Deutsches Elektronen-Synchrotron

Suppression of Quantum Decoherence in a Driven Two-Level System

DESY Summer Project

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Date of Project: 16.07-05.09.2013
Date of Submission: 04.09.2013

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Abstract

Method for suppression of decoherence in a driven two-level system is studied by proposing a new type of system coupling. A spin-boson system with added two-level system driven by external field is studied for the purpose. Suppression of decaying coherence in a dissipative two-level system is successfully done by use of sequential π −pulse. Also, coherence is reinstated in a complete relaxed system, through sequential $\frac{\pi}{2}$ −pulse.

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Introduction

In recent time, the quantum computing and quantum information science has been a major field of interest. The wide possible application in solving mathematically and computationally complex problems and also the possibility of decrypting many cryptographic systems in use today, has led a wide field of research for development of quantum computers. However, there lie several challenges to this feat and one of the major obstacle is controlling or removing quantum decoherence from the qubits (quantum bits).

Quantum decoherence is the consequence of Quantum theory and unavoidable interaction of system with the environment. This loss of coherence or decoherence invalidates the quantum superposition principle and a classical system emerges from the quantum one. Thus, the quantum computer which basically relies on quantum properties of bits, at best, turns into a classical computer.

Also in research for superconductors, maintaining coherence is important. Electrons pair up into Cooper pairs and collectively form a coherent state at low temperatures in superconductors. If this coherent state can be controlled well and not relaying in low temperatures, room temperature superconductors could well be in grasp.

There has been several studies and research on control of decoherence, however, a concrete scalable way for removal of decoherence has still not been realised. In this project, we investigate the possibility of decoherence control in a two-level system (TLS) coupled to a bath through the application of coherent light pulses interacting with another TLS coupled to the one of interest.

The TLS can be imagined as a dipole that can point in two directions, either left or right. And the set of harmonic oscillators according to their position can either stabilizes the Left pointing or the Right pointing state. We take this simple model of TLS-bath coupling and add another coupling to the reference TLS. This new coupling is another two-level system coupled to the reference TLS (TLS2). The new TLS (TLS1) is then driven by pulsed electric field and attempt is made to control the decoherence in the reference TLS2. This is better illustrated in the figure below:

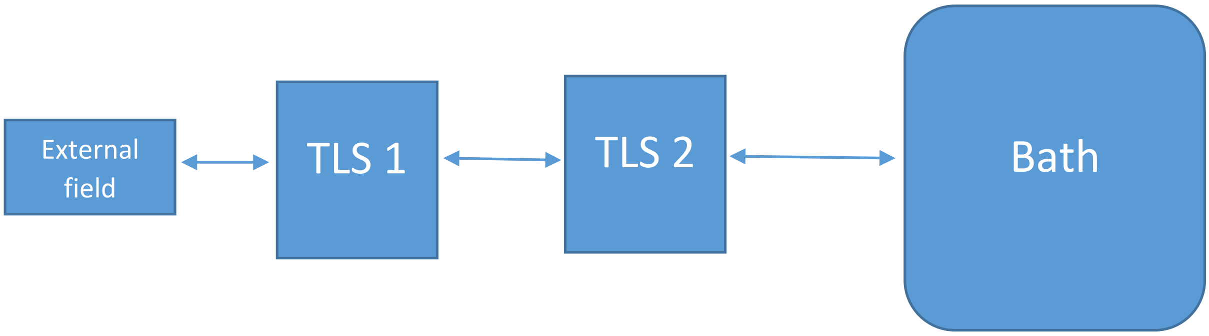


Fig. 1: Coupling denoted by the double headed arrow. The external field coupled to TLS1, TLS1 in turn coupled to TLS2 and the TLS2 coupled to the bath represented in schematic way.

The individual system of TLS2 and bath is called Spin-Boson problem and has been studied extensively.

MCTDH

We use MCTDH (multi-configuration time dependent Hartree) program for the numerical propagation of the Schrodinger equation for the whole system. A small description of the program is given in the following text.

Study of a time dependent quantum mechanical system can be done by propagation of wave packet. The multi-configuration time dependent Hartree is a general algorithm for solving time dependent Schrodinger equation for multi dynamical systems consisting of distinguishable particles. The MCTDH program was created to perform the MCTDH general algorithm and is capable of propagation of wave packet. The initial parameters and Hamiltonian should be defined for the calculation and for the result, several analysis programs included in MCTDH package can be used to analyse it.

In general, the algorithm expresses the wave function with many degrees of freedom as a sum of Hartree product basis set,

$$\Psi(S_1, S_2, Q_1, \dots Q_a, t) = \sum_J A_J(t)\Phi_J(t) \ , \qquad J = (j_1, \dots j_a)$$

$$\Phi_J(t) = \phi_{S_1}^{(1)}\phi_{S_2}^{(2)} \prod_{k=1}^{N} \phi_{j_k}^{k}\,(Q_k, t)$$

Where a is the number of degrees of freedom, $Q_1, \dots Q_a$ are the coordinates, A_J is the time-dependent expansion coefficients, and the single particle functions ϕ_{j_k} form the time dependent basis functions of degree of freedom k. They in turn are defined on time-independent primitive grid. A discrete variable representation (DVR) is used as primitive basis to present these single-particle functions. For large system, or even certain degrees of freedom are strongly coupled, it may be advantageous to combine degrees of freedom together and use multi-mode single-particle functions, doing so the number of the expansion coefficients A_J is significantly reduced.

Hamiltonian

The Hamiltonian of overall system can be given as the sum of individual Hamiltonians of TLSs and bath along with the sum of coupling Hamiltonians.

$$H\ (t) = H_{TLS1} + H_{TLS2} + H_{Bath} + H_{TLS1\_TLS2} + H_{Bath\_TLS2} + H_{Field\_TLS1}$$

Where, H_{TLS1}, H_{TLS2} and H_{Bath} represents the Hamiltonian for the individual uncoupled TLS1, TLS2 and Bath and H_{TLS1_TLS2}, H_{Bath_TLS2}, and H_{Field_TLS1} represents the coupling between the systems with subscripts representing the individual system involved in coupling.

The individual terms in the Hamiltonian are expressed as

$$H_{TLS1} = \frac{1}{2}\Delta_1\sigma_x$$

$$H_{TLS2} = \frac{1}{2}\Delta_2\sigma_x$$

$$H_{Bath} = \sum_{a}^{N}\frac{\omega}{2}\left(-\frac{\partial^2}{\partial {Q_a}^2} + {Q_a}^2\right)$$

$$H_{TLS1\_TLS2} = \gamma\sigma_z^{(1)}\sigma_z^{(2)}$$

$$H_{Bath\_TLS2} = \sum_{a}^{N}\lambda_a Q_a\sigma_z^{(2)}$$

$$H_{Field\_TLS1} = E(t).\sigma_z$$

The $\Delta_1$and Δ_2 in the Hamiltonian of the TLSs are the tunnelling constant of the TLS1 and TLS2 respectively.

The σ_x, σ_z and σ_y are the Pauli matrices which are defined as

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix};\ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix};\ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

The H_{Bath} gives the Hamiltonian of the environment/bath as a set of harmonic oscillators in bath coordinates $Q_a = \sqrt{\omega_a m_a}x_a$. H_{Bath_TLS2} represents the linear coupling of bath and the TLS2 with λ_a as coupling parameter. The system of only the TLS2 and bath coupled would be called as spin-boson model.

The coupling of TLS1 and TLS2 is given by the term H_{TLS1_TLS2} with γ as the coupling parameter. And finally H_{Field_TLS1} gives the coupling of TLS1 with the field where $E(t)$ is the external field given as

$$E(t) = A.\cos(\omega_f t + \varphi_0)\,; A = \mu\varepsilon$$

The Hamiltonian is written in atomic units ($\hbar = 1$).

The Hamiltonian above represents each TLS interms of two states termed $|L\rangle$ and $|R\rangle$ mixed by the tunnelling constant Δ_k, $k = 1{,}2$ for TLS1 and TLS2 respectively.

These states can be expressed as the linear combination of the eigenstates of the uncoupled TLS.

$$|L\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle);\ |R\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)$$

The Hamiltonian can be transformed using the transformation matrix given below to the eigenbasis of the uncoupled TLS1 and TLS2.

$$U^{(k)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}; k = 1, 2$$

The Hamiltonian in the eigenbasis looks like

$$H_{TLS1} = -\frac{1}{2}\Delta_1\sigma_z$$

$$H_{TLS2} = -\frac{1}{2}\Delta_2\sigma_z$$

$$H_{Bath} = \sum_{a}^{N}\frac{\omega}{2}\left(-\frac{\partial^2}{\partial {Q_a}^2} + {Q_a}^2\right)$$

$$H_{TLS1\_TLS2} = \gamma\sigma_x^{(1)}\sigma_x^{(2)}$$

$$H_{Bath\_TLS2} = \sum_{a}^{N} -\lambda_a Q_a \sigma_x^{(2)}$$

$$H_{Field\_TLS1} = -E(t).\sigma_x$$

This shows the coupling in $|L\rangle$ and $|R\rangle$ basis is through $\sigma_z$and the coupling in the $|+\rangle$ and $|-\rangle$ basis is through σ_x. This means the coupling can stabilize one state and destabilize another state in $|L\rangle/|R\rangle$ basis, where as in $|+\rangle/|-\rangle$ basis, the coupling causes a flip or transfer in population from one state to another. If we imagine a dipole point left or right for $|L\rangle/|R\rangle$, then coupling, which can be imagined as a field acting on dipole, depending upon the direction of the field may favour and stabilize one of the $|L\rangle/|R\rangle$ and destabilize another. However, in $|+\rangle/|-\rangle$ eigenstate, the coupling will cause a flip or transition from one state to another, say a field will cause the dipole pointing up ($|+\rangle$) to point down ($|-\rangle$).

From the solution of the time-dependent Schrodinger equation

$$i\frac{d\Psi(t)}{dt} = H\Psi(t)$$

We can calculate the density matrix

$$\rho^{(2)}_{s_2 s_2'} = \sum_{s_1} \int dQ_1 \dots dQ_N \Psi^*(s_1, s_2, Q_1 \dots Q_N, t)\, \Psi(s_1, s_2', Q_1 \dots Q_N, t)$$

The appropriate density matrix $\rho_{s_2 s_2'}$ is a 2 x 2 matrix elements where s_2 and s_2' are 0 or 1. We denoted the diagonal matrix elements of the density matrix as ρ_{00} and ρ_{11}, the off-diagonal elements (so-called coherences terms) as ρ_{01} and ρ_{10}. The diagonal elements describe the population of the states of TLS. The off-diagonal elements give information about the coherence in quantum system. We express the coherence as

$$c = \sqrt{\langle \sigma_x \rangle^2 + \langle \sigma_y \rangle^2} = 2|\rho_{01}|$$

which can have maximum value of 1 ($|\rho_{01}| \leq \frac{1}{2}$).

The expectation values of the operators σ_x and σ_y give the information about off-diagonal coherences terms. It is easy to show that

$$\langle \sigma_x \rangle = 2\, Re\{\rho_{01}\}$$

$$\langle \sigma_y \rangle = 2\, Im\{\rho_{01}\}$$

Bath Model

The main property of the bath is its spectral density.

$$J(\omega) = \frac{\pi}{2} \sum_j \frac{c_j^2}{\omega_j} \delta(\omega - \omega_j)$$

It characterizes the effect of the bath on the TLS. Here, in this project, we choose an Ohmic bath with direct cutoff which can be expressed as

$$J_o = \frac{\pi}{2} \alpha\omega . \theta(\omega - \omega_c)$$

The coupling parameter for Bath and TLS2, λ_a, can be expressed as

$$\lambda_a = \sqrt{a} . \Delta c$$

Where, $\Delta c = \sqrt{\alpha} . \frac{\omega_c}{N}$, ω_c is the cutoff frequency of the bath and N is the total number of bath modes in the environment.

For the calculations in this project, we have used the total number of bath modes, $N = 16$, and the parameter $\alpha = 0.032$. The cutoff frequency is kept at $\omega_c = 2.5$, with 16 bath modes this means that the recurrence time of the bath is at $t = 40$.

π and $\frac{\pi}{2}$ Pulse:

The coherence control in TLS2 was attempted through use of π and $\frac{\pi}{2}$ pulses to the TLS1. The π pulse causes the complete population transfer from the ground state to the upper state and the $\frac{\pi}{2}$ pulse causes a half-transfer of population and causes a complete 50-50 mixture of the two states.

To derive the pulse duration of the π and $\frac{\pi}{2}$ pulses, we need to know about the Rabi frequency. Rabi frequency is defined as the frequency of oscillation for a given transition in a given field. The Generalised Rabi frequency which includes the detuning of the field and the system frequency is expressed as

$$\Omega = \sqrt{\Delta^2 + (\overline{\Omega})^2}$$

With external field as $V = -\mu\varepsilon\cos(\omega t)$ and $\varDelta = \omega - \omega_o$ as the detuning of system frequency ω_o with field frequency ω. $\overline{\Omega} = \frac{\mu\varepsilon}{\hbar}$ is then the Rabi frequency of the system.

After solving Schrodinger equation, we can get for resonance case, the population of the states as

$$a(t) = \cos\left(\frac{\overline{\Omega}\mathrm{t}}{2}\right); \qquad b(t) = i\sin\left(\frac{\overline{\Omega}\mathrm{t}}{2}\right)$$

Where $a(t)$ represents the population of ground state and $b(t)$ represents the population of excited state in eigenbasis (+/- basis). Considering the resonance case, when detuning $\varDelta = 0$, Rabi frequency then becomes $\Omega = \frac{\mu\varepsilon}{\hbar} = \overline{\Omega}$. As the calculation are in atomic units ($\hbar = 1$), Rabi frequency is then just the amplitude of the external field $\Omega = \mu\varepsilon$.

So we can see, for the pulse of duration, $\mathrm{t} = \frac{\pi}{\overline{\Omega}}$, all the population is in excited state. And the pulse of this duration is called the π −pulse. Whereas for the pulse of duration $\mathrm{t} = \frac{\pi}{2\overline{\Omega}}$, one can transfer half of the population to excited state and can create a 50-50 superposition of the states. This pulse is then called $\frac{\pi}{2}$ −pulse. We use these pulses at different time of evolution and at varying sequence to check the effectiveness on control of the coherence.

Results

At first, one needs to learn the pattern and rate of coherence decay in a simple dissipative system. Thus, we took the TLS2 uncoupled it from TLS1 and with only Bath coupling ran the calculation to check the decoherence. For the TLS2, the system was kept in $|L\rangle$ state at initial time and was let to decay over time with interaction of Bath. This gave as expected an exponential decay of the coherence with some oscillation and for the population, it oscillated and damped to 50-50 mixture of the two states ($|L\rangle/|R\rangle$).

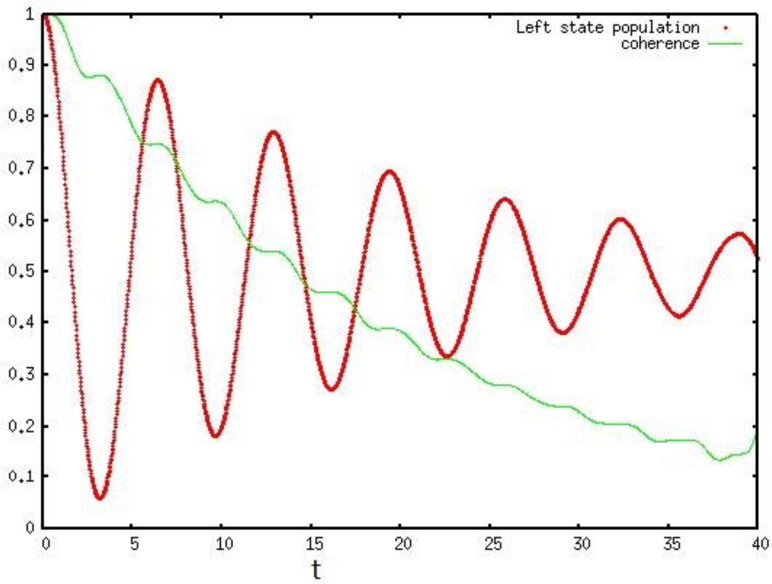


Fig. 2: The coherence decay of TLS2 with only Bath coupling.

Thus we started with this reference decay and tried to control it with sequence of different pulses. We established the coupling of TLS1 with TLS2 and completed the whole system of external field, TLS1, TLS2 and bath. Then we tried supplying TLS1 with different pulses at different times and tried to supress the decay. Through calculation the best control in loss of decay was found out with sequence of π −pulses acting on TLS1.

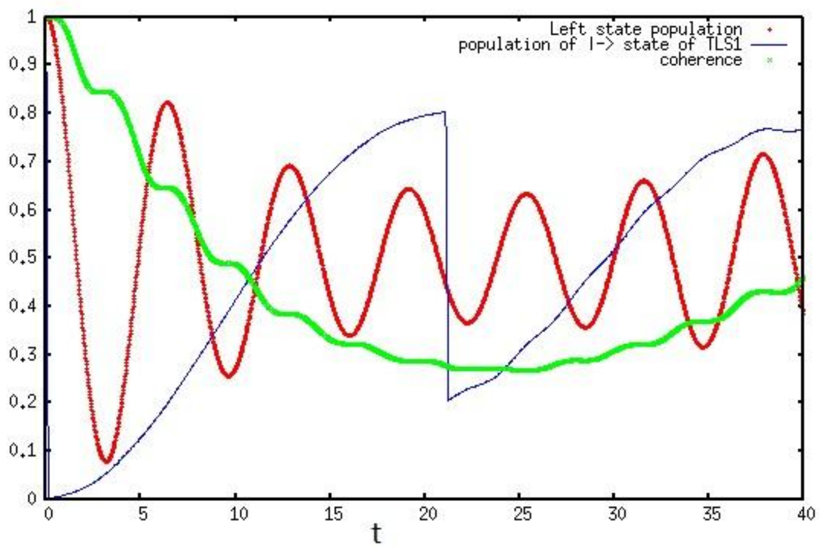
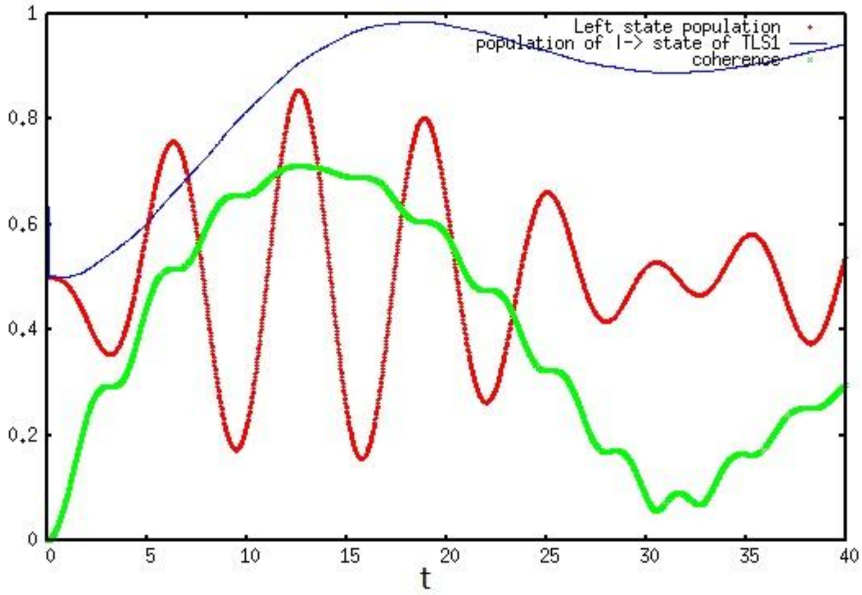


Fig. 3: The loss of coherence controlled through π −pulses acting on TLS1.

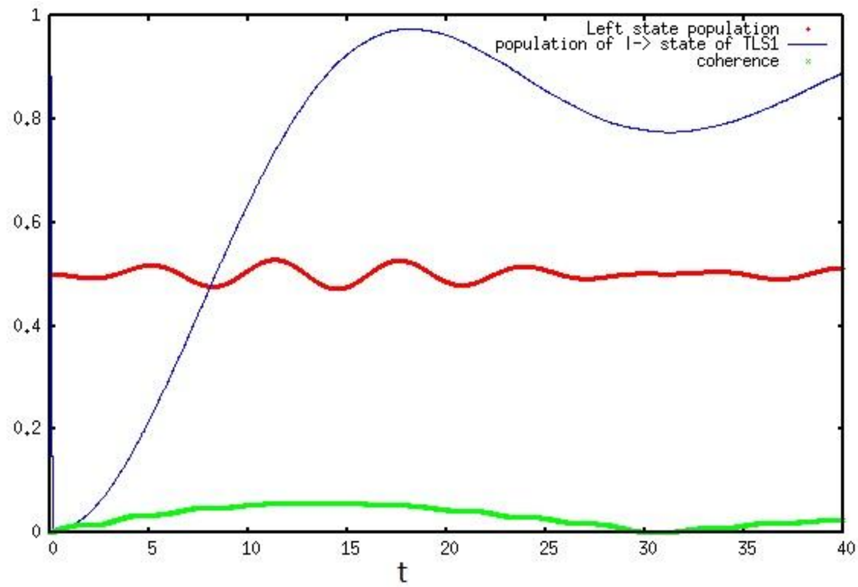
The two timed π −pulses with intensity $A = 4\pi$ and calculated π −pulse duration, $t_\pi = 0.125$, were supplied to TLS1. This sequence of pulses were able to control the decay of coherence and re-establish the coherence till about 0.5. It also was able to control the decay of population and reinstate higher oscillation in TLS2.

After the successful control of decay of coherence, we tried investigating also if coherence can be restored in system which had completely been decohered. For that purpose, we took TLS2 with all its population in ground state, this way its coherence is at 0 at $t = 0$. Then we supplied the TLS1 with different pulses and different sequence to check the best coherence gain.

We deduce from the calculation that initial $\frac{\pi}{2}$ −pulse is better at coherence gain then an initial π −pulse.



(a)



(b)

Fig 4: (a) $\frac{\pi}{2}$ *−pulse at* $t = 0$ *on TLS1 and resulting coherence gain and population oscillation. (b)* π *−pulse at* $t = 0$ *on TLS1.*

From the figure, we see with the supply of initial $\frac{\pi}{2}$ −pulse, there is big oscillation in the population of $|L\rangle/|R\rangle$ states and the coherence gain is at nearly 0.7. However, the similar oscillation and gain and coherence cannot be gained through a π −pulse at $t = 0$. Thus, it is worth noting that further investigation is better when conducted with an initial $\frac{\pi}{2}$ −pulse. Taking this case of $\frac{\pi}{2}$ −pulse, we investigated ways to keep the coherence up and prevent the decay. And from the results of calculation, we found out the best coherence regain and control can be achieved through use of $\frac{\pi}{2}$ −pulse sequence.

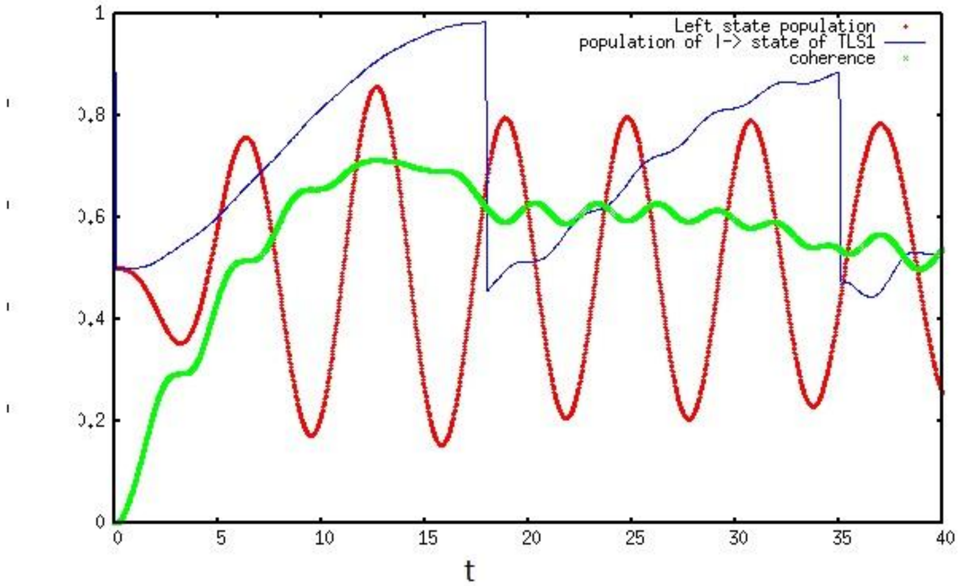


Fig 5: Use of sequence of $\frac{\pi}{2}$ −pulses to regain and control coherence in TLS2.

From figure 5, it can be seen that sequence of $\frac{\pi}{2}$ −pulses is able to regain high coherence and keep it from decaying for longer time. Also, it excites an oscillation in the population of $|L\rangle/|R\rangle$states and keeps the oscillation nearly constant through propagation.

Conclusion

In this project, we demonstrated coherence decay in a quantum system can be controlled through field pumping in another system coupled to the quantum system. This also has importance as the studied quantum system is not directly in exposure to the external field pumping but is only passively driven by the coupling with the pumped system.

In two parts, we showed firstly by pumping sequence of periodic π −pulses to coherent quantum system, the decay of coherence due to interaction with the environment can be controlled and even regained. And secondly, we also concluded that one can reinstate the coherence in completely decohered quantum system by supplying mentioned sequence of pulses. This means we can re-establish quantum properties in a system which has completely lost them and effectively is a classical system.

There is of course room for improvement in the results. One can increase the number of bath modes and double the propagation time, which was limited to $t = 40$ for us for 16 bath modes. This would allow to insert more pulses and to check if one can completely regain the decayed coherence.

Also the pulses used in this project were almost a delta pulse. It was high intensity and extremely short timed pulse. In further investigations, one can check for pulses which would last little longer and this would also account for other dynamics during the pulse time.

In short, we achieved what we set out for at the start of the project. We gave a method to control and regain coherence through use of short pulses. There were limitations and room for improvement but in the short time, we have a result that could be further built upon.

Acknowledgement

I would like to express my heartful gratitude towards my supervisor Dr. Oriol Vendrell, who kept faith on me and granted me a chance to work on this wonderful and significant project. Without his thorough guidance, the project would not have been successful. I would also like to offer my special thanks to my head supervisor Prof. Robin Santra for his support and comments on the project. My thanks also goes to all the CFEL theory group members for their support, and also for making my stay at DESY warm and welcoming.

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