QCD description of multiparticle production

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6.2.2001
DESY

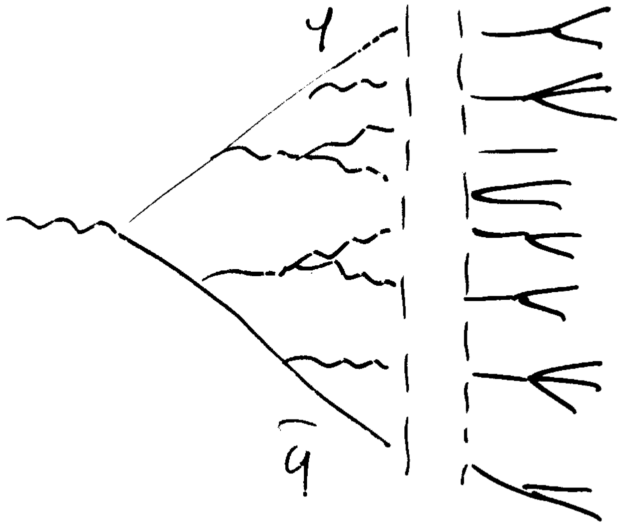
1. Theoretical approach

2. Results
   - multiplicity : mean and moments
   - inclusive distribution
     → soft limit
   - multijet events
   - angular correlation
   - rapidity gaps
   - outlook

3. Conclusions

Theoretical approach

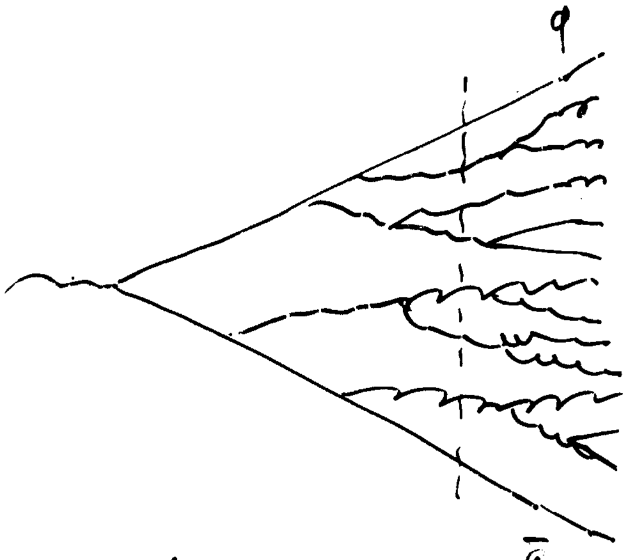
Hadronization model



Hadrons
Resonances

parameter Λ $\qquad$ $Q_0 \sim 1\,\mathrm{GeV}$

"Parton Hadron Duality"



parameter Λ $\qquad$ $k_T > Q_0 \gtrsim \Lambda$ (few 100 MeV)

cascade evolves towards lower scale

is QCD relevant here?

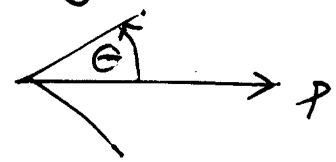
study:
- $\alpha_s \gtrsim 1$
- coherence effects of soft gluons
- color factors C_A, C_F

Theoretical approach

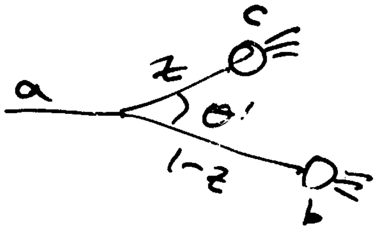
Parton cascade

Master equation for generating functional

$Z(P, \Theta, \{u(p_i)\})$



$$a \text{—}O\!\equiv \; = \; \overline{\quad a \quad} \; + \sum_{a\to bc} \iint dz\, d\Theta' \; a$$



$Z_a(P,\Theta) \qquad e^{-w_a(P,\Theta)}\, u_a \qquad \frac{\alpha_s(k_\perp)}{2\pi} P_a^{bc}(z)\, Z(zP,\Theta')\, Z((1-z)P,\Theta')$

- $P_{a\to bc}(z)$ DGLAP splitting function
- angular ordering $\Theta' < \Theta$ (coherence)
- initial condition: absolute normalization
- running coupling $\alpha_s(k_\perp)$ at 1-loop
- non-perturbative cut-off $k_\perp > Q_0 > \Lambda_{QCD}$
- approximations in leading log:  $dn \sim \frac{d\omega\, d\Theta}{\omega\, \Theta}$

LL (DLA): $\alpha_s^n \ln^{2n} y \to e^{1/\sqrt{\alpha_s}} \sim e^{\sqrt{\ln Q}}$

NLL (MLLA): " $+ \alpha_s^n \ln^{2n-1} y \to e^{1/\sqrt{\alpha_s} + p \ln \alpha_s}$

numerical solution of e.eq. $\to$ yet higher order but not complete

- match with matrix element $O(\alpha_s)$, $O(\alpha_s^2)$

- numerical method MC:
  ARIADNE parton level $k_\perp > Q_0$, $\alpha_s(k_\perp)$

Parameters: Λ_{QCD}

$Q_0 \sim R^{-1}$ non-perturbative

(K normalization)

Hadrons

Parton – Hadron – Duality

$$O(x_1 \cdots x_n)\Big|_{\text{hadron}} = K\, O(x_1 \cdots x_n, Q_0, \Lambda)\Big|_{\text{parton}}$$

$$K \simeq 1 \cdots 2$$

Azimov
Dokshitzer
Khoze
Troyan
for inclusive spectra
in MLLA

mismatch near kinematic border

minimal K_T:

$K_T\big|_{\text{hadron}} \to 0 \qquad K_T\big|_{\text{parton}} \to Q_0$

simple model:

a hadron is like a parton with mass Q_0

partons $\qquad$ hadrons

$E_p^2 = k_p^2 > Q_0^2 \qquad E_h^2 = k_h^2 + Q_0^2 > Q_0^2$

compare partons and hadrons at $E_p = E_h$

Results:

Mean multiplicity

a) high energy approximation (MLLA)

$$N \sim \exp\left(\frac{c_1}{\sqrt{\alpha_s(Y)}} + c_2 \ln \alpha_s(Y) + \ldots \right)$$

$$c_1 = \sqrt{96\pi}/b, \quad c_2 = \frac{1}{4} + \frac{10}{27}\frac{n_f}{b}, \quad b = \frac{11}{3}N_c - \frac{2}{3}n_f$$

b) full solution with initial condition $N = 1$

transition jets $\rightarrow$ hadrons

Durham $k_{T\,ij} > Q_c$ $\qquad$ $k_T > Q_0$

$$y_c = \left(\frac{Q_c}{Q}\right)^2 \rightarrow \quad y_c = \left(\frac{Q_c}{Q}\right)^2$$

hadrons $\qquad$ jets $\qquad$ jets

$y_c \sim 1$

$y_c \to 0$ $\qquad$ $0 < y_c < 1$

$y_c \to y_0$

c) gluon jet multiplicity

$$r = \frac{N_g}{N_q} = \frac{C_A}{C_F}\left(1 - r_1\gamma_0 - r_2\gamma_0^2 - r_3\gamma_0^3\right)$$

$$\gamma_0 = \left(\frac{2C_A\alpha_s}{\pi}\right)^{1/2}$$

Multiplicity moments

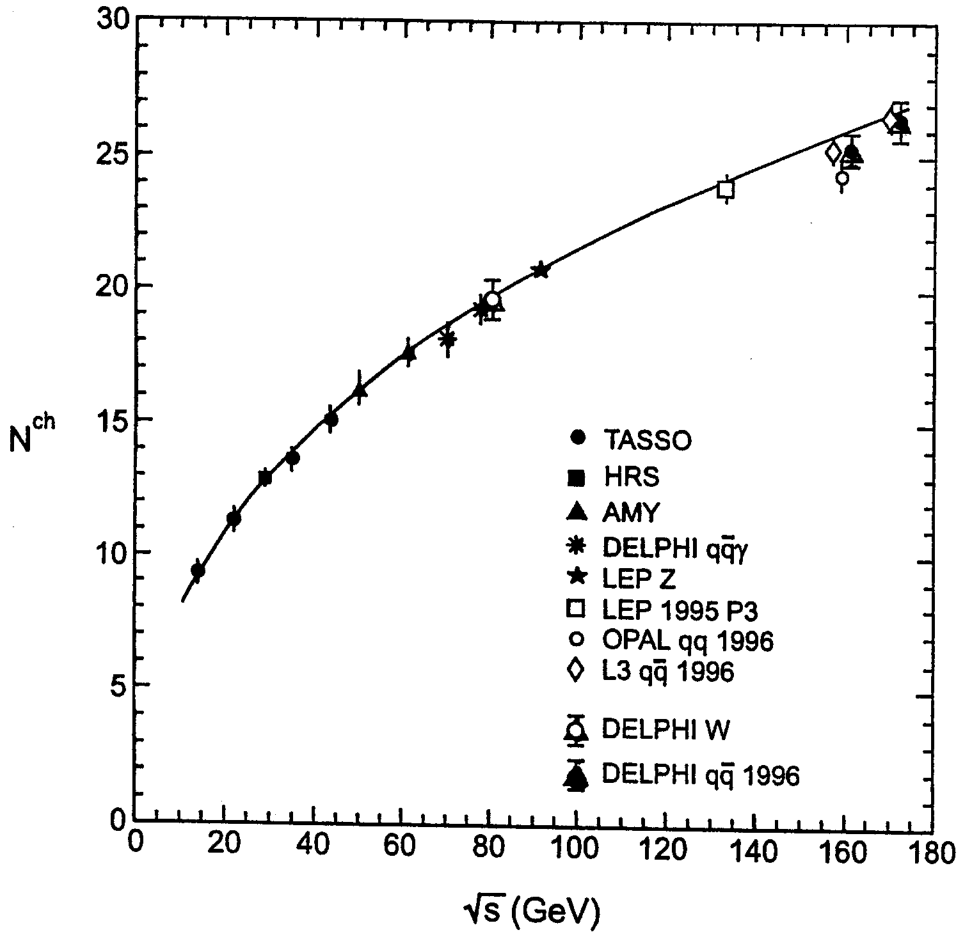
factorial $F_q = \langle n(n-1)\cdots(n-q+1)\rangle = \int P(p_1 \cdots p_q)\, dp_1 \cdots dp_q$

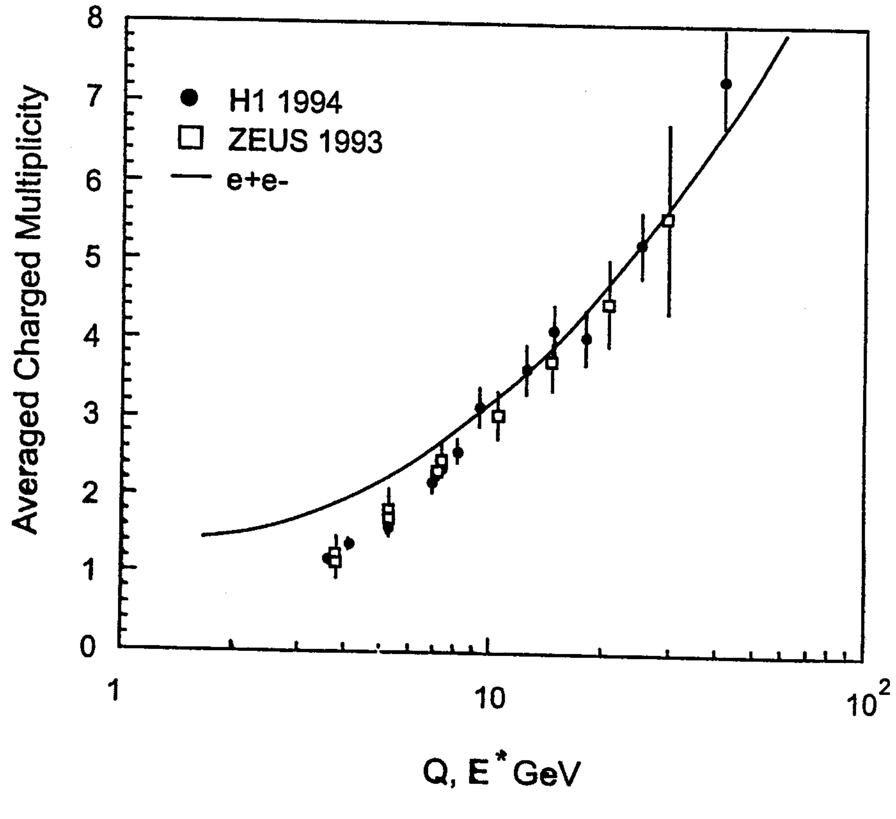
Kumulant $K_2 = F_2 - 1, \ldots$

genuine correlations

mean particle multiplicity

MLLA
K free

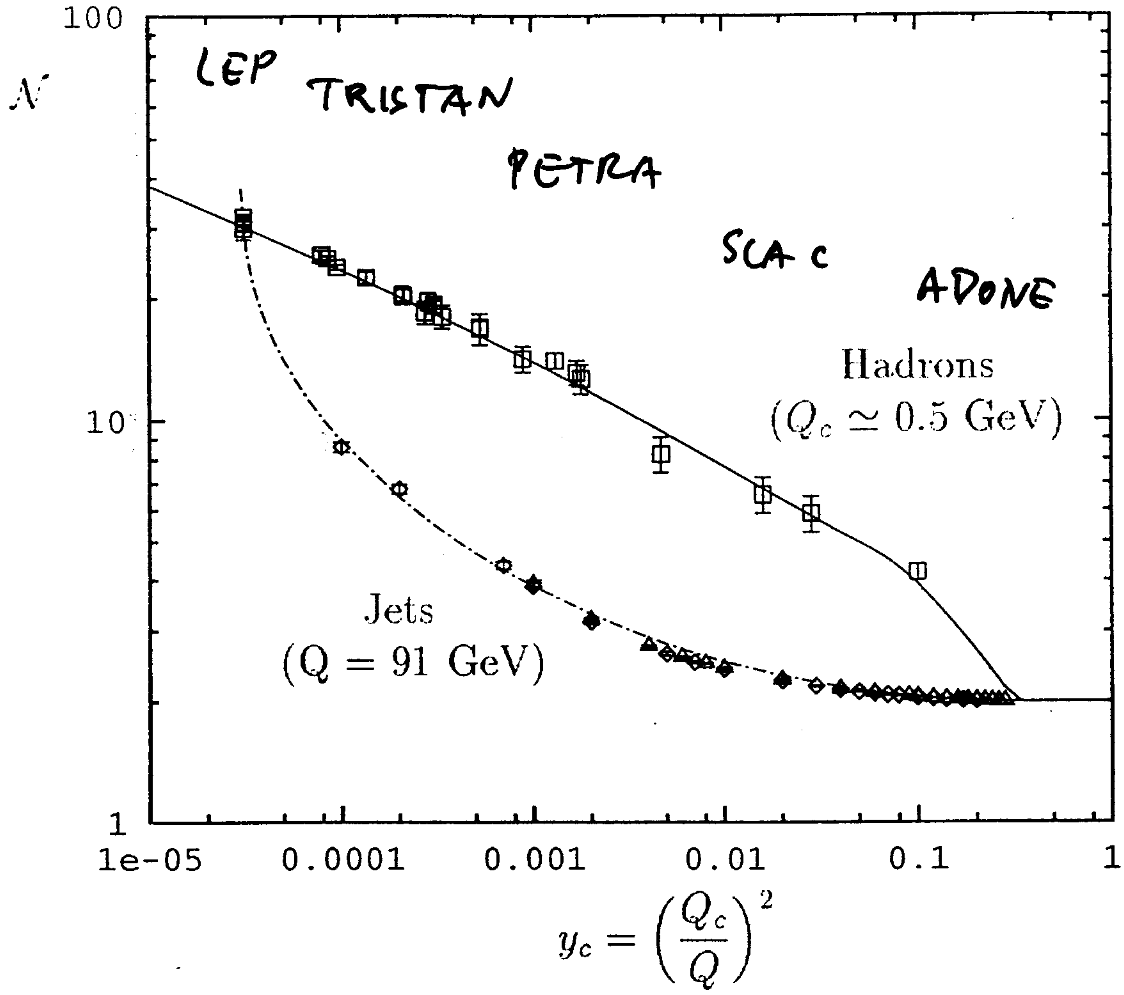




Hadron and Jet multiplicities

numerical solution of MLLA evolution eqn. + $O(\alpha_s)$ ME

Lupia W.O. '98



Durham algorithm

$K_T > Q_c$

$\Rightarrow$ strong particle production if $\alpha_s(K_T)$ large

<u>Parameters</u> $\Rightarrow K = 1$

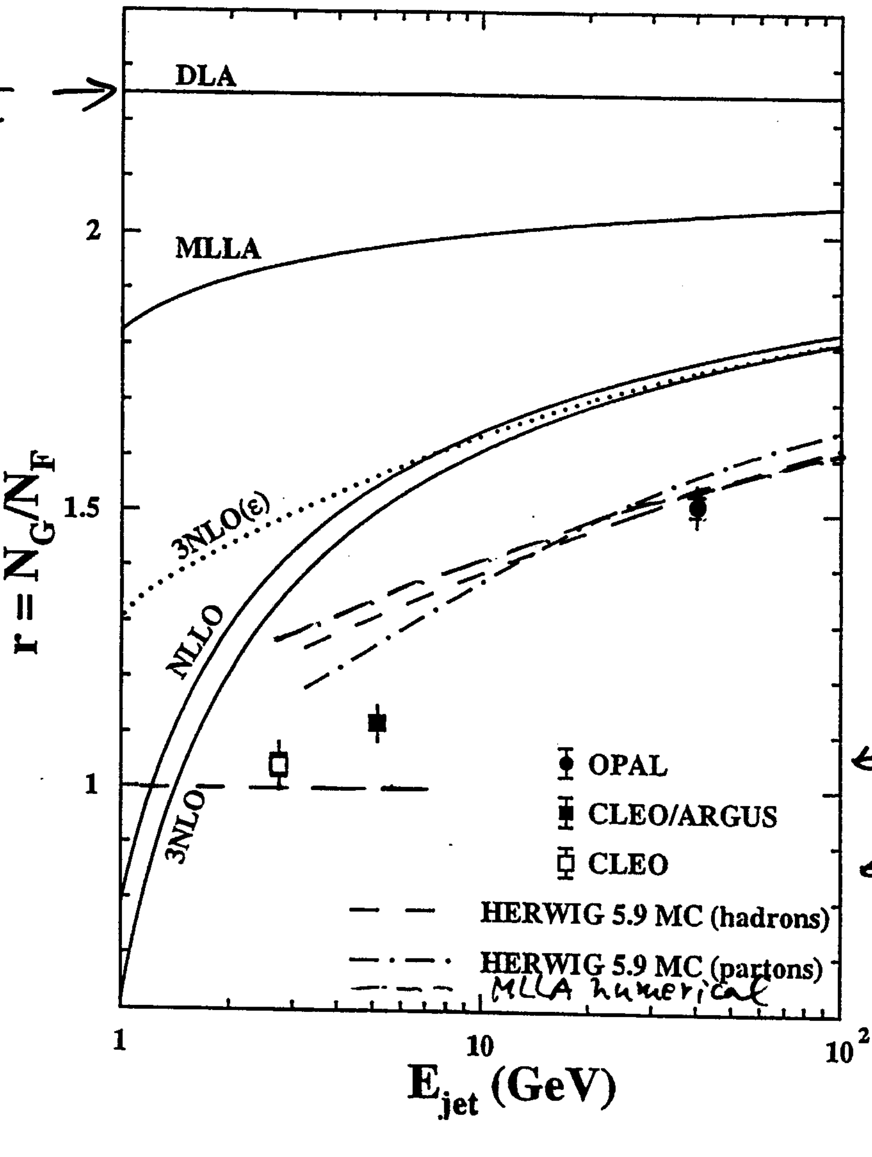
$\Lambda = 500$ MeV , $\lambda = 0.015$ (Durham K_T)

$\Lambda = 350$ MeV , $\lambda = 0.015$ (standard K_T $K_T = z(1-z)Q$)

$$\lambda = \ln \frac{Q_0}{\Lambda}$$

$$r = \frac{\text{multiplicity in } g\text{-jet}}{\text{multiplicity in } q\text{-jet}}$$

$r = \frac{C_A}{C_F} = \frac{9}{4} \rightarrow$



$ee < \begin{matrix} b \\ \bar{b} \end{matrix}$

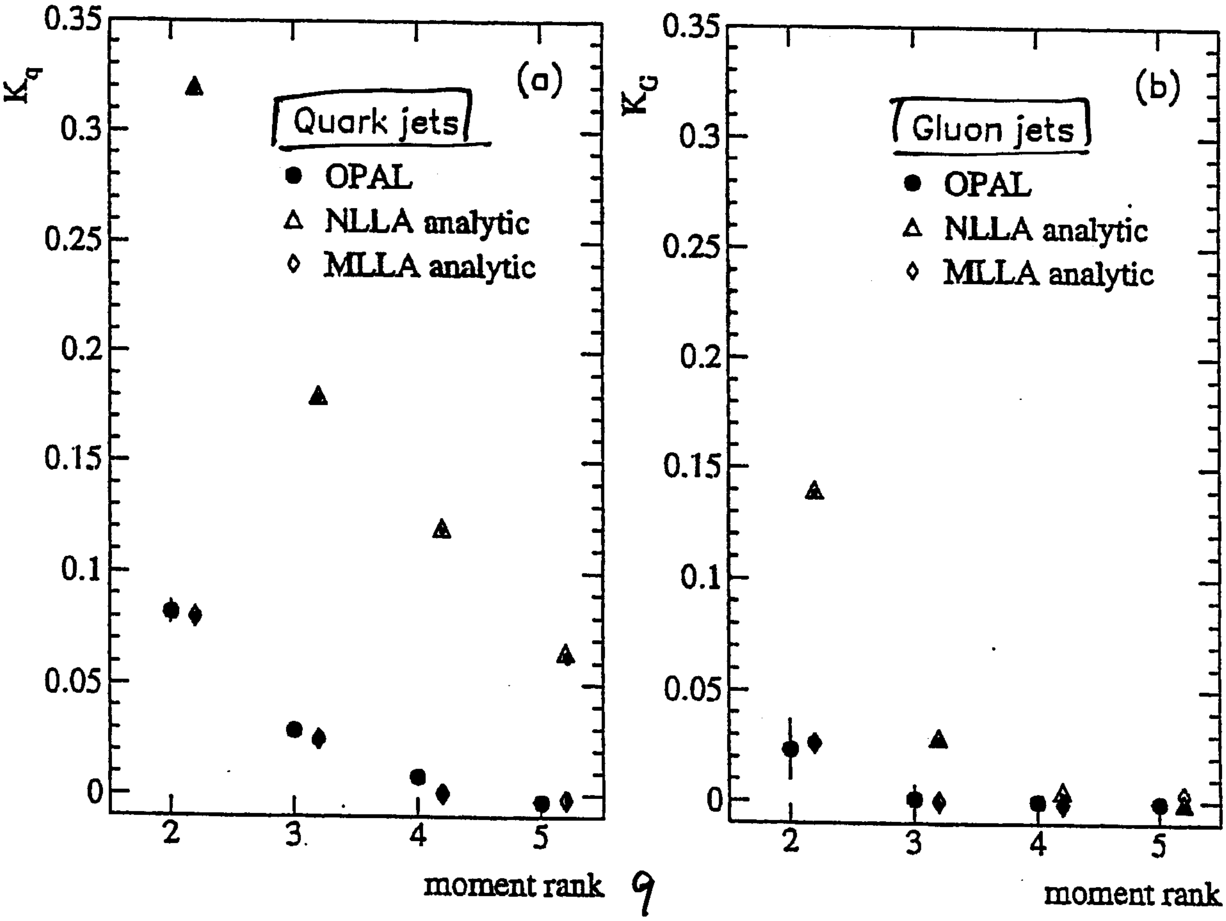
$\leftarrow Z \rightarrow g(b\bar{b})$

$\leftarrow \Upsilon(1S) \rightarrow \gamma g g$

Kumulant moments of multiplicity distribution

O. Biebel / Physics Reports 340 (2001) 165–289

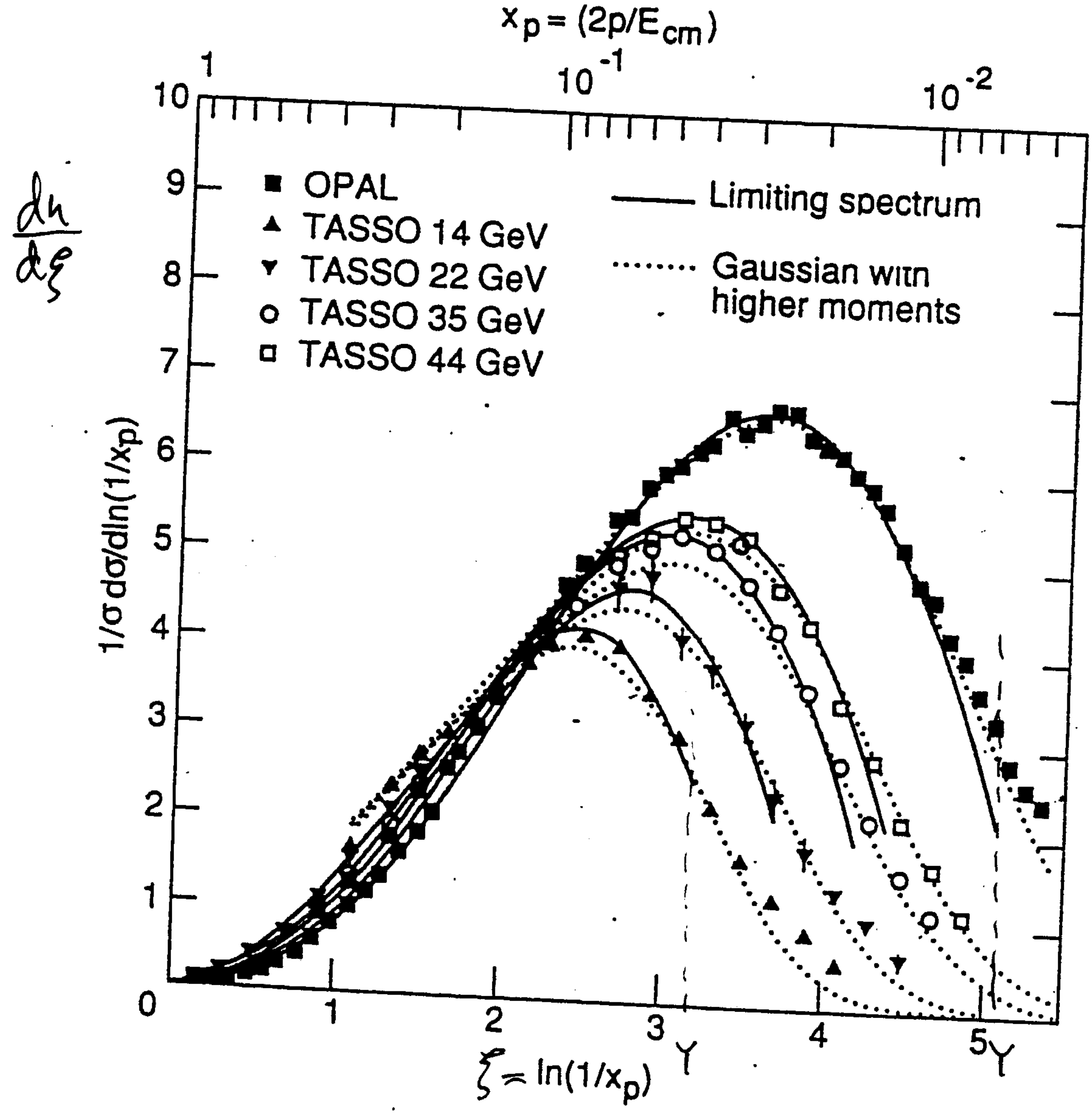
K_q



▲ NLLO (Dremin et al.)

⧫ MLLA computer solution (Lupia)

Inclusive spectra in $\xi = \ln \frac{1}{x}$



$Y = \ln \frac{P}{Q_0}$

"hump-backed plateau"

Shape parameters of ξ-distribution

Dokshitzer, Khoze, Troyan '92

Moments in MLLA

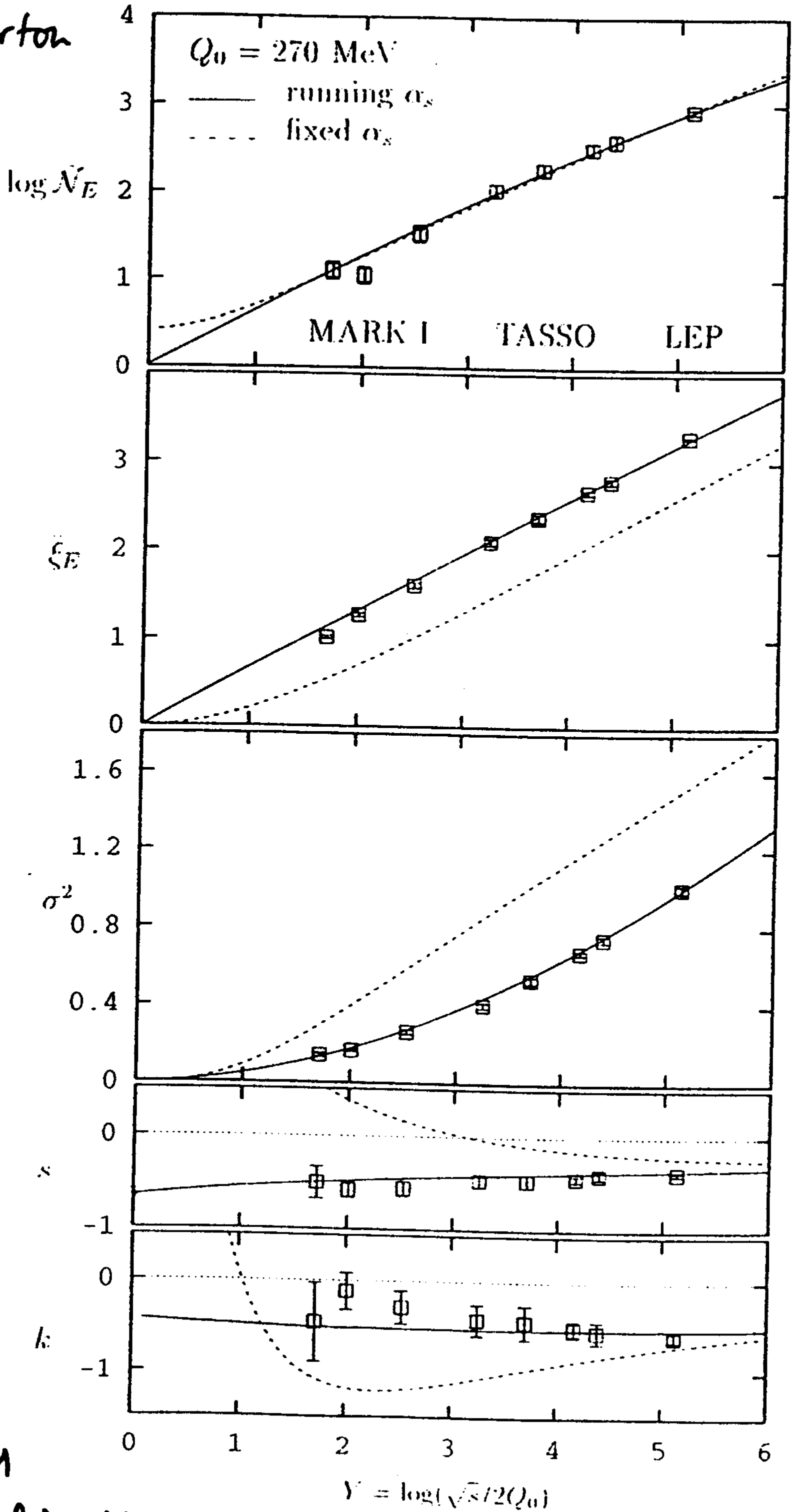
$$\frac{\langle \xi^q \rangle}{Y^q} = P_0^{(q)}(B,z) + \frac{2}{z}\frac{I_{B+2}(z)}{I_{B+1}(z)} P_1^{(q)}(B,z)$$

$$z \sim \sqrt{Y}$$

$N_{hadron} = K\, N_{parton}$

$K \sim 2$

Lupia W.O.



NLO LO

Skewness

Kurtosis

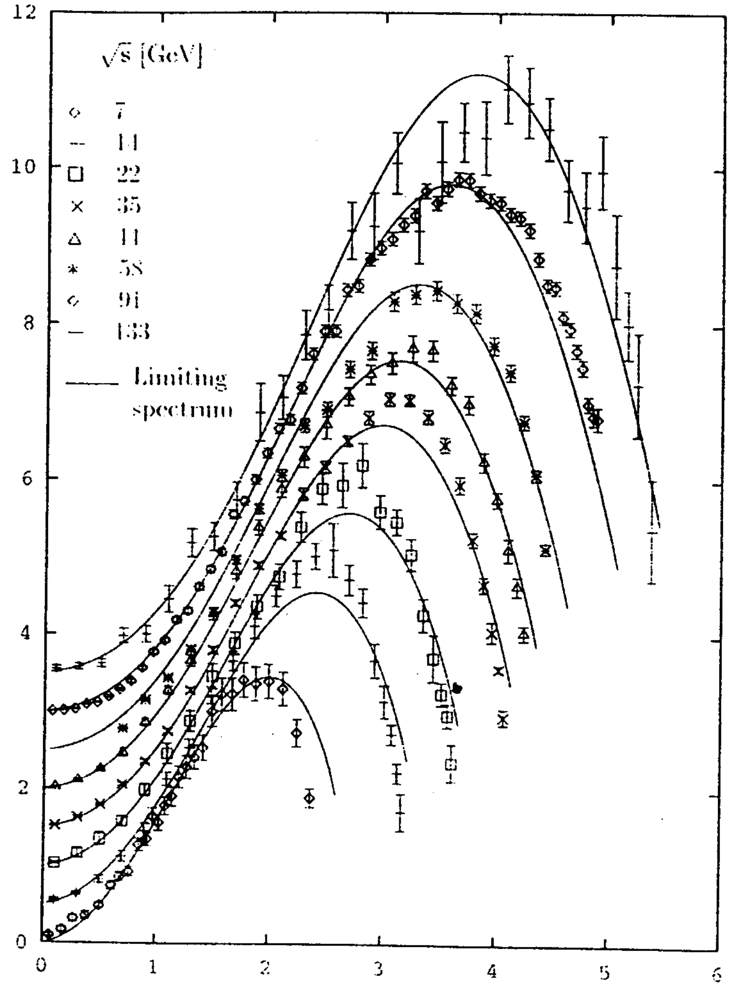
1) $Q_0/\Lambda \lesssim 1.1$
2) Threshold limit
3) running α_s

$$Y = \ln\left(\frac{\sqrt{s}}{2Q_0}\right)$$

Inclusive Momentum spectrum

$$D(\xi, Y) \simeq \frac{N(Y)}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\xi-\bar{\xi})^2}{2\sigma^2}}, \quad \xi = \ln\frac{1}{x}, \; Y = \ln\frac{E}{Q_0}$$

moments: $\bar{\xi} = \frac{1}{2} Y + \dots$, $\sigma^2 = \frac{1}{3}\sqrt{\frac{bY^3}{16 N_c}} + \dots$

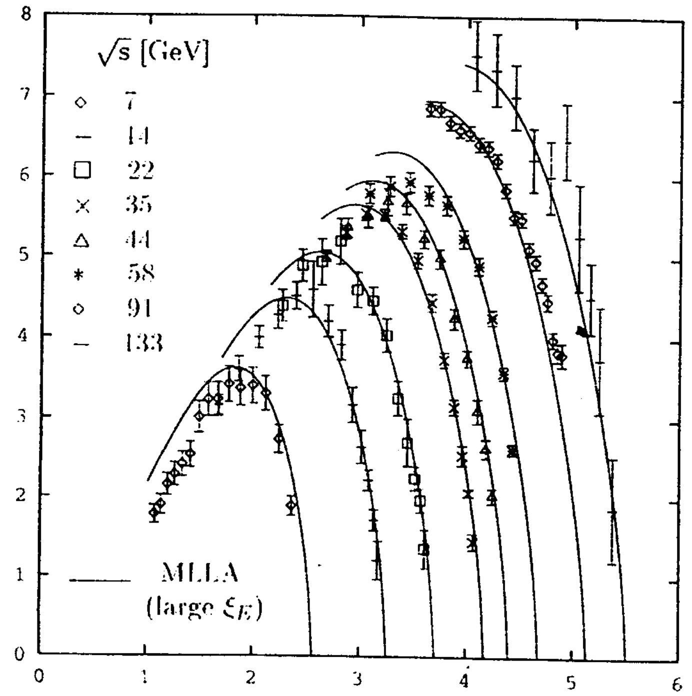


"hump-backed plateau"
Dokshitzer et al.
Bassetto et al
Mueller (82)

Limiting spectrum
$(Q_0 \sim \Lambda)$
Dokshitzer, Khoze, Troyan

$\xi_E = \ln \frac{E_{jet}}{E}$, $E = \sqrt{p^2 + Q_0^2}$

Q_0 as eff. hadron mass



MLLA, large ξ
$(Q_0 > \Lambda)$

expansion $O(\alpha_s^2)$ around soft limit $\xi \sim Y$

hard $\leftarrow$ ξ_E $\rightarrow$ soft

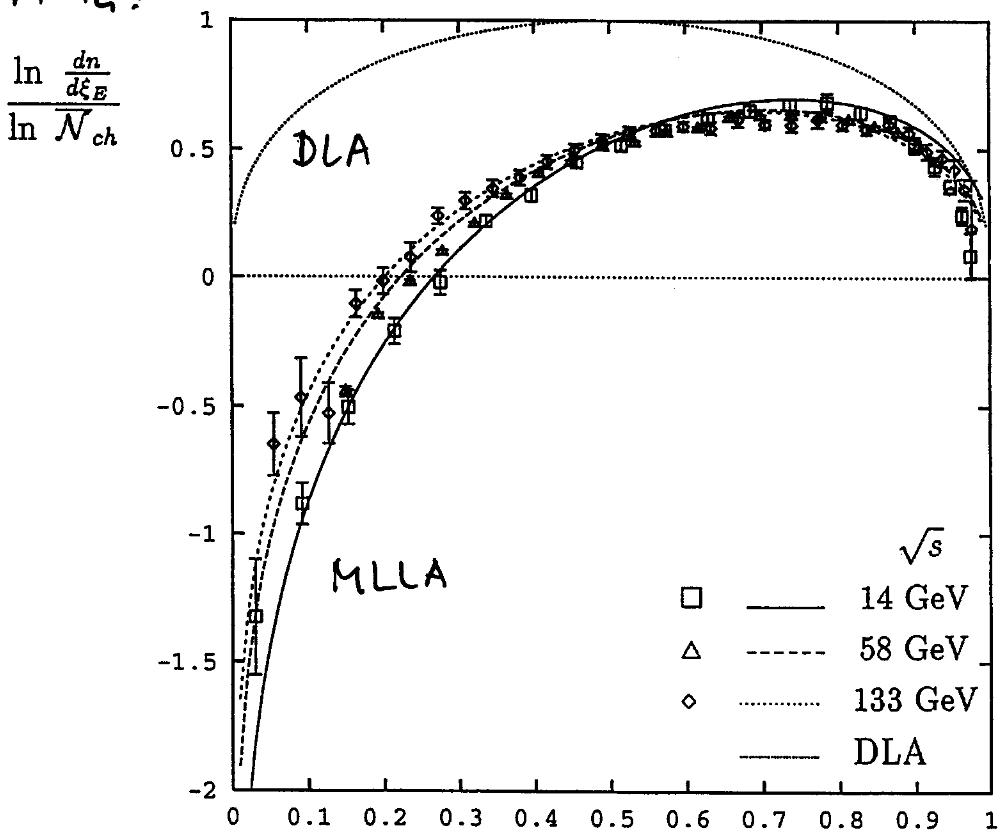
Lupia w.o. EPJ-C (98)

Asymptotic (DLA) limit of particle spectra

$s \to \infty$ $$\frac{dn}{d\xi} \sim \bar{N}\, e^{-\frac{(\xi-\bar{\xi})^2}{2\sigma^2}}$$

$$\bar{N} \sim e^{c\sqrt{Y}}, \quad \bar{\xi} = \frac{1}{2} Y, \quad \sigma^2 \sim Y^{3/2}.$$

scaling limit in:



$$\zeta_E \equiv \frac{\xi_E}{Y} \equiv \frac{\ln \frac{E_{jet}}{E}}{\ln \frac{E_{jet}}{Q_0}}$$

hard ← → soft

⇒ slow approach of asymptotic limit

but: soft particles already asymptotic ($\xi \sim Y$)

Soft limit of particle spectrum

soft gluon emission in $q\bar{q}$ jet system

gluon with large wavelength "sees" only total charge
(coherent emission from all sources)

→ Born term dominates $O(\alpha_s)$
like QED Bremsstrahlung

$$\frac{dn}{dy\,dp_T} \approx C_{A,F}\,\frac{\alpha_s(p_T)}{p_T}\left(1 + O\left(\ln\frac{\ln p_T/\Lambda}{\ln Q_0/\Lambda}\,\ln\frac{\ln \frac{p_T/x}{\Lambda}}{\ln p_T/\Lambda}\right)\right)$$

↑ energy independent — ↑ vanishes for $p_T \to Q_0$

Khoze
Lupia
W-O.

Predictions

1) $\left.\frac{dn}{dy\,dp_T^2}\right|_{p\to 0 \text{ or } p_T \to 0}$ – energy independent
– flat plateau

2) $\frac{dn}{dy\,dp_T^2} \sim \begin{cases} C_A = 3 & \text{gluon jet} \\ C_F = \frac{4}{3} & \text{quark jet} \end{cases}$ $\quad \frac{N_g}{N_q} = \frac{9}{4} = 2.25$

(total multiplicity $\frac{N_g}{N_q} \sim 1.5$ at LEP)

3) multiplicity distribution of particles at small p_T:
Poissonian

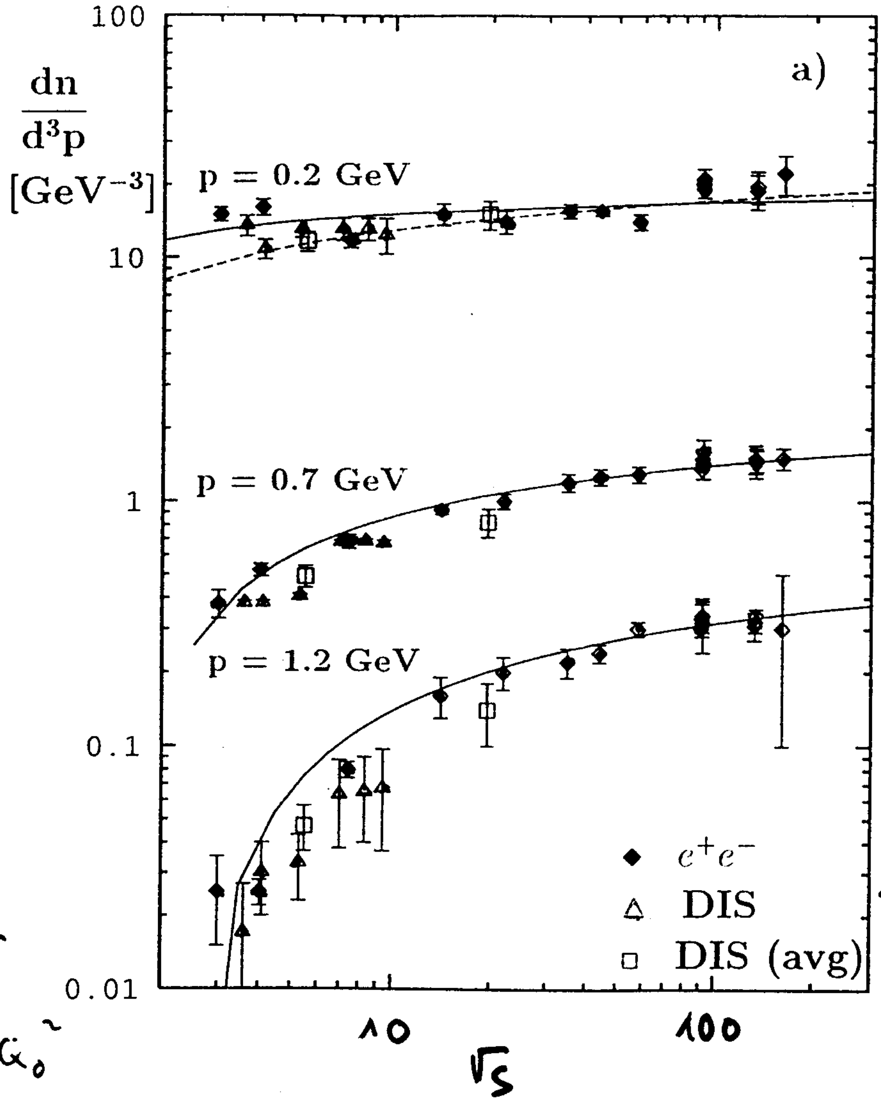
Realistic tests

e^+e^- 3 jet events

DIS – resolved and unresolved Photoproduction

inclusive particle spectrum

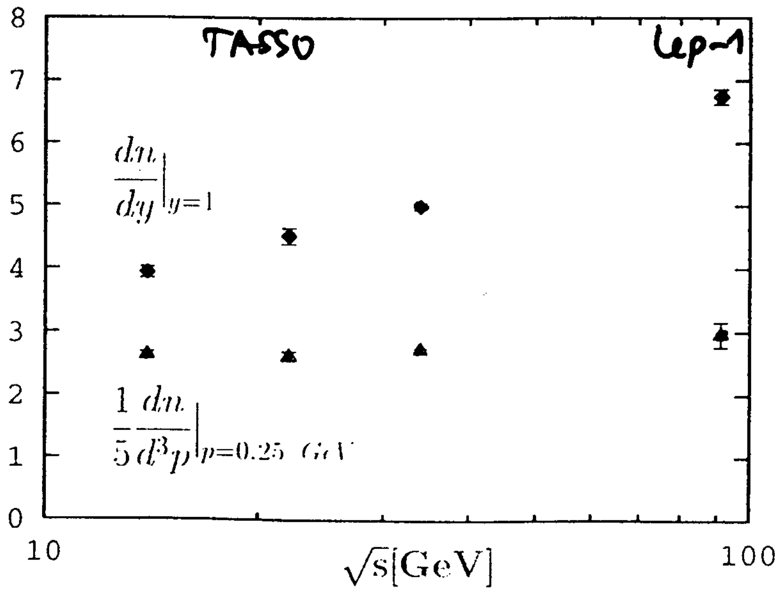
Khoze
Leupin
Ochs



} H1 (Breit)

$O(\alpha_s^2)$ solution

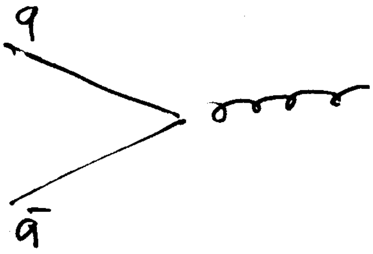
$E_p^2 = p_H^2 + Q_0^2 \geq Q_0^2$



y small
(p arbitrary)

p small

the spectrum tends to become energy independent for $p \to 0$



B. Gary
OPAL

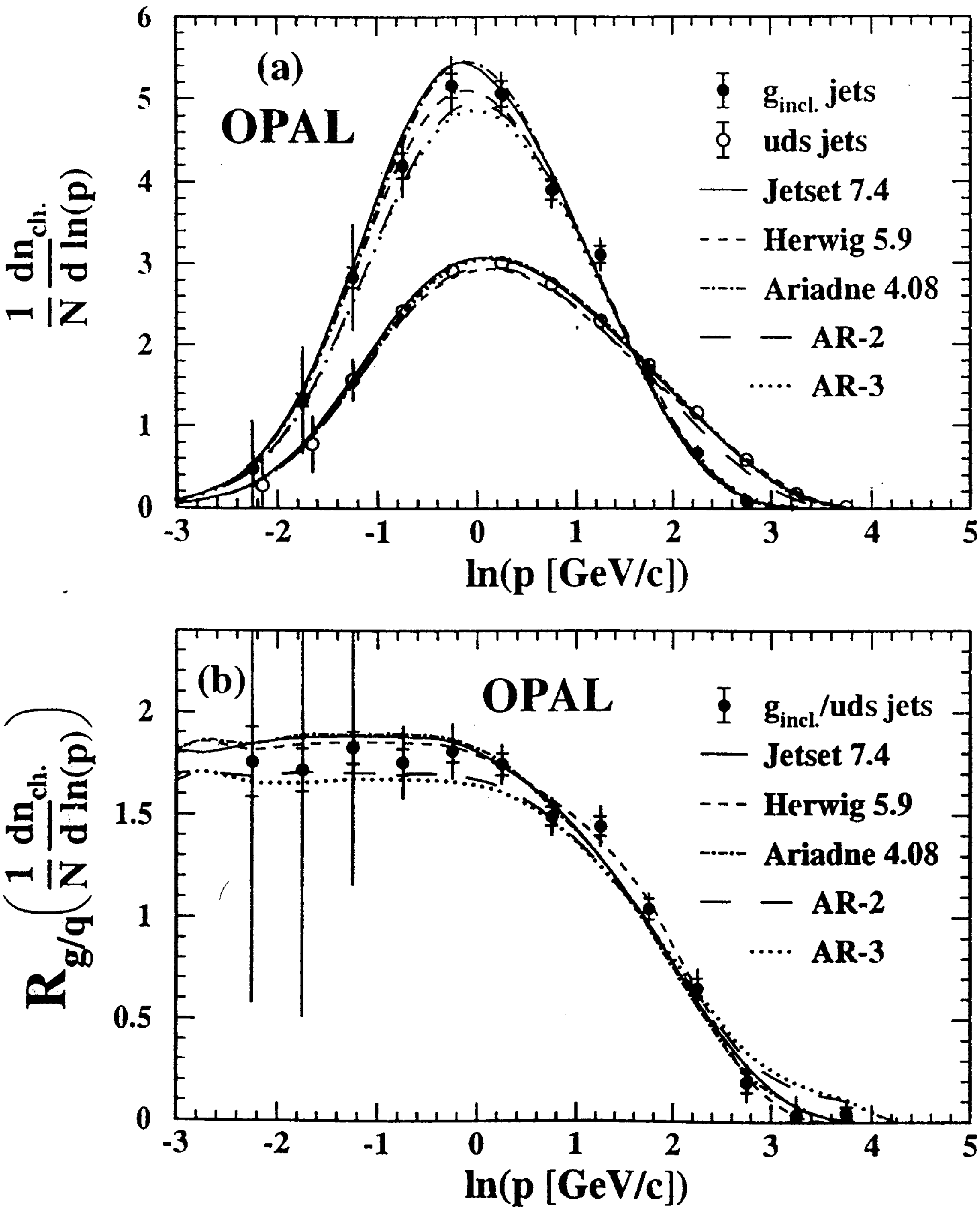
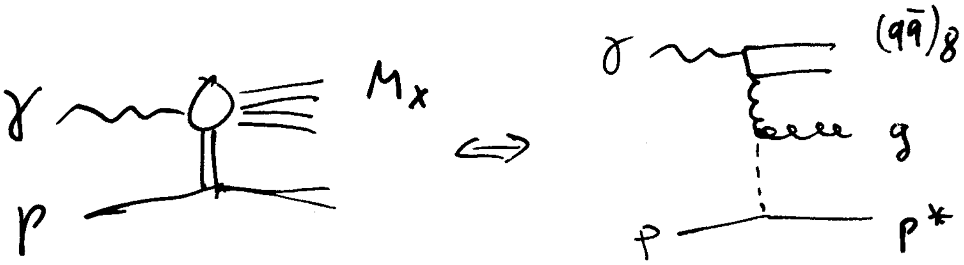


Figure 7: (a) Corrected distributions of the logarithm of charged particle momentum, ln (p), for 40.1 GeV $g_{incl.}$ gluon jets and 45.6 GeV uds quark jets. (b) The ratio of the gluon to quark jet ln (p) distributions for 40.1 GeV jets. The total uncertainties are shown by vertical lines. The experimental statistical uncertainties are indicated by small horizontal bars. (The statistical uncertainties are too small to be seen for the uds jets.) The predictions of various parton shower Monte Carlo event generators are also shown. These data are tabulated in Table 3.

Diffraction in DIS



$\frac{dn}{dy}$

$\frac{N_g}{N_q}$ at $y \sim 0$

$M_X = 5.3$ GeV

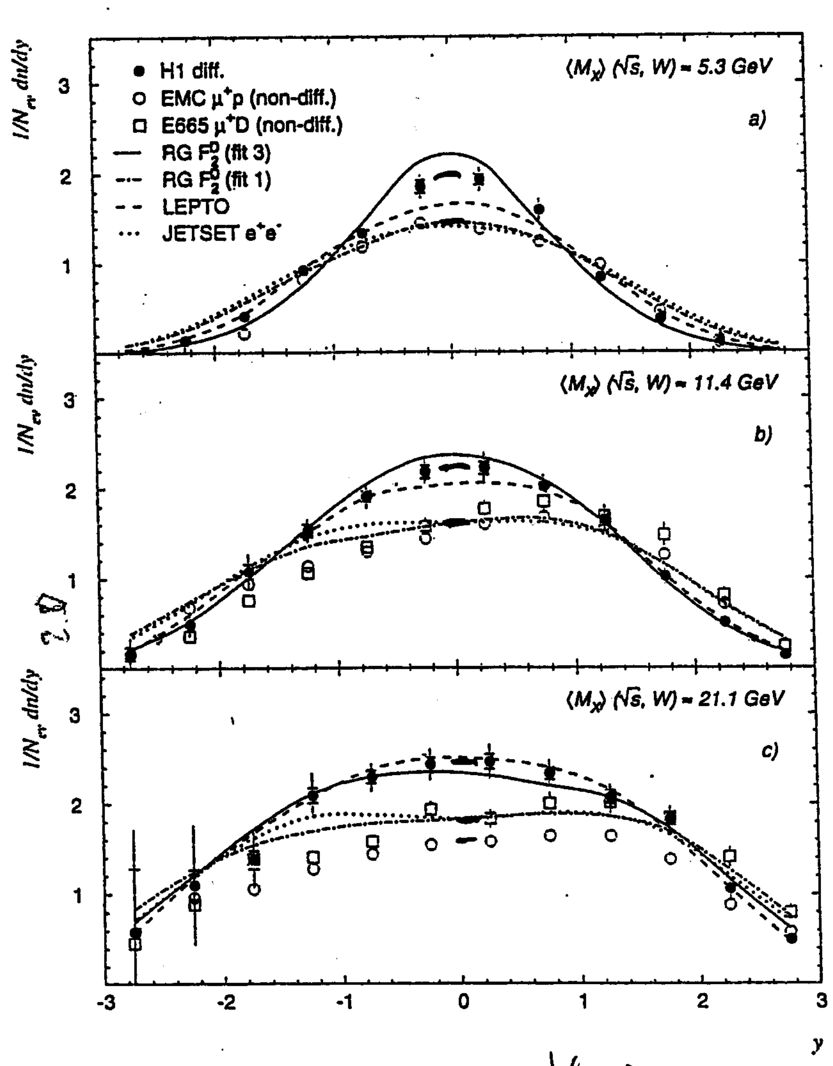
$r \sim 1.4$

$= 11.4$ GeV

1.4

$= 21.1$ GeV

1.3–1.6



$y \rightarrow$

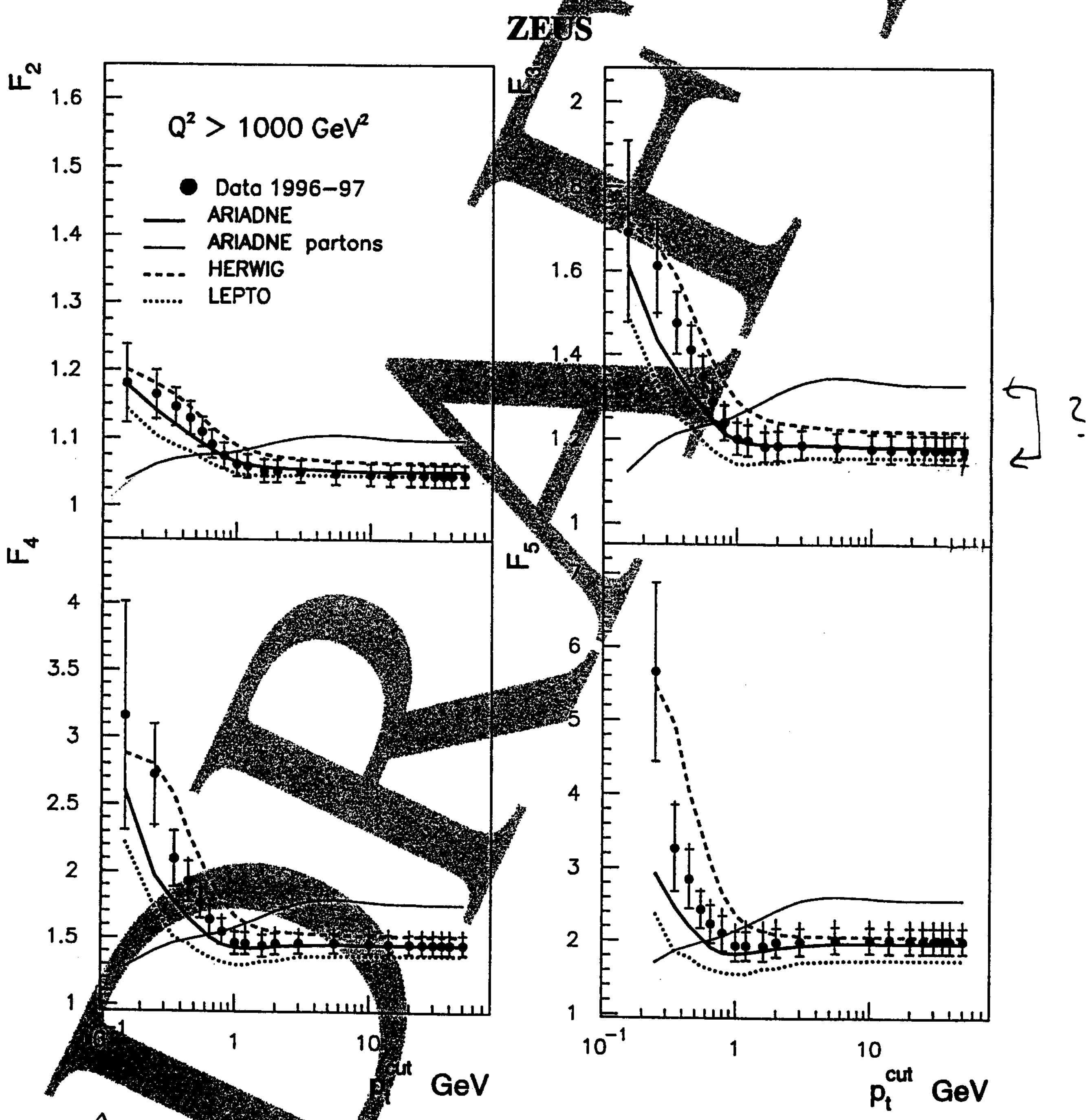
further increase for $p \to 0$?

$p_T \to 0$

Born term → Poissonian

p_t^{cut} Moments

ZEUS Coll., IHEP2000, Osaka, Abst 893, Plen 3, Par. 7



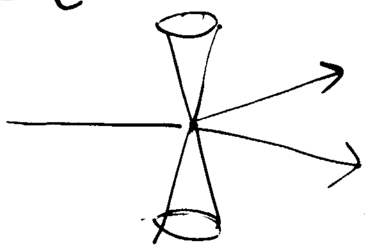
expect Poisson distribution $F_q \to 1$

Study soft radiation perpendicular to event plane

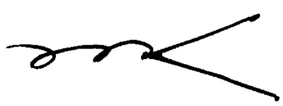
Test of duality picture for small scales

Khoze Lupia ...

a) e^+e^-

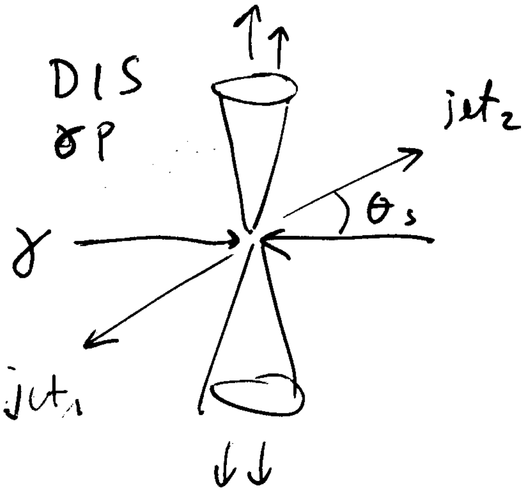


$\sim C_F$



$\sim N_c$

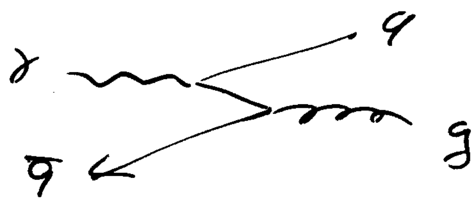
b) DIS γp



photoproduction of dijets

small θ_s

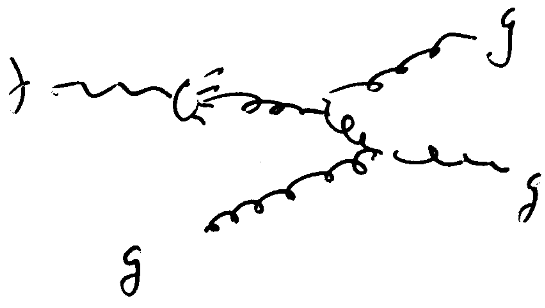
- direct



q - exchange

$\sim C_F$

- indirect

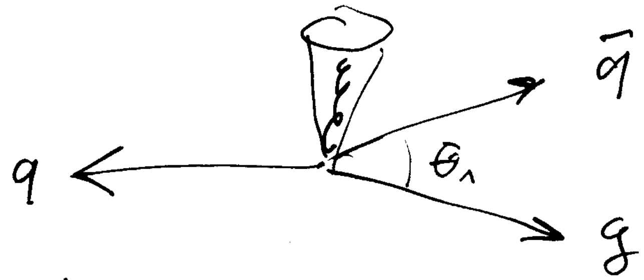


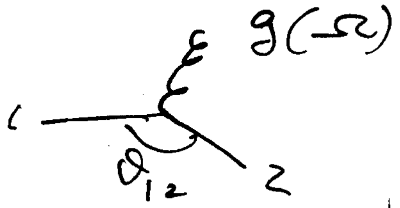
g - exchange

$\sim N_c$

Phys. Lett. B394 (97) 179

EPJ C5 (1998) 77



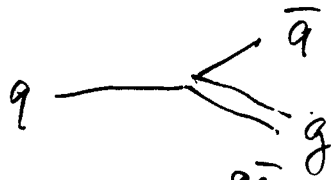


radiation from 1 dipole

$$\frac{dn}{d\Omega} \simeq \frac{1-\cos\vartheta_{12}}{(1-\cos\vartheta_{1g})(1-\cos\vartheta_{2g})}$$

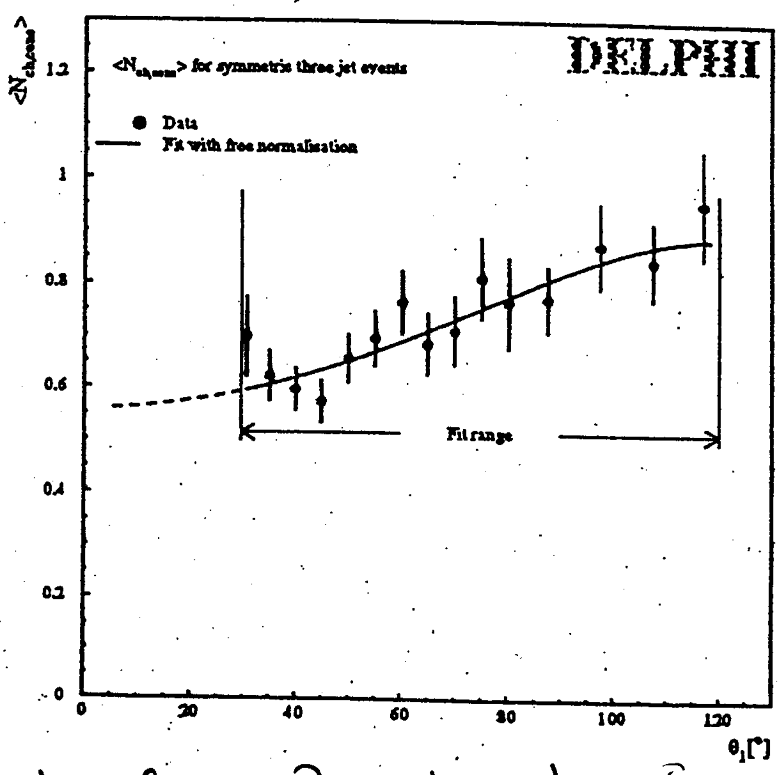
$$\to 1-\cos\vartheta_{12} \text{ for perpendicular radiation}$$
$$\left(\vartheta_{1g}=\vartheta_{2g}=\frac{\pi}{2}\right)$$

radiation from $(q\bar{q}g)$



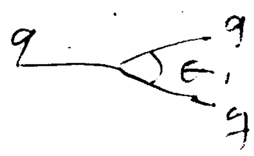
$$\frac{dN_+^{q\bar{q}g}}{dN_+^{q\bar{q}}} = \frac{N_C}{4C_F}\left[(1-\cos\vartheta_{gq})+(1-\cos\vartheta_{g\bar{q}})-\frac{1}{N_C^2}(1-\cos\vartheta_{q\bar{q}})\right]$$

$N_\perp$ in cone



$r \sim 1.64$

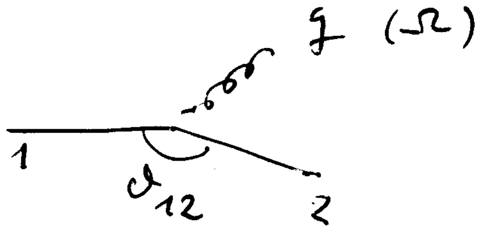
- coherent emission from 3 partons! Θ_1



- multiplicity depends on eff. color charge
- momentum dependence?

Radiation perpendicular to scattering plane

$q\bar{q}$ dipole



$$\frac{dn}{d\Omega} \simeq \frac{1-\cos\vartheta_{12}}{(1-\cos\vartheta_{1g})(1-\cos\vartheta_{2g})} \xrightarrow[\text{perp.}]{} 1-\cos\vartheta_{12} \xrightarrow[\vartheta_{12}\to\pi]{} 2.$$

$$\text{Ratio } R_\perp = \frac{\frac{dn}{d\Omega}}{\left(\frac{dn}{d\Omega}\right)_{\text{Dipole at } \vartheta_{12}=\pi}}$$

Direct production

$\gamma g \to q\bar{q}$: $\quad R_\perp = 1$

$\gamma q \to q g$: $\quad R_\perp = \frac{N_c}{4C_F}\left[3-\cos\theta_s - \frac{1}{N_c^2}(1+\cos\theta_s)\right]$

$R_\perp \to 1$ small angles θ_s

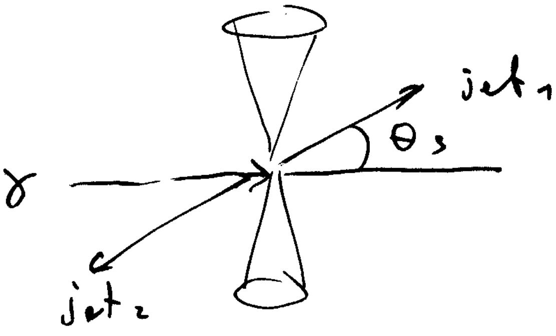
Indirect production

$g g \to g g$: $\quad R_\perp = \frac{N_c}{4C_F}(5-\cos\theta_s)$

$g q \to g q$: $\quad R_\perp = \frac{N_c}{4C_F}\left(4-\frac{1}{N_c^2}(1-\cos\theta_s)\right)$

etc.

$R_\perp \to \frac{N_c}{C_F}$ small angles θ_s



Measure Ratio

$$R(p_T,\theta_s) = \frac{\left(\frac{dn}{dp_T}\right)_{\text{resolved at } \theta_s}}{\left(\frac{dn}{dp_T}\right)_{\text{direct at } \theta_s}}$$

$R(p_T,\theta_s) \to \frac{9}{4}$ for $\theta_s \to 0$

Test run with HERWIG
(4.5 pb^{-1})

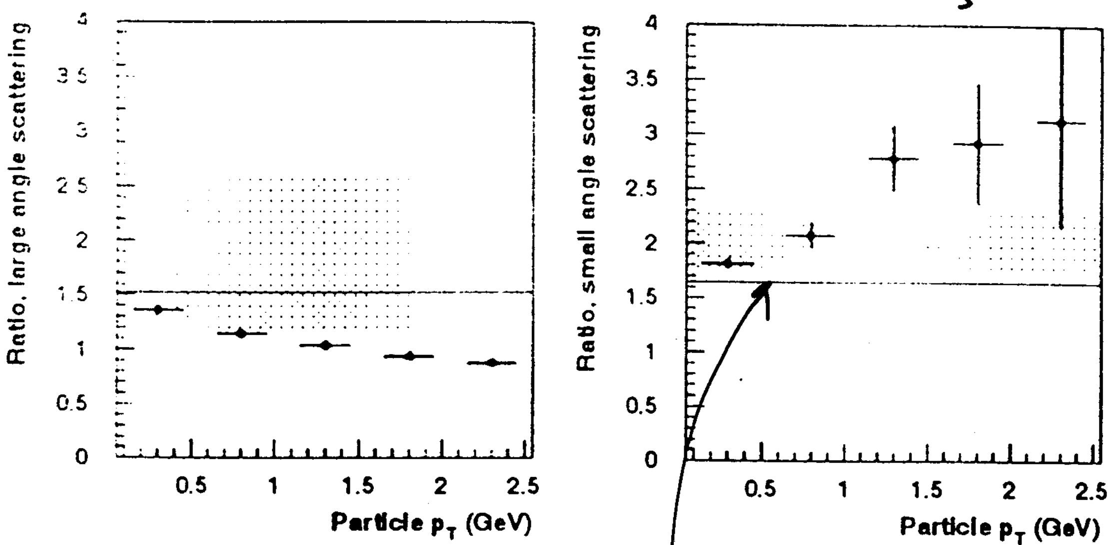
Butterworth
Khoze
Ochs
hep-ph/9901419
(Durham)

direct : $x_{obs} > 0.9$

resolved : $x_{obs} < 0.75$

$R(p_T, \theta_s)$ $\frac{\text{resolved}}{\text{direct}}$

$0 < \cos\theta_s < 0.64$ $\qquad$ $0.64 < \cos\theta_s < 0.96$



$\rightarrow p_T$ $\qquad$ $\rightarrow p_T$

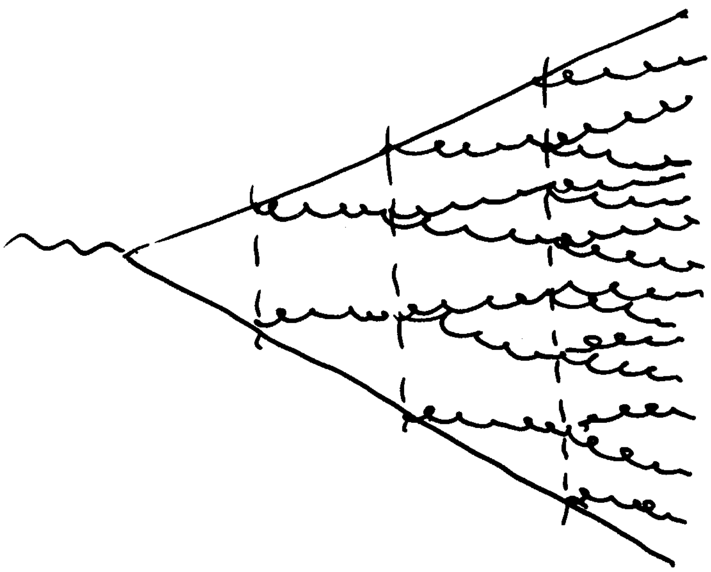
theor. expectation using HERWIG structure fcts.

→ "data" approach expectation for small p_T

→ studies towards small angles θ_s appear feasible

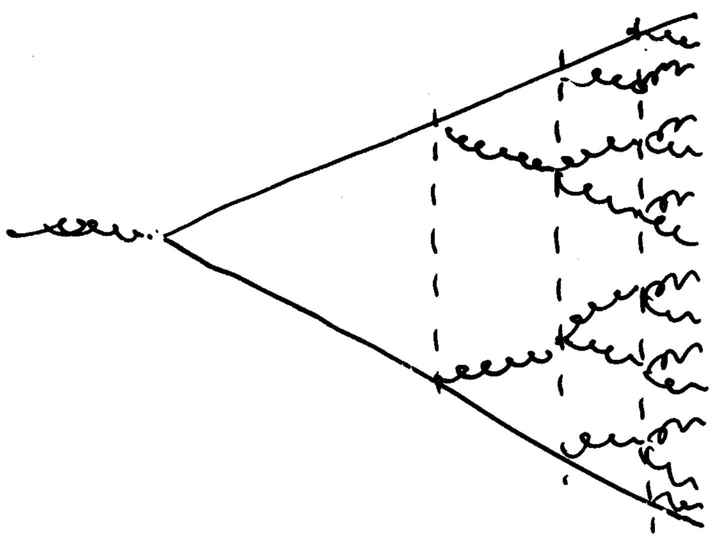
Multiparticle correlations

selfsimilar cascade : fractal



$O(\delta) \sim \delta^{\varphi}$

QCD cascade : selfsimilarity broken by $\alpha_s(Q^2)$

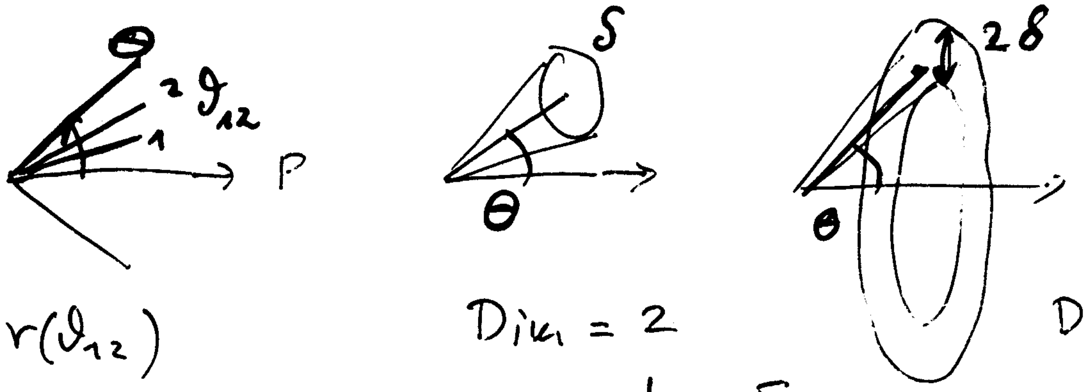


→ resolution power

$\alpha_s(Q^2) \to$ increasing

angular correlations

W.O.-E. Wosiek
Dokshitzer, Dremin
Brax, Meunier
Peschanski



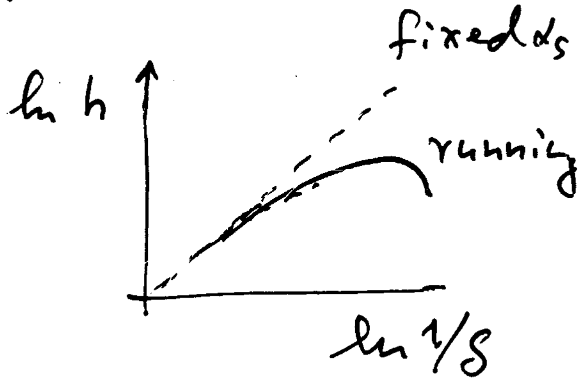
$r(\vartheta_{12})$ Dim = 2 Dim = 1

moments F_q

Analysis in DLA of QCD

$$h_q(\delta, \Theta, P) \sim \left(\frac{\Theta}{\delta}\right)^{\varphi_q}$$ for fixed α_s

running α_s



- bending at small angle
- running of initial slope

moments:

$$F_q \sim \left(\frac{\Theta}{\delta}\right)^{\phi_q}, \quad \phi_q \approx (q-1)D - \left(q - \frac{1}{q}\right)\gamma_0$$

$$\gamma_0^2 = \frac{6\alpha_s(P\Theta)}{\pi}$$

- asymptotic scaling law (ε-scaling)

rescaled $\ln h_q$ depends only on ε

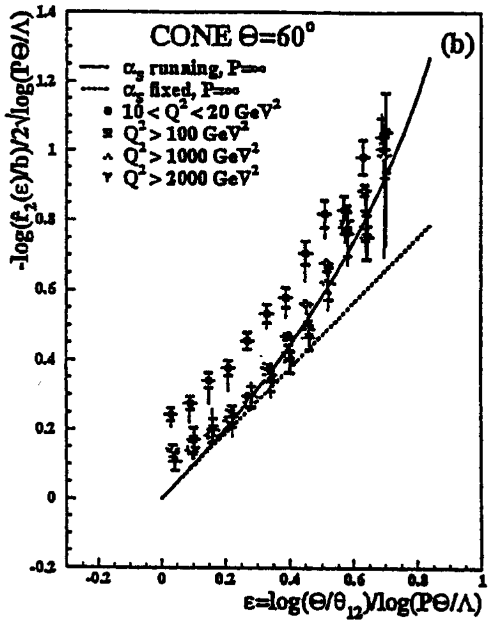
$$\varepsilon = \frac{\ln \Theta/\delta}{\ln P\Theta/\Lambda} \qquad (0 < \varepsilon < 1)$$

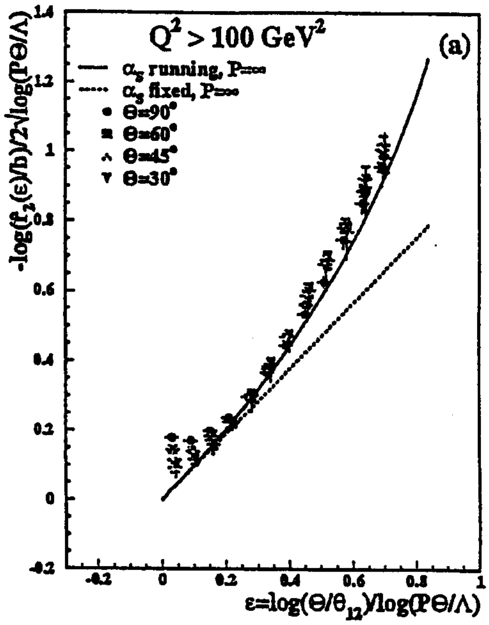
ξ - scaling

Q varies
θ fixed

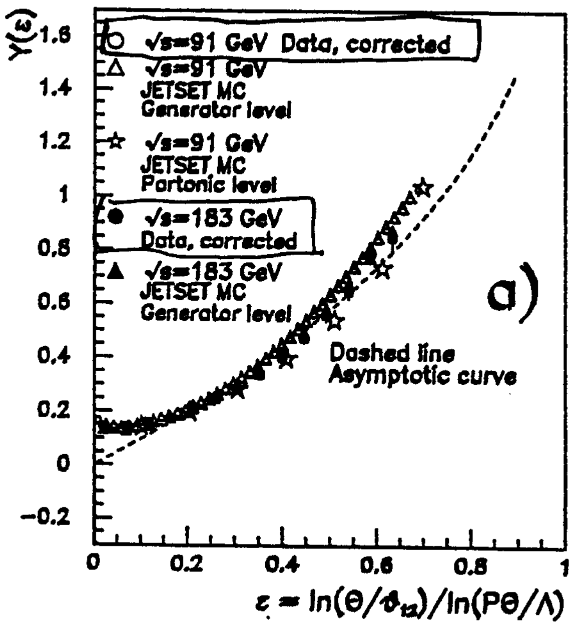
θ varies
Q fixed

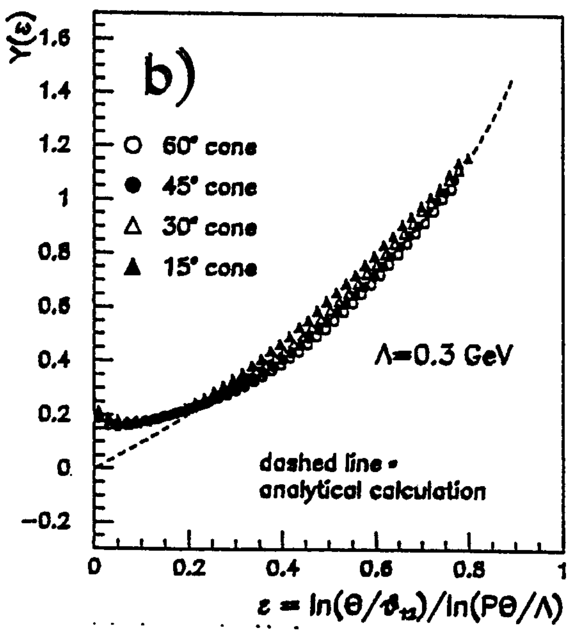
Zeus '98 (Vancouver)





DELPHI '98





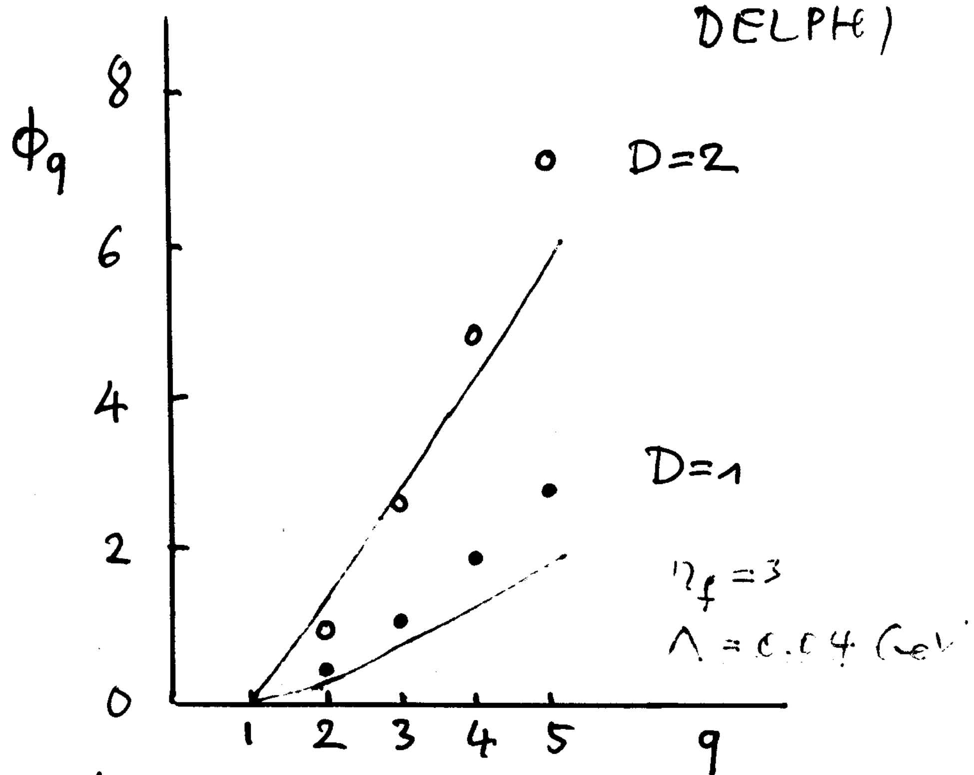
$$\xi = \frac{\ln \theta/\vartheta_{12}}{\ln P\theta/\Lambda}$$

- early approach to asymptopia
- Dendons.

Initial slopes

moments F_q: dependence on q and D

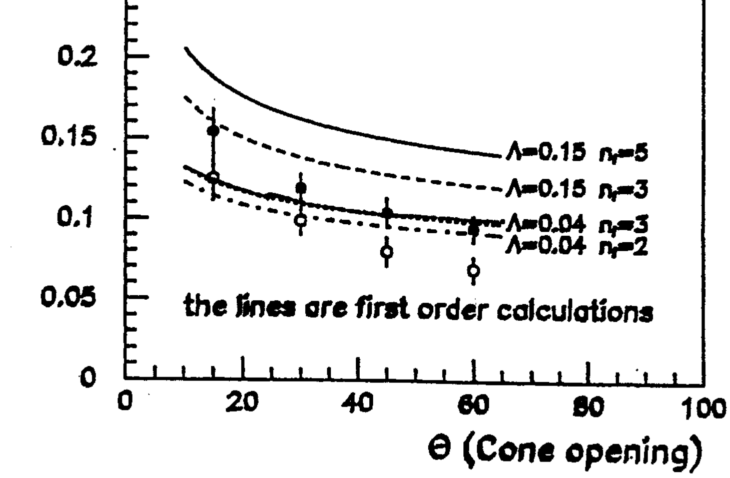
$$\Phi_q \simeq (q-1)\,D - \left(q - \frac{1}{q}\right)\gamma_0 \qquad \text{(DLA)}$$



correlation function: dependence on jet opening Θ

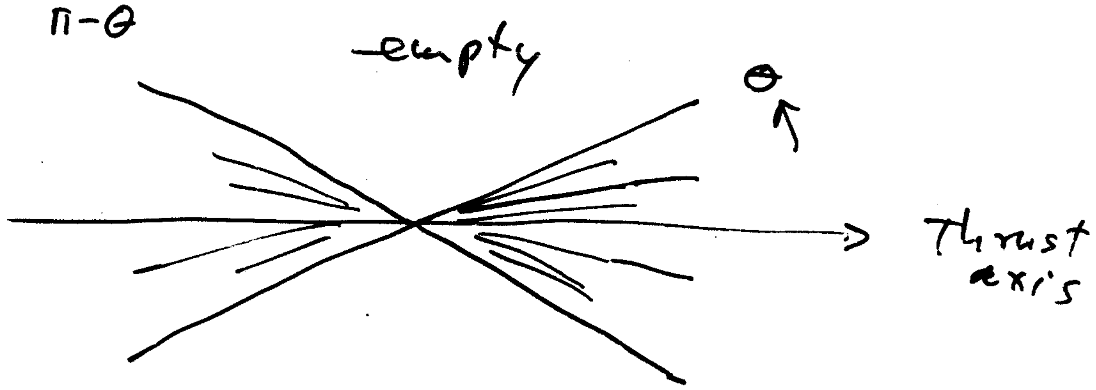
$$r(\vartheta_{12}) = \left(\frac{\Theta}{\vartheta_{12}}\right)^{\gamma_0/2}$$

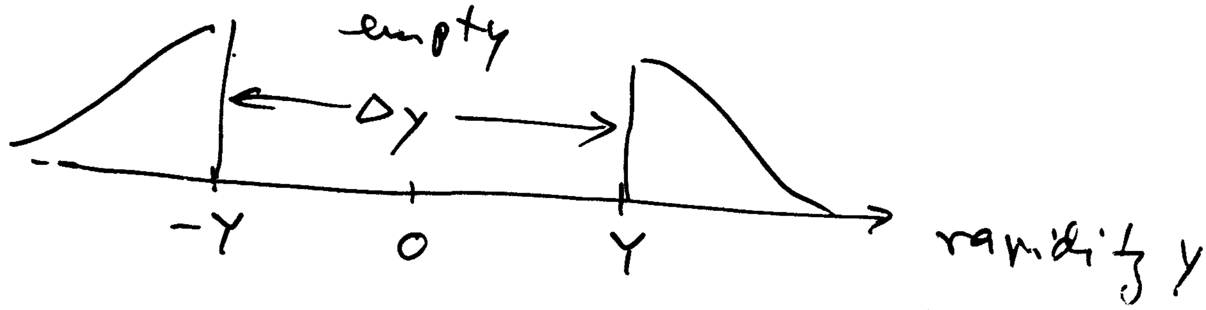
$$\gamma_0 = \left(\frac{6\,\alpha_s(P\Theta)}{\pi}\right)^{1/2}$$



Rapidity gaps in e^+e^- annihilation

with
T. Shimada
ISMD '99



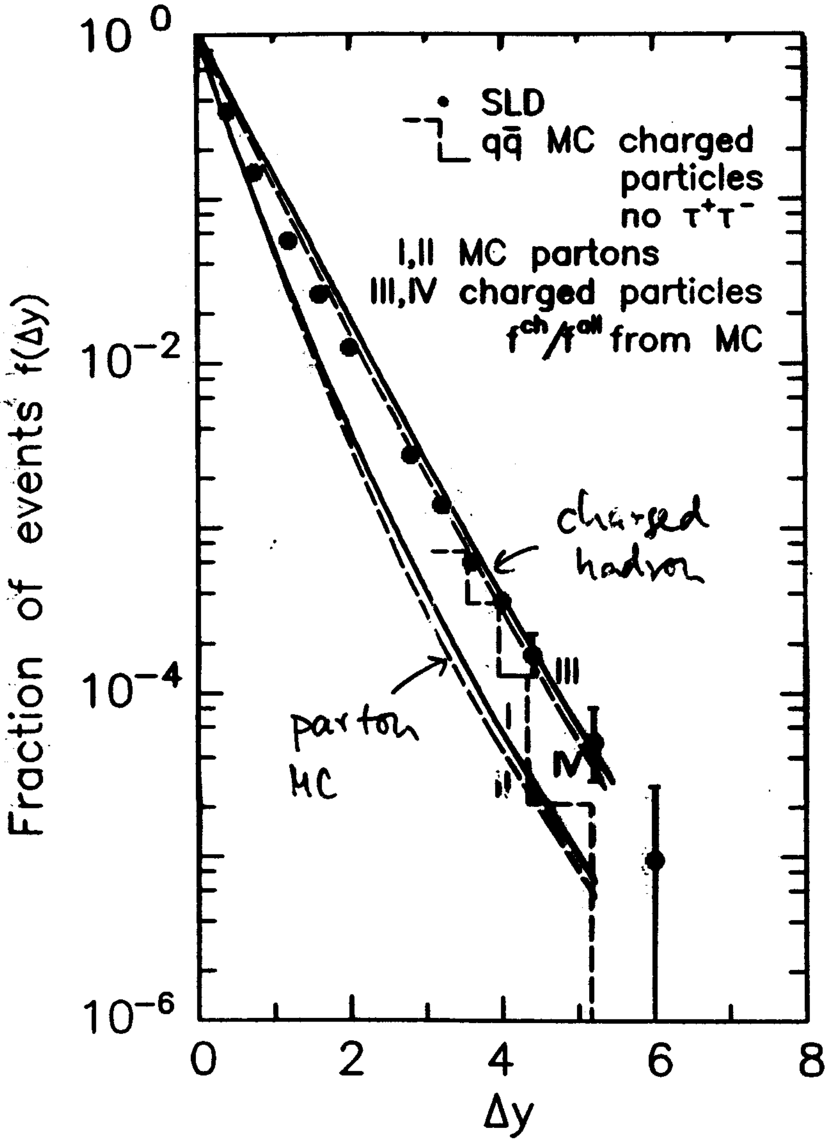


$$\text{rapidity} = -\ln \operatorname{tg} \frac{\theta}{2}$$

study effect of color neutralization through gap

Rapidity gaps in e^+e^- annihilation

w.o.
T. Shimada
'99



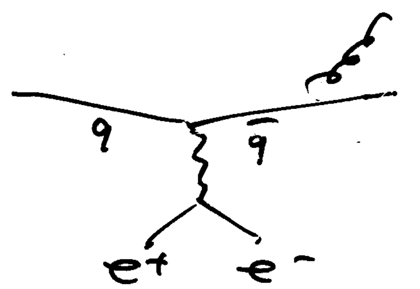
Ariadne MC parton with parameters Q_0, Λ fixed by jet e hadron multiplicity

DLA: $f_{gap} \cong e^{-A_p \Delta y}$, $A_p = \frac{4C_{A/F}}{b} \ln \frac{Y}{\lambda}$

$Y = \ln \frac{P}{Q_0}$, $\lambda = \ln \frac{Q_0}{\Lambda}$

Puzzle

space-time picture

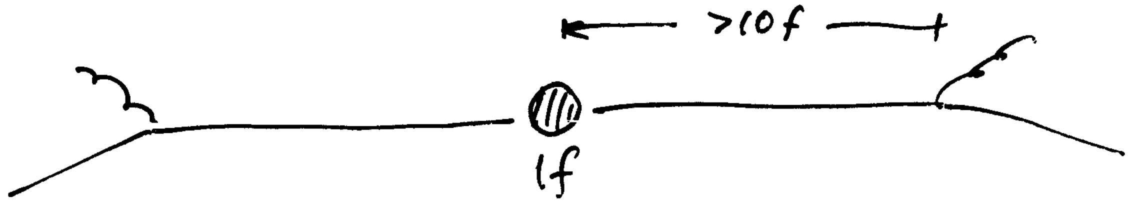


lifetime of quark: $\tau \sim \frac{1}{m_{\bar q}} \cdot \frac{E}{m_{\bar q}} \sim \frac{k}{k_\perp^2}$ Dokshitzer Khoze ...

average lifetime of quark in gap events:

$$\left\langle \frac{k}{k_\perp^2} \right\rangle_{DLA} \sim 10-20\, f \qquad (\Delta y = 3)$$

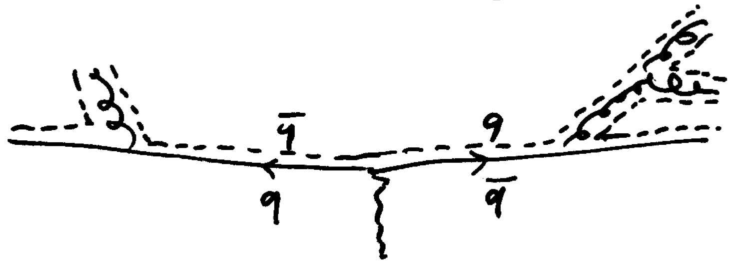
First gluon emission far outside confinement region $R \sim \frac{1}{Q_0} \sim 1f$



$\Rightarrow$ global color blanching should not produce new soft hadrons

Possible mechanisme

soft colour blanching by $q\bar q$ pairs with $k_\perp < Q_0$



Dual hadron cascade

Cohen Tannoudji – w.o. '88

Summary

hadronic jets represented by QCD cascade with low cutoff $Q_0 \gtrsim \Lambda$.

- running coupling $\alpha_s \sim 1$
- soft gluon coherence
- colour factors C_A, C_F

} also for soft particles

* simplicity & analytic predictions

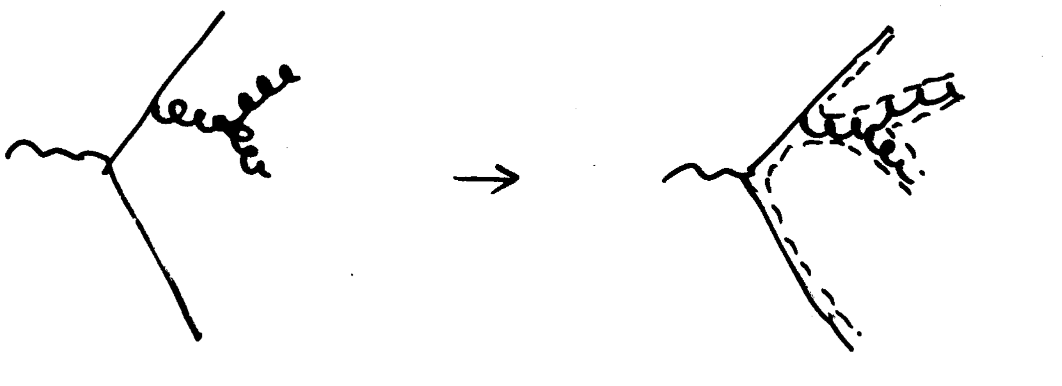
* successes of perturbative picture

- global multiplicities and moments
  MLLA & computer solutions
  almost quantitative
- spectra – take into account
  MLLA kinematic corrections
- angular distributions & correlations
  DLA : at least qualitative
- rapidity gaps in e^+e^-
  quark travels ~1 of before g emission
  → soft confinement
- outlook: MC for shapes

* failure

- soft multiparticle correlations (no Poisson limit)

* picture of soft hadronization



colour neutralitation by soft $q\bar{q}$ pairs (possibly also gluons)