

Theory of Elementary Particles

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1. Introduction

central questions in particle physics

elementary constituents of matter

fundamental forces

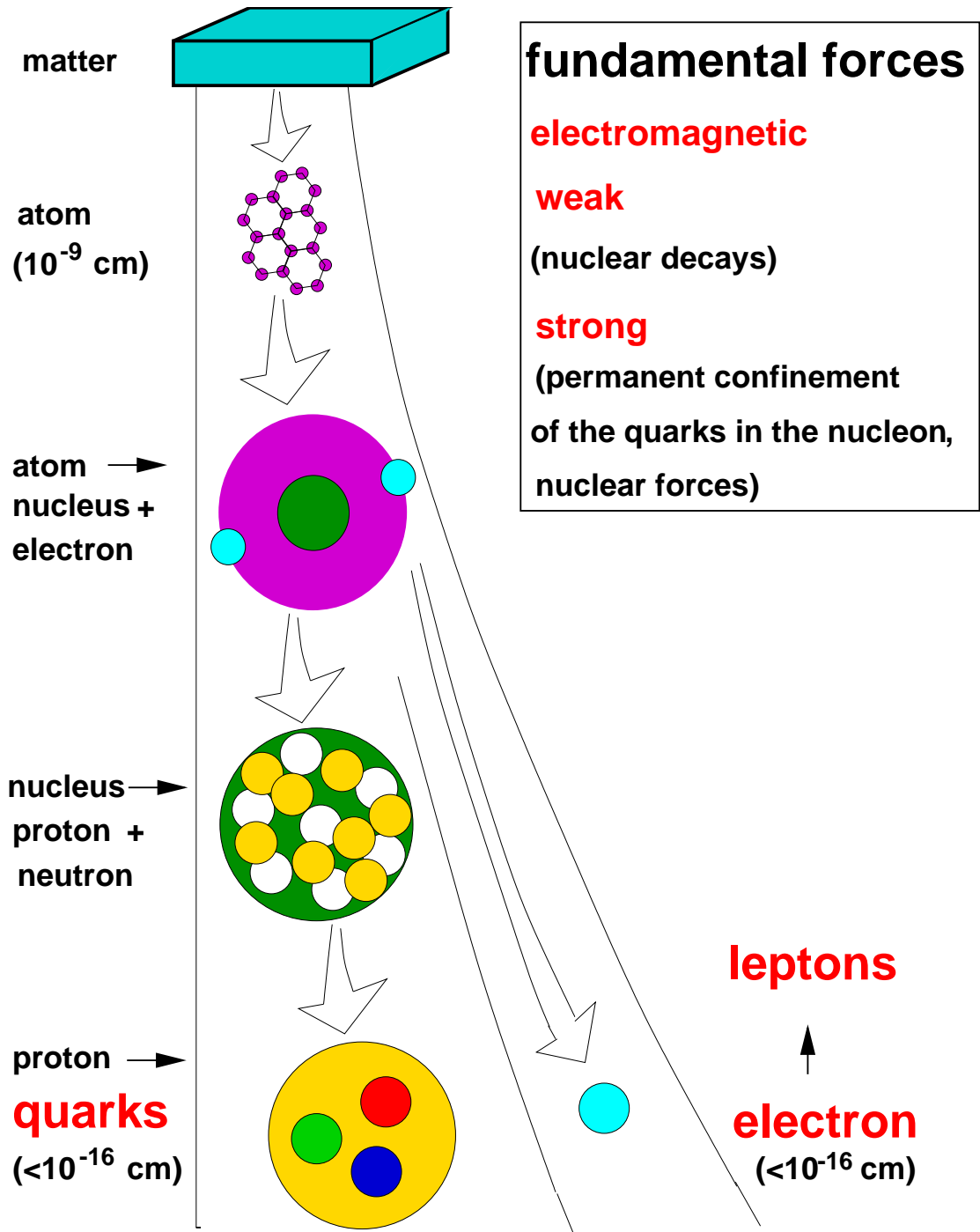
present status

STANDARD MODEL

quarks and leptons

gauge forces as a consequence of the

**gauge principle =
symmetrie requirement**



Elementary particles of the Standard Model

spin 1/2

matter particles, in three generations

electric charge

| | | | | |
|--------------------|--|--|--|---|
| leptons (l) | $\begin{pmatrix} \nu_e \\ e \end{pmatrix}$ | $\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$ | $\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$ | $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ |
| quarks (q) | $\begin{pmatrix} u \\ d \end{pmatrix}$ | $\begin{pmatrix} c \\ s \end{pmatrix}$ | $\begin{pmatrix} t \\ b \end{pmatrix}$ | $\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$ |

spin 1

gauge bosons (mediators of the fundamental interactions)

photon (γ)

gluons (g)

W^\pm , Z bosons

spin 0

Higgs boson (H) ?

- no free quarks and gluons
- confinement' in **hadrons**
- **indirect** evidence

Fundamental forces = gauge forces

| interaction | theory | participating matter particles | mediator | examples |
|---------------------------|---|--------------------------------|---------------------|---|
| electro-magnetic | QED unified electroweak gauge theory | electrically charged l, q | photon (γ) | $e^+e^- \rightarrow e^+e^-e^+e^-$ $e^+e^- \rightarrow \mu^+\mu^-$ $\pi^0 \rightarrow \gamma\gamma$ |
| weak | | all l, q (in pairs) | W^\pm, Z, γ | \rightarrow decay of nuclei $n \rightarrow p e \bar{\nu}_e$ $\mu \rightarrow \nu_\mu e \bar{\nu}_e$ $K^0 \rightarrow \pi^+\pi^-$ $e^+e^- \rightarrow Z$ |
| strong resp. colour | QCD | coloured q | gluons | \rightarrow nuclear forces $\rho \rightarrow \pi^+\pi^-$ in $e^\pm p \rightarrow e^\pm +$ anything in $e^+ + e^- \rightarrow$ three jets |

Plan of the lecture

2. quantum field theory (QFT) why? how formulated?

3. gauge interactions

- **all interactions** $\left\{ \begin{array}{l} \text{electromagnetic (QED)} \\ \text{weak} \\ \text{strong (QCD)} \end{array} \right.$ gained from a simple, elegant principle, the so-called

gauge principle in form of gauge interactions

- – perturbation theory (for small couplings) \rightarrow **Feynman diagrams**
 - (lattice physics for large couplings)
- the Higgs boson, **spontaneous symmetry breakdown** and masses for W^\pm, Z, l, q

4. quantum effects, some applications and key precision tests

- quantum effects and precision tests in QED
- running couplings in QED and QCD →

qualitative understanding of
quark confinement
at large distances

asymptotic freedom
of quarks
at small distances

- test of **asymptotic freedom** and of **three colours**
- **HERA**: deep inelastic scattering and the nucleon structure functions
as test of perturbative QCD
- quantum effects in electroweak interactions and the indirect determination of
 m_t and m_H at LEP, SLD

5. physics beyond the Standard Model

- some open questions in the Standard Model
- brief remark on neutrino masses
- composite quarks and leptons
- new particles, examples: leptoquarks and leptogluons
- new gauge interactions
- grand unification
- supersymmetry
- brief remarks on
supergravity, superstrings, baryon asymmetry, cosmology, extra dimensions, non-commutative geometry

2. Quantum field theory



QFT - why?

non-relativistic quantum mechanics

$\Delta x \searrow$ uncertainty principle $\Delta x \cdot \Delta p \gtrsim O(\hbar)$

$\Delta x \searrow \rightarrow \Delta p \nearrow$

implying $p \nearrow$ implying $v \nearrow c$ (c = speed of light)

special relativity

- $E = \sqrt{\vec{p}^2 c^2 + m^2 c^4}$ with $E_{\text{rest}} = mc^2$
- conservation of energy E
- **kinetic energy** $\xleftrightarrow{\text{transformation}}$ **mass**
- **no conservation of particle number and particle species**

relativistic quantum mechanics is **insufficient**

quantum field theory

allows description of

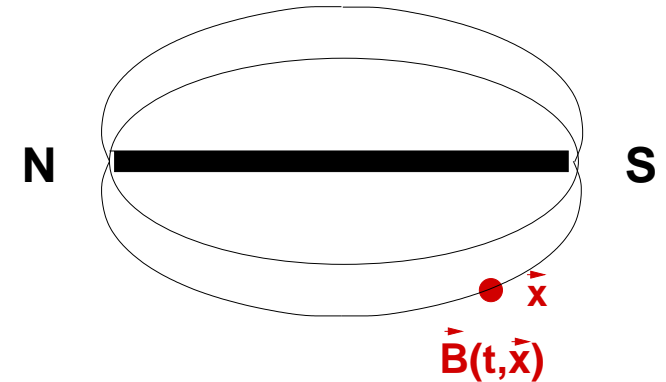
particle production and annihilation



QFT - formulation

step 1

each particle species \longleftrightarrow association field $\Phi(t, \vec{x})$



role model: electric and magnetic fields $\vec{E}(t, \vec{x})$, $\vec{B}(t, \vec{x})$

classical mechanics

1 mass point (in 1 dim.), described by 1 generalized coordinate

$$q(\textcolor{red}{t}), \dot{q}(\textcolor{red}{t})$$

Lagrangefunction $L = T - V$ (V contains interaction)

$$\textcolor{red}{L}(q(t), \dot{q}(t))$$

classical field theory

1 **field**, described by 1 generalized coordinate in **each space point** \vec{x}

$$\Phi(\textcolor{red}{t}, \textcolor{violet}{\vec{x}}), \dot{\Phi}(\textcolor{red}{t}, \textcolor{violet}{\vec{x}}), \vec{\nabla} \Phi(\textcolor{red}{t}, \textcolor{violet}{\vec{x}})$$

$$L = \int d^3x \underbrace{\mathcal{L}(\Phi(t, \vec{x}), \dot{\Phi}(t, \vec{x}), \vec{\nabla} \Phi(t, \vec{x}))}_{\text{Lagrange density}}$$

action $S = \int dt L$, Hamiltonprinciple of extremal action $\delta S = 0 \rightarrow$

equation of motion

$$\frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} = 0$$

field equations

$$\frac{\partial \mathcal{L}}{\partial \Phi} - \frac{\partial}{\partial t} \frac{\partial \mathcal{L}}{\partial \dot{\Phi}} + \vec{\nabla} \frac{\partial \mathcal{L}}{\partial (\vec{\nabla} \phi)} = 0$$

step 2

establish \mathcal{L} for each of the **fundamental interactions** among the relevant **fields** (see sect.3)

For **free** fields, i.e. **no** interaction:

spin 0: $\Phi(t, \vec{x})$ field equations = Klein-Gordon equation

= relativistic generalization of the Schrödinger equation

$$(E^2 - c^2 \vec{p}^2 - m^2 c^4 = 0; \quad E \rightarrow i\hbar \frac{\partial}{\partial t}, \quad \vec{p} \rightarrow -i\hbar \vec{\nabla})$$

$$\hbar = c = 1$$

$$(\square + m^2)\Phi(t, \vec{x}) = 0 \quad \text{with } \square = \left(\frac{\partial}{\partial t}\right)^2 - \vec{\nabla}^2$$

$$\rightarrow \mathcal{L}_{\text{free}}^{\Phi} = \frac{1}{2} \left(\left(\frac{\partial}{\partial t} \Phi \right)^2 - (\vec{\nabla} \Phi)^2 \right) - \frac{1}{2} m^2 \Phi^2$$

spin $\frac{1}{2}$: $\psi(t, \vec{x})$ field equations = Dirac equation

with $\partial_\mu = \left(\frac{\partial}{\partial t}, \vec{\nabla} \right)$, $\mu=0,1,2,3$
 ψ = 4-spinor = $(\psi_1, \psi_2, \psi_3, \psi_4)$,
 γ_μ = 4 \times 4-matrices

$$(i\gamma^\mu \partial_\mu - m)\psi(t, \vec{x}) = 0 \rightarrow \mathcal{L}_{\text{free}}^\psi = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

spin 1, m=0: $A^\mu(t, \vec{x})$ $\mu = 0, 1, 2, 3$ electromagnetic field

$$A^\mu(t, \vec{x}) = \begin{pmatrix} V(t, \vec{x}) = \text{scalar potential} \\ \vec{A}(t, \vec{x}) = \text{vector potential} \end{pmatrix} \quad \vec{B} = \vec{\nabla} \times \vec{A}, \quad \vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}$$

electromagnetic field strength tensor

$$F^{\mu\nu}(t, \vec{x}) = \partial^\mu A^\nu(t, \vec{x}) - \partial^\nu A^\mu(t, \vec{x}) \quad \text{with} \quad \partial_\mu = \left(\frac{\partial}{\partial t}, \vec{\nabla} \right)$$

field equations = Maxwell equations (in absence of charge and current densities)

$$\partial_\mu F^{\mu\nu}(t, \vec{x}) = 0 \rightarrow \mathcal{L}_{\text{free}}^{A^\mu} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

step 3

field quantization

quantum mechanics

conjugate coordinate

$$p(\textcolor{red}{t}) := \frac{\partial L}{\partial \dot{q}(\textcolor{red}{t})}$$

$$[q(t), p(t)] = i\hbar$$

$$[A, B] = A \cdot B - B \cdot A$$

$$\{A, B\} = A \cdot B + B \cdot A$$

q, p \rightarrow operators

quantum field theory

spin 0: $\Phi(t, \vec{x})$

conjugate field

$$\pi(\textcolor{red}{t}, \textcolor{violet}{\vec{x}}) := \frac{\partial \mathcal{L}}{\partial \dot{\Phi}(\textcolor{red}{t}, \textcolor{violet}{\vec{x}})}$$

$$[\Phi(t, \textcolor{violet}{\vec{x}}), \pi(t, \textcolor{violet}{\vec{x}}')] = i\hbar \delta^{(3)}(\textcolor{violet}{\vec{x}} - \textcolor{violet}{\vec{x}}')$$

scalar boson field quantization

spin 1/2:

$$\pi_\alpha(\textcolor{red}{t}, \textcolor{violet}{\vec{x}}) := \frac{\partial \mathcal{L}}{\partial \dot{\psi}_\alpha(\textcolor{red}{t}, \textcolor{violet}{\vec{x}})} \quad \psi(t, \vec{x}): \psi_\alpha(t, \vec{x}), \alpha = 1, \dots, 4$$

$$\{\psi_\alpha(t, \textcolor{violet}{\vec{x}}), \pi_\beta(t, \textcolor{violet}{\vec{x}}')\} = \delta_{\alpha\beta} i\hbar \delta^{(3)}(\textcolor{violet}{\vec{x}} - \textcolor{violet}{\vec{x}}')$$

Dirac fermion field quantization

Φ, π resp. $\psi_\alpha, \pi_\alpha \rightarrow$ field operators

quantum mechanics

quantum field theory

spin 1, m=0: $A^\mu(t, \vec{x})$ $\mu = 0, 1, 2, 3$

$$\pi^\mu(\mathbf{t}, \vec{x}) := \frac{\partial \mathcal{L}}{\partial \dot{A}_\mu(\mathbf{t}, \vec{x})}$$

$$\left[A^\mu(t, \vec{x}), \pi^\nu(t, \vec{x}') \right] = g^{\mu\nu} i\hbar \delta^{(3)}(\vec{x} - \vec{x}')$$

electromagnetic field quantization

(modulo complications due to gauge invariance)

$g^{\mu\nu}$ = metric tensor

$A^\mu, \pi^\mu \rightarrow$ field operators

♣ QFT for a free scalar field and particle interpretation

- field equation $(\square + m^2)\Phi(t, \vec{x}) = 0$, with $\square = (\frac{\partial}{\partial t})^2 - \vec{\nabla}^2$

general solution

$$\Phi(t, \vec{x}) \propto \int dE d^3p \delta(E^2 - \vec{p}^2 - m^2) \times \left(\mathbf{a}(\mathbf{E}, \vec{p}) e^{-i(Et - \vec{p}\vec{x})} + \mathbf{a}^\dagger(\mathbf{E}, \vec{p}) e^{+i(Et - \vec{p}\vec{x})} \right)$$

energy 3-momentum relativistic energy-momentum relation

- Lagrange density $\mathcal{L}_{\text{free}}^\Phi = \frac{1}{2}((\frac{\partial}{\partial t}\Phi)^2 - (\vec{\nabla}\Phi)^2) - \frac{1}{2}m^2\Phi^2$

conjugate field $\pi(t, \vec{x}) := \frac{\partial \mathcal{L}}{\partial \dot{\Phi}(t, \vec{x})} = \dot{\Phi}(t, \vec{x})$

- field quantization $[\Phi(t, \vec{x}), \pi(t, \vec{x}')] = i\hbar \delta^{(3)}(\vec{x} - \vec{x}') \quad \longleftrightarrow$

$$[\mathbf{a}(p), \mathbf{a}^\dagger(p')] = 2E \hbar \delta^{(3)}(\vec{p} - \vec{p}'), \quad [a(p), a(p')] = 0, \quad [a^\dagger(p), a^\dagger(p')] = 0 \quad \text{with } p = \begin{pmatrix} E \\ \vec{p} \end{pmatrix}$$

$a(p) = a(E, \vec{p})$ is field operator, $a^\dagger(p)$ is its hermitian conjugate field operator

- **Hamiltonoperator** (measures total energy in the field)

$$H = \int d^3x (\pi \dot{\Phi} - \mathcal{L}) = \dots + \int dE d^3p \delta(E^2 - \vec{p}^2 - m^2) \underbrace{E a^\dagger(p) a(p)}_{\text{number operator } N(p)}$$

- **eigenbasis of $N(p)$** (Fock space of multiparticle states)

$$N(p) |n(p)\rangle = \mathbf{n(p)} |n(p)\rangle$$

eigenvalue eigenstate

$n(p)$ = number of particles with spin 0, mass m with energy between E and $E + dE$ and momentum between \vec{p} and $\vec{p} + d\vec{p}$, $E = +\sqrt{\vec{p}^2 + m^2}$

$$N(p) \mathbf{a(p)}^{(\dagger)} |n(p)\rangle = (n(p) \mathbf{(\mp)} 1) \mathbf{a(p)}^{(\dagger)} |n(p)\rangle \longleftrightarrow$$

$$\mathbf{a^\dagger(p)} |n(p)\rangle \propto |(n+1)(p)\rangle \quad \mathbf{a^\dagger(p)} = \text{particle creation operator}$$

$$\mathbf{a(p)} |n(p)\rangle \propto |(n-1)(p)\rangle \quad \mathbf{a(p)} = \text{particle annihilation operator}$$

provides basis for particle production and particle annihilation in QFT

Normalize the energy of the ground state $|0\rangle$ to zero \rightarrow eigenvalue spectrum $\mathbf{n(p)} = 0, 1, 2, \dots$

consequence of the field quantization! **field \longleftrightarrow particle**

- multiparticle states **Bose-Einstein statistics**

$$|n_1(p_1), \dots, n_m(p_m)\rangle \propto (a^\dagger(p_1))^{n_1} \cdot \dots \cdot (a^\dagger(p_m))^{n_m} |0\rangle$$

automatically: total **symmetry** with respect to the exchange of any two particles

♣ QFT for a free Dirac field

- $[,] \rightarrow \{, \}$ for a Dirac fermion field the arguments runs analogously, also leading to particle creation and annihilation operators. Due to the anticommutator $(\{a^\dagger(p), a^\dagger(p')\} = 0, \text{ implying } (a^\dagger(p))^2 = 0)$ the multiparticle states obey

Fermi-Dirac statistics resp. the **Pauli principle**

$$|p_1, \dots, p_m\rangle \propto a^\dagger(p_1) \cdot \dots \cdot a^\dagger(p_m) |0\rangle \quad (\text{for simplicity the spin degrees of freedom have been suppressed})$$

automatically: total **antisymmetry** with respect to the exchange of any two particles

- existence of antiparticles in QFT
- the field energy is bounded from below

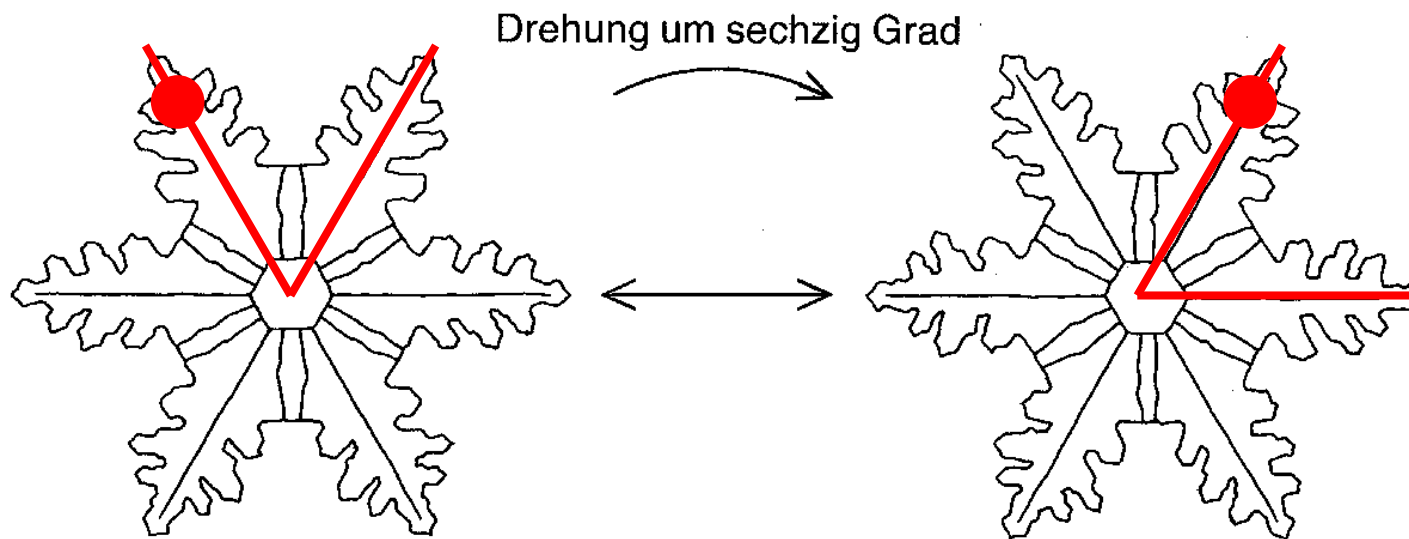
3. Local gauge interactions



preexercise in symmetries

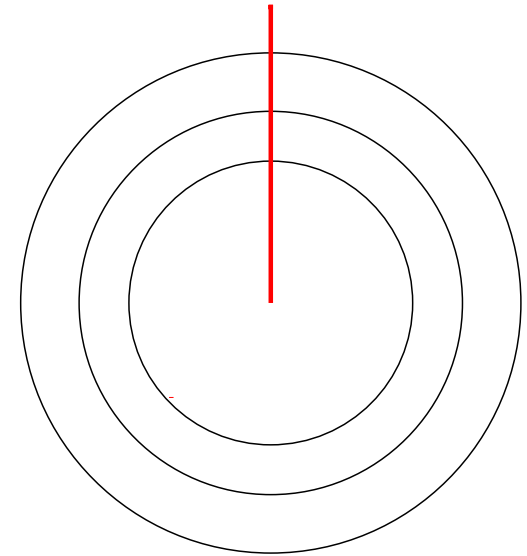
by means of examples from daily life

snowflake



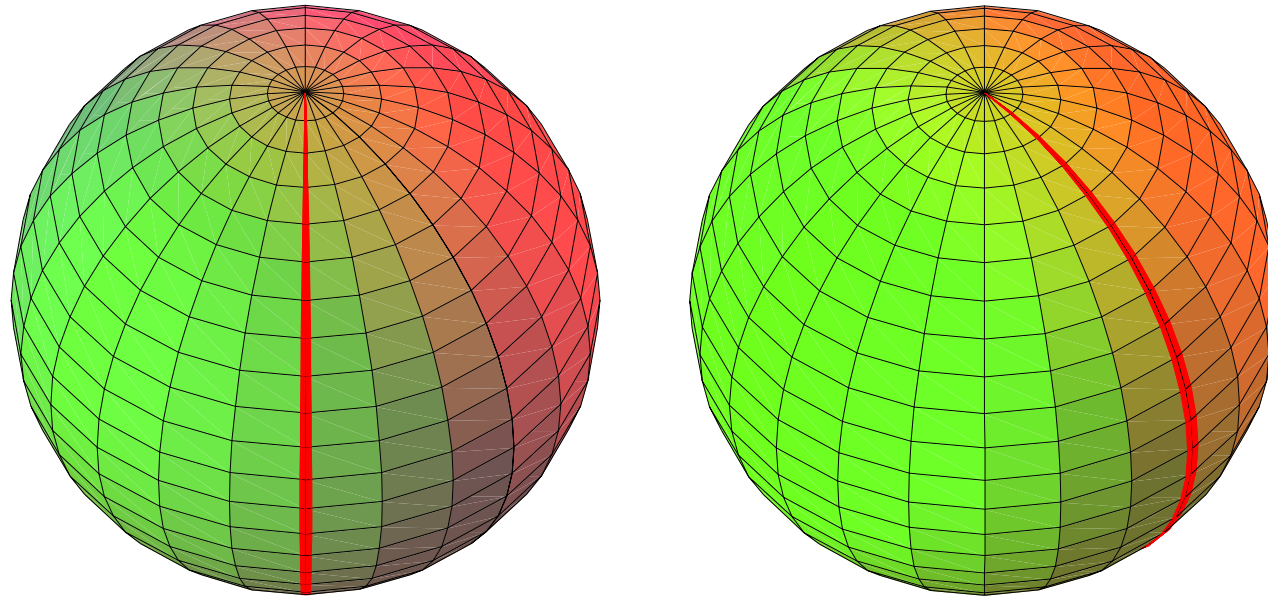
- **invariance** with respect to common, i.e. **global** rotations by 60°
- section can be chosen by convention

throwing a stone into the water



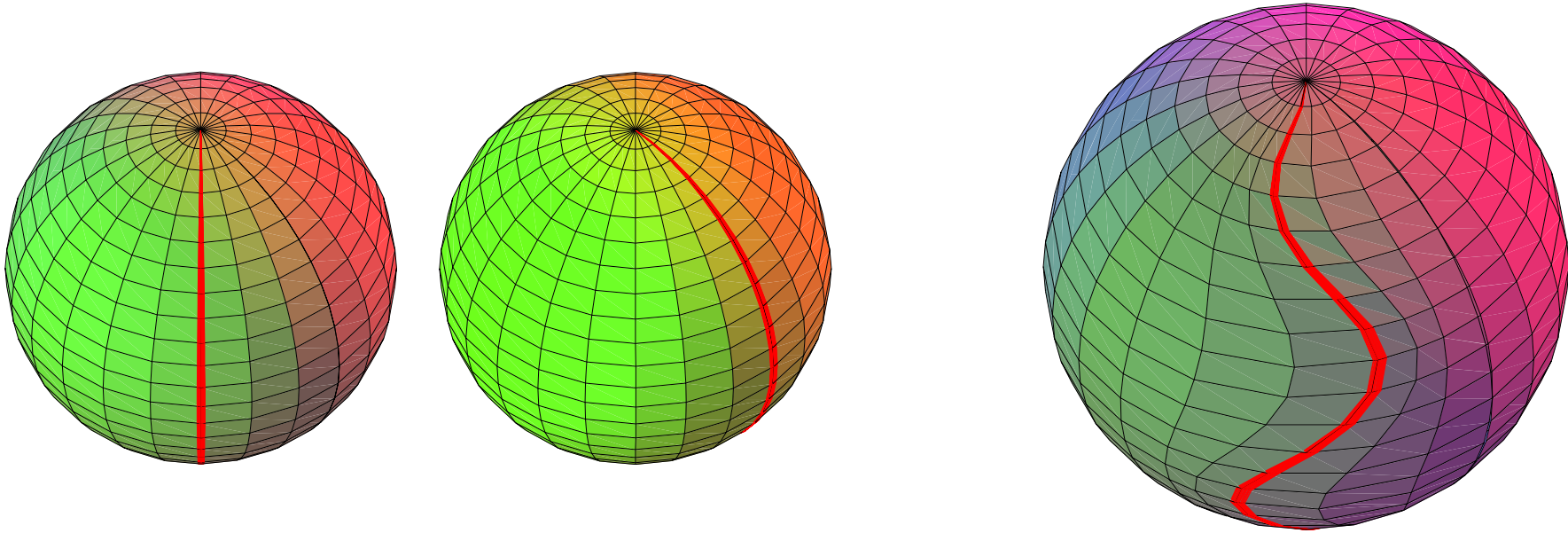
- **invariance** with respect to common, i.e. **global** rotations of all points by an arbitrary angle
- the line can be chosen by convention

balloon



- **invariance** with respect to common, i.e. **global** rotations of all points of the surface by an arbitrary angle around the given axis
- longitudinal circle can be chosen by convention

requirement of local symmetry



- the balloon is required to keep its form, if each point of the surface is allowed to be rotated by an arbitrary angle - independently of the other points - i.e. if the surface remains **invariant** with respect to **local** rotations
- angular convention can be chosen arbitrarily **for each point** of the surface

the local symmetry is only possible
in the presence of **forces**

'derivation' of quantum electrodynamics (QED) from the gauge principle

♣ **history** electromagnetic interactions (Maxwell equations, QED) have a local gauge invariance → generalizable → put on the level of a principle → access to the understanding of strong and weak interactions

♣ **starting point** **free matter particle** e.g. electron (with electric charge $Q_\psi = -1$),

described by the Dirac equation $(i\frac{\partial}{\partial t}\gamma^0 - i\vec{\nabla}\vec{\gamma} - m)\psi(t, \vec{x}) = 0$

♣ **global symmetry** the **absolute phase** of the field $\psi(t, \vec{x})$ is **not measurable**

invariance with respect to **global** phase transformations

$$\psi(t, \vec{x}) \rightarrow e^{i\alpha} \psi(t, \vec{x}),$$

where α is an **arbitrary constant**.

The absolute phase can be fixed by **convention**. However, the convention has to be **identical** at all times and at all space points.



requirement of local symmetry

(conceptual analogy to general relativity)

GAUGE PRINCIPLE

- **invariance** with respect to **local** phase transformations

$$\psi(t, \vec{x}) \rightarrow e^{i\alpha(t, \vec{x})} \psi(t, \vec{x}),$$

where α is an **arbitrary function** of t, \vec{x} .

- The **phase convention** can be chosen **arbitrarily** at **each** time and at **each** space point **without effect on observables**



symmetry group

the transformations $\psi(t, \vec{x}) \rightarrow e^{i\alpha(t, \vec{x})} \psi(t, \vec{x})$

build a group of unitary transformations: $U(1)_{\text{em}}$



The requirement of local symmetry is **not** fulfilled for the **free** electron, since

$$\left(\frac{\partial}{\partial t} - \vec{\nabla} \right) (e^{i\alpha(t, \vec{x})} \psi(t, \vec{x})) = e^{i\alpha(t, \vec{x})} \left(\frac{\partial}{\partial t} - \vec{\nabla} \right) \psi(t, \vec{x}) + \underbrace{ie^{i\alpha(t, \vec{x})} \psi(t, \vec{x}) \left(\frac{\partial}{\partial t} - \vec{\nabla} \right) \alpha(t, \vec{x})}_{\text{additional term}}$$



Force as a consequence of the gauge principle

in order to implement the local gauge invariance, the four additional terms require the introduction of **four** fields, the so-called **gauge fields** with **spin 1, mass 0**

$$\underbrace{\left(\frac{\partial/\partial t}{\vec{\nabla}} \right)}_{\partial_\mu} \rightarrow \underbrace{\left(\frac{\partial/\partial t}{\vec{\nabla}} \right)}_{\partial_\mu} + i \underbrace{e \begin{pmatrix} V(t, \vec{x}) \\ -\vec{A}(t, \vec{x}) \end{pmatrix}}_{A_\mu(t, \vec{x})} =: \mathcal{D}_\mu$$

covariant
derivative

local gauge invariance with respect to the **simultaneous local gauge transformations**

$$\psi(t, \vec{x}) \rightarrow e^{i\alpha(t, \vec{x})} \psi(t, \vec{x})$$

$$\underbrace{\begin{pmatrix} V(t, \vec{x}) \\ -\vec{A}(t, \vec{x}) \end{pmatrix}}_{A_\mu} \rightarrow \underbrace{\begin{pmatrix} V(t, \vec{x}) \\ -\vec{A}(t, \vec{x}) \end{pmatrix}}_{A_\mu} - \frac{1}{e} \underbrace{\begin{pmatrix} \partial/\partial t \\ \vec{\nabla} \end{pmatrix}}_{\partial_\mu} \alpha(t, \vec{x})$$

$$(\partial_\mu + ieA_\mu) \psi \rightarrow (\partial_\mu + ie(A_\mu - \frac{1}{e} \partial_\mu \alpha)) (e^{i\alpha} \psi) = \partial_\mu(e^{i\alpha} \psi) + ie(A_\mu - \frac{1}{e} \partial_\mu \alpha)(e^{i\alpha} \psi) =$$

$$e^{i\alpha} \partial_\mu \psi + i(\partial_\mu \alpha) e^{i\alpha} \psi - i(\partial_\mu \alpha) e^{i\alpha} \psi + iee^{i\alpha} A_\mu \psi = e^{i\alpha} (\partial_\mu + ieA_\mu) \psi$$



gauge field=electromagnetic field



results: field equations and Lagrange density of QED

– Dirac equation

field equation for electron field $\psi(t, \vec{x})$

$$(i\gamma^\mu \partial_\mu - m)\psi(t, \vec{x}) = e \gamma^\mu A_\mu(t, \vec{x}) \psi(t, \vec{x})$$

interaction term

– Maxwell equation

field equation for the gauge field = electromagnetic field $A_\mu(t, \vec{x})$

$$\partial_\mu F^{\mu\nu}(t, \vec{x}) = e \bar{\psi}(t, \vec{x}) \gamma^\nu \psi(t, \vec{x}) = \begin{pmatrix} \rho(t, \vec{x}) \\ \vec{j}(t, \vec{x}) \end{pmatrix}$$

interaction term

charge density ρ , current density \vec{j}

– Lagrange density

$$\mathcal{L}_{\text{QED}} = \mathcal{L}_{\text{free}}^\psi + \mathcal{L}_{\text{free}}^{A_\mu} - e \bar{\psi}(t, \vec{x}) \gamma^\mu \psi(t, \vec{x}) A_\mu(t, \vec{x}) \text{ local interaction}$$

formulation of QED

$$\mathcal{L}_{\text{QED}} = \mathcal{L}_{\text{free}} + e \sum_{\psi} Q_\psi \bar{\psi}(t, \vec{x}) \gamma^\mu \psi(t, \vec{x}) A_\mu(t, \vec{x})$$

\mathcal{L}_{int} for all matter fields ψ with electric charges Q_ψ

– satisfying **local gauge invariance** w.r. to $\psi \rightarrow e^{-iQ_\psi \alpha(t, \vec{x})} \psi$ for all ψ , $A_\mu \rightarrow A_\mu - \frac{1}{e} \partial_\mu \alpha(t, \vec{x})$

– **quantization** of the fields ψ and A_μ





coupling

The gauge principle fixes the form of the electromagnetic interaction completely **except for** a constant **e** , the electromagnetic **coupling constant**, which is a measure for the interaction strength and is related to the

fine structure constant

$$\alpha_{\text{em}} = e^2/(4\pi) \quad \text{experimentally } \approx 1/137 \ll 1$$



multiparticle states

matterfields $\psi \rightarrow \text{multi-}\psi$ and $\text{multi-}\bar{\psi}$ states, for $\psi = e, \mu, \tau$, quarks
electromagnetic field $A_\mu \rightarrow \text{multi-photon states}$



formal solution of QED

and multi-photon states

scattering operator S , acting in the space of multi- ψ , multi- $\bar{\psi}$

$$|t = +\infty\rangle = \mathbf{S} |t = -\infty\rangle$$

scattering operator

$$\mathbf{S} = T \left[1 + \underbrace{i \int dt d^3x \mathcal{L}_{\text{int}}}_{\propto e} + \frac{i^2}{2} \underbrace{\left(\int dt d^3x \mathcal{L}_{\text{int}} \right)^2}_{\propto e^2} + \dots \right]$$

\uparrow
 time ordering



perturbation theory

cutting the series off after an appropriate number of terms

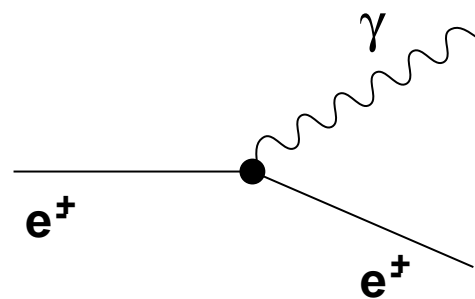


Feynman diagrams

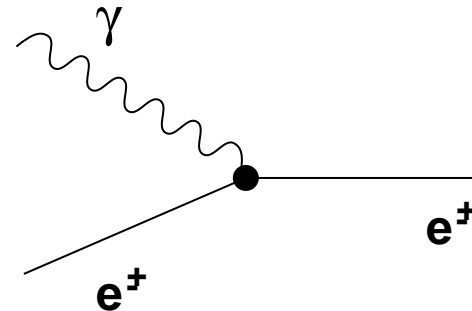
the transition probability for any QED reaction between electrically charged l , q , \bar{l} , \bar{q} and/or photons can be calculated in perturbation theory. The contributions may be represented by Feynman diagrams with the basic building blocks (only electrons (e^-), positrons (e^+) and photons (γ) are considered)

- e^\pm propagator 
- photon propagator 
- **interaction vertex** from \mathcal{L}_{int}

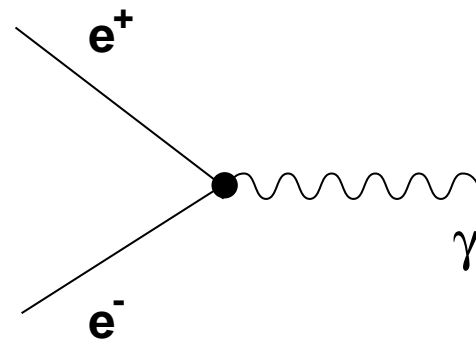
time t

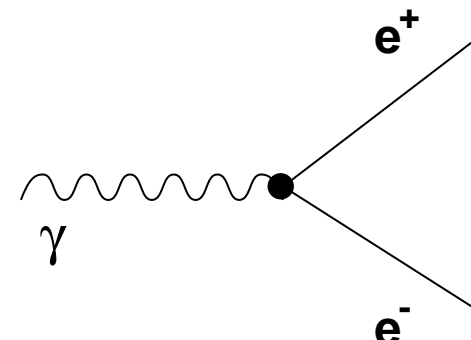
emission of a photon



absorption of a photon



pair annihilation



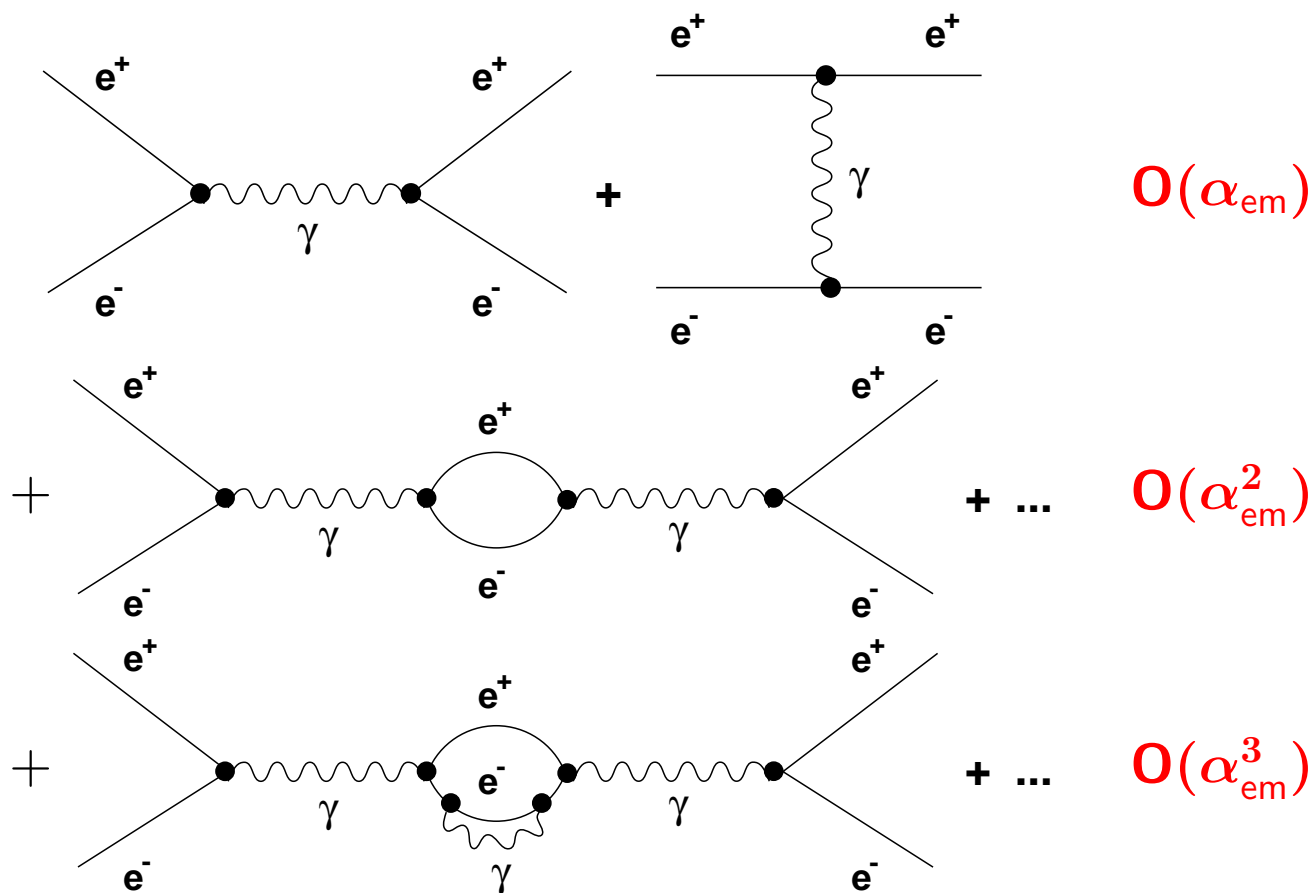
pair production

the same vertex with different orientation of its legs with respect to the time arrow; $e^\pm \rightarrow e^\mp$ if a line changes its direction with respect to the time arrow.



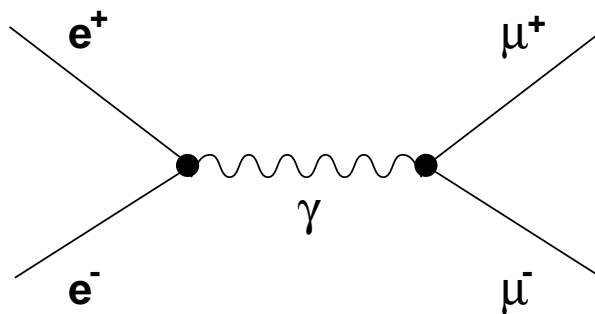
examples

– $e^+e^- \rightarrow e^+e^-$



– $e^+e^- \rightarrow \mu^+\mu^-$

to (α_{em})



'derivation' of quantum chromodynamics (QCD) from the gauge principle

♣ global symmetry in analogy to QED

electric charge

color (charge)

quarks appear in three different colour charges q_{red} , q_{green} , q_{blue}
for each quark flavour $q = u, d, c, s, t, b$

starting point **free particles**

electrically charged particles

field $\psi(t, \vec{x})$

coloured quarks

fields $\psi_{\text{red}}(t, \vec{x})$, $\psi_{\text{green}}(t, \vec{x})$, $\psi_{\text{blue}}(t, \vec{x})$

global symmetry with respect to the global transformations, which leave invariant

$\bar{\psi}\psi$

$\bar{\psi}_{\text{red}}\psi_{\text{red}} + \bar{\psi}_{\text{green}}\psi_{\text{green}} + \bar{\psi}_{\text{blue}}\psi_{\text{blue}}$

$\psi \rightarrow e^{-i\alpha Q_\psi} \psi$

$\psi_i \rightarrow \sum_{j=1}^3 U_{ij} \psi_j \quad i, j = \text{red, green, blue}$

one real constant α

3×3 complex, unitary, **constant** matrix U with $UU^\dagger = U^\dagger U = \mathbb{1}$, $\det U = 1$



requirement of local symmetry

GAUGE PRINCIPLE

- **invariance** with respect to **local** phase transformations

$$\psi_i \rightarrow \sum_{j=1}^3 U_{ij}(t, \vec{x}) \psi_j$$

$i, j = \text{red, green, blue},$

with **arbitrary functions** $U_{ij}(t, \vec{x})$ of t, \vec{x} satisfying $UU^\dagger = U^\dagger U = \underline{1}$, $\det U = 1$.

QED

QCD

symmetry group

$U(1)_{\text{em}}$ | $SU(3)_c$ group of special unitary transformations

QED

QCD

♣ The requirement of local symmetry is **not fulfilled for free particles** → interactions with

gauge fields resp. gauge particles

1× $A^\mu(t, \vec{x}) = \begin{pmatrix} V(t, \vec{x}) \\ \vec{A}(t, \vec{x}) \end{pmatrix}$

electromagnetic field

photon spin 1, mass 0

photons are electrically neutral

$(3 \times 3 - 1) \times G^{\mu, A}(t, \vec{x}), A = 1, \dots, 8$

gluon fields

gluons spin 1, mass 0

gluons carry colour: decisive difference

| | | |
|-------------|-------------|-------------|
| $r \bar{r}$ | $r \bar{g}$ | $r \bar{b}$ |
| $g \bar{r}$ | $g \bar{g}$ | $g \bar{b}$ |
| $b \bar{r}$ | $b \bar{g}$ | $b \bar{b}$ |

'minus' $r \bar{r} + g \bar{g} + b \bar{b}$

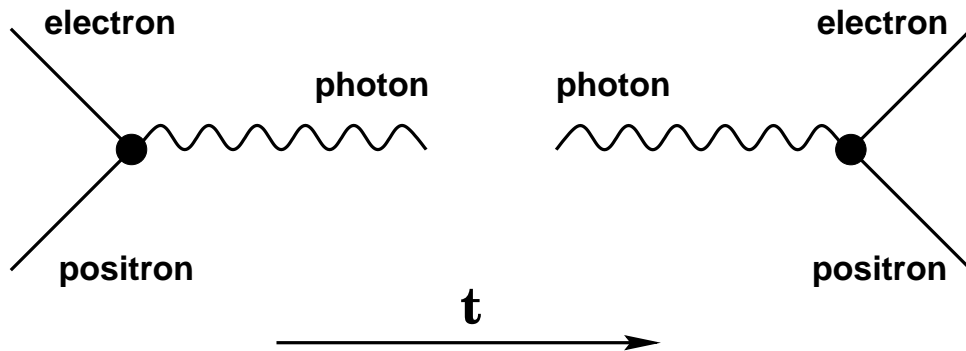
♣ Local gauge invariance fixes all interactions in \mathcal{L}_{QCD} in terms of a single unknown

coupling constant g_c of QCD

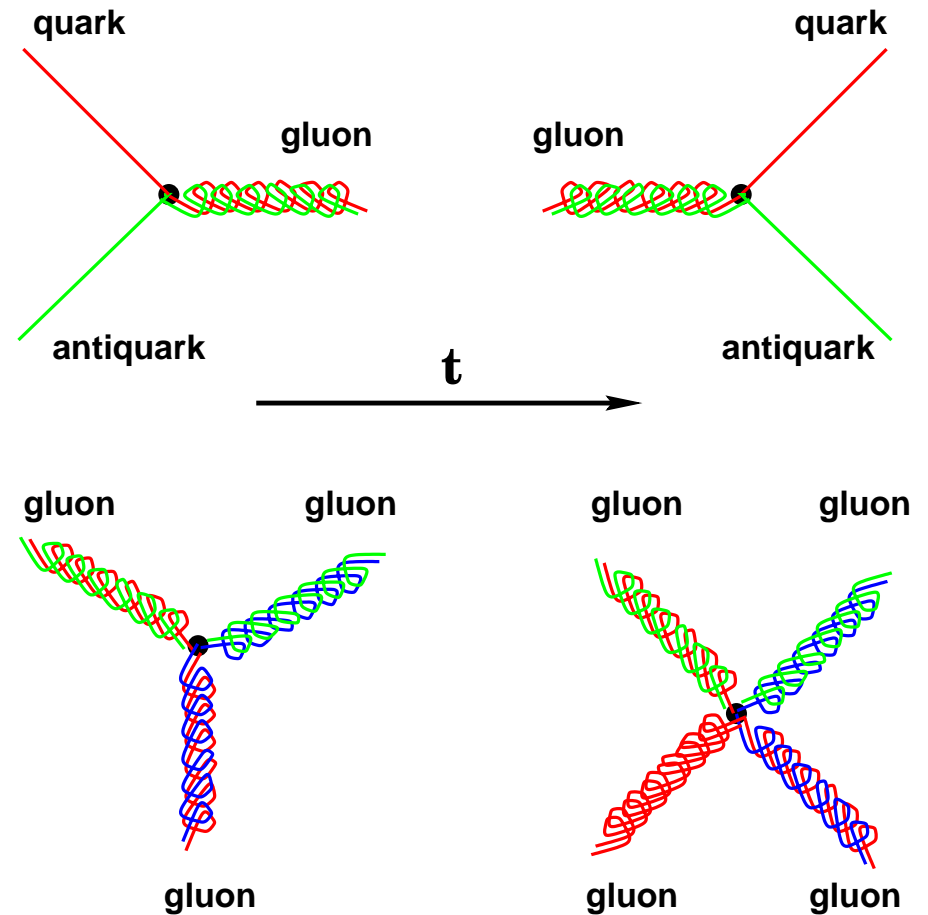


Feynman diagrams

QED



QCD



All couplings are completely determined in terms of a single unknown parameter, the QCD coupling constant g_c

unified electroweak gauge theory from the gauge principle

♣ parity violation in weak interactions

experimentally weak interaction processes violate the invariance with respect to

space reflections $\vec{x} \rightarrow -\vec{x}$

For each fermion $\psi(t, \vec{x}) = (\psi_L(t, \vec{x}), \psi_R(t, \vec{x}))$ with $\psi_L(t, \vec{x}) \rightarrow \psi_R(t, -\vec{x})$ and $\psi_R(t, \vec{x}) \rightarrow \psi_L(t, -\vec{x})$. Thus parity violation is implemented into the theory by treating differently the left-handed (ψ_L) and right-handed (ψ_R) components of the lepton and quark fields (see below). Since the handedness is only Lorentz invariant for massless fermions this implies as a

♣ starting point: massless leptons and quarks

♣ global symmetries to be gauged later on

The global symmetry of the system of **massless free** quarks and leptons is large (symmetry group $U(12)_L \times U(12)_R$). In nature only an $SU(2) \times U(1)$ subgroup appears to be gauged; the following selection leads to success

- $SU(2)_L$ weak isospin symmetry group

the left-handed leptons and quarks are arranged in doublets

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L$$

each described by a doublet of fields $\begin{pmatrix} \psi_{u.c.}^L(t, \vec{x}) \\ \psi_{l.c.}^L(t, \vec{x}) \end{pmatrix}$ $I_3 = +1/2$
 $I_3 = -1/2$

(u.c. for upper component, l.c. for lower component) with assigned quantum numbers I_3 . The two quantum numbers $I_3 = \pm \frac{1}{2}$ play the role of generalized charges, in analogy to the three colours in QCD

invariance with respect to **global $SU(2)_L$** transformations which leave invariant

$$\bar{\psi}_{u.c.}^L \psi_{u.c.}^L + \bar{\psi}_{l.c.}^L \psi_{l.c.}^L : \quad \psi_i^L \rightarrow \sum_j U_{ij} \psi_j^L, \quad i, j = u.c., l.c.$$

with $UU^\dagger = U^\dagger U = \underline{1}$, $\det U = 1$ for the 2×2 complex matrix U

Right-handed leptons and quarks

$e_R, \mu_R, \tau_R, u_R, d_R, c_R, s_R, t_R, b_R$ are assigned zero weak isospin, **$I_3 = 0$**

($\nu_{eR}, \nu_{\mu R}, \nu_{\tau R}$ are not existent in the SM)

- $U(1)_Y$ hypercharge symmetry group

Each l.-h. lepton and quark doublet and each r.-h. lepton and quark is assigned a so-called hypercharge quantum number Y with

$$Q = I_3 + Y/2$$

where Q is the electric charge

invariance with respect to the **global $U(1)_Y$** transformations which leave $\bar{\psi}\psi$ invariant, i.e.

$$\psi(t, \vec{x}) \rightarrow e^{i\alpha Y_\psi} \psi(t, \vec{x})$$

| ψ | Q_ψ | $I_{3\psi}$ | Y_ψ |
|--|---|--|----------|
| $\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$ $\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$ $\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$ | $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ | $\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$ | -1 |
| $\begin{pmatrix} u \\ d \end{pmatrix}_L$ $\begin{pmatrix} c \\ s \end{pmatrix}_L$ $\begin{pmatrix} t \\ b \end{pmatrix}_L$ | $\begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix}$ | $\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$ | 1/3 |
| e_R, μ_R, τ_R | -1 | 0 | -2 |
| u_R, c_R, t_R | 2/3 | 0 | 4/3 |
| d_R, s_R, b_R | -1/3 | 0 | -2/3 |

- global $SU(2)_L \times U(1)_Y$ and $U(1)_{\text{em}}$ symmetries

Because of $Q = I_3 + Y/2$, i.e. $e^{-i\alpha I_3\psi} e^{-i\alpha Y_\psi/2} = e^{-i\alpha Q_\psi}$

$U(1)_{\text{em}}$ is subgroup: $SU(2)_L \times U(1)_Y \supset U(1)_{\text{em}}$



requirement of local symmetry

GAUGE PRINCIPLE

- **invariance** with respect to **local** $SU(2)_L \times U(1)_Y$ transformations \rightarrow
unified electroweak gauge interactions \supset QED

♣ 2 undetermined gauge couplings

$$\begin{array}{ccccc}
 SU(2)_L & \times & U(1)_Y & \supset & U(1)_{\text{em}} \\
 \updownarrow & & \updownarrow & & \updownarrow \\
 g & & g' & & e \quad \rightarrow \quad \boxed{\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2}}
 \end{array}$$

with $e = g \sin \theta_W = g' \cos \theta_W$, θ_W = Weinberg angle

2 parameters **not fixed by the gauge principle**
 $g, g' \longleftrightarrow e, \sin \theta_W$

♣ gauge fields resp. gauge bosons

$$\begin{array}{ccccc}
 SU(2)_L & \times & U(1)_Y & \supset & U(1)_{\text{em}} \\
 \updownarrow & & \updownarrow & & \updownarrow \\
 \underbrace{W_\mu^{1,2,3}(t, \vec{x})} & & B_\mu(t, \vec{x}) & & A_\mu(t, \vec{x})
 \end{array}$$

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \pm iW_\mu^2) \quad (\text{with electric charge } \pm 1)$$

$$A_\mu = \sin \theta_W W_\mu^3 + \cos \theta_W B_\mu = \text{electromagnetic field}$$

$$Z_\mu = \cos \theta_W W_\mu^3 - \sin \theta_W B_\mu = \text{orthogonal field combination}$$

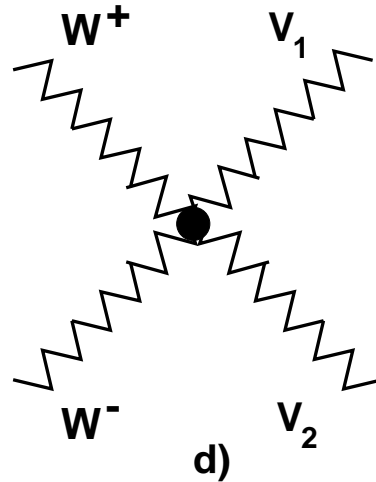
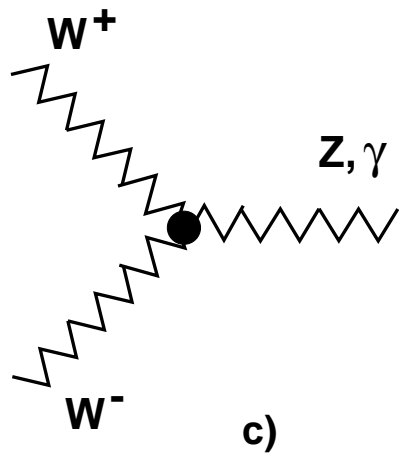
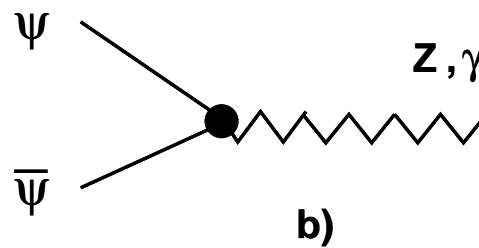
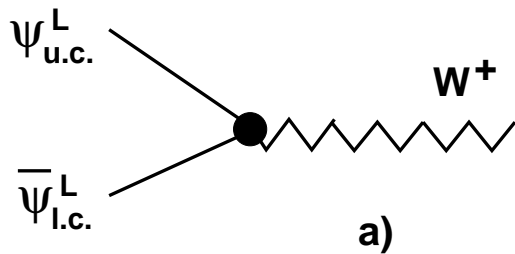
gauge bosons **W^\pm, Z, γ**

with spin 1, mass 0 (so far)



interaction vertices of the electroweak gauge theory

all couplings are determined in terms of the two parameters e , $\sin \theta_W$



$$\text{a) } \begin{pmatrix} \psi_{u.c.}^L \\ \bar{\psi}_{l.c.}^L \end{pmatrix} = \begin{pmatrix} \nu_{eL} \\ e_L^+ \end{pmatrix}, \begin{pmatrix} \nu_{\mu L} \\ \mu_L^+ \end{pmatrix},$$

$$\begin{pmatrix} \nu_{\tau L} \\ \tau_L^+ \end{pmatrix}, \begin{pmatrix} u_L \\ \bar{d}_L \end{pmatrix}, \begin{pmatrix} c_L \\ \bar{s}_L \end{pmatrix}, \begin{pmatrix} t_L \\ \bar{b}_L \end{pmatrix}$$

b) $\psi = \nu_e, \nu_\mu, \nu_\tau, e, \mu, \tau, u, d, c, s, t, b$; no coupling of $\nu \bar{\nu}$ to γ

c) three gauge boson vertices

d) four gauge boson vertices

$$V_1 V_2 = W^+ W^-, Z Z, Z \gamma, \gamma \gamma$$

spontaneous symmetry breakdown



aim **masses**

- for the gauge bosons W^\pm, Z (experimentally $m_W \approx 80 \text{ GeV}$, $m_Z \approx 91 \text{ GeV}$)
- for the quarks and charged leptons

without explicitly breaking the local $SU(2)_L \times U(1)_Y$ gauge symmetry

('explicit' \rightarrow on the level of the forces, i.e. of the Lagrange density)



characteristics of spontaneous symmetry breakdown [SSB]

- symmetry is **unbroken** on the level of the forces
- **groundstate breaks** the symmetry

SSB appears in (classical and quantum) systems with infinitely many degrees of freedom



classical example

- elastic rod, length l , radius r , Young elasticity modul E
- force \vec{K} in direction of the rod axis
 - **cylindrical symmetry** with respect to the rod axis
- critical value of the force $K_{\text{crit}} = \frac{\pi^3}{4} \frac{r^4}{l^2} E$



$$K < K_{\text{crit}}$$



$$K > K_{\text{crit}}$$



SSB in scalar field theory with global $U(1)$ symmetry

- global symmetry on the level of the Lagrange density

a complex scalar (spin 0) field $\Phi(t, \vec{x}) = e^{i\xi(t, \vec{x})} \rho(t, \vec{x})$ i.e. two real scalar fields ξ, ρ

$$\mathcal{L} = \frac{1}{2} \partial_\mu \Phi^\dagger \partial^\mu \Phi - \underbrace{\left(\frac{\mu^2}{2} \Phi^\dagger \Phi + \frac{\lambda}{4} \Phi^\dagger \Phi \Phi^\dagger \Phi \right)}$$

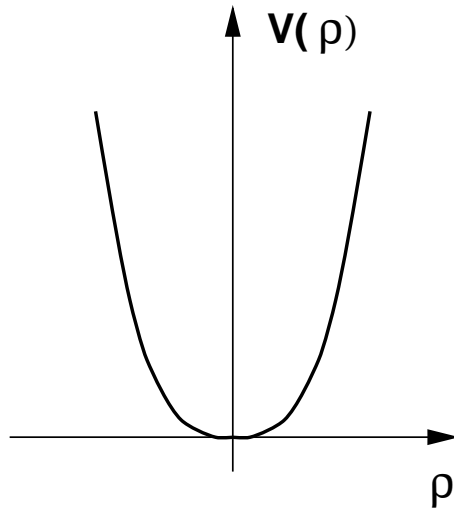
potential $V(\Phi) = V(\rho) = \frac{\mu^2}{2} \rho^2 + \frac{\lambda}{4} \rho^4, \lambda > 0$

global $U(1)$ symmetry w.r. to $\Phi(t, \vec{x}) \rightarrow e^{i\alpha} \Phi(t, \vec{x})$

$\alpha = \text{arbitrary constant}$

– ground state $\frac{\partial V(\rho)}{\partial \rho} = 0 \iff \mu^2 \rho + \lambda \rho^3 = 0$

$\mu^2 > 0$

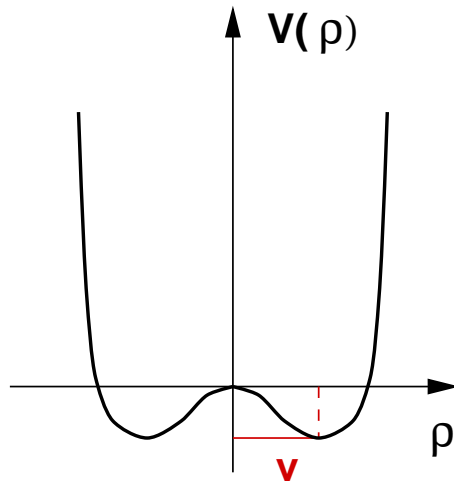


minimum at

$\rho(t, \vec{x}) = 0$

no SSB

$\mu^2 < 0$



minimum at

$\rho(t, \vec{x}) = \pm v$

$v = \sqrt{\frac{|\mu|^2}{\lambda}}$

SSB

expectation value of the field ρ in the ground state $|0\rangle$ $\langle 0 | \rho(t, \vec{x}) | 0 \rangle = v \neq 0$

field shift $\rho(t, \vec{x}) = v + \eta(t, \vec{x})$, $\langle 0 | \eta(t, \vec{x}) | 0 \rangle = 0$



SSB in scalar field theory with local $U(1)$ gauge symmetry Higgs mechanism

- local gauge symmetry on the level of the Lagrange density

$$\mathcal{L} = \frac{1}{2} (\mathcal{D}_\mu \Phi)^\dagger \mathcal{D}^\mu \Phi - V(\Phi) + \mathcal{L}_{\text{free}}^{A_\mu}$$

where $A_\mu(t, \vec{x})$ is the $U(1)$ gauge field
and g the $U(1)$ gauge coupling

$$\mathcal{D}_\mu = \partial_\mu + i g A_\mu(t, \vec{x})$$

local $U(1)$ gauge symmetry with respect to

$$\Phi(t, \vec{x}) \rightarrow e^{i\alpha(t, \vec{x})} \Phi(t, \vec{x})$$

$\alpha(t, \vec{x}) = \text{arbitrary function of } (t, \vec{x})$

- Higgs mechanism

$$\mu^2 < 0, \text{ i.e. SSB: } \Phi(t, \vec{x}) = e^{i\xi(t, \vec{x})} (v + \eta(t, \vec{x}))$$

special gauge transformation to **unitary gauge**

$$\alpha(t, \vec{x}) = -\xi(t, \vec{x}) \rightarrow \Phi(t, \vec{x}) \rightarrow (v + \eta(t, \vec{x}))$$

$$\frac{1}{2} (\mathcal{D}_\mu \Phi)^\dagger \mathcal{D}^\mu \Phi \text{ in } \mathcal{L} \text{ contains } \frac{1}{2} (vg)^2 A_\mu A^\mu =: \frac{1}{2} \mathbf{m}_A^2 A_\mu A^\mu$$

the ξ -field is 'eaten' by A_μ

the gauge field A_μ acquires a mass $m_A = gv$

| balance of number of fields | | | |
|-----------------------------|---|-------------------------------------|---|
| before field shift | | after field shift, in unitary gauge | |
| A_μ , spin 1, mass=0 | 2 | A_μ , spin 1, mass $\neq 0$ | 3 |
| ξ, η | 2 | η | 1 |

- the physical Higgs field $\eta(t, \vec{x})$ with spin 0 and mass $m_H = \sqrt{2} |\mu|$



SSB in the Standard Model

- Higgs sector 4 scalar fields (= 1 complex $SU(2)_L$ doublet field with hypercharge $Y = +1$)
- local $SU(2)_L \times U(1)_Y$ gauge invariance in \mathcal{L}
 - * including the Higgs sector
 - * including gauge invariant Yukawa couplings of l.h. and r.h. lepton and quark fields to the Higgs doublet field

– additional parameters

* μ^2, λ in $V(\text{scalar fields})$, **SSB** for $\mu^2 < 0 \quad \longleftrightarrow$

$$v = \sqrt{\frac{|\mu|^2}{\lambda}}, \quad m_H = \sqrt{2} |\mu|$$

* G_ψ , a Yukawa coupling for each quark and charged lepton

– spontaneous symmetry breakdown is arranged such that

$$SU(2)_L \times U(1)_Y \xrightarrow{\text{spontaneously broken to}} U(1)_{\text{em}}$$

– Higgsmechanism \rightarrow massive gauge fields W^\pm, Z

3 of the 4 scalar fields are eaten by the W^\pm, Z gauge fields \rightarrow

$$m_{W^\pm} = \frac{gv}{2}, \quad m_Z = \frac{m_{W^\pm}}{\cos \theta_W}$$

$m_\gamma = 0$ remains

- masses for quarks and charged leptons from the Yukawa couplings

$$m_\psi = \frac{v}{\sqrt{2}} G_\psi \quad \text{for } \psi = \text{quarks and charged leptons}$$

- one physical Higgs boson spin 0, $m_H = \sqrt{2} |\mu|$

- additional interactions

- * Higgs boson selfinteractions
- * gauge interactions of Higgs bosons with gauge bosons W^\pm, Z
- * Yukawa interactions of Higgs bosons with quarks and leptons

- additional parameters

4 parameters for quark mass mixing

→ violation of invariance under time reversal $t \rightarrow -t$

summary on the gauge theory of the Standard Model

the Standard Model is a **local gauge theory** with gauge group

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

and spontaneous symmetry breakdown

$$SU(2)_L \times U(1)_Y \xrightarrow{\text{spontaneously broken to}} U(1)_{\text{em}}$$

All gauge interactions are fixed by the gauge principle in terms of the three parameters

- g_c , the gauge coupling of the $SU(3)_c$ colour gauge interactions
- e , the gauge coupling of the $U(1)_{\text{em}}$ electromagnetic gauge interactions
- $\sin \theta_W$, relating e by $e = g \sin \theta_W$ and $e = g' \cos \theta_W$ to the gauge couplings g and g' of the $SU(2)_L \times U(1)_Y$ unified electroweak gauge interactions

4. quantum effects, some applications and key precision tests

♣ quantum effects and precision tests in QED

- determination of α_{em} from quantum hall effect $\alpha_{\text{em}} = 1/137.03599911(46)$
- precision test **magnetic moment of the electron**

the **electron** has spin = intrinsic angular momentum ($= \frac{1}{2}\hbar$) and electric charge (-1)
→ it has a **magnetic moment**

$$\mu_e = (1 + a_e)\mu_B \quad \mu_B = \text{Bohr magneton}$$

$$2002: a_{e \text{ exp}} = 0.0011596521859(38)$$

$$a_{e \text{ theo}} = 0.0011596521535(240)$$

$$a_{e \text{ theo}} = \frac{1}{2} \frac{\alpha_{\text{em}}}{\pi} + C_2 \left(\frac{\alpha_{\text{em}}}{\pi} \right)^2 + C_3 \left(\frac{\alpha_{\text{em}}}{\pi} \right)^3 + C_4 \left(\frac{\alpha_{\text{em}}}{\pi} \right)^4 + \dots$$

C_2, C_3, C_4 calculated in QED with
 $m_e = 0.510998918(44) \text{ MeV} \rightarrow$

– precision test **magnetic moment of the muon**

2004 including 2001 data: $a_{\mu \text{ exp}} = 0.0011659208(6)$

with α_{em} and $m_e \rightarrow m_{\mu} = 105.6583692(94) \text{ MeV}$ up to $O((\alpha_{\text{em}}^5))$ as well as including weak and hadronic quantum corrections!

$$2002: a_{\mu \text{ theor}} = \left\{ \begin{array}{l} 0.00116591669 \pm 74 \text{ Hagiwara et al} \\ 0.00116591693 \pm 109 \text{ Davier et al} \\ (e^+e^- \text{ based}) \\ 0.001169591936 \pm 98 \text{ Davier et al} \\ (\tau \text{ based}) \end{array} \right.$$

2005: τ -based analysis in doubt

deviation up to 2.7σ

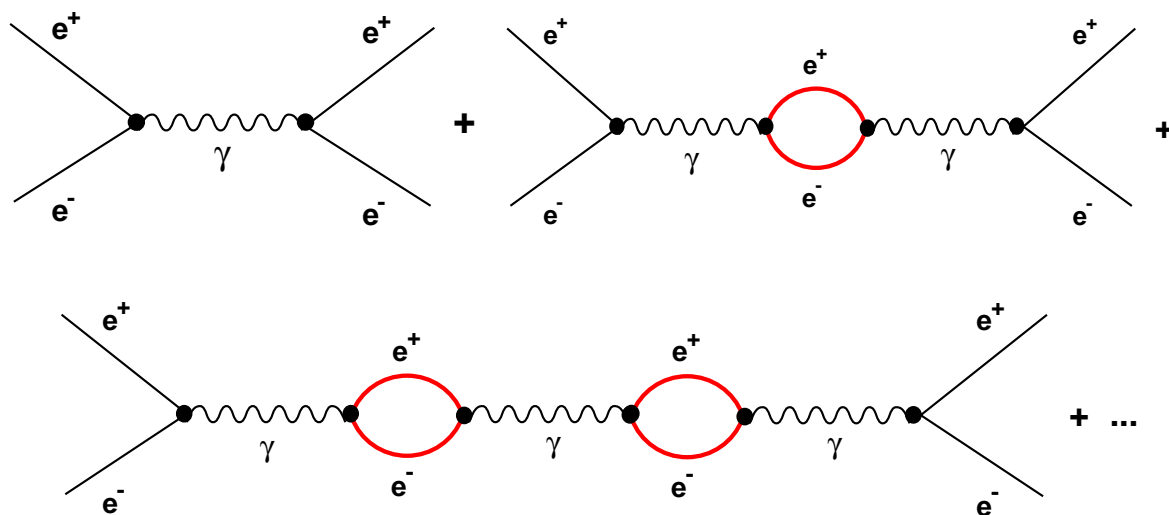
improved theoretical investigations under way



running couplings in QED and QCD – confinement and asymptotic freedom

- **QED** for simplicity for electrons only

evaluation of an important class of diagrams to all orders in perturbation theory of QED leads to



running coupling of QED

$$\alpha_{\text{em}}(Q^2) = \frac{\alpha_{\text{em}}(Q_0^2)}{1 - \frac{\alpha_{\text{em}}(Q_0^2)}{3\pi} \log \frac{Q^2}{Q_0^2}}$$

$\sqrt{Q^2}$ = momentum transfer

uncertainty principle:

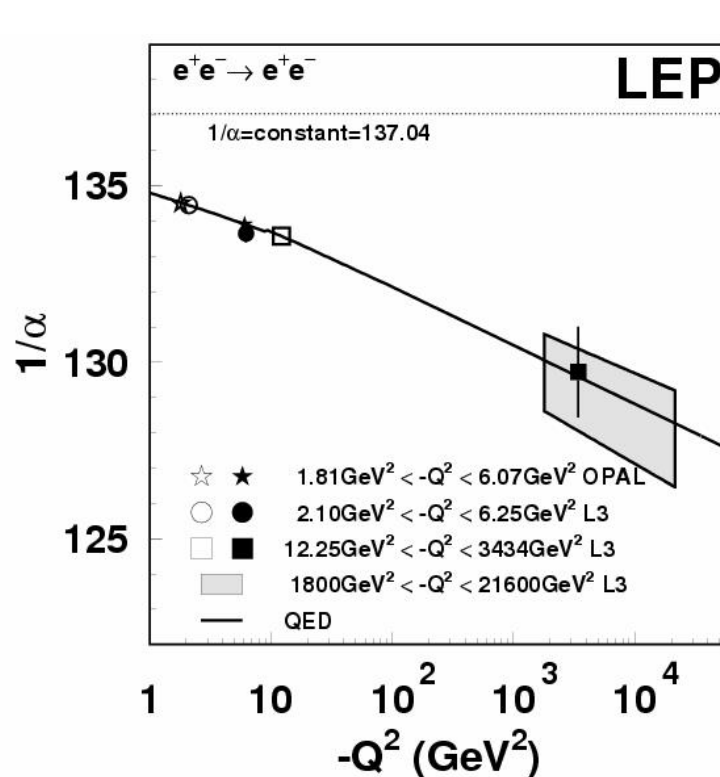
$$\sqrt{Q^2} \Delta x \gtrsim \hbar$$

$$\alpha_{\text{em}}(0) = 1/137.03599976(50)$$

increasing distance $\Delta x \nearrow$, i.e. $Q^2 \searrow$: $\alpha_{\text{em}}(Q^2) \searrow$
screening of electric charge

2005: running $\alpha_{\text{em}}(Q^2)$ in a plot $1/\alpha_{\text{em}}(Q^2)$ versus Q^2 from LEP

Another LEP textbook plot?



$$\alpha(Q^2) = \frac{\alpha_0}{1 - C \cdot \Delta\alpha(Q^2)}$$

$C = 1.05 \pm 0.15$ for $1800\text{GeV}^2 < -Q^2 < 21600\text{GeV}^2$
 $26^\circ < \theta < 90^\circ$ $\sqrt{s} = 189\text{--}209\text{GeV}$ L3, hep-ex/0507078

$\Delta\alpha(-6.07\text{GeV}^2) - \Delta\alpha(-1.81\text{GeV}^2) = (440 \pm 78) \times 10^{-5}$
 $1.4^\circ < \theta < 3.4^\circ$ $\sqrt{s} \approx 91\text{GeV}$ OPAL, CERN-PH-EP/2005-014

$\alpha^{-1}(-2.10\text{GeV}^2) - \alpha^{-1}(-6.25\text{GeV}^2) = 0.78 \pm 0.26$
 $1.8^\circ < \theta < 3.1^\circ$ $\sqrt{s} \approx 91\text{GeV}$ L3, PLB 476(2000)40

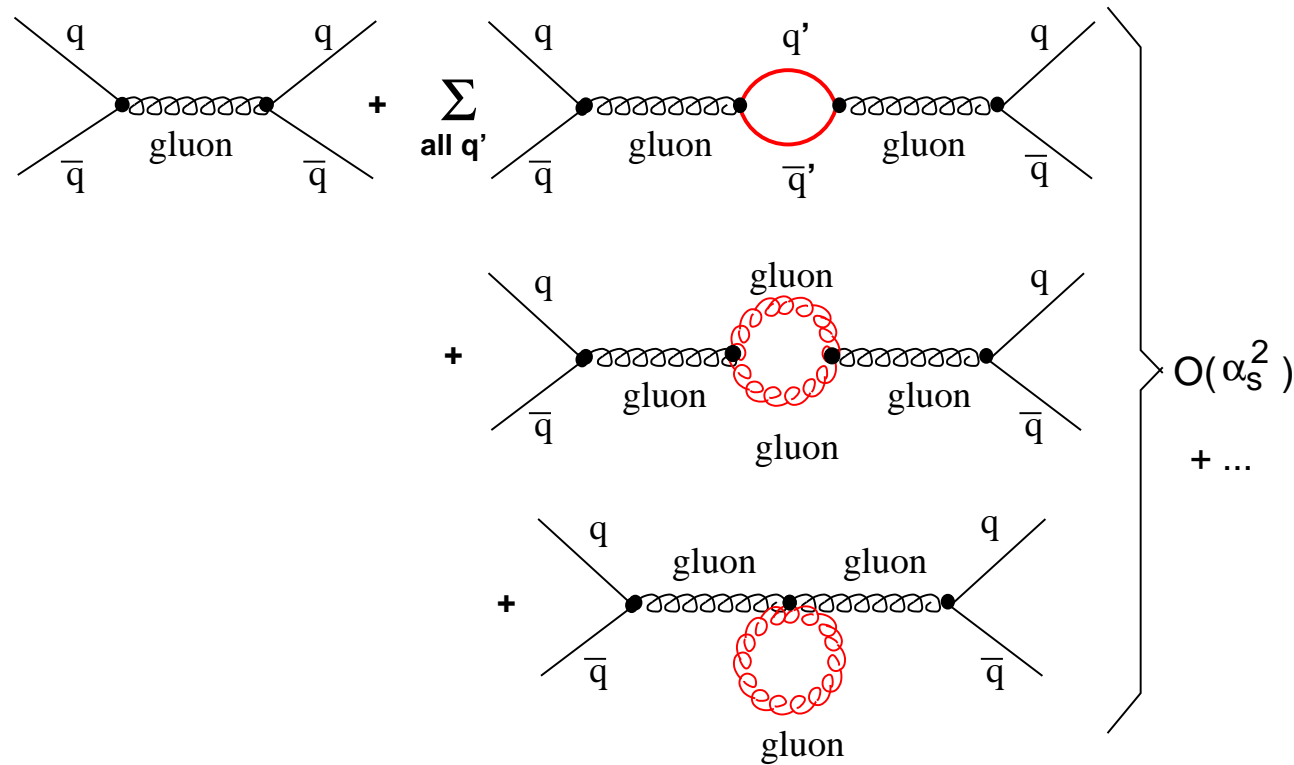
$\alpha^{-1}(-12.25\text{GeV}^2) - \alpha^{-1}(-3434\text{GeV}^2) = 3.80 \pm 1.29$
 $20^\circ < \theta < 36^\circ$ $\sqrt{s} = 189$ L3, PLB 476(2000)40

The last three measurements need an *anchor* to plot the evolution of $\alpha(Q^2)$!

Fix the low- Q^2 sides of the ranges to the QED predictions
 (Burkhardt&Pietrzyk PLB 513(2001)46) and derive the high- Q^2 values

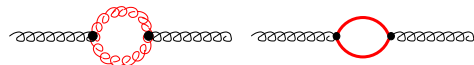
– QCD

evaluation of the corresponding class of diagrams to all orders in perturbation theory of QCD – provided $\alpha_s = g_c^2/(4\pi) \ll 1$ – leads to



running coupling of QCD

$$\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 + \left(11 - \frac{2}{3}n_q\right) \frac{\alpha_s(Q_0^2)}{4\pi} \log \frac{Q^2}{Q_0^2}}$$



antiscreening **screening**

valid for $\alpha_s \ll 1$

n_q = number of quark flavours, $11 - \frac{2}{3}n_q > 0$ for $n_q \leq 6$

the antiscreening is the **consequence of the gluon selfinteraction**, which in turn is the consequence of the gauge principle!

$$\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 + (11 - \frac{2}{3}n_q) \frac{\alpha_s(Q_0^2)}{4\pi} \log \frac{Q^2}{Q_0^2}}$$

increasing distance $\Delta x \nearrow$, i.e. $Q^2 \searrow$: $\alpha_s(Q^2) \nearrow$

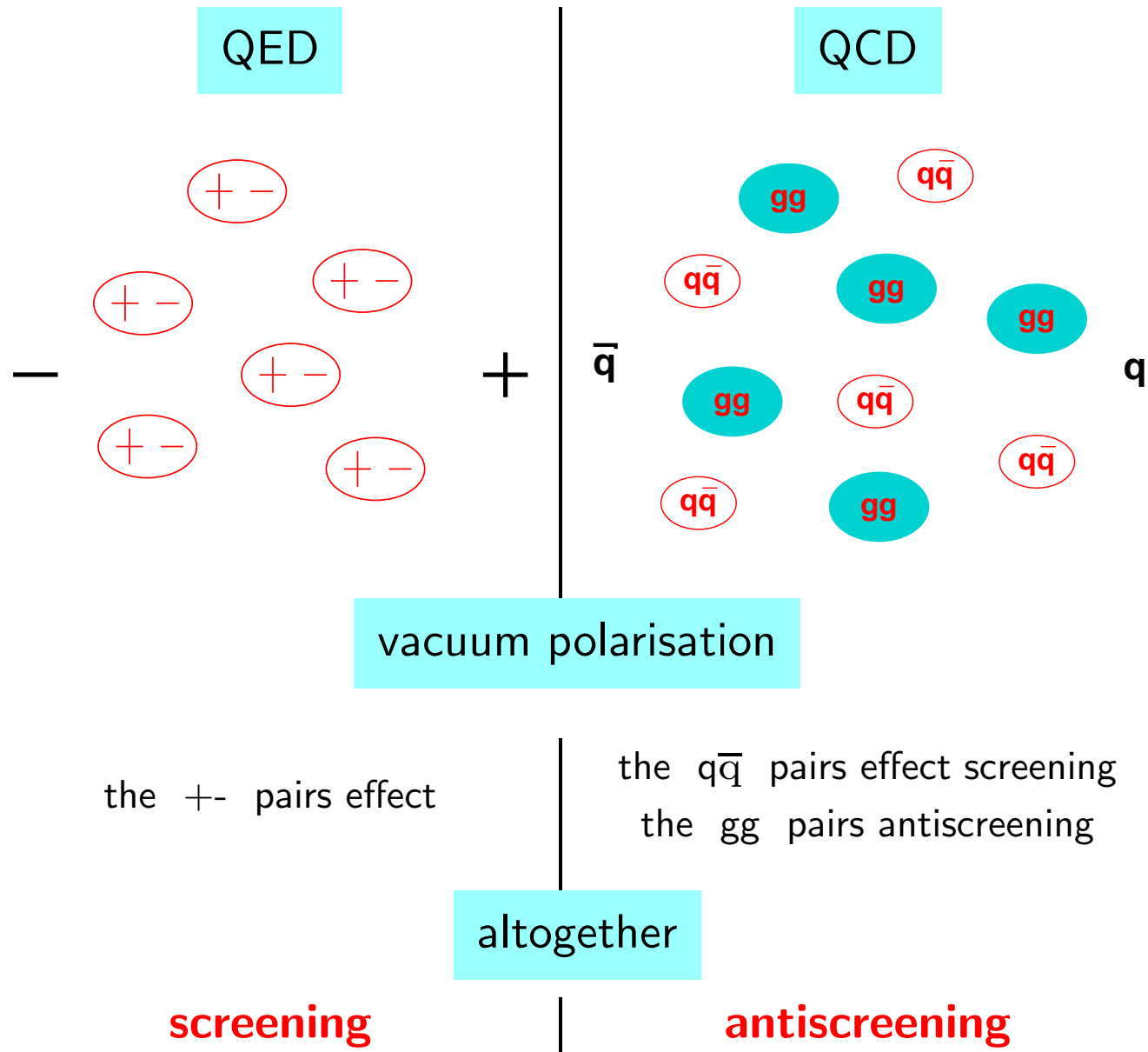
antiscreening of colour

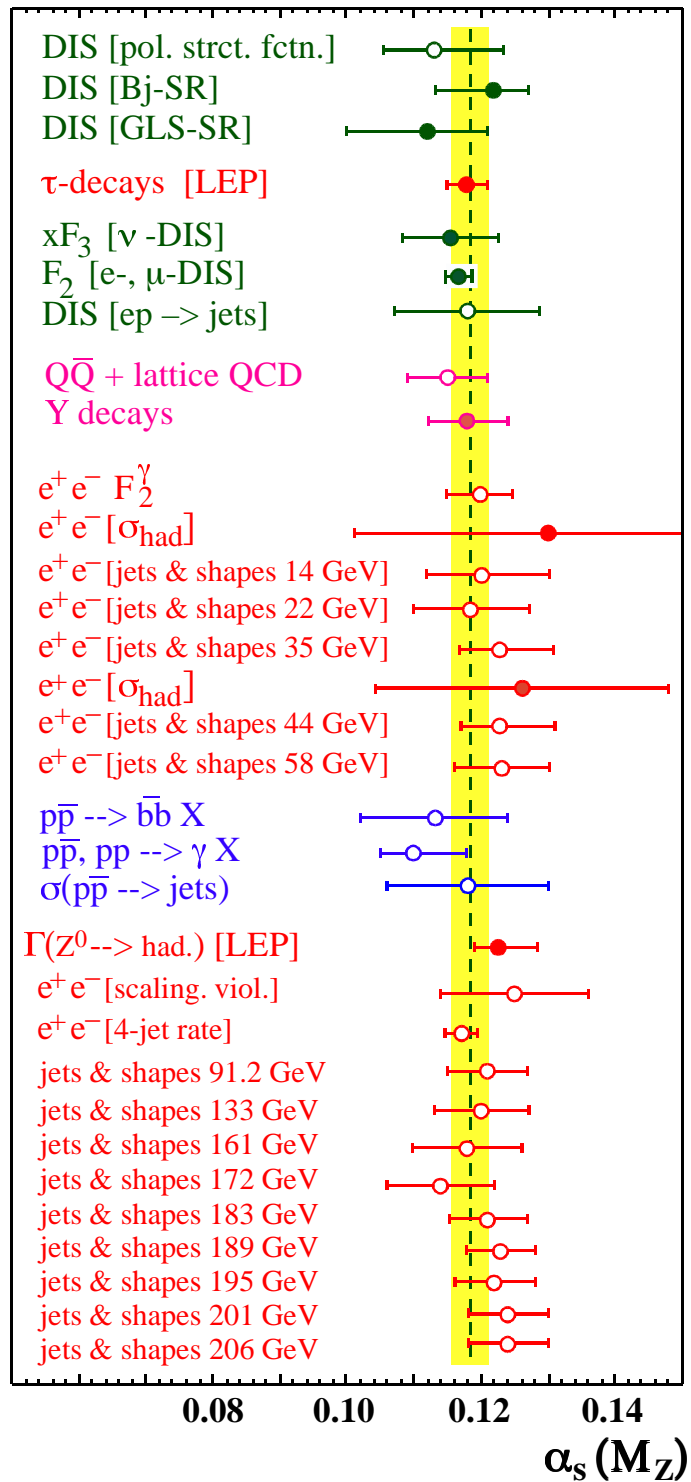
\rightarrow suggests **confinement**

decreasing distance $\Delta x \searrow$, i.e. $Q^2 \nearrow$: $\alpha_s(Q^2) \searrow 0$

\rightarrow **asymptotic freedom** (no interaction for $Q^2 \rightarrow \infty$)

– qualitative discussion

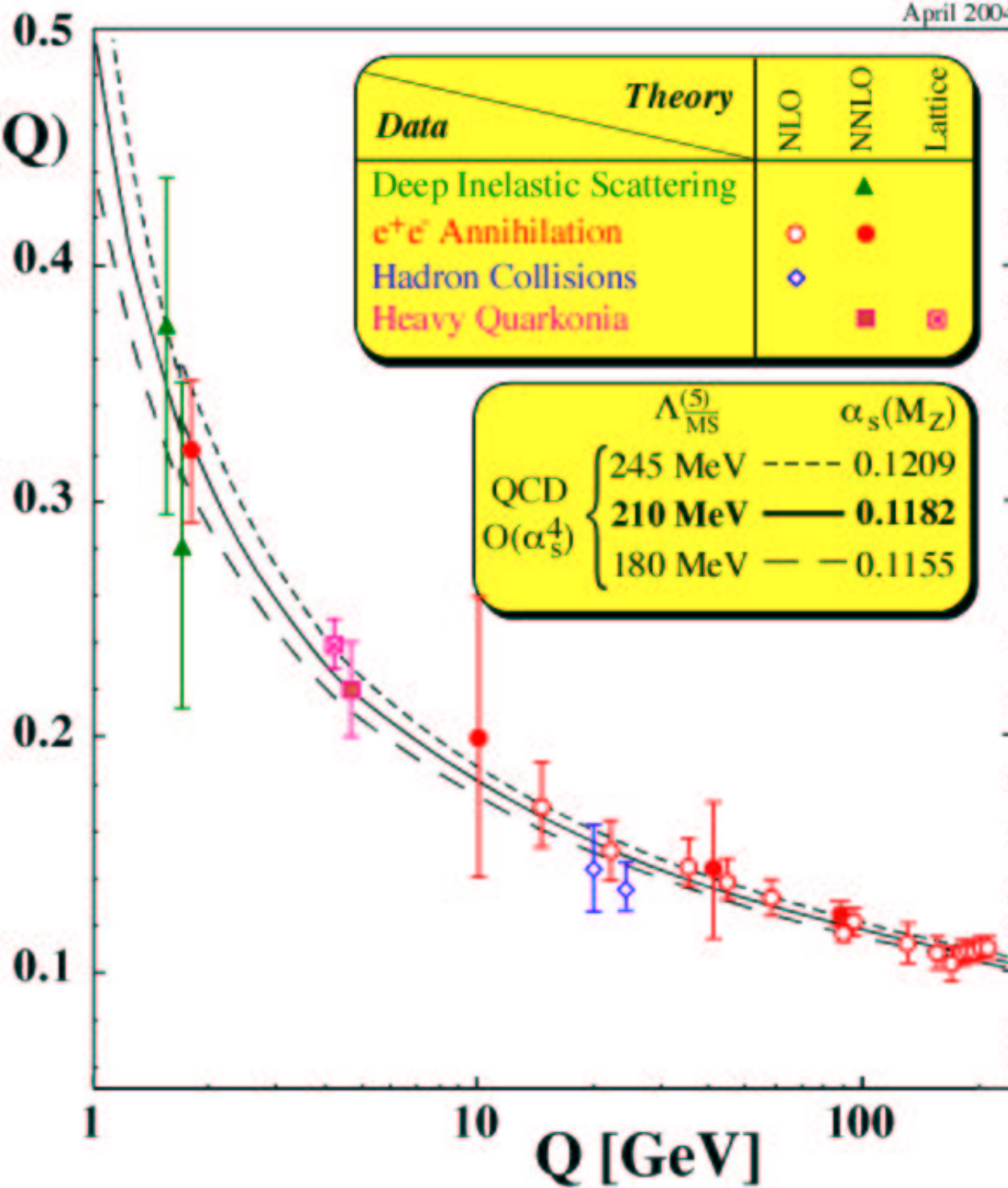




world average 2004: $\alpha_s(M_Z) = 0.1182 \pm 0.0027$

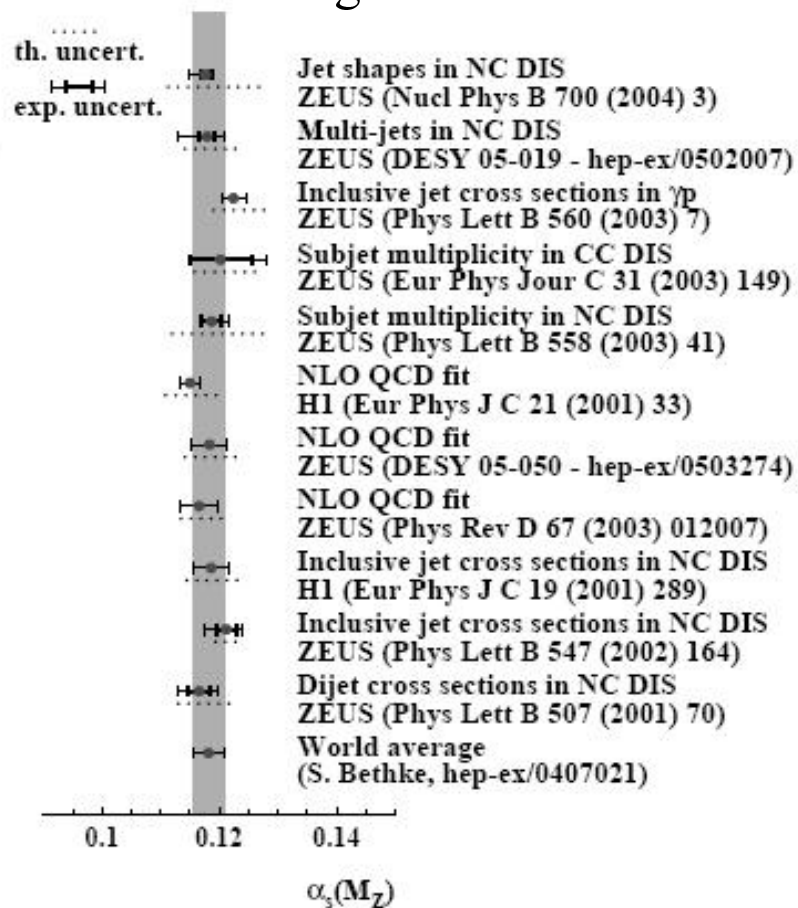
CDF 2005: $\alpha_s(M_Z) = 0.1188 \pm 0.0027$

HERA 2005: $\alpha_s(M_Z) = 0.1186$
 $\pm 0.0011 \text{ exp } \pm 0.0050 \text{ theor.}$

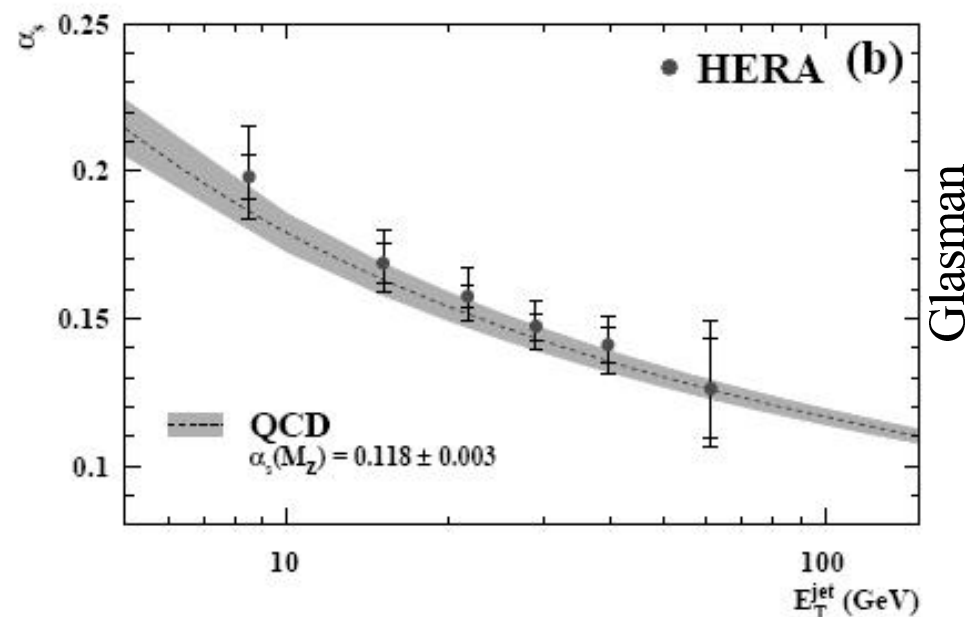
running $\alpha_s(Q)$ 

HERA α_s summary

- C.f. HERA α_s measurements and world average:



- α_s measurements from jets.



- Preliminary HERA average:
 $\alpha_s(m_Z^2) = 0.1186 \pm 0.0011(\text{exp.})$
 $\pm 0.0050(\text{th.})$



test of asymptotic freedom and of three colours

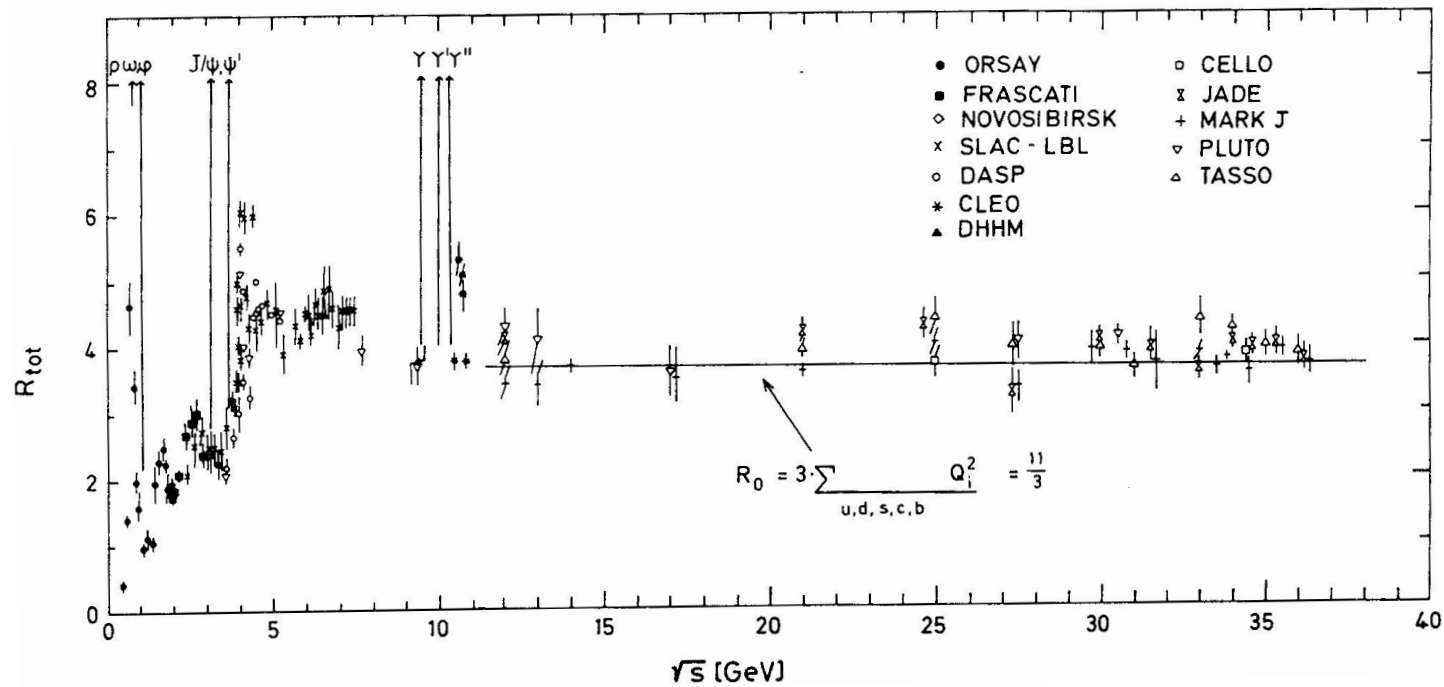
process of interest:

$e^+e^- \rightarrow$ **all hadronic final states** at small distances, i.e. at **large Q^2** ($m_b^2 \ll Q^2 \ll m_Z^2$)

parton model = 'zeroth order' QCD, asymptotic freedom approximated by **$\alpha_s = 0$: no colour interaction** between the q and \bar{q} , i.e. no exchange or radiation of gluons, etc.

$$R_{e^+e^-} = \frac{\sum_{\text{all hadronic final states}} |e^+e^- \rightarrow \text{hadrons}|^2}{|e^+e^- \rightarrow \mu^+\mu^-|^2} = \frac{\sum_{\text{all quarks } q} \left| \begin{array}{c} e^+ \\ e^- \end{array} \begin{array}{c} \gamma \\ \end{array} \begin{array}{c} q \\ Q_q e \\ \bar{q} \end{array} \right|^2}{\left| \begin{array}{c} e^+ \\ e^- \end{array} \begin{array}{c} \gamma \\ \end{array} \begin{array}{c} \mu^+ \\ Q_\mu e \\ \mu^- \end{array} \right|^2} \times \underbrace{\left\{ \begin{array}{l} \text{probability that the } q\bar{q} \text{ final state turns} \\ \text{at large distances into some hadronic final state} \end{array} \right\}}_{\equiv 1 \text{ due to confinement}}$$

$$= \sum_{\text{all quarks } q} Q_q^2 = \underset{\text{colour}}{\underset{\uparrow}{3}} \sum_{q=u,d,s,c,b} Q_q^2 = 3 \left(\underset{u}{\frac{4}{9}} + \underset{d}{\frac{1}{9}} + \underset{s}{\frac{1}{9}} + \underset{c}{\frac{4}{9}} + \underset{b}{\frac{1}{9}} \right) = \frac{11}{3} = 3.67$$



sensitive to

- asymptotic freedom
- number of colours
- electric charges of the quarks



deep inelastic scattering and proton structure functions as test of perturbative QCD

HERA!

– process

$$e^\pm p \rightarrow e^\pm \text{ all hadronic final states } (X)$$

at small distances, i.e. at **large Q^2**

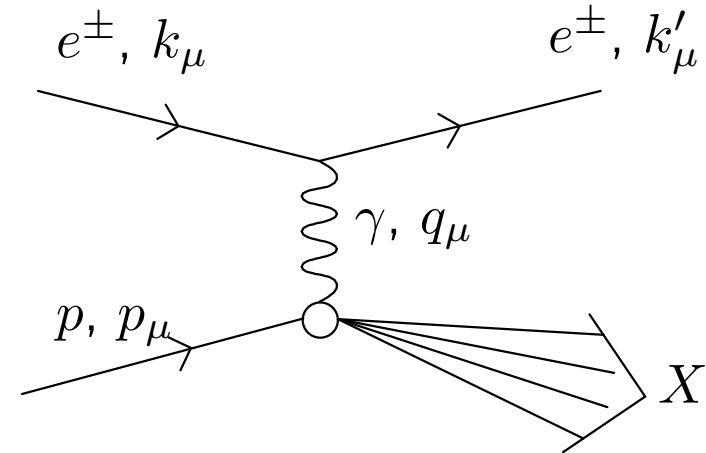
– two variables

* $\boxed{Q^2} = -q^2 = -(\text{momentum transfer})^2$ carried by the photon

$Q^2 \nearrow$:

- the **resolution increases** with which the photon probes the (electrically charged) constituents of the proton, i.e. the quarks
- $\alpha_s(Q^2) \searrow$, which allows to treat the interactions between the quarks and gluons in the proton within the framework of QCD perturbation theory

* $\boxed{x} = \frac{Q^2}{2p \cdot q} = \text{fraction of the proton momentum carried by the quark interacting with the photon}$
 $(0 \leq x \leq 1)$



- quark distribution functions

$$q_i(x, Q^2)$$

= probability to find the quark q_i in the proton with proton momentum fraction x , probed by the photon carrying Q^2 .

- parton model = 'zeroth' order QCD asymptotic freedom approximated by $\alpha_s = 0$

$$\rightarrow q_i(x, Q^2) = q_i(x), \quad Q^2 \text{ independence} \rightarrow \text{scaling}$$

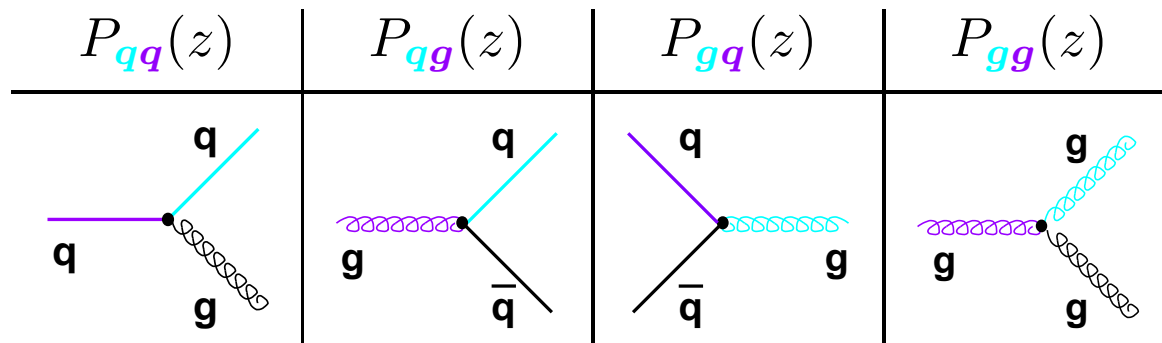
- first order QCD DGLAP equations (Dokshitzer, Gribov, Lipatov, Altarelli, Parisi)

coupled integro-differential equations for the quark distribution functions $q_i(x, Q^2)$ and the gluon distribution function $g(x, Q^2)$

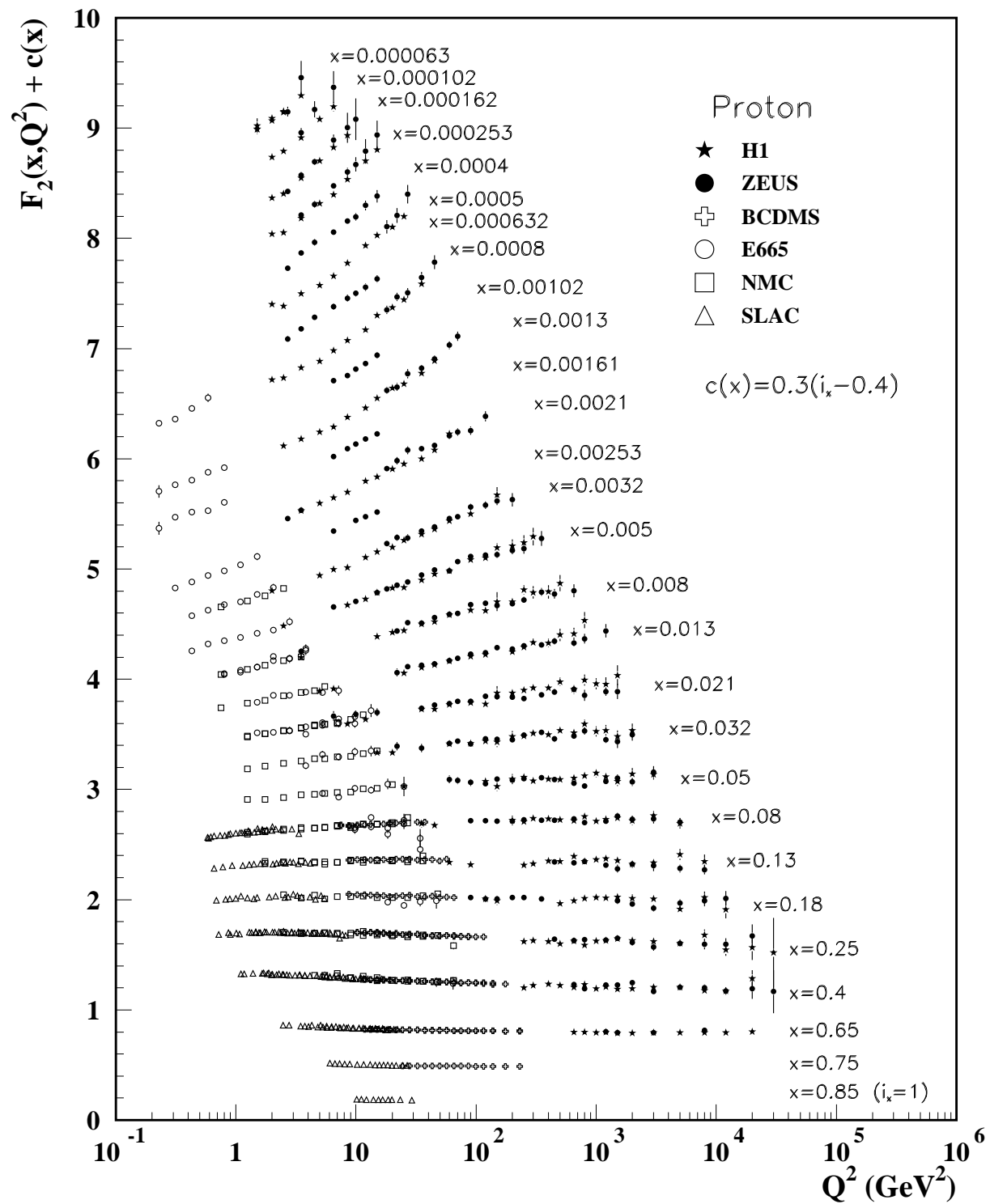
$$Q^2 \frac{\partial q_i(x, Q^2)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} (q_i(y, Q^2) P_{qq}\left(\frac{x}{y}\right) + g(y, Q^2) P_{qg}\left(\frac{x}{y}\right))$$

$$Q^2 \frac{\partial g(x, Q^2)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} (\sum_i q_i(y, Q^2) P_{gq}\left(\frac{x}{y}\right) + g(y, Q^2) P_{gg}\left(\frac{x}{y}\right))$$

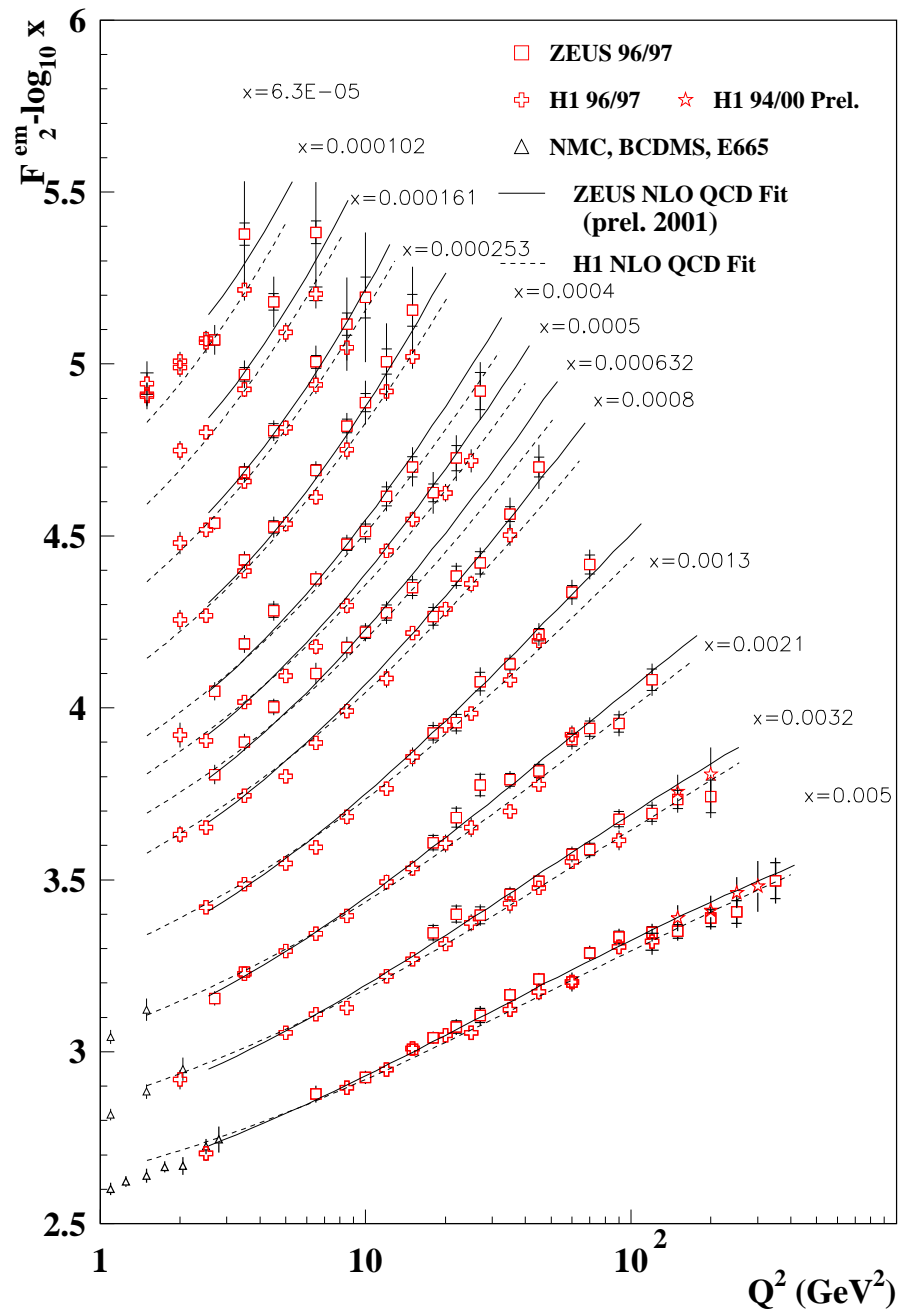
the splitting functions P are known from QCD



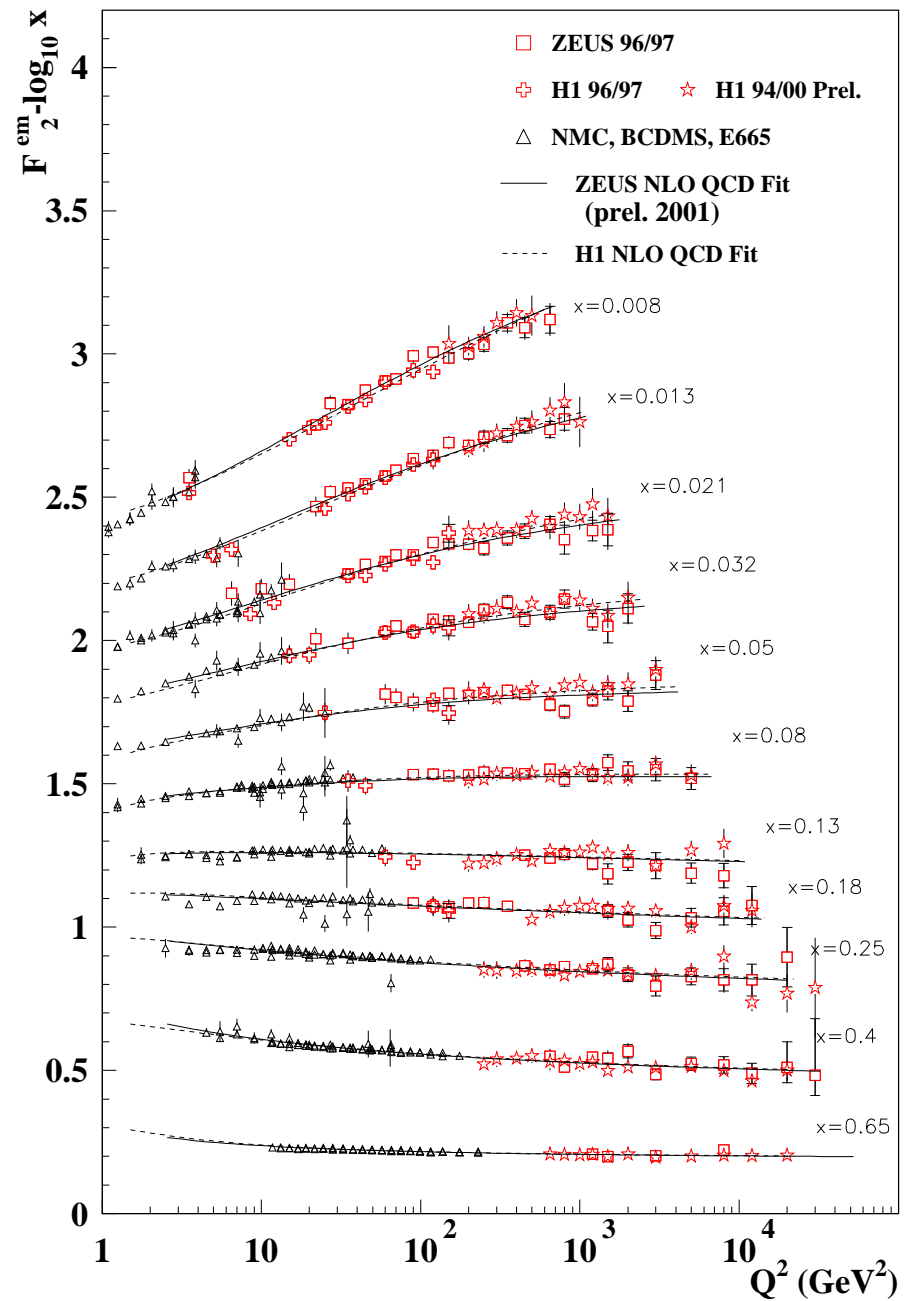
prediction of **scaling violation** as function of Q^2



ZEUS+H1



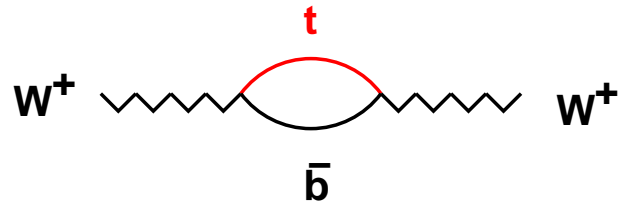
ZEUS+H1





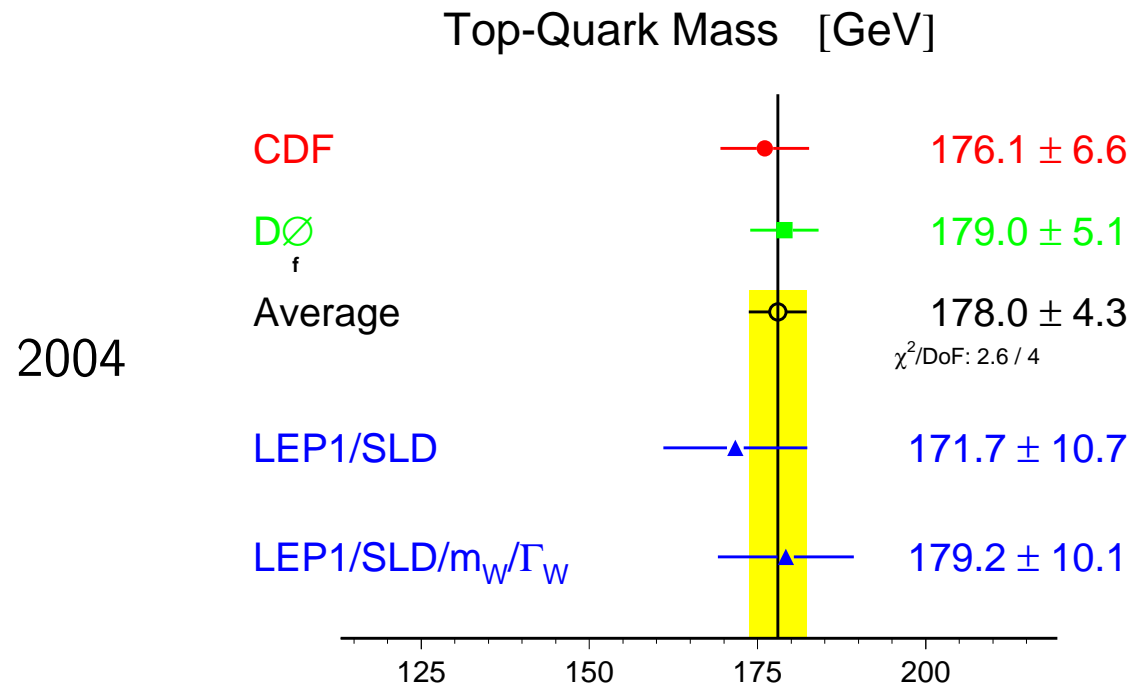
quantum effects in electroweak interactions

- indirect determination of m_t through its effect in loops at LEP and SLD, eg.



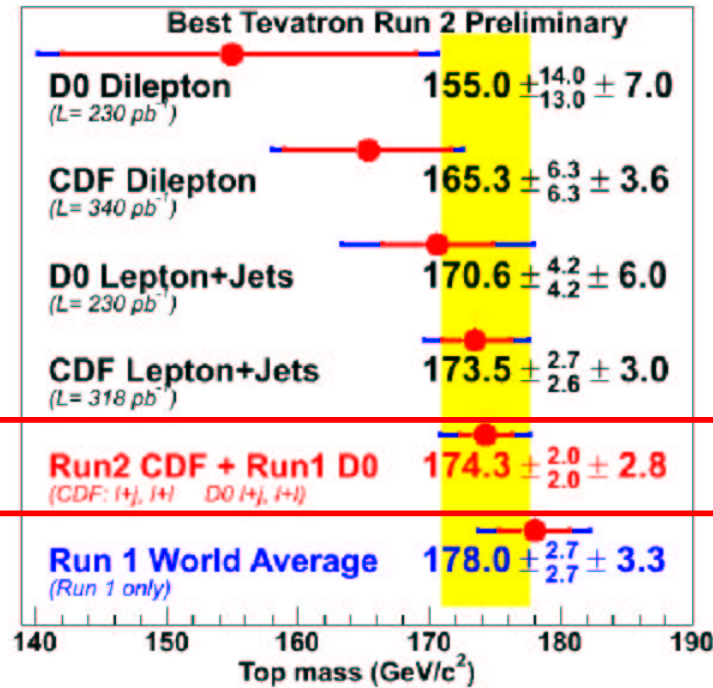
assumption: no effects from new physics beyond the Standard Model

- comparison with direct measurement of m_t at FERMILAB (D0 and CDF)



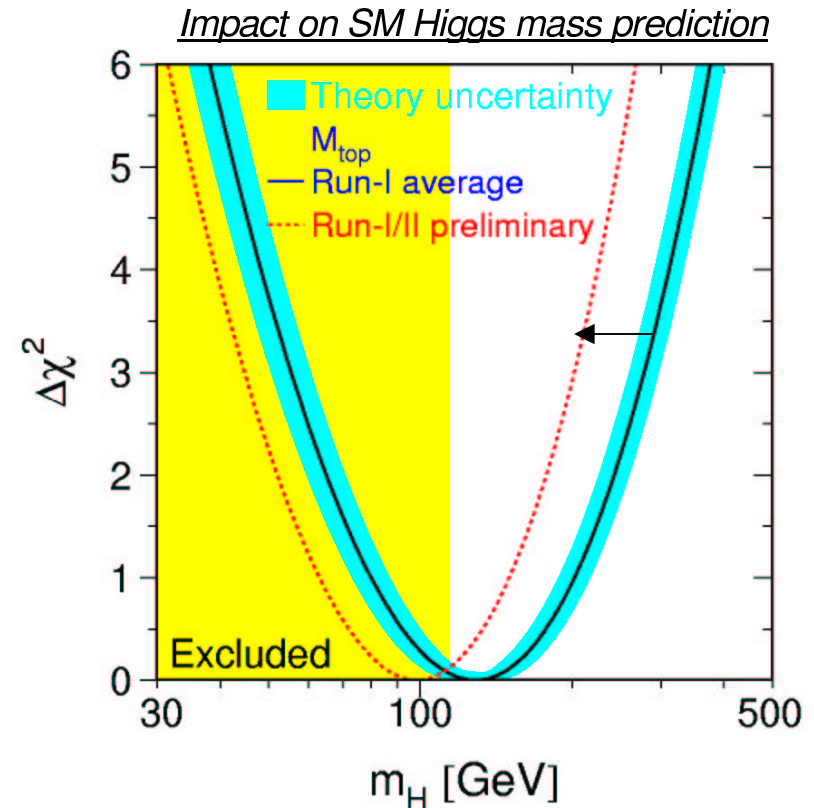
Top Quark Mass: Summary

2005



- New Run II single measurements achieving uncertainties comparable to/better than current Run I world average.
- BREAKING NEWS:** New preliminary world average combining CDF Run II and DØ Run I.

$$m_t = 174.3 \pm 3.4 \text{ GeV}; \chi^2 / \text{dof} = 3.6/3$$



$$M_H = 129^{+74}_{-49} \text{ GeV}; M_H < 285 \text{ GeV @ 95\% CL}$$

$$M_H = 98^{+52}_{-36} \text{ GeV}; M_H < 208 \text{ GeV @ 95\% CL}$$

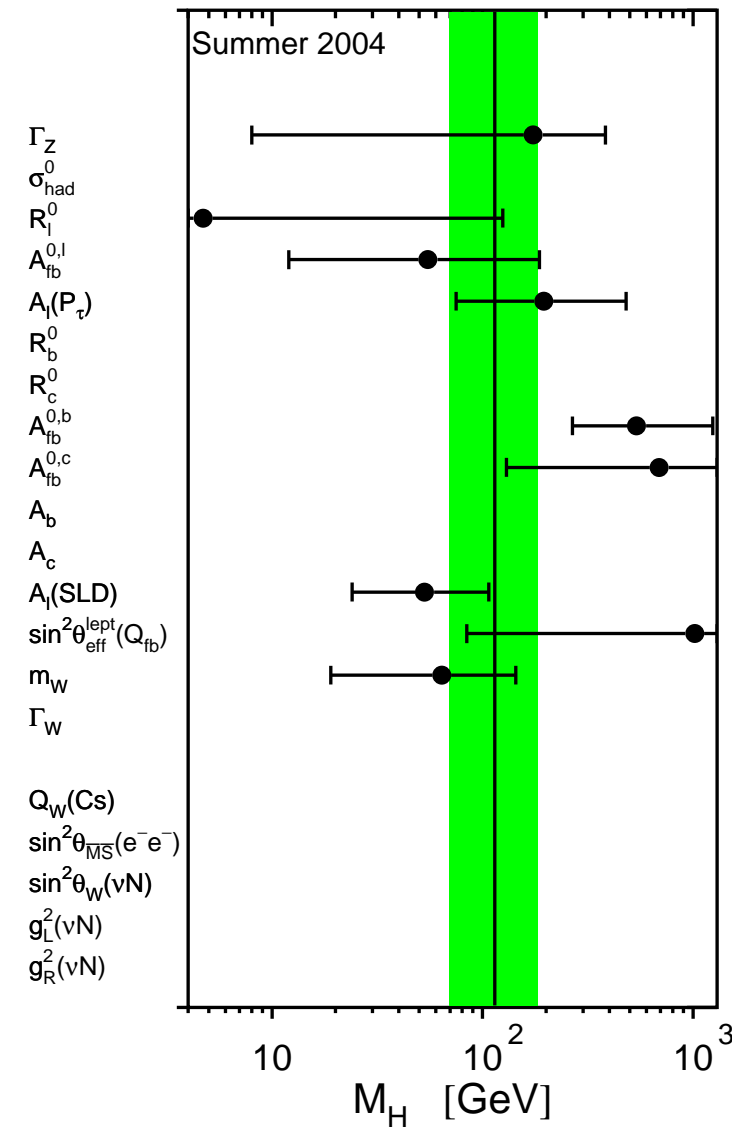
– indirect determination of m_H

lower bound on Higgs mass $m_H > 114$, resp. 117 GeV
at 95% CL from LEP resp. FERMILAB

best fit for the Higgs mass $m_H = 98^{+52}_{-36}$ GeV

at 68% CL from LEP, using the 2005 FERMILAB value for m_t

upper bound on Higgs mass $m_H < 208$ GeV at 95% CL



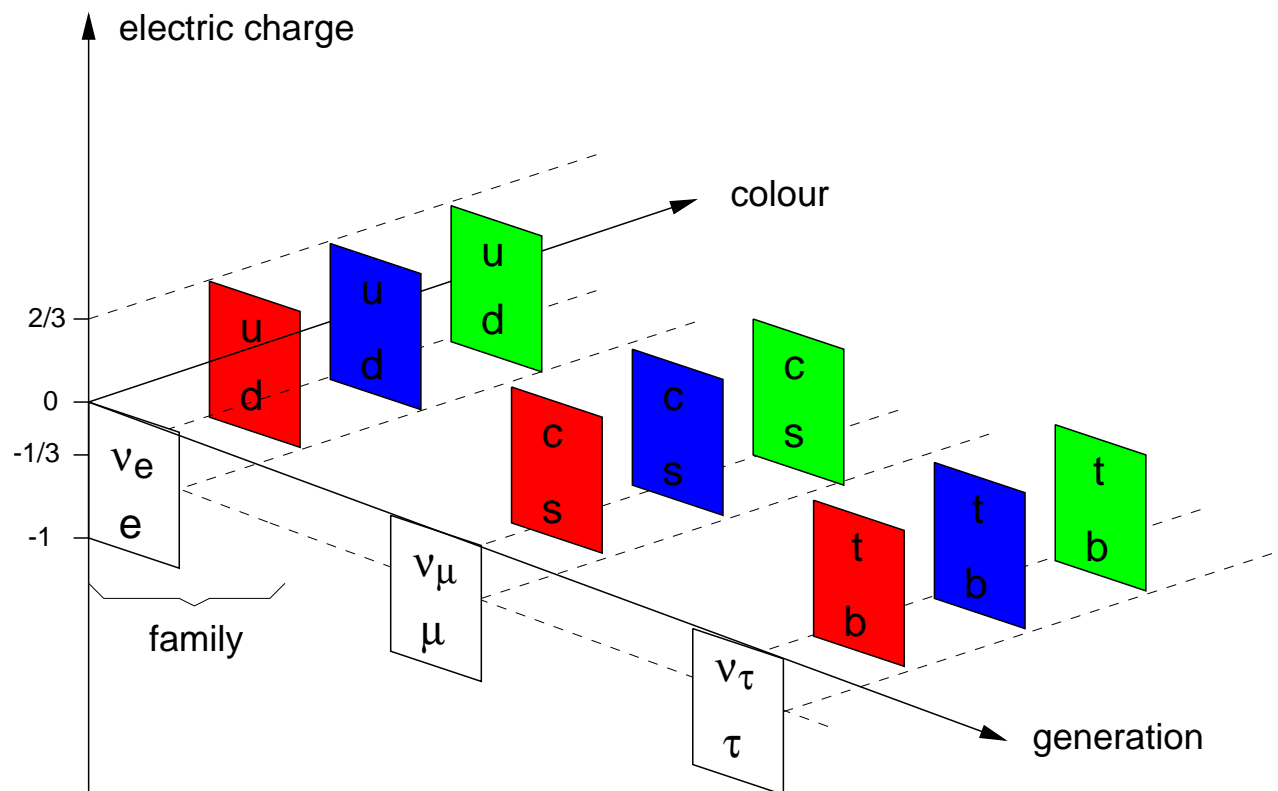
| Data | \hat{s}_Z^2 | s_W^2 | $\alpha_s(M_Z)$ | M_H |
|--|---------------|------------|-----------------|---------------------|
| All data | 0.23120(15) | 0.2228(4) | 0.1213(18) | 113^{+56}_{-40} |
| All indirect (no m_t) | 0.23116(17) | 0.2229(4) | 0.1213(18) | 79^{+95}_{-38} |
| Z pole (no m_t) | 0.23118(17) | 0.2231(6) | 0.1197(28) | 79^{+94}_{-38} |
| LEP 1 (no m_t) | 0.23148(20) | 0.2237(7) | 0.1210(29) | 140^{+192}_{-74} |
| SLD + M_Z | 0.23067(28) | 0.2217(6) | 0.1213 (†) | 43^{+38}_{-23} |
| $A_{FB}^{(b,c)} + M_Z$ | 0.23185(28) | 0.2244(8) | 0.1213 (†) | 408^{+317}_{-179} |
| $M_W + M_Z$ | 0.23089(37) | 0.2221(8) | 0.1213 (†) | 67^{+77}_{-45} |
| M_Z | 0.23117(15) | 0.2227(5) | 0.1213 (†) | 117 (†) |
| DIS (isoscalar) | 0.2359(16) | 0.2274(16) | 0.1213 (†) | 117 (†) |
| Q_W (APV) | 0.2292(19) | 0.2207(19) | 0.1213 (†) | 117 (†) |
| polarized Møller | 0.2292(42) | 0.2207(43) | 0.1213 (†) | 117 (†) |
| elastic $\nu_\mu(\overline{\nu}_\mu)e$ | 0.2305(77) | 0.2220(77) | 0.1213 (†) | 117 (†) |
| SLAC eD | 0.222(18) | 0.213(19) | 0.1213 (†) | 117 (†) |
| elastic $\nu_\mu(\overline{\nu}_\mu)p$ | 0.211(33) | 0.203(33) | 0.1213 (†) | 117 (†) |

$\sin^2 \theta_W$ in two next to leading order variants

5. Physics beyond the Standard Model

♣ open questions in the Standard Model

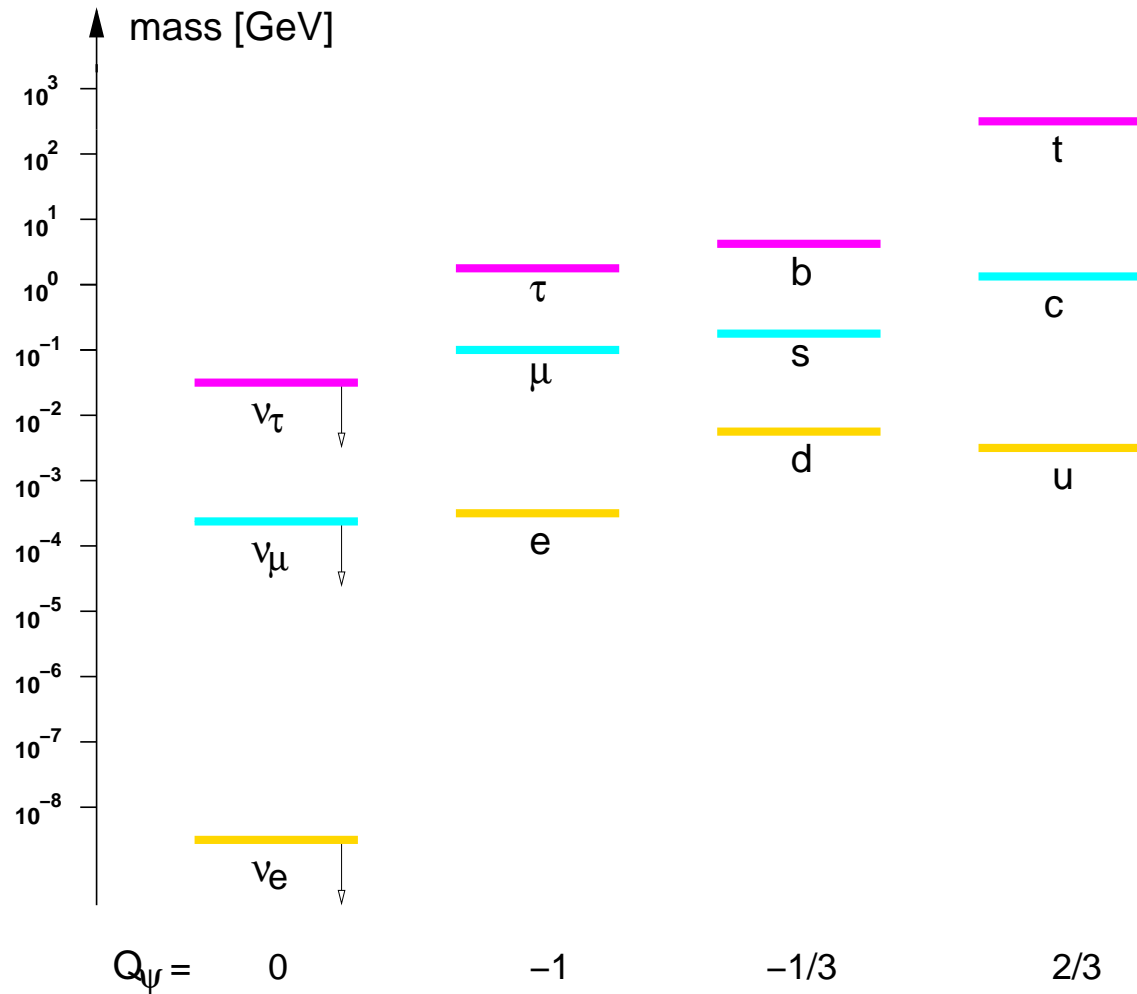
– 'periodic system' of elementary particles



* more than 3 generations?
no, if they have neutrinos lighter than $m_Z/2$

* if 3 generations, why 3?

– unknown parameters



- * why is $m_1 \ll m_2 \ll m_3$? (1,2,3 denote generation indices)
- * why is $m_{\frac{2}{3}} > m_{-\frac{1}{3}} > m_{-1} > m_0$ for each generation except for $m_u < m_d$ (the indices denote the electric charge)

– further questions

- * where is the Higgs Boson?
- * why three gauge forces
(\rightarrow three undetermined gauge couplings)?
why the gauge groups $SU(3)_c \times SU(2)_L \times U(1)_Y$?
- * origin of parity violation?

– expectation

answers to these questions from measurements at smaller distances, i.e. at higher momenta.

♣ experimental signatures for neutrino masses

Experimental signatures suggesting neutrino masses, neutrino mass mixing, neutrino oscillations.

This issue leads beyond the SM; it is discussed in a separate DESY summerschool lecture.



substructure

– hypothesis

increasing
heresy
↓

Higgs boson
leptons and quarks
((W^\pm , Z bosons))



are composite particles, built from smaller
common constituents
= preons

– Standard Model charges

electroweak and colour forces **remain gauge forces**
if preons carry appropriate electroweak and colour charges

– model building

atoms are electrically neutral, but bound states of the electrically charged electrons and

remember: nucleus

protons and all **hadrons are colour neutral**,
but bound states of coloured quarks

– basic assumptions

- * **preons carry hypercolour**, a new conserved quantum number →
bound states of preons (among them quarks and leptons) are hypercolour neutral
- * there exists a **local hypercolour gauge theory** leading to confinement of preons in their bound states

– basic question and constraint

* radius of quarks and leptons $\lesssim 10^{-16}$ cm \rightarrow expected from uncertainty principle

mass of bound states of preons $\gtrsim O(200 \text{ GeV})$

* theory has to provide a natural explanation, why the composite quarks and leptons are so light in comparison to this scale \rightarrow

chiral symmetry, a strong constraint on model building

– prediction of new exotic particles

suitable combinations of preons lead to the bound state quarks and leptons etc.

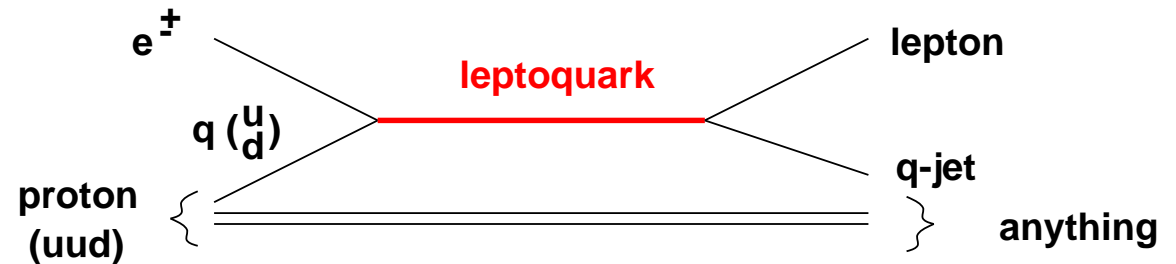
depending on the specific model, further allowed bound states of preons lead to the

prediction of **new particles**
with exotic electroweak and colour charges and
masses $\gtrsim O(200 \text{ GeV})$

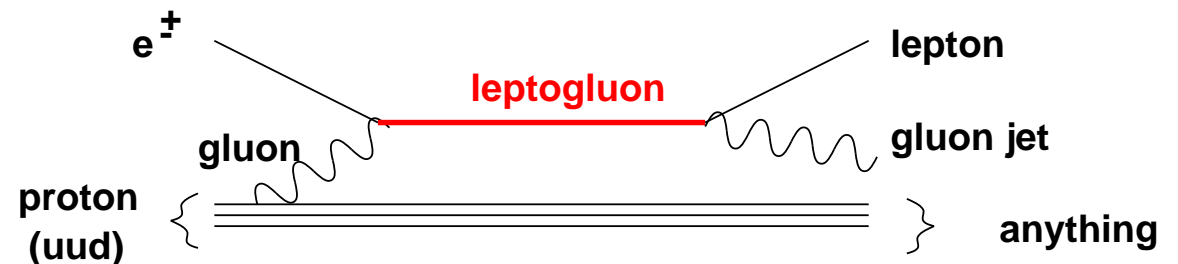
- examples of new (composite) particles

HERA

- * leptoquarks (bosons)



- * leptogluons (fermions)



Additional gauge groups

- an additional $U(1)$ gauge group

$$\underbrace{SU(3)_c \times SU(2)_L \times U(1)_Y}_{\text{standard model}} \times \mathbf{U(1)} \quad \text{simplest example}$$

SSB of $U(1) \rightarrow \text{massive } Z' \text{ gauge boson}$

- a left-right symmetric gauge theory above $p \gtrsim m_{W_R}$: **parity conserving theory**

$$SU(3)_c \times SU(2)_L \times \underbrace{SU(2)_R \times U(1)_{B-L}}$$

SSB to $U(1)_Y \rightarrow$ **massive W_R, Z_R gauge bosons**

- ♣ grand unification of the electroweak and colour forces

- assume the “grand desert”, i.e. no new physics for

$$10^{-16} \text{ cm} \gtrsim \text{distance } d \gtrsim \mathbf{10^{-29} \text{ cm}},$$

i.e. according to the uncertainty principle for

$$10^2 \text{ GeV} \lesssim \text{momentum } p \lesssim \mathbf{10^{15} \text{ GeV}}$$

- extrapolation of the running couplings to higher momenta p from experimentally determined initial values at $p = m_Z$

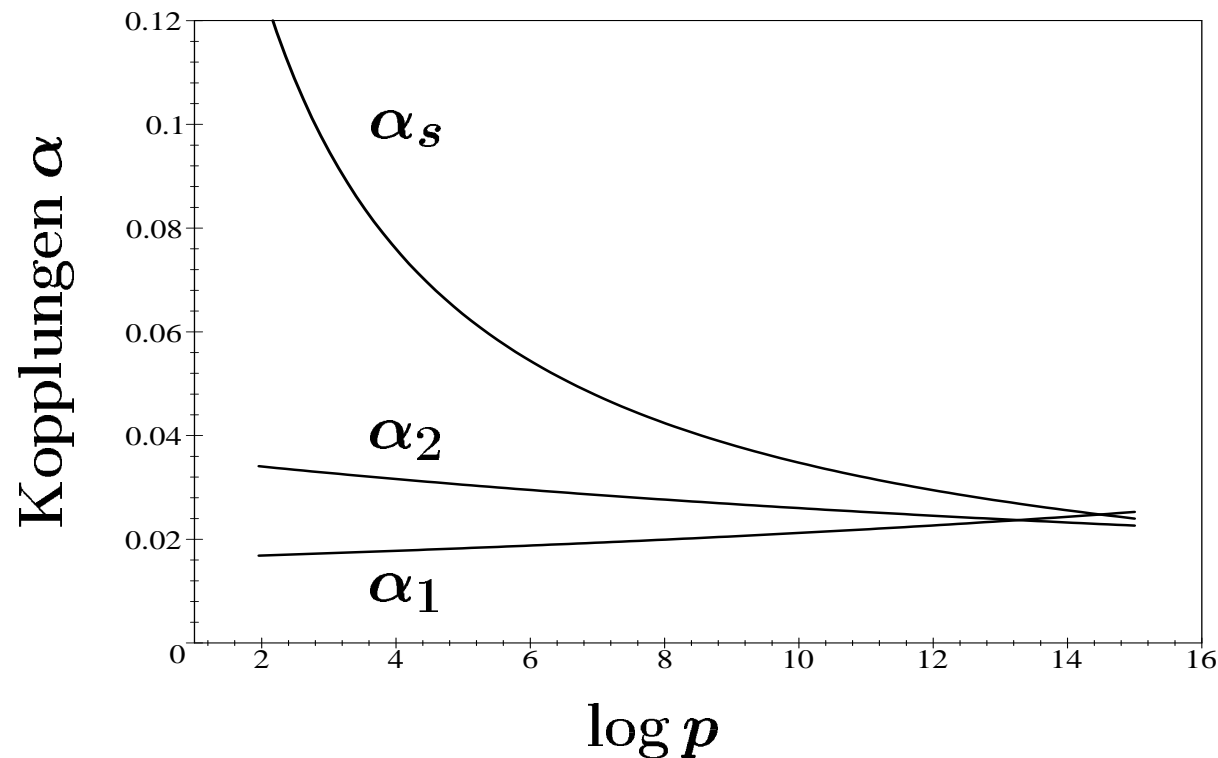
$$\alpha_s(p) = g_c^2/4\pi, \quad \alpha_1(p) = (5/3) g'^2/4\pi, \quad \alpha_2(p) = g^2/4\pi,$$

- unification of gauge couplings

$$\alpha_s(p) \approx \alpha_1(p) \approx \alpha_2(p)$$

at $p \approx 10^{15}$ GeV (i.e. $d \approx 10^{-29}$ cm)

with slight mismatch



- unification of gauge forces suggesting

one **single fundamental force, unifying** the electroweak and colour forces in terms of a single (undetermined) coupling

– model scenario

* single fundamental force = **gauge force**

* gauge group contains $SU(3)_c \times SU(2)_L \times U(1)_Y$ smallest group: **$SU(5)$**

number of gauge bosons: $5 \times 5 - 1 = 24$, among them 8 gluons, 3 W^\pm , Z , 1 γ .

→ 12 of the 24 $SU(5)$ gauge bosons have to be heavy

* via **spontaneous symmetry breakdown** $SU(5) \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$

at **$p \approx 10^{15} \text{ GeV}$** → **$m_{\text{gauge boson}} \approx 10^{15} \text{ GeV}$**

* heavy gauge bosons mediate

proton decay, e.g. $p \rightarrow e^+ \pi^0$

problem!

predicted lifetime $\tau_p \approx 10^{31}$ years, experiment for $p \rightarrow e^+ \pi^0$: $\tau_p > 5 \cdot 10^{33}$ years.

* possible solution: grand unification with supersymmetry



unification with gravity?

- gravitational forces are described by general relativity (classical theory)
- gravitational forces become of comparable size as electroweak and color forces at the

$$\text{Planck scale } p \approx 10^{19} \text{ GeV} \hat{=} d \approx 10^{-33} \text{ cm}$$

- problem: **NO** renormalizeable quantum field theory
- substantial amelioration by **supersymmetry** an extended space-time symmetry

each standard model particle has a
supersymmetric partner

- leading to particle multiplets
(**fermion, boson**) with $\Delta \text{ spin} = \frac{1}{2}$

| | | | |
|--------------|---|-----------------|------------|
| leptons | → | sleptons | (spin 0) |
| quarks | → | squarks | (spin 0) |
| gauge bosons | → | gauginos | (spin 1/2) |
| Higgs boson | → | higgsino | (spin 1/2) |

- → **supergravity**
- **superstring theory** (mainly relevant for $p \geq 10^{19}$ GeV)
 - * elementary fields → **strings with length 10^{-33} cm**
 - * leptons, quarks, gauge bosons, Higgs boson are lowest string excitations

♣ grand unification with supersymmetry

- Implement

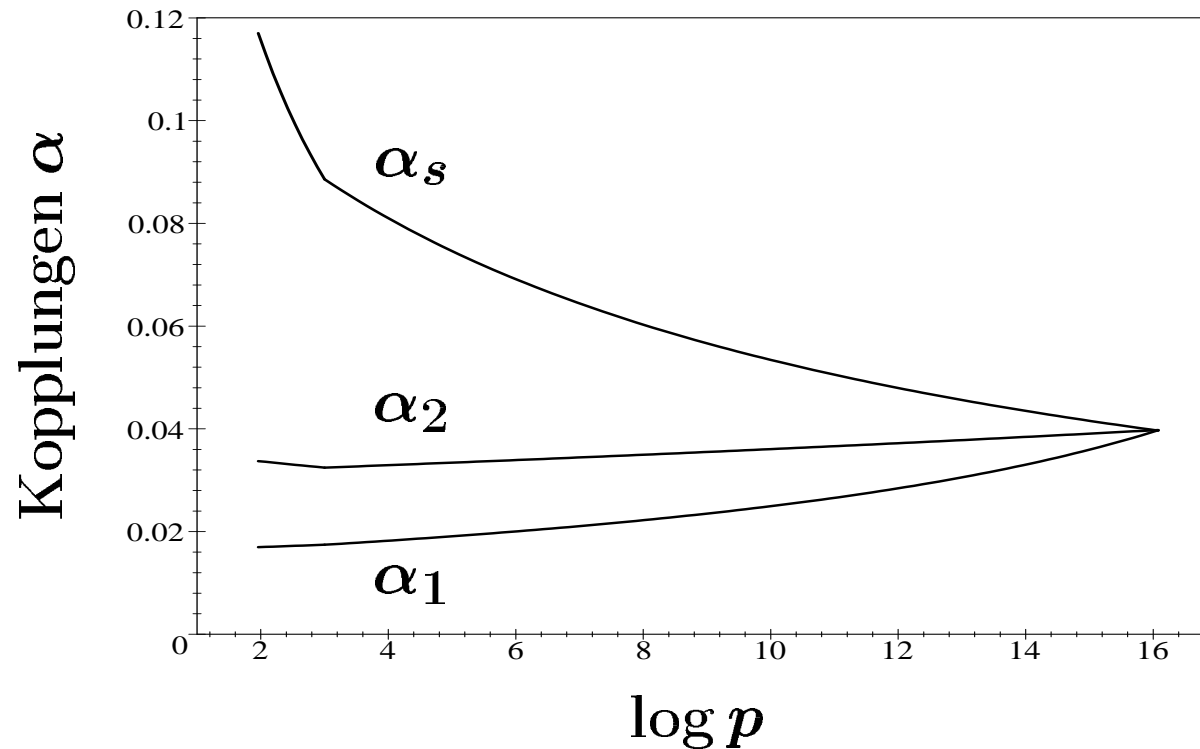
supersymmetry into the Standard Model
→ **minimal supersymmetric Standard Model**

→ improved renormalizability properties

- soft supersymmetry breaking (necessary since $m_{\text{particle}} \neq m_{\text{sparticle}}$)
at scale $M_{SUSY} \approx 200\text{-}1000$ GeV →

$m_{\text{sparticles}} \approx 200\text{-}1000$ GeV

- grand unification in the supersymmetric framework (e.g. with $SU(5)$ unifying gauge group)
unification of gauge couplings at $p \approx 2 \cdot 10^{16} \text{ GeV}$

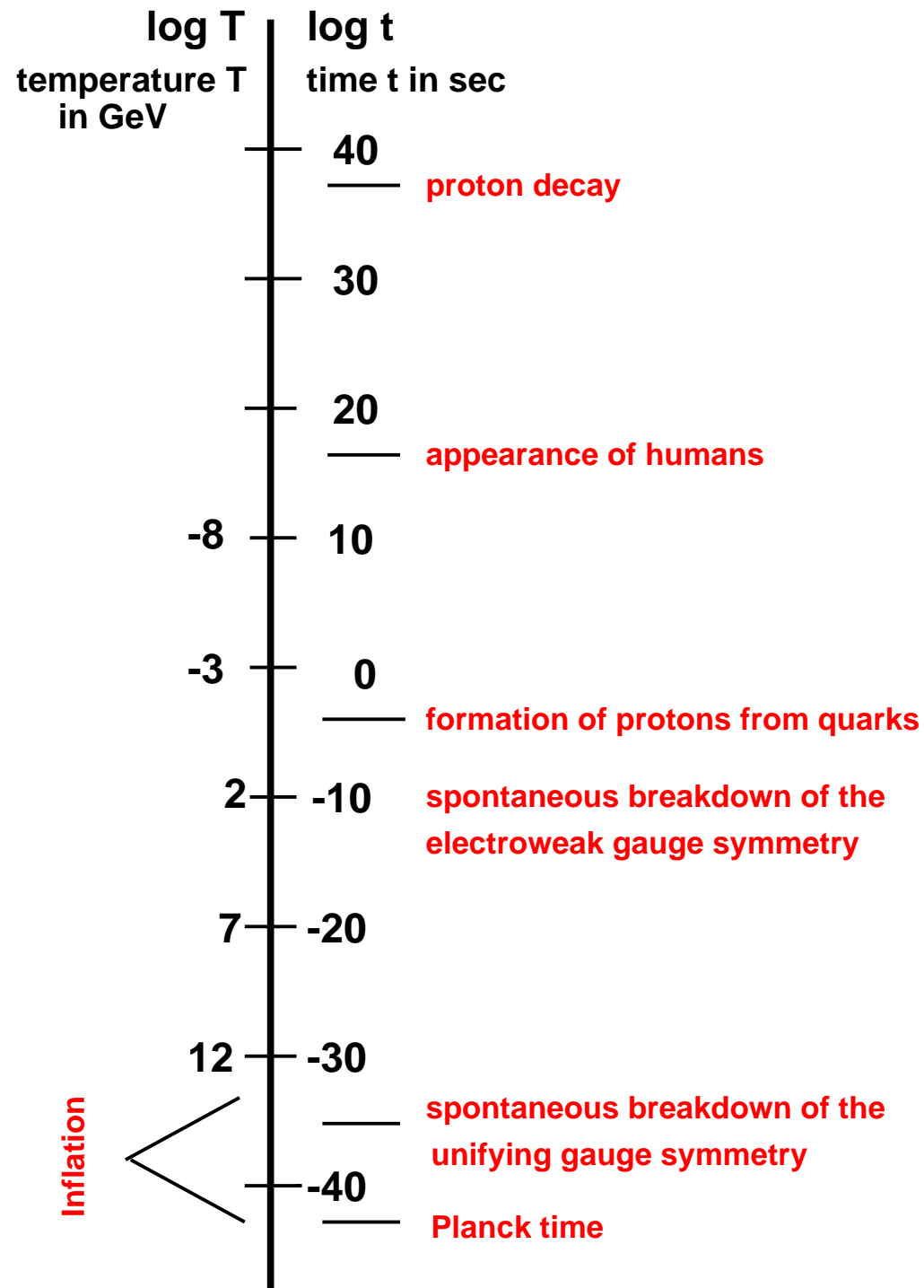


(marginally) no problem with
proton decay

The unifying theory implies violation of baryon number and of time reversal invariance
which – together with thermal inequilibrium – allows to explain

the baryon asymmetry of the universe

→ of interest for the cosmology of the early universe





Extra dimensions

- Theoretical developments based on the idea that there are extra dimensions in addition to the 4 space-time dimensions.
- Idea with the most immediate implications for future experiments:
while the Standard Model gauge interactions “live” in our habitual four dimensions, gravitational forces act in a higher dimensional space with the result that gravitational forces become comparable in strength to the Standard Model gauge forces at a momentum scale as low as

$$\mu \approx 1000 \text{ GeV.}$$

- Grand unification as discussed above has then to be reconsidered under the new circumstances; it is not straightforwardly recovered.



Noncommutative Geometry

Noncommuting space-time coordinates are assumed. Effects can be looked for at future Accelerators.