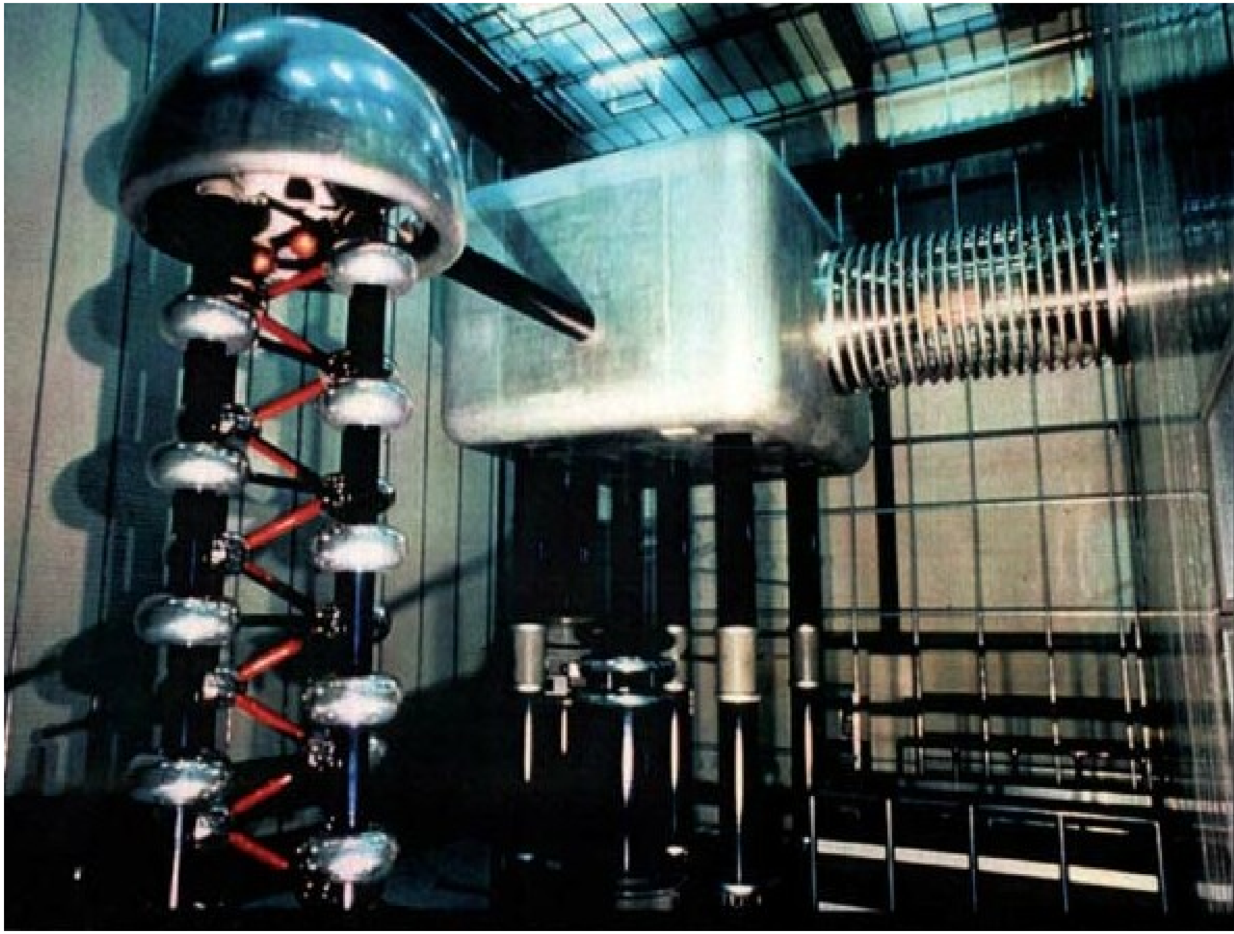
Energy: $$\Delta W = \int_{r1}^{r2} \vec{F}\, d\vec{r} = \int_{r1}^{r2} q * \vec{E}\, d\vec{r} = q * U$$

usefull energy unit: „eV“ ...

...1 eV = 1.6*10^{-19} J

1932: First particle beam (protons) produced for nuclear reactions: splitting of Li-nuclei with a proton beam of 400 keV

Particle source: Hydrogen discharge tube on a 400 kV level
Accelerator: evacuated glas tube
Target: Li-Foil on earth potential

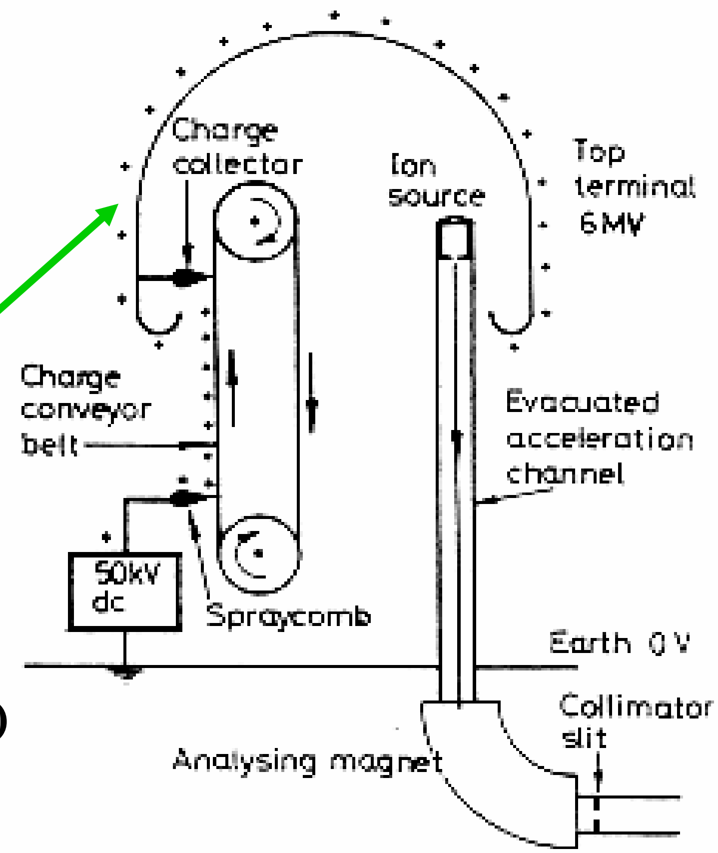


Example:
Pre-accelerator for Protons at PSI (Villingen)

Electrostatic Machines

III.) (Tandem –) van de Graaff Accelerator

creating high voltages by mechanical transport of charges



*** Terminal Potential: U ≈ 12 ...28 MV using high pressure gas to suppress discharge (SF_6)**

Problems: * Particle energy limited by high voltage discharges

*** high voltage can only be applied once per particle ...**

... or twice ?

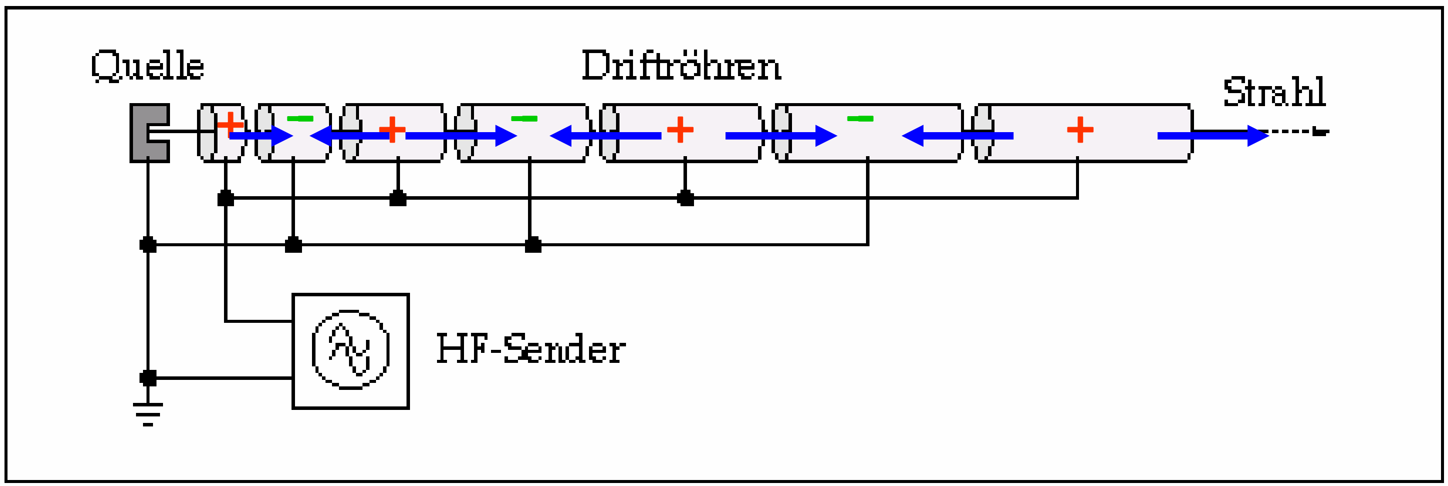
* The „Tandem principle“: Apply the accelerating voltage twice ...
... by working with negative ions (e.g. H^-) and
stripping the electrons in the centre of the structure

Example for such a „steam engine“: 12 MV-Tandem van de Graaff Accelerator at MPI Heidelberg

IV.) Linear Accelerators

1928, Wideroe: **how can the acceleration voltage be applied several times to the particle beam**

schematic Layout:



* **acceleration of the proton in the first gap**
* **voltage has to be „flipped“ to get the right sign in the second gap → RF voltage**
→ shield the particle in drift tubes during the negative half wave of the RF voltage

Beam Energy: Acceleration in the Wideroe Structure

1.) Energy gained after *n* acceleration gaps

$$E_n = n * q * U_0 * \sin\psi_s$$

n number of gaps between the drift tubes
q charge of the particle
U_0 Peak voltage of the RF System
Ψ_S synchronous phase of the particle

2.) kinetic energy of the particles

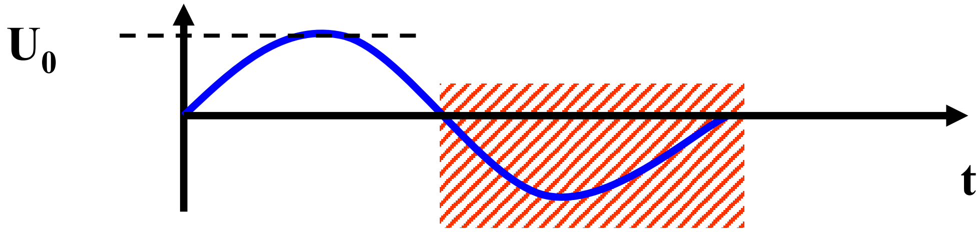
$$E_n = \frac{1}{2} m * v_n^{\,2}$$

valid for non relativistic particles ...

velocity of the particle (from (1) and (2))

$$v_n = \sqrt{\frac{2E_n}{m}} = \sqrt{\frac{2 * n * q * U_0 * \sin\psi_S}{m}}$$

3.) shielding of the particles during the negative half wave of the RF



Time span of the negative half wave: $\tau_{RF}/2$

Length of the n-th drift tube:

$$l_n = v_n * \frac{\tau_{RF}}{2} = v_n * \frac{1}{2\nu_{RF}}$$

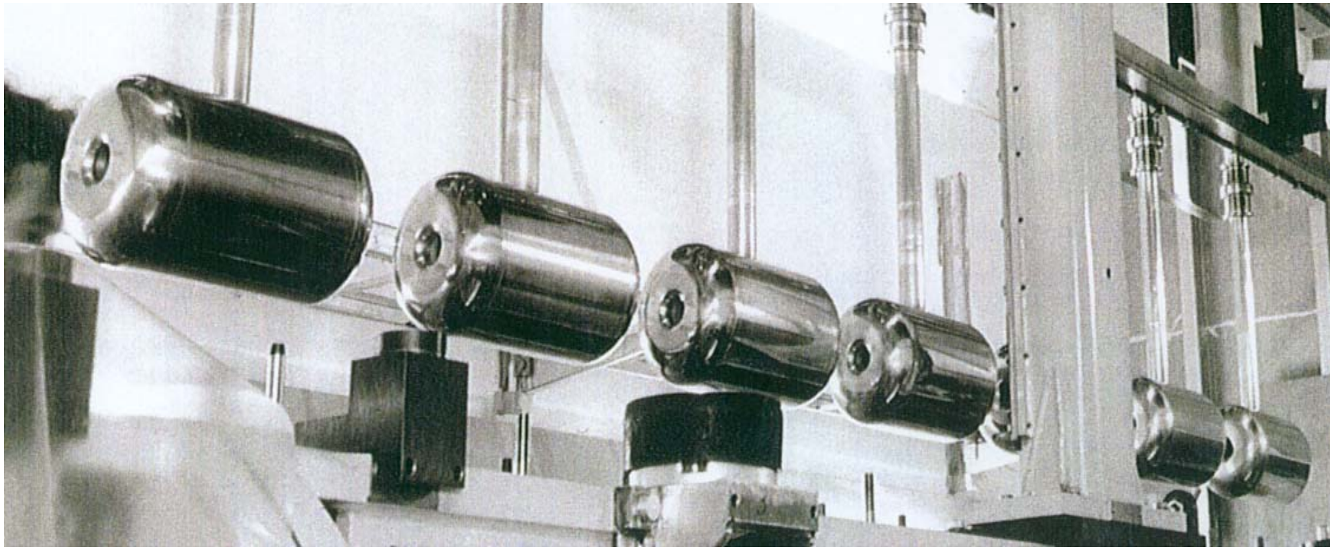
! high RF frequencies make small accelerators

length of the *n-th* drift tube ... or ... distance between two accelerating gaps:

$$l_n = \frac{1}{\nu_{RF}} \sqrt{\frac{n * q * U_0 * \sin\psi_S}{2m}}$$

Example: DESY Accelerating structure of the Proton Linac

$$E_{total} = 988\,MeV$$



reminder of some relativistic formula

rest energy $E_0 = m_0 c^2 = 938\ MeV$

total energy $E = \gamma * E_0 = \gamma * m_0 c^2$

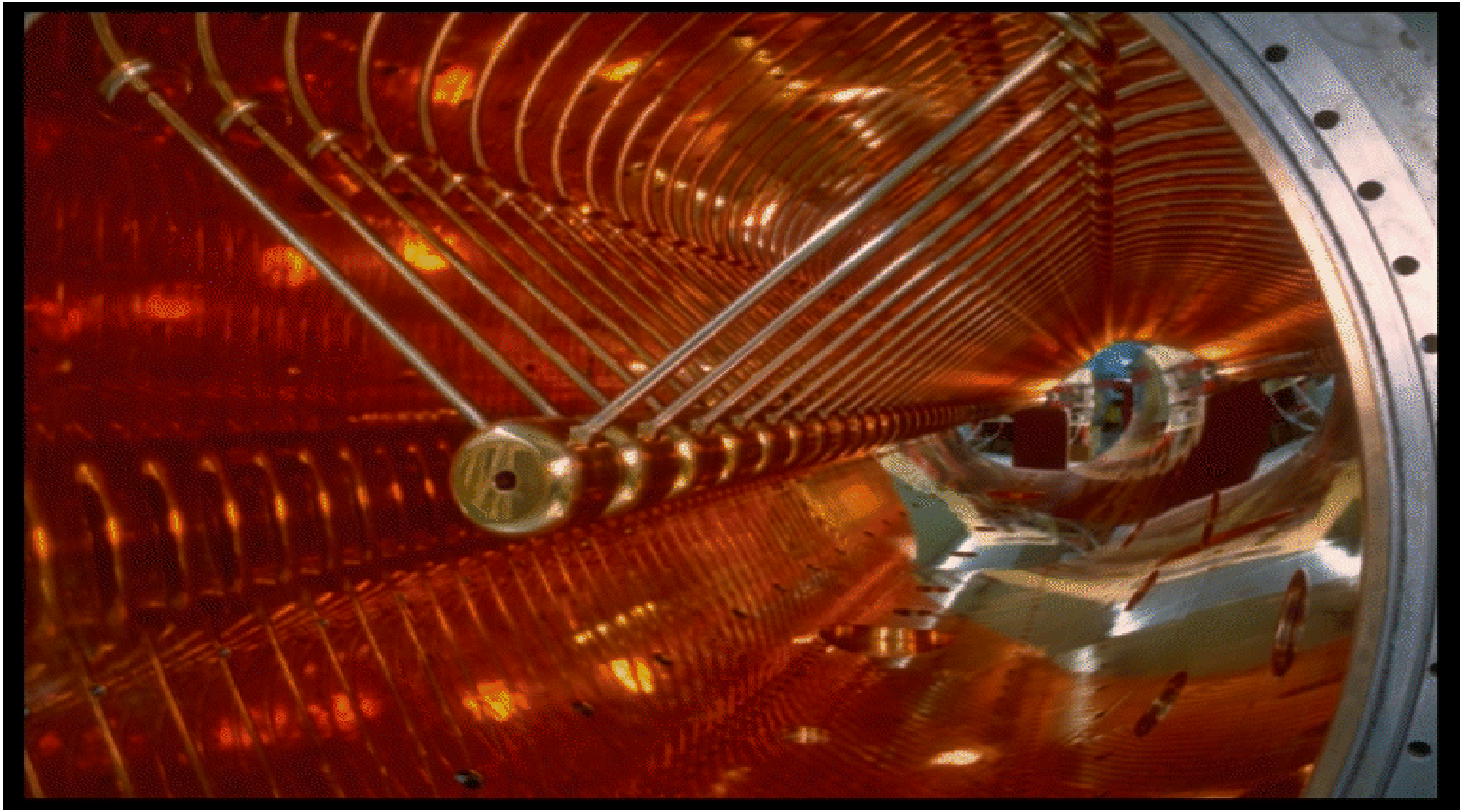
$$E_{kin} = E_{total} - m_0 c^2$$

$$E_{kin} = 50\,MeV$$

momentum $E^2 = c^2 p^2 + m_0{}^2 c^4$

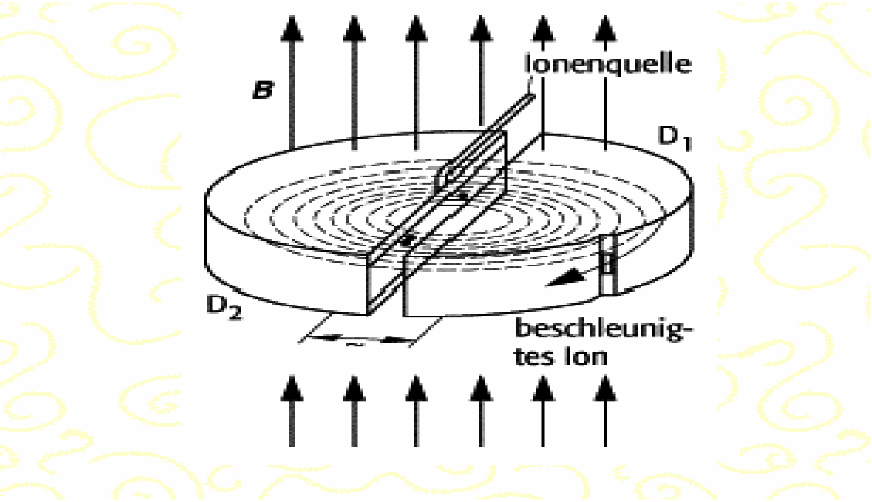
$$p = 310\,MeV / c$$

GSI: ***Unilac, typical Energie ≈ 20 MeV per Nukleon, $\beta \approx 0.04 \ldots 0.6$, Protons/Ions, $\nu = 110$ MHz***



V.) The Cyclotron: (~1930)

circular accelerator with a constant magnetic field
B = const



Lorentz force:

$$\vec{F} = q * (\vec{v} \times \vec{B})$$

centrifugal force:

$$F = \frac{m * v^2}{\rho}$$

condition for a circular particle orbit:

$$q * v * B = \frac{m * v^2}{\rho} \quad \rightarrow \quad B * \rho = p / q$$

$$\rightarrow \quad \frac{\rho}{v} = \frac{m}{q * B}$$

time for one revolution: $T = 2\pi \frac{\rho}{v} = 2\pi \frac{m}{q * B_z}$

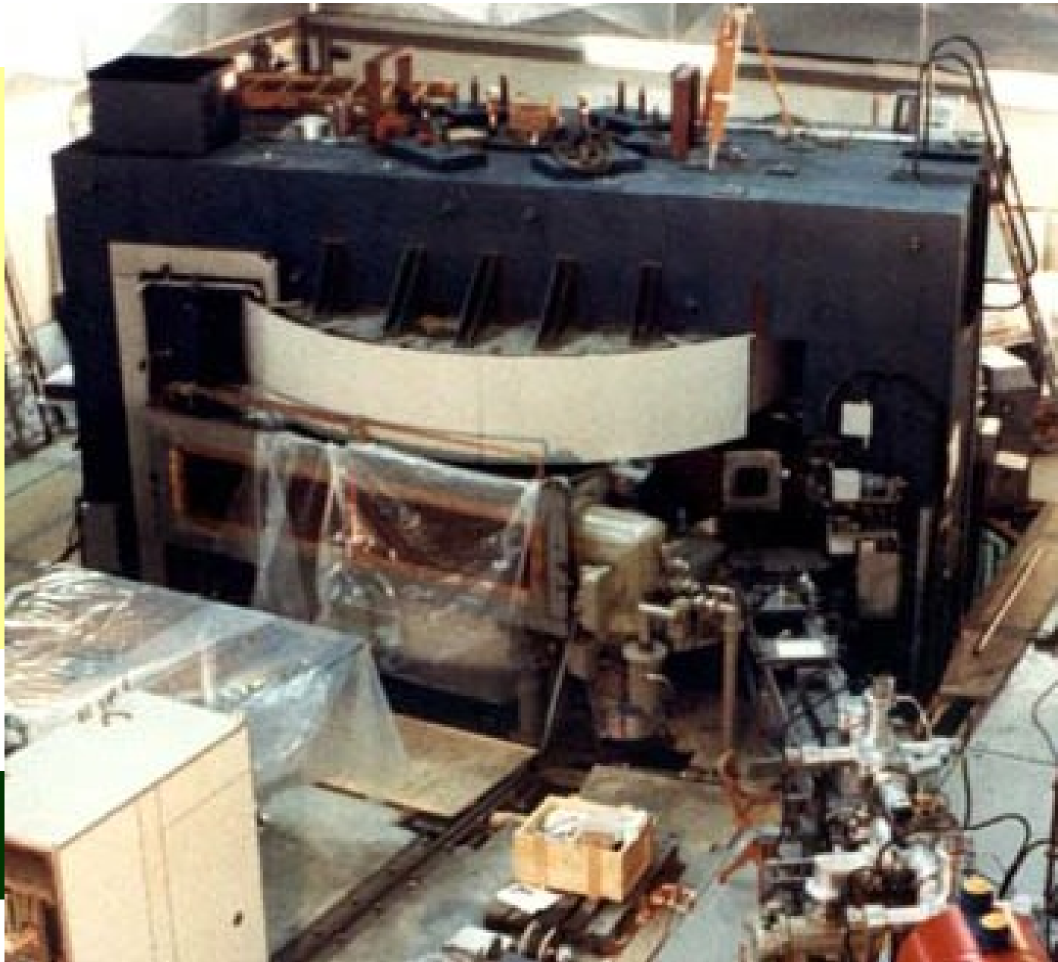
revolution frequency $\omega_z = 2\pi \frac{1}{T} = \frac{q}{m} * B_z \rightarrow \omega_z = const.$

! ω is constant for a given q & B

!! B*ρ = p/q
large momentum → huge magnets

!!!! ω ~ 1/m ≠ const.
works properly only for non relativistic particles

Example: cyclotron at PSI



The state of the art in high energy machines:

The synchrotron: linear beam optics

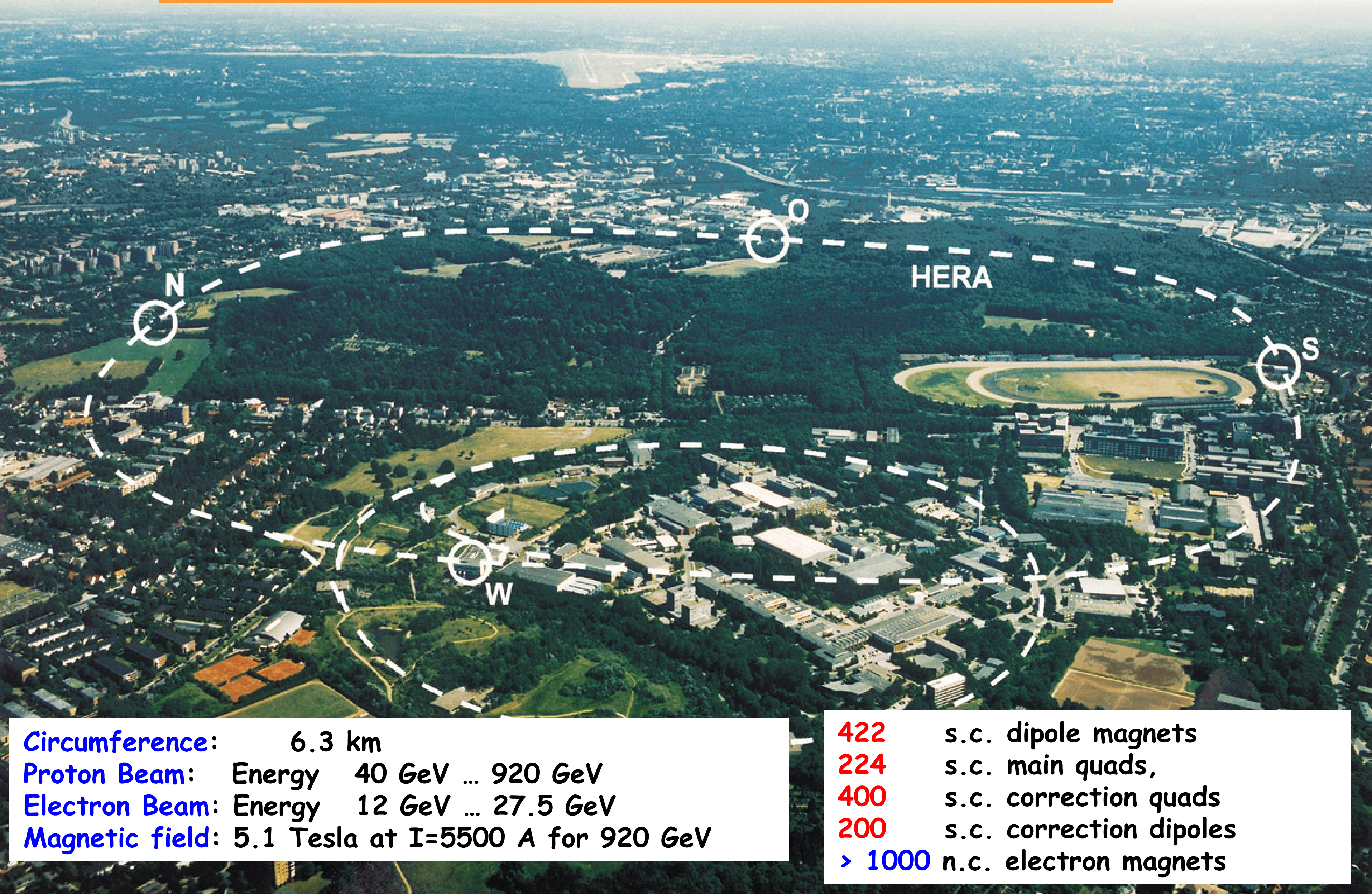
colliding beams,

luminosity

„... how does it work ?"

„...does it ?"

The state of the art in high energy acceleration: The Synchrotron



Circumference: 6.3 km
Proton Beam: Energy 40 GeV … 920 GeV
Electron Beam: Energy 12 GeV … 27.5 GeV
Magnetic field: 5.1 Tesla at I=5500 A for 920 GeV

422	s.c. dipole magnets
224	s.c. main quads,
400	s.c. correction quads
200	s.c. correction dipoles
> 1000	n.c. electron magnets

Design Principles of a Synchrotron

I.) the bending magnets … guide the particles

„ ... in the end and after all it should be a kind of circular machine“

→ need transverse deflecting force

... again ... the Lorentz force

$$\vec{F} = q * (\cancel{\vec{E}} + \vec{v} \times \vec{B})$$

typical velocity in high energy machines: $v \approx c \approx 3 * 10^8 \, m/s$

old greek dictum of wisdom:

if you are clever, you use magnetic fields in an accelerator where ever it is possible.

But remember: magn. fields act allways perpendicular to the velocity of the particle

→ only bending forces, → no „beam acceleration“

Lattice Design: Prerequisites

Lorentz force $\vec{F} = q * (\vec{v} \times \mathbf{B})$

High energy accelerators → circular machines
somewhere in the lattice we need a number of dipole magnets, that are bending the design orbit to a closed ring

In a constant external magnetic field the particle trajectory will be a part of a circle and ... the centrifugal force will be equal to the Lorentz force

$$e * v * B = \frac{m v^2}{\rho} \qquad \rightarrow e * B = \frac{mv}{\rho} = p / \rho$$

$$\rightarrow B * \rho = p / e$$

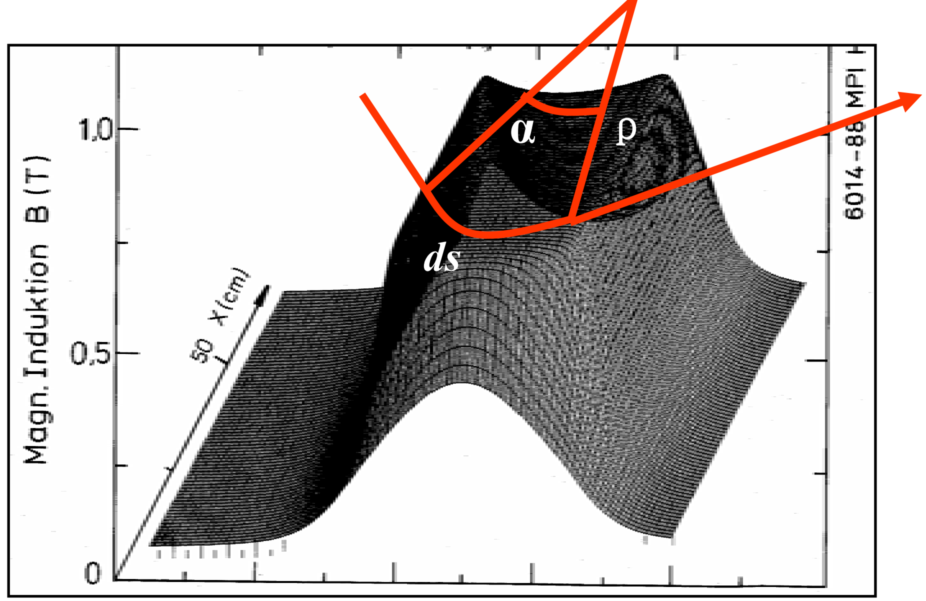
p = momentum of the particle,
ρ = curvature radius

B*ρ is called the "beam rigidity"

Example:
heavy ion storage ring: TSR
8 dipole magnets of equal bending strength

Circular Orbit:

$$\alpha = \frac{s}{\rho} \approx \frac{l}{\rho} \qquad \alpha = \frac{B*l}{B*\rho}$$



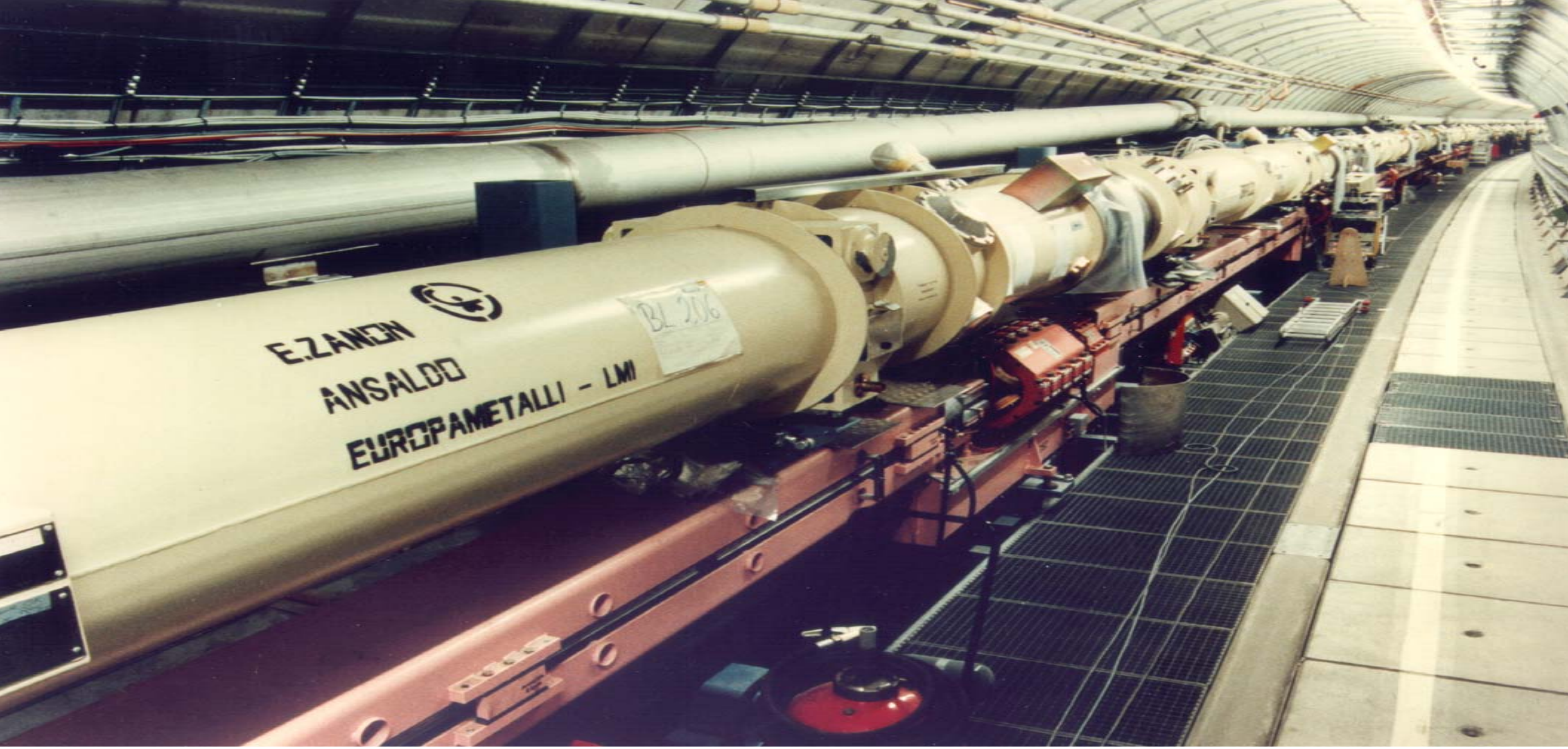
field map of a storage ring dipole magnet

The angle swept out in one revolution must be 2π, so

$$\alpha = \frac{\int B\,dl}{B*\rho} = 2\pi \quad \text{... for a full circle}$$

$$\rightarrow B*L = 2\pi * \frac{p}{q}$$

The overall length of all dipole magnets multiplied by the dipole field corresponds to the momentum (≈ energy) of the beam !



Example HERA:

920 GeV Proton storage ring

number of dipole magnets N = 416

l = 8.8m

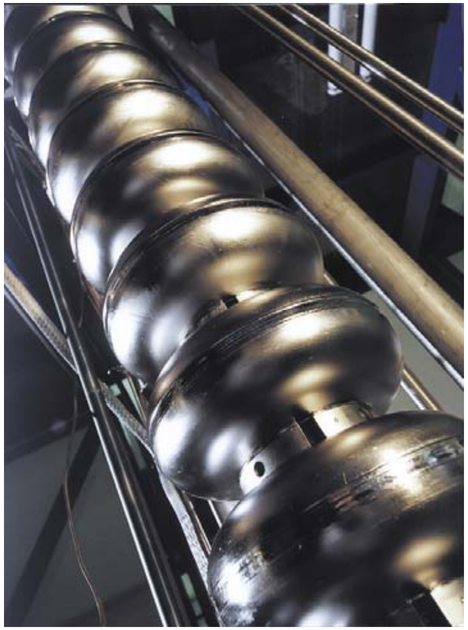
q = +1 e

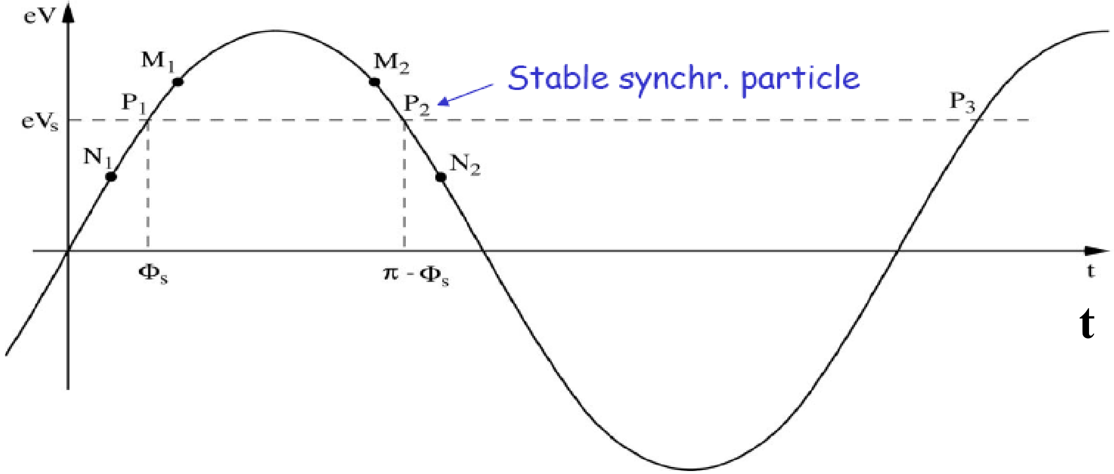
$$\int B dl \approx N * l * B = 2\pi \, p / q$$

$$B \approx \frac{2\pi * 920 * 10^9 eV}{416 * 3 * 10^8 \frac{m}{s} * 8.8m * e} \approx 5.15 \, Tesla$$

II.) The Acceleration

Where is the acceleration?
Install an RF accelerating structure in the ring:



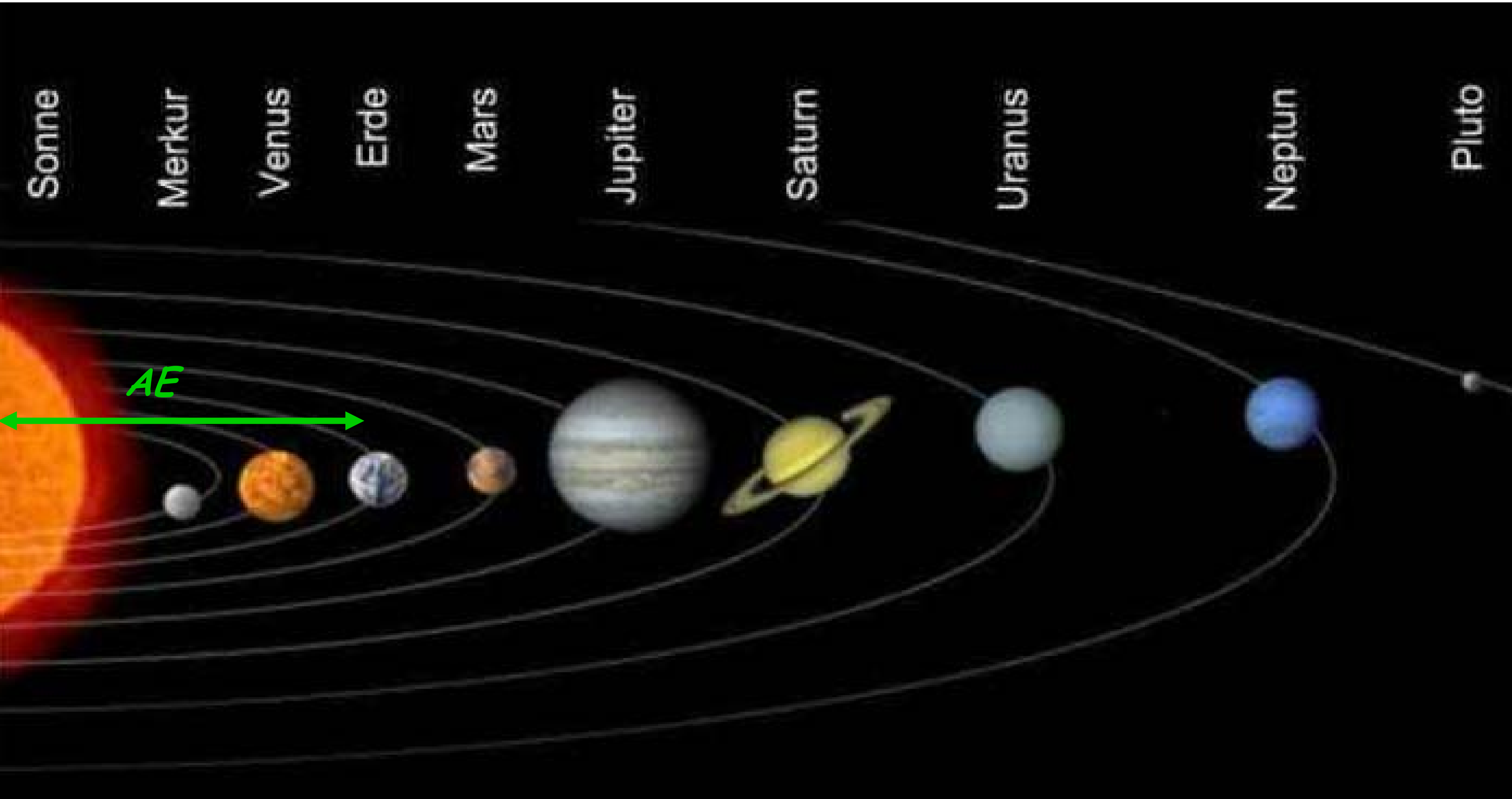


$$B * \rho = p / e$$

$$\rho = \frac{p / e}{B}$$

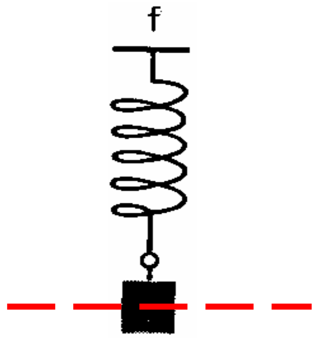
Largest storage ring: The Solar System

astronomical unit: average distance earth-sun
$1AE \approx 150 * 10^6$ km
Distance Pluto-Sun $\approx$ 40 AE



III.) Focusing Properties – Transverse Beam Optics

classical mechanics: pendulum



there is a restoring force, proportional to the elongation x:

$$m * \frac{d^2x}{dt^2} = -c * x$$

general solution: free harmonic oszillation

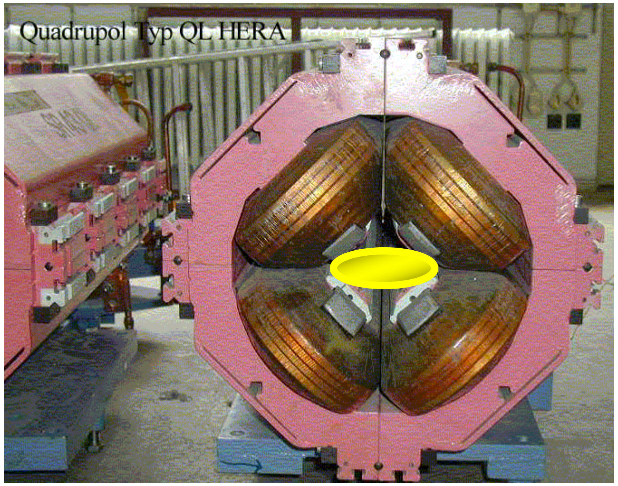
$$x(t) = A * \cos(\omega t + \varphi)$$

Storage Ring: we need a Lorentz force that rises as a function of the distance to .... ?

.... the design orbit

$$F(x) = q * v * B(x)$$

Quadrupole lenses to focus the beam



four iron pole shoes
of hyperbolic contour

linear increasing magnetic field

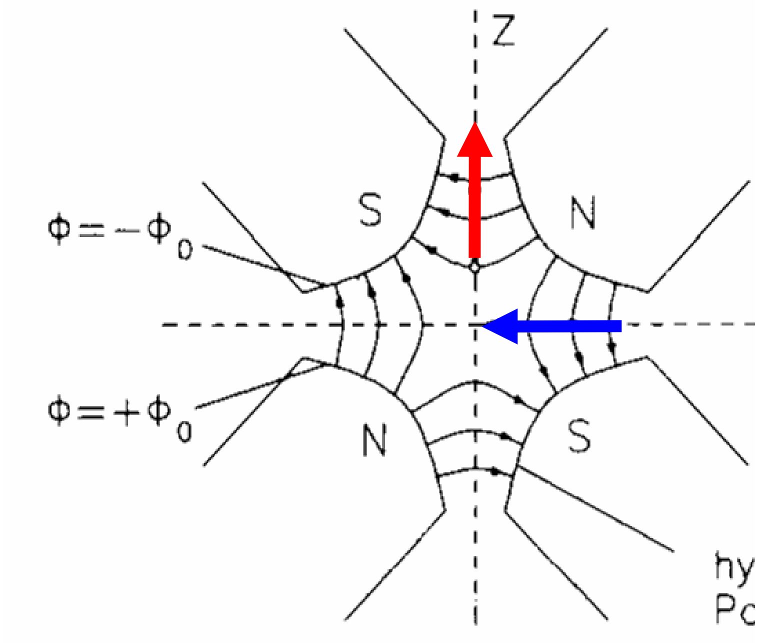
$$B_z = g * x \,, \quad B_x = g * z$$

Maxwell's equation at the location of the beam
... no current, no electr. field

$$\vec{\nabla} \times \vec{B} = 0$$

→ the B field can be expressed as gradient of a scalar potential ***V:***

$$\vec{B} = -\vec{\nabla} V \,, \quad V(x,z) = g\,x\,z$$



Focusing forces and particle trajectories:

normalise magnet fields to momentum
(remember: $B*\rho = p / q$)

Dipole Magnet

$$\frac{B}{p/q} = \frac{B}{B\rho} = \frac{1}{\rho}$$

Quadrupole Magnet

$$k := \frac{g}{p / q}$$

Example: HERA Ring

Momentum:

p = 920 GeV/c

Bending field:

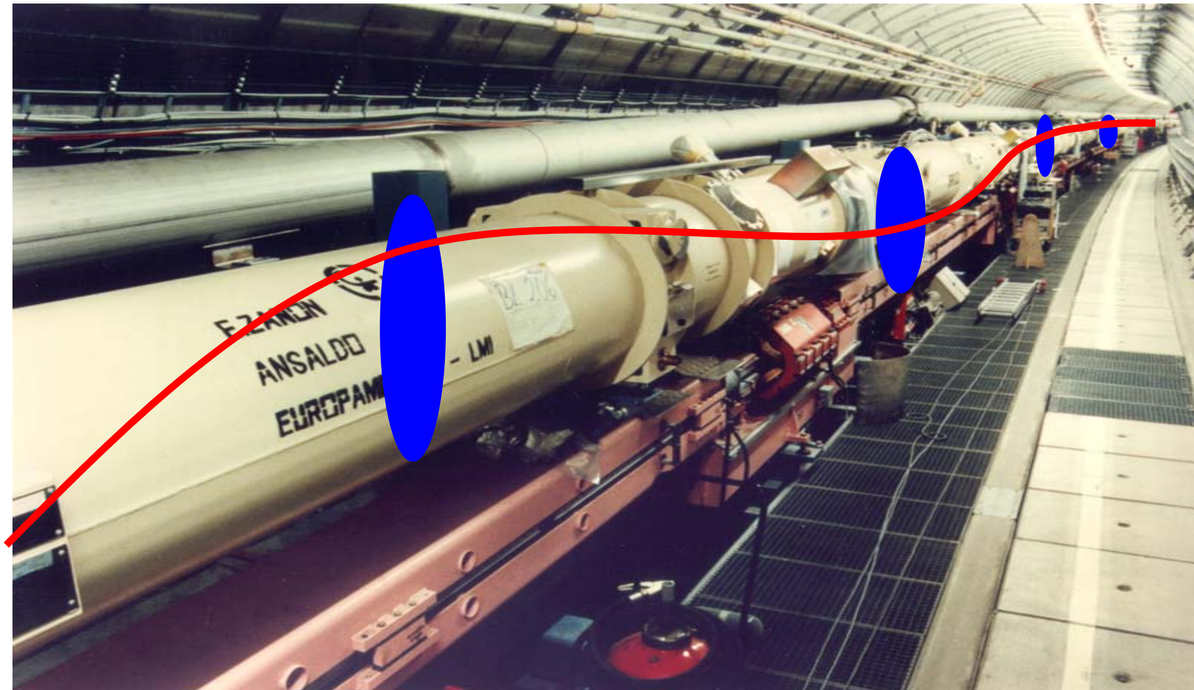
B = 5.5 Tesla

Quadrupol Gradient

G= 110 T/m

→ **k** $= 33.64 * 10^{-3} /m^2$

→ $1/\rho = 1.7 * 10^{-3} /m$



the focusing properties - Equation of motion

Under the influence of the focusing and defocusing forces the differential equation of the particles trajectory can be developed:

$$x'' + k * x = 0$$ **horizontal plane**

x = distance of a single particle to the center of the beam

$$x' := \frac{dx}{ds}$$

vert. plane: $k \Rightarrow -k$

if we assume ....

* linear retrieving force
* constant magnetic field
* first oder terms of displacement ***x***

... we get the general solution (hor. focusing magnet):

$$x(s) = x_0 * \cos(\sqrt{k}s) + \frac{x_0'}{\sqrt{k}} * \sin(\sqrt{k}s)$$

$$x'(s) = -x_0\sqrt{k} * \sin(\sqrt{k}s) + x_0' * \cos(\sqrt{k}s)$$

More elegant description: Matrix formalism

$$\begin{pmatrix} x \\ x' \end{pmatrix}_s = M * \begin{pmatrix} x \\ x' \end{pmatrix}_0$$

Matrices of lattice elements

Hor. focusing Quadrupole Magnet

$$M_{QF} = \begin{pmatrix} \cos(\sqrt{K} * l) & \frac{1}{\sqrt{K}} \sin(\sqrt{K} * l) \\ -\sqrt{K} \sin(\sqrt{K} * l) & \cos(\sqrt{K} * l) \end{pmatrix}$$

Hor. defocusing Quadrupole Magnet

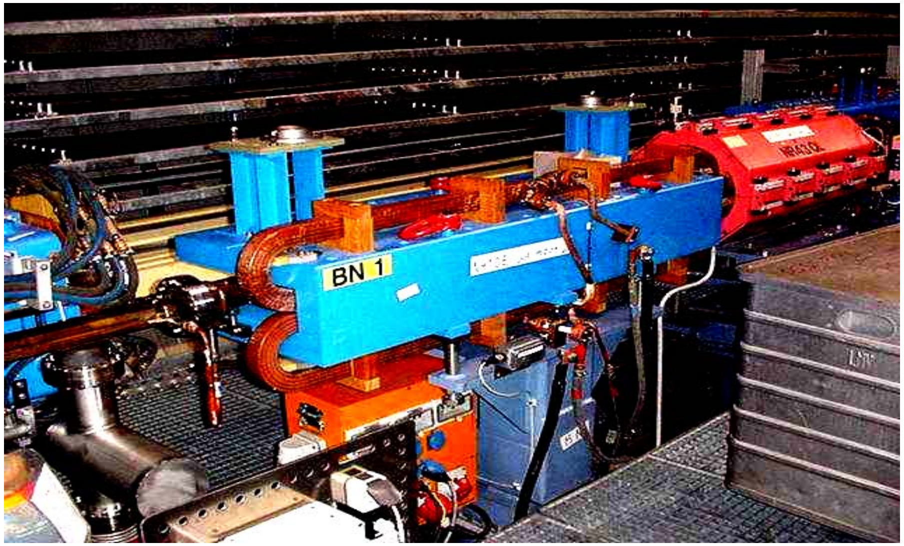
$$M_{QD} = \begin{pmatrix} \cosh(\sqrt{K} * l) & \frac{1}{\sqrt{K}} \sinh(\sqrt{K} * l) \\ \sqrt{K} \sinh(\sqrt{K} * l) & \cosh(\sqrt{K} * l) \end{pmatrix}$$

Drift space

$$M_{Drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

Nota bene:
formalism is only valid within one lattice element where ***k = const***

in reality: ***k = k(s)***



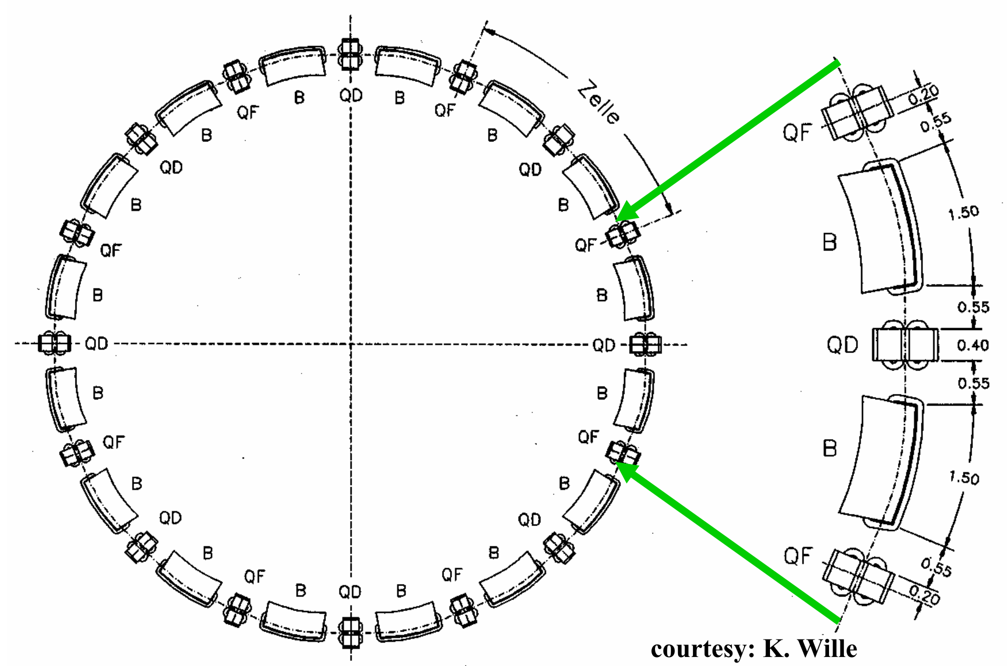
„veni vidi vici ...“ .... or in english .... „we got it !“

* we can calculate the trajectory of a single particle within a single lattice element
* for any starting conditions $x_{0,}\ x'_0$
* we can combine these piecewise solutions together and get the trajectory for the complete storage ring.

$$M_{lattice} = M_{QF1} * M_{D1} * M_{QD} * M_{D1} * M_{QF2} \ldots\ldots$$

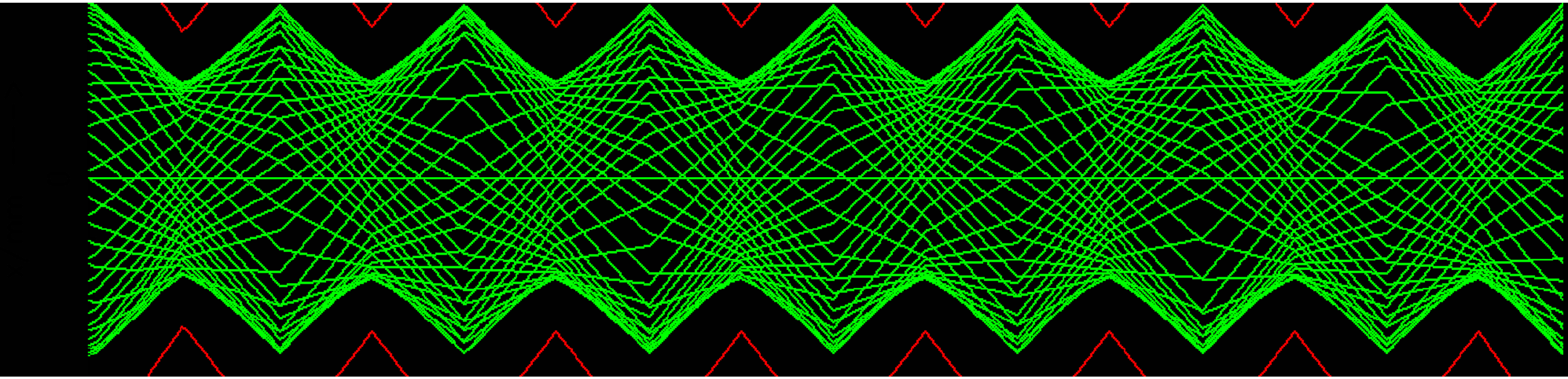
Example:
storage ring for beginners

Dipole magnets and QF & QD quadrupole lenses



courtesy: K. Wille

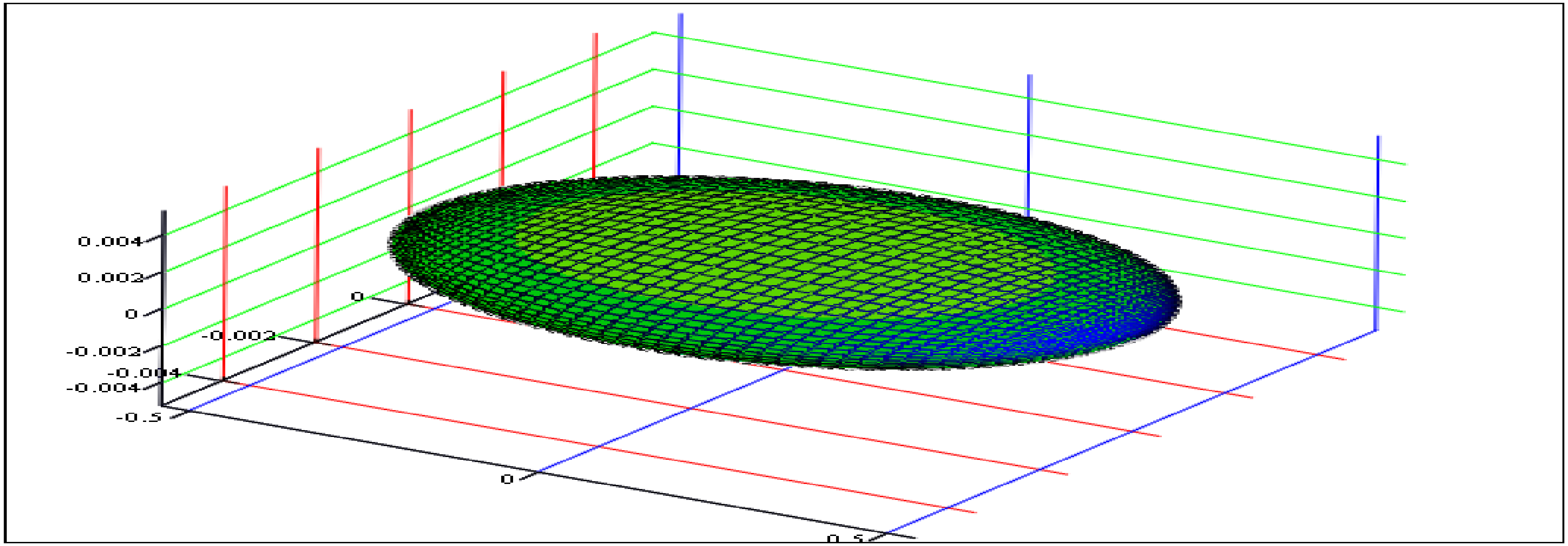
tracking one particle through the storage ring



Contour of a particle bunch given by the external focusing fields (arc values)

	e	*p*
σ_x	*1.0 mm*	*0.75 mm*
σ_z	*0.2 mm*	*0.46 mm*
σ_s	*10.3 mm*	*190 mm*
N_p	$3.5*10^{10}$	$7.3*10^{10}$

Example: HERA Proton Bunch
3d contour calculated for the design parameters



(Z , X , Y)

Twiss Parameters

Astronomer Hill:

differential equation for motions with periodic focusing properties: „Hill's equation"

Example: particle motion with periodic coefficient

equation of motion:

$$x''(s) - k(s)x(s) = 0$$

restoring force $\neq$ ***const,***
k(s) = depending on the position s
k(s+L) = k(s),   periodic function

we expect a kind of quasi harmonic oscillation: amplitude & phase will depend on the position s in the ring.

The Beta Function

in this case the solution can be written in the form:

Ansatz:

$$x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)} * \cos(\psi(s) + \phi)$$

ε, Φ = integration constants determined by initial conditions

β(s) given by focusing properties of the lattice ↔ quadrupoles

ε beam emittance = woozilycity of the particle ensemble,
intrinsic beam parameter,
cannot be changed by the foc. properties.

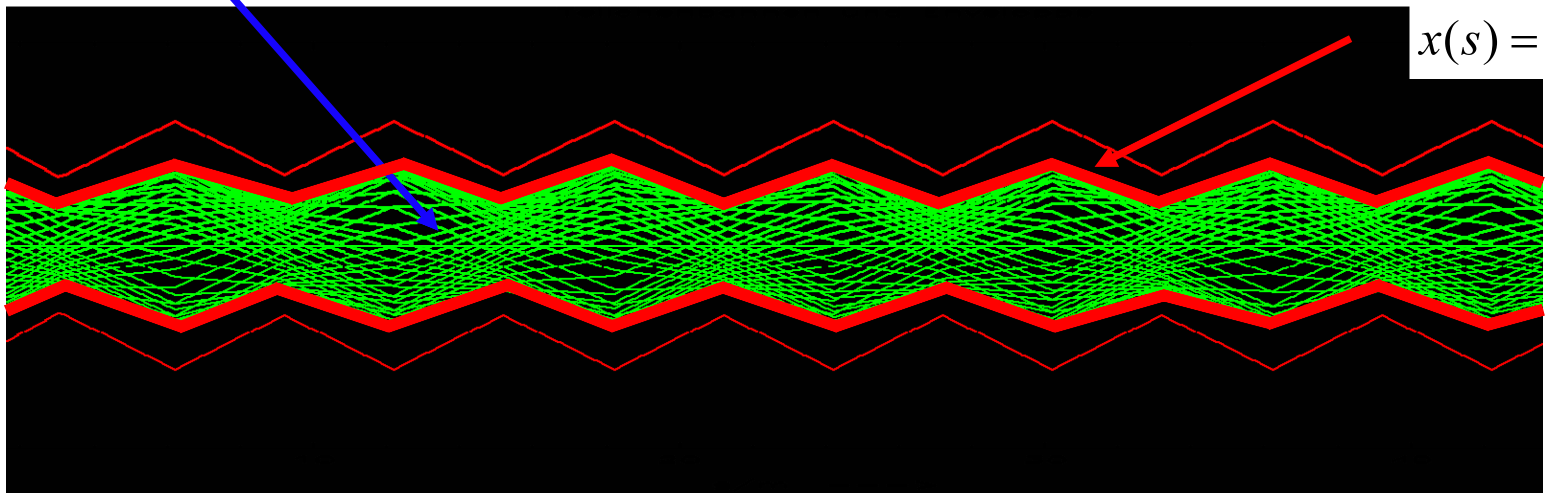
Scientifiquely spoken: area covered in transverse x, x´ phase space
… and it is constant !!!

Ensemble of many (...all) possible particle trajectories

x(s) of a single particle

max. amplitude of all particle trajectories

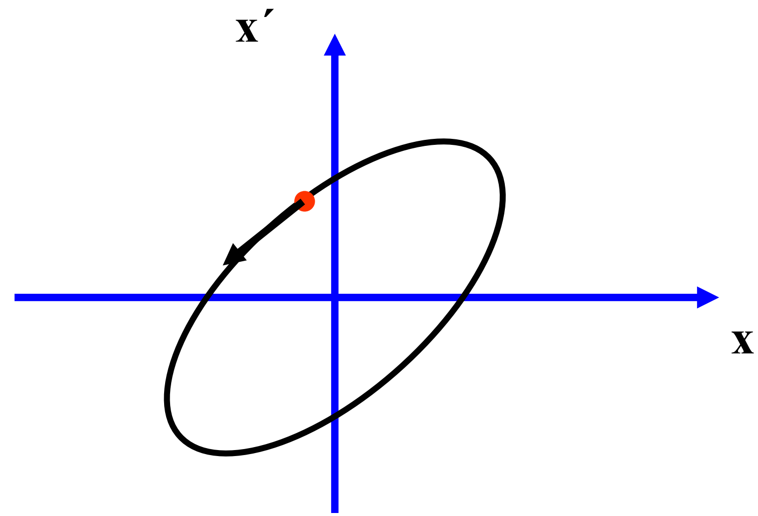
$$x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)}$$



Beam Dimension:

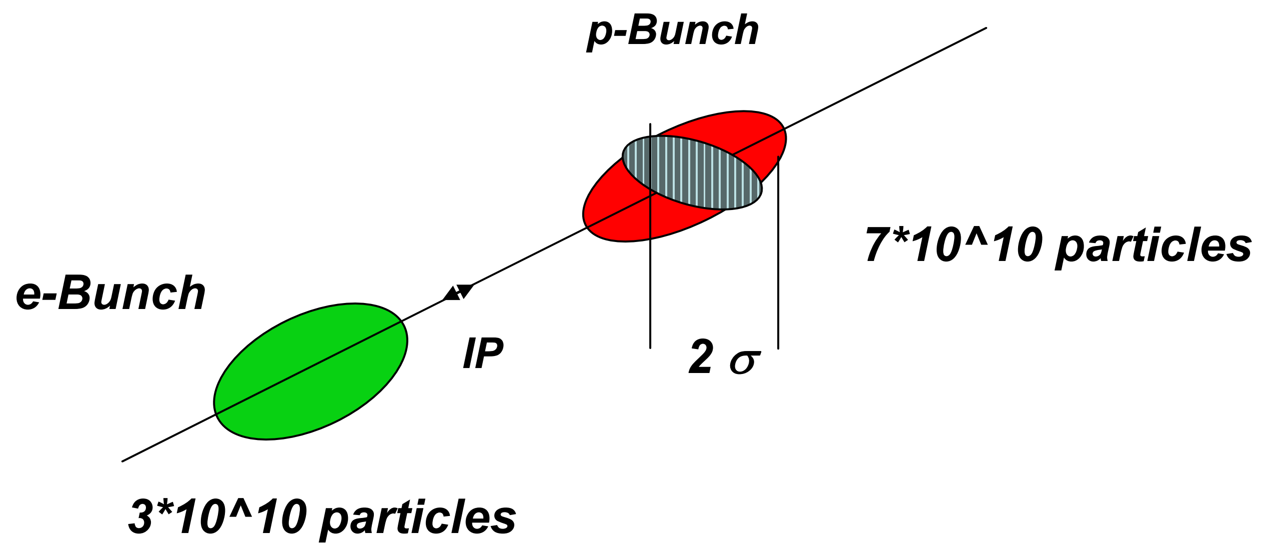
determined by two parameters

$$\sigma = \sqrt{\varepsilon * \beta}$$



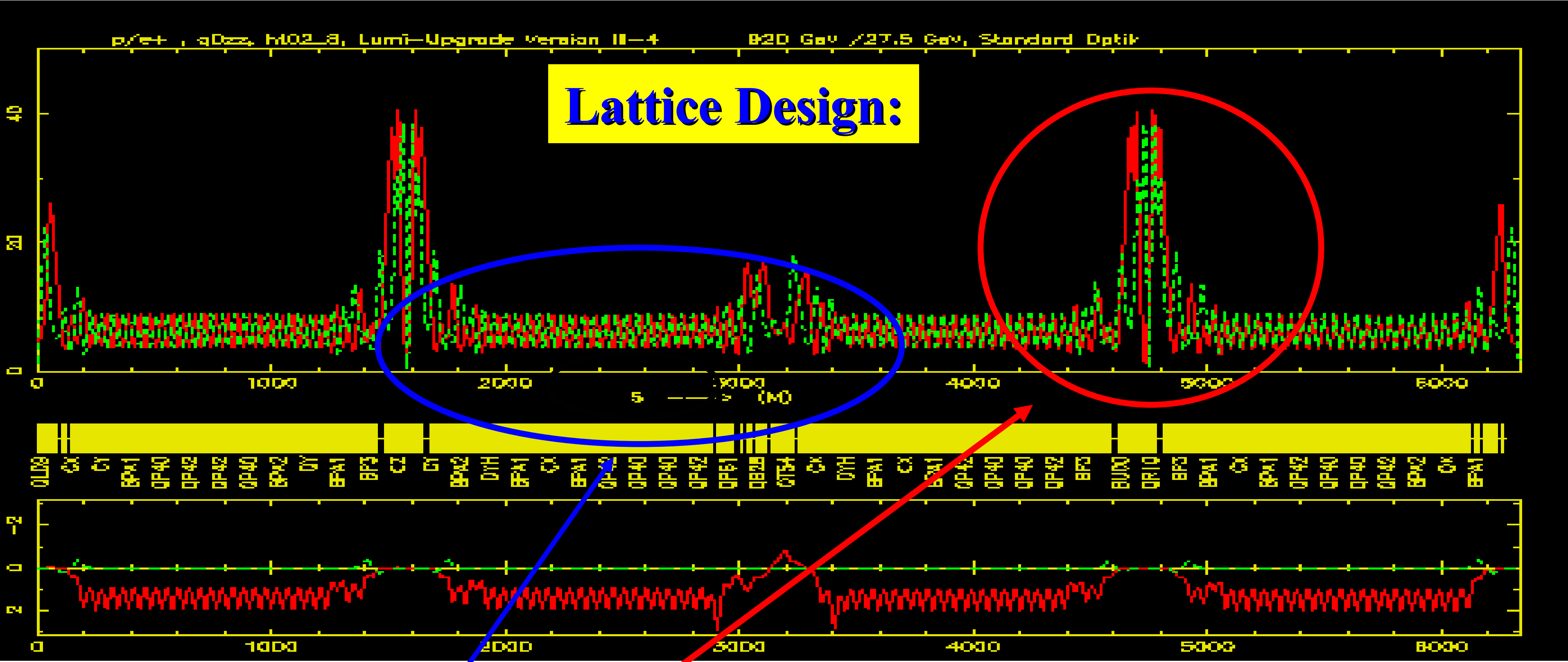
ε = area in phase space

Luminosity



$$L = \frac{1}{4\pi e^2 f_0 n_b} * \frac{I_e I_p}{\sigma_x^* \sigma_y^*}$$

$$\sigma = \sqrt{\varepsilon * \beta}$$



Arc: regular **(periodic) magnet structure:**

bending magnets → define the energy of the ring

main focusing & tune control, chromaticity correction,

multipoles for higher order corrections

Straight sections: drift spaces for injection, dispersion suppressors,

low beta insertions, RF cavities, etc....

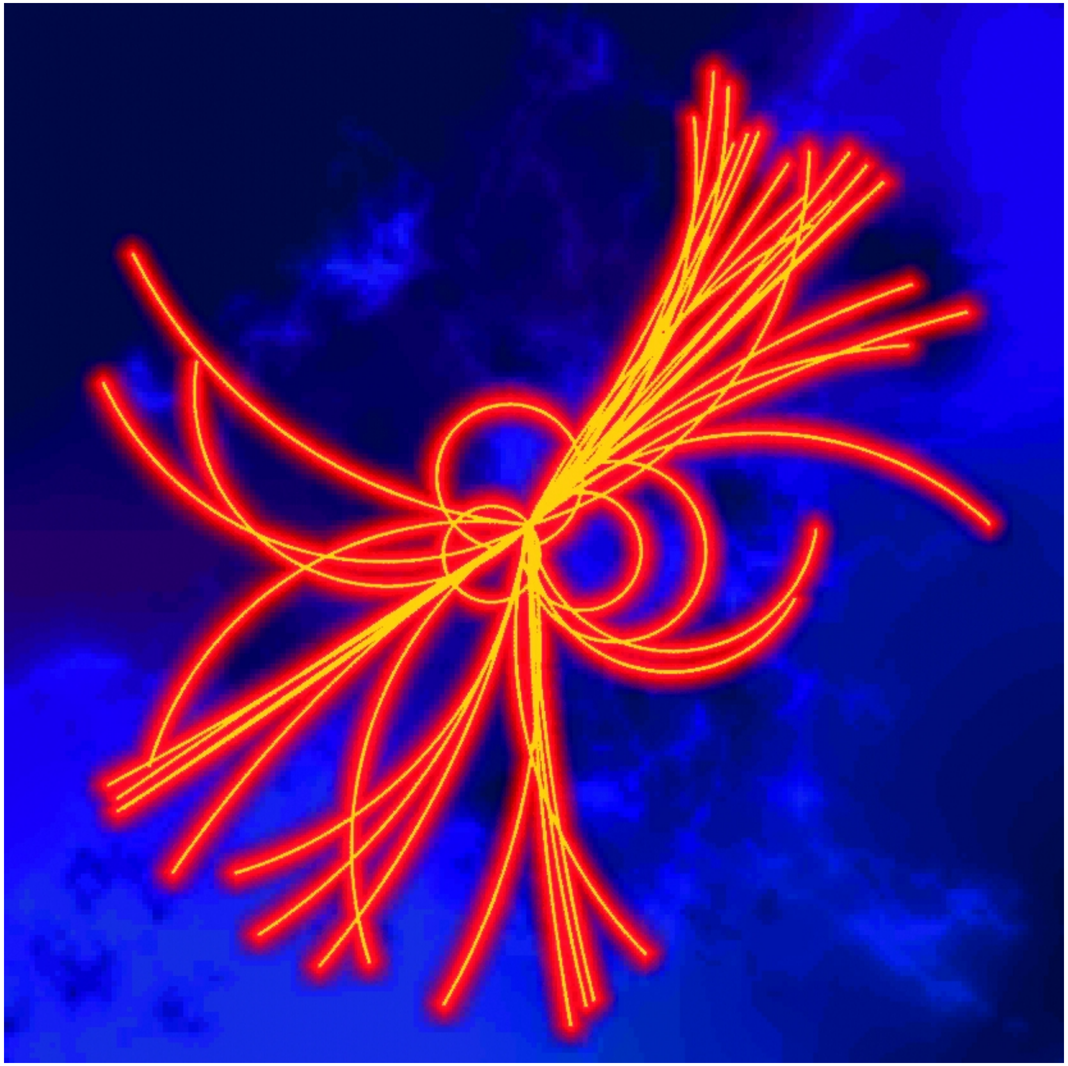
... and the high energy experiments if they cannot be avoided

Luminosity

$$R = L * \Sigma_{react.}$$

production rate of (scattering) events is determined by the cross section Σ_{react} and a parameter L that is given by the design of the accelerator:
… the luminosity

$$L \propto \frac{1}{\sigma_x^* * \sigma_y^*}$$



H1 detector: inelastic scattering event of e+/p

Bibliography

1.) Edmund Wilson: Introd. to Particle Accelerators
Oxford Press, 2001

2.) Klaus Wille: Physics of Particle Accelerators and Synchrotron
Radiation Facilicties, Teubner, Stuttgart 1992

3.) Peter Schmüser: Basic Course on Accelerator Optics, CERN Acc.
School: 5th general acc. phys. course CERN 94-01

4.) Bernhard Holzer: Lattice Design, CERN Acc. School: Interm.Acc.phys course,
http://cas.web.cern.ch/cas/ZEUTHEN/lectures-zeuthen.htm

5.) Herni Bruck: Accelerateurs Circulaires des Particules,
presse Universitaires de France, Paris 1966 (english / francais)

6.) M.S. Livingston: J.P. Blewett, Particle Accelerators,
Mc Graw-Hill, New York,1962

7.) Frank Hinterberger: Physik der Teilchenbeschleuniger, Springer Verlag 1997

8.) Mathew Sands: The Physics of e+ e- Storage Rings, SLAC report 121, 1970

9.) D. Edwards, M. Syphers : An Introduction to the Physics of Particle
Accelerators, SSC Lab 1990