A. Caldwell
M. Soares

Vector Meson production in the Golec-Biernat Wüsthoff Model

Abstract

In this work we apply the Golec-Biernat Wüsthoff Model in the calculation of Vector Meson production processes. Starting from very simple non-relativistic wave function we show that the model provides a very good description of J/Ψ cross sections in a wide Q^2 range. Also for the light mesons one obtains the correct W dependence and ratio longitudinal/transverse cross sections, although in this case the normalization, affected mainly by the wave function employed, is not in good agreement with data.

1 Introduction

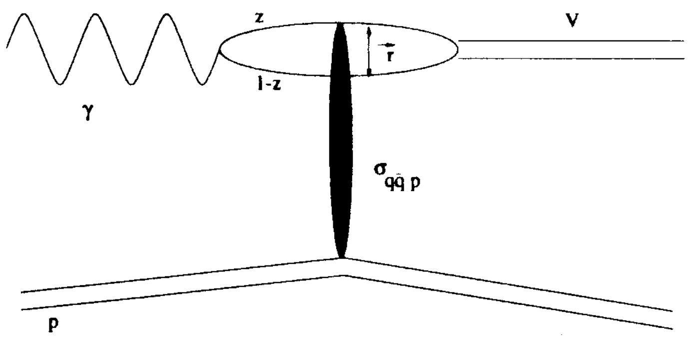


Figure 1: Scheme of Vector Meson Production in the Color Dipole Model

The model is built in the proton rest frame, where the pair $q\bar{q}$ can have a lifetime big enough to allow its interaction with the proton. In this frame the $q\bar{q}$ longitudinal momentum is quite big so that $\vec{r}$, the bi-dimensional separation of the pair doesn't change significantly during the process. For this reason one chooses to work in the $(\vec{r}, z)$ representation.

The informations about the of the interaction pair-proton are contained in the dipole cross section, $\sigma_{q\bar{q}-p}$.

Within the Color Dipole approach the so called "saturation model" was proposed by K. Golec-Biernat & M. Wüsthoff Model. In this model in the inicial state and the same in the final state and the interaction cross section, $A = < V \mid \sigma_{q\bar{q}-p} \mid \gamma^{(*)} >$. In the (r, z) representation it can be approximated by

$$A_{L,T} = \int_0^1 dz \int d^2\vec{r}\ \Psi_V^{*L,T}(r,z)\ \sigma_{q\bar{q}-p}(Q,W,r)\ \Psi_\gamma^{L,T}(Q,r,z) \quad (5)$$

where $\Psi_V(r,z)$ is the vector meson wave function, $\Psi_\gamma(Q,r,z)$ is the photon wave function, $\sigma_{q\bar{q}-p}(Q,W,r)$ is the cross section for the pair-proton interaction. The amplitude normalization follows

$$\frac{d\sigma}{dt}|_{t=0} = \frac{|A|^2}{16\pi}. \quad (6)$$

Since the model does not incorporate any t dependence we assume the ordinary exponencial t dependence, $d\sigma/dt \simeq Ce^{-B|t|}$ observed in the data, so that the total cross section is given by

$$\sigma(\gamma p \to Vp) = \frac{1}{B}\frac{d\sigma}{dt}|_{t=0} = \frac{1}{B}\frac{|A|^2}{16\pi}. \quad (7)$$

The photon wave function it is quite well known [5]. For photons with longitudinal polarization we have

$$\Psi_\gamma^L = \frac{\sqrt{N_c}}{(2\pi)^{3/2}}\sqrt{4\pi\alpha}\ e_q\ 2Qz(1-z)\phi^\gamma(z,r)\delta_{\lambda_1,-\lambda_2} \quad (8)$$

and for transverse polarization

$$\Psi_\gamma^T = \frac{\sqrt{N_c}}{(2\pi)^{3/2}}\sqrt{4\pi\alpha}\ e_q\{m_q \begin{pmatrix} \mp 1 \\ -i \end{pmatrix} \phi^\gamma(r,z)\delta_{\lambda_1,\lambda_2} + \begin{pmatrix} i(2z-1)\hat{b}_x \mp \hat{b}_y \\ \pm\hat{b}_x + i(2z-1)\hat{b}_y \end{pmatrix} \frac{\partial\phi^\gamma(r,z)}{\partial r}\delta_{\lambda_1,-\lambda_2}\}. \quad (9)$$

In the expressions above N_c is the number of colors, $e_q(m_q)$ is the quark charge (mass), $\lambda_{1,2}$ are the helicities of q, $\bar{q}$ and $\phi^\gamma(r,z)$ is the photon spatial wave function given by

$$\phi^\gamma(r,z) = K_0(r\sqrt{Q^2 z(1-z) + m_q^2}\ ). \quad (10)$$

In analogy with the photon wave function one can write for the longitudinal polarized vector mesons

$$\Psi_V^L = -2M_V\phi^V(z,r)\delta_{\lambda_1,-\lambda_2} \quad (11)$$

and in the transverse polarization

$$\Psi_V^T = \frac{m_q}{z(1-z)}\begin{pmatrix} \pm 1 \\ -i \end{pmatrix}\phi^V(z,r)\delta_{\lambda_1,\lambda_2}. \quad (12)$$

However in the mesons case the spatial part of the wave function is not known, and some hypothesis must be taken in consideration. In the following sections we discuss the vector meson wave functions.

3 J/Ψ wave function

Since for the heavy quarkonium the internal motion is small, we must built the wave function in the the non-relativistic approach. For transverse polarization final state total angular momentum is $J_z = 1$. But in the non-relativistic case, since the transverse momentum is $k_t^2 \sim 0$ the orbital angular momentum must be $L_z = 0$. Therefore the spin is necessarily $S = \pm 1$ which implies $\lambda_1 = \lambda_2$, and the second term in Eq. (9) disappears.

The wave function in the (z, k_t) space is related to $\phi_V(z, r)$ throught a 2-D Fourier transform,

$$\phi_V(r, z) = \int \frac{d^2 k_t}{4\pi^2} \phi_V(z, k_t) e^{ir\cdot k_t}. \tag{13}$$

So we analyse some possible relativistic situations for (z, k_t) from wich we write the wave function.

3.1 Delta Function

The simplest case one can have is to consider that q and $\bar{q}$ have the same longitudinal momentum fraction and that the transverse momentum is null. This hypothesis can be translated as a Wave Funtion in the (z, k_t) space,

$$\phi_V(z, k_t) = K\, \delta(z - 1/2)\delta^2(k_t). \tag{14}$$

The only free parameter is the normalization. It can be set if we relate the wave funtion with the decay width,

$$\Gamma^V_{e^+e^-} = \frac{32\pi\alpha^2 e_q^2}{M_V} \left| \int dz \int \frac{d^2 k_t}{8\pi^{3/2}} \Psi_V(z, k_t) \right|^2 \tag{15}$$

resulting

$$K = \frac{1}{2M_V} \frac{8\pi^{3/2}}{e_q \alpha} \sqrt{\frac{\Gamma^V_{e^+e^-} M_V}{32\pi}}. \tag{16}$$

The wave function in the (r, z) space representation, obtained via eq. (13), is

$$\phi_V(r, z) = \frac{1}{2M_V} \frac{\sqrt{4\pi}}{e_q \alpha} \sqrt{\frac{\Gamma^V_{e^+e^-} M_V}{32\pi}} \delta(z - 1/2). \tag{17}$$

The full expression for longitudinal and transverse wave functions is obtained inserting the expression above in Eqs. 11 and 12.

3.2 Gaussian function

Another possibility is to consider again the same longitudinal momentum fraction for both q and $\bar{q}$ but now we assume that some small transverse momentum is allowed inside the J/Ψ. Following Ref.[8] instead of a delta function we use for k_t a Gaussian distribution around zero and the wave function reads

$$\phi_V(z, k_t) = K\, \delta(z - 1/2) \exp(-a \frac{k_t^2}{m_c^2}). \tag{18}$$

We take for a the value obtained from lattice QCD calculations, $a = 4.68$ and $m_c = 1.43$ GeV.

Again the normalization is obtained from the decay width,

$$K = \frac{a}{m_c^2} \frac{1}{2M_V} \frac{8\pi^{3/2}}{e_q \alpha} \sqrt{\frac{\Gamma^V_{e^+e^-} M_V}{32\pi}} \tag{19}$$

and the (r, z) representation of the Wave Function comes from the Fourier transform.

3.3 "Double-Gaussian" function

We propose now that not only the transverse momentum may have some distribution but also the longitudinal momentum fraction has a Gaussian distribution around 1/2, resulting

$$\phi_V(z, k_t) = K \; \exp[-c^2(z - 1/2)^2] \exp(-a \frac{k_t^2}{m_c^2}). \tag{20}$$

Now the wave function has one extra parameter, c, however one extra constraint also comes up since now the wave function becomes normalizable.

So, once again we have from lattice QCD calculations $a = 4.68$ (and $m_c = 1.43$ GeV), from the decay width

$$K = \frac{a}{m_c^2} \frac{1}{2M_V} \frac{8\pi^{3/2}}{e_q \alpha} \sqrt{\frac{\Gamma^V_{e^+e^-} M_V}{32\pi}} \frac{c}{\sqrt{\pi}\, \mathrm{erf}(c/2)}. \tag{21}$$

and from the orthogonality condition,

$$\frac{N_c}{2\pi} \int_0^1 \frac{dz}{z^2(1 - z)^2} \int d^2\vec{r} \{ m_c^2 \phi_i(r, z) \phi_j(r, z) \} = \delta_{ij} \tag{22}$$

we get $c^2 = 27.22126$. In this way the (r, z) representation of the wave function is obtained via the Fourier transform with no free parameters.

4 Light Mesons wave function

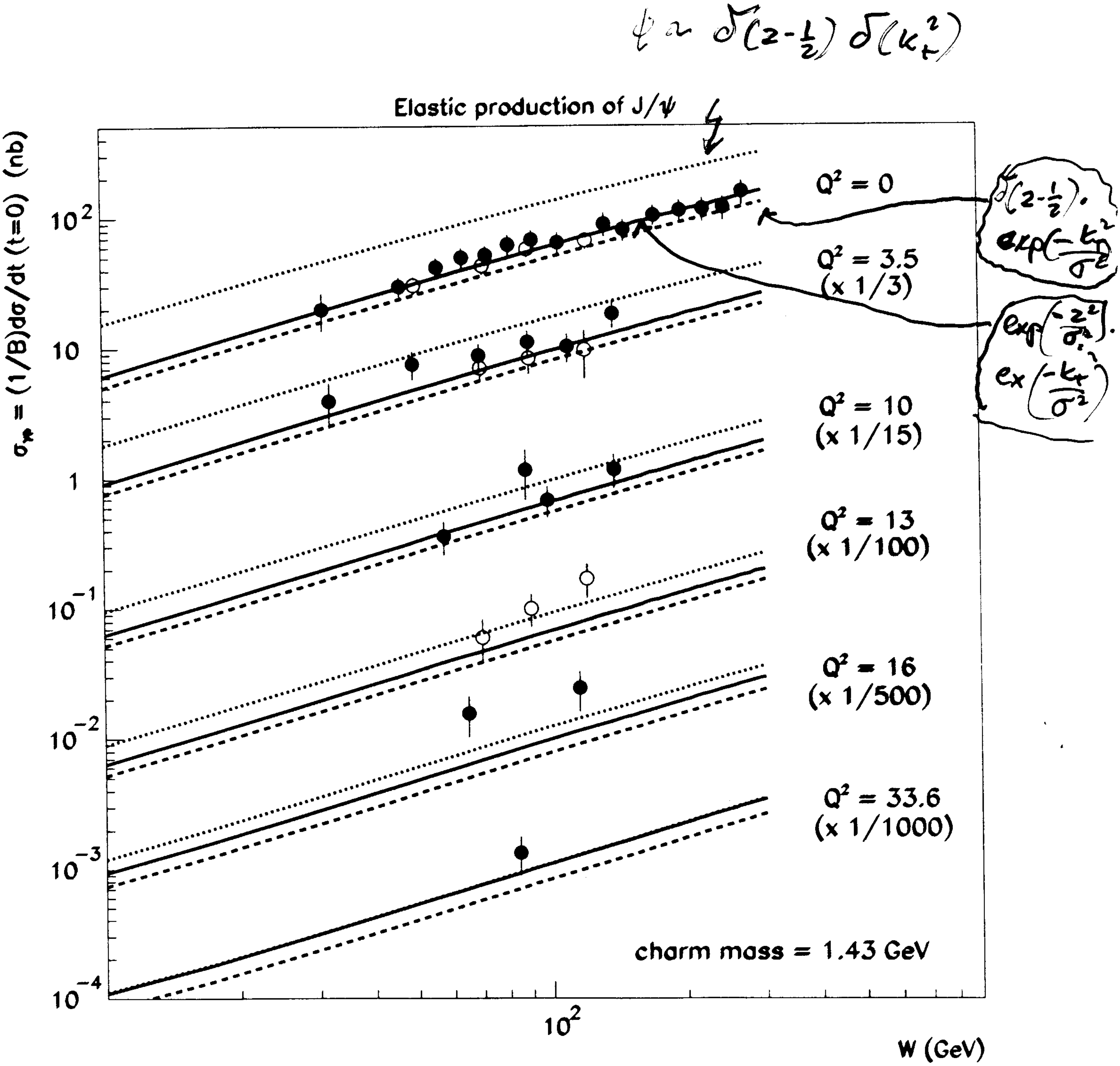
Now for the light mesons the relativistic approach must be used. Using the same relativization procedure as in Refs. [6, 3] one writes the wave function in terms of the lightcone invariant variable $\vec{p}^{\,2}$ (3-momentum of the quark in the non-relativistic limit),

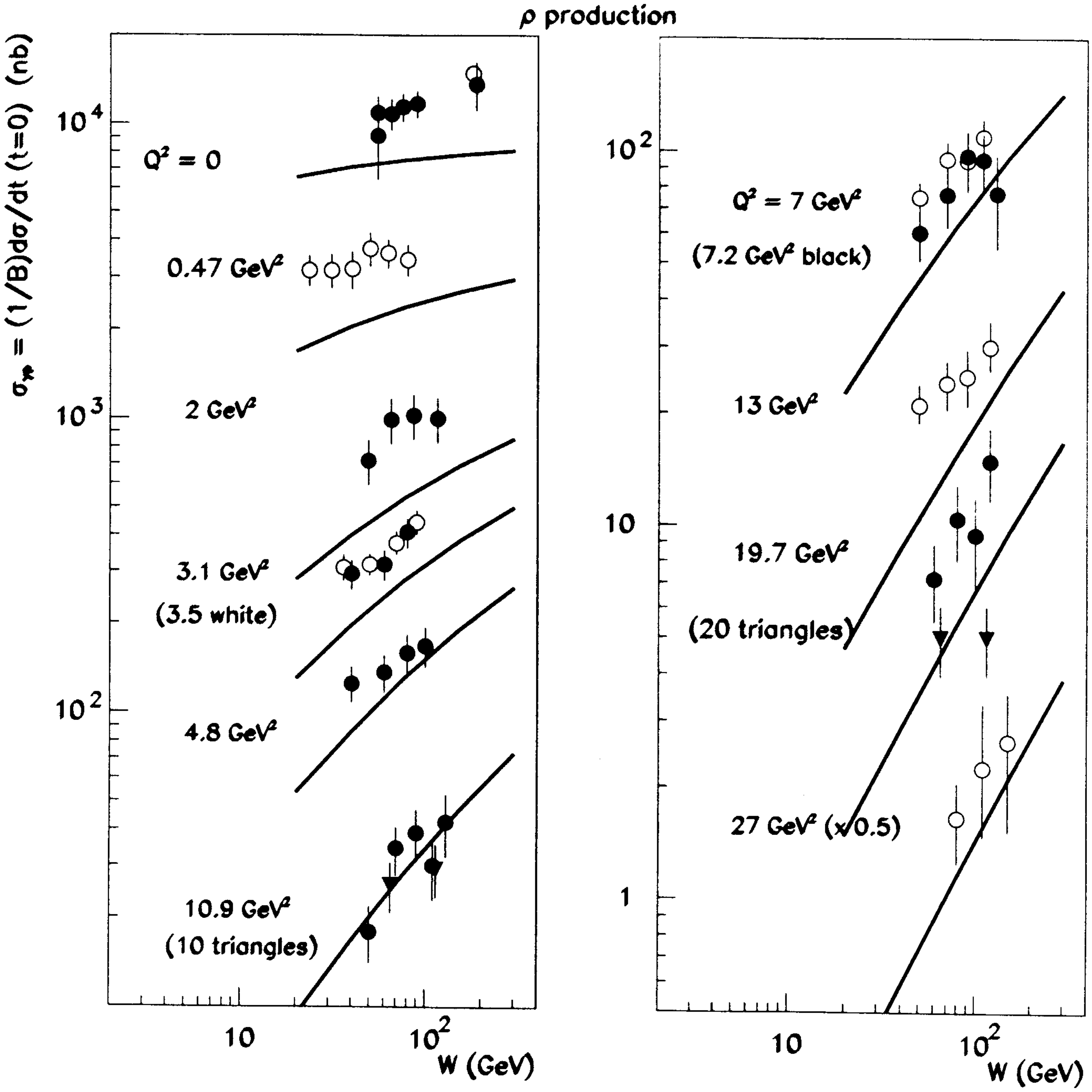
$$\vec{p}^{\,2} = \frac{1}{4}(M^2 - 4m_q^2), \qquad M^2 = \frac{m_q^2 + k_t^2}{z(1 - z)}. \tag{23}$$

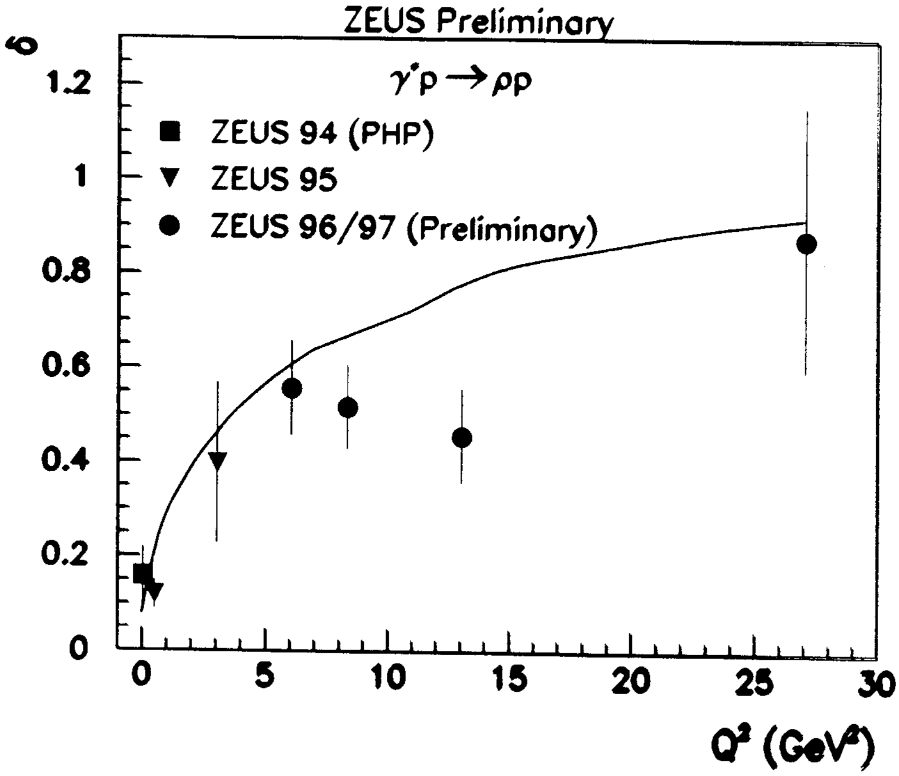
In this expression M is invariant mass of $q\bar{q}$ system. Again a Gaussian form is applied, now in the $\vec{p}$ variable,

$$\phi(\vec{p}) = K \; (2\pi R^2)^{3/2} \exp\left[-\frac{1}{2} p^2 R^2 \right] ; \tag{24}$$

from $M_\rho = 0.77$ GeV and $M_\phi = 1.01$ GeV we have


Elastic production of J/ψ
σ_ψ = (1/B)dσ/dt (t=0) (nb)
Q² = 0
Q² = 3.5
(x 1/3)
Q² = 10
(x 1/15)
Q² = 13
(x 1/100)
Q² = 16
(x 1/500)
Q² = 33.6
(x 1/1000)
charm mass = 1.43 GeV
W (GeV)


ρ production
σ_γp = (1/B)dσ/dt (t=0) (nb)
Q² = 0
0.47 GeV²
2 GeV²
3.1 GeV²
(3.5 white)
4.8 GeV²
10.9 GeV²
(10 triangles)
10
10²
W (GeV)
Q² = 7 GeV²
(7.2 GeV² black)
13 GeV²
19.7 GeV²
(20 triangles)
27 GeV² (x 0.5)
1
10
10²
W (GeV)

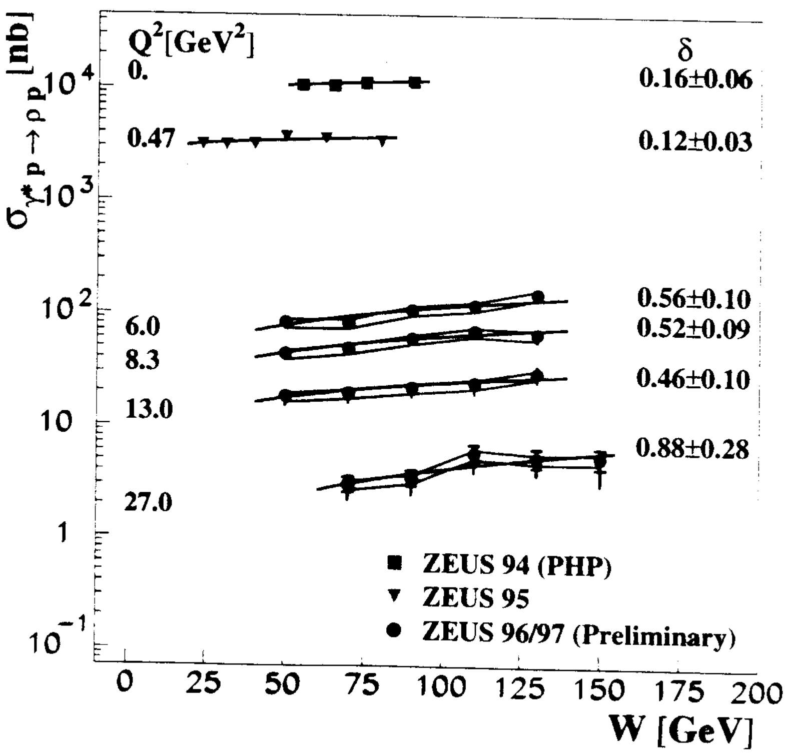


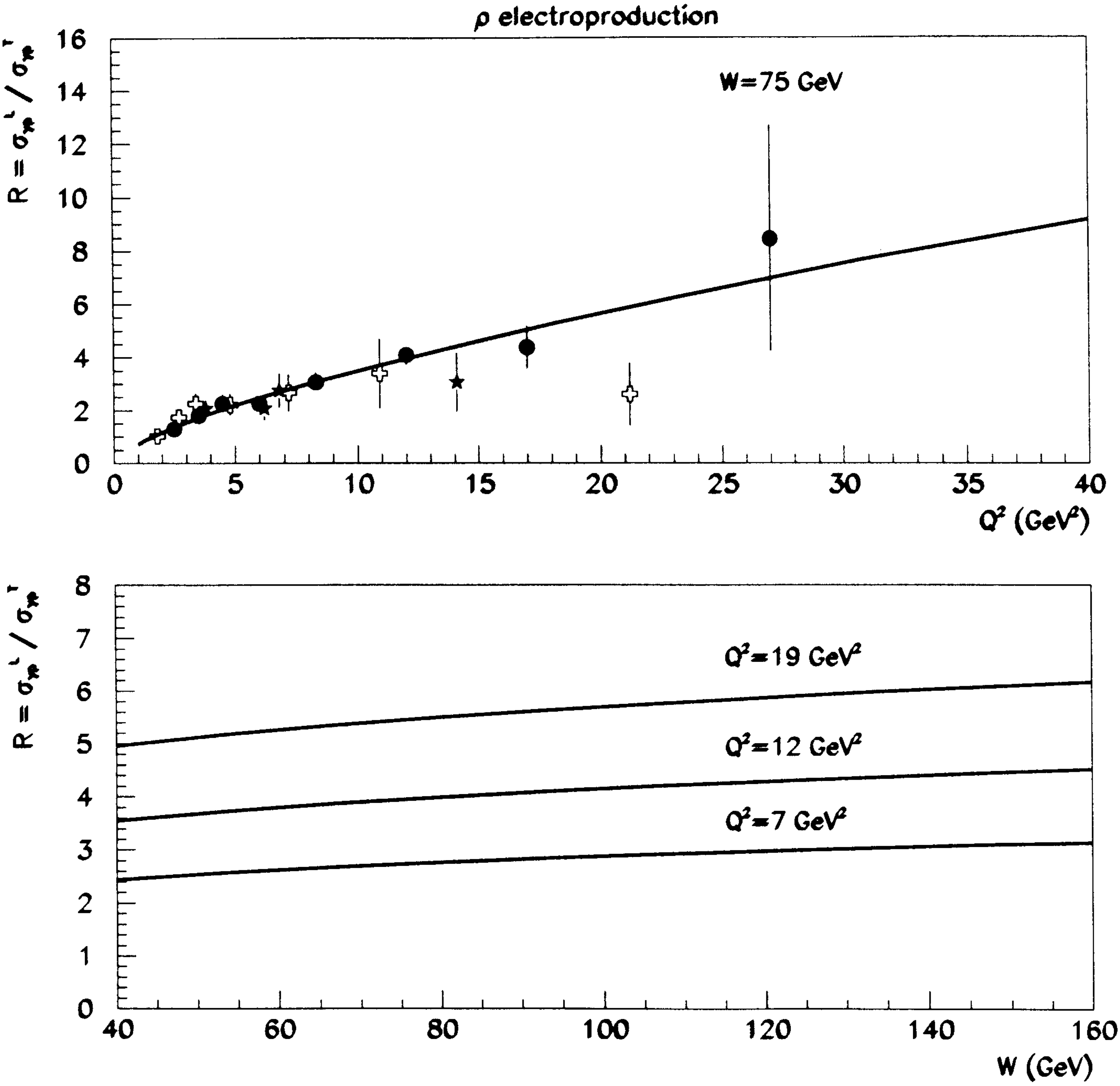
$\sigma \propto W^{\delta}$

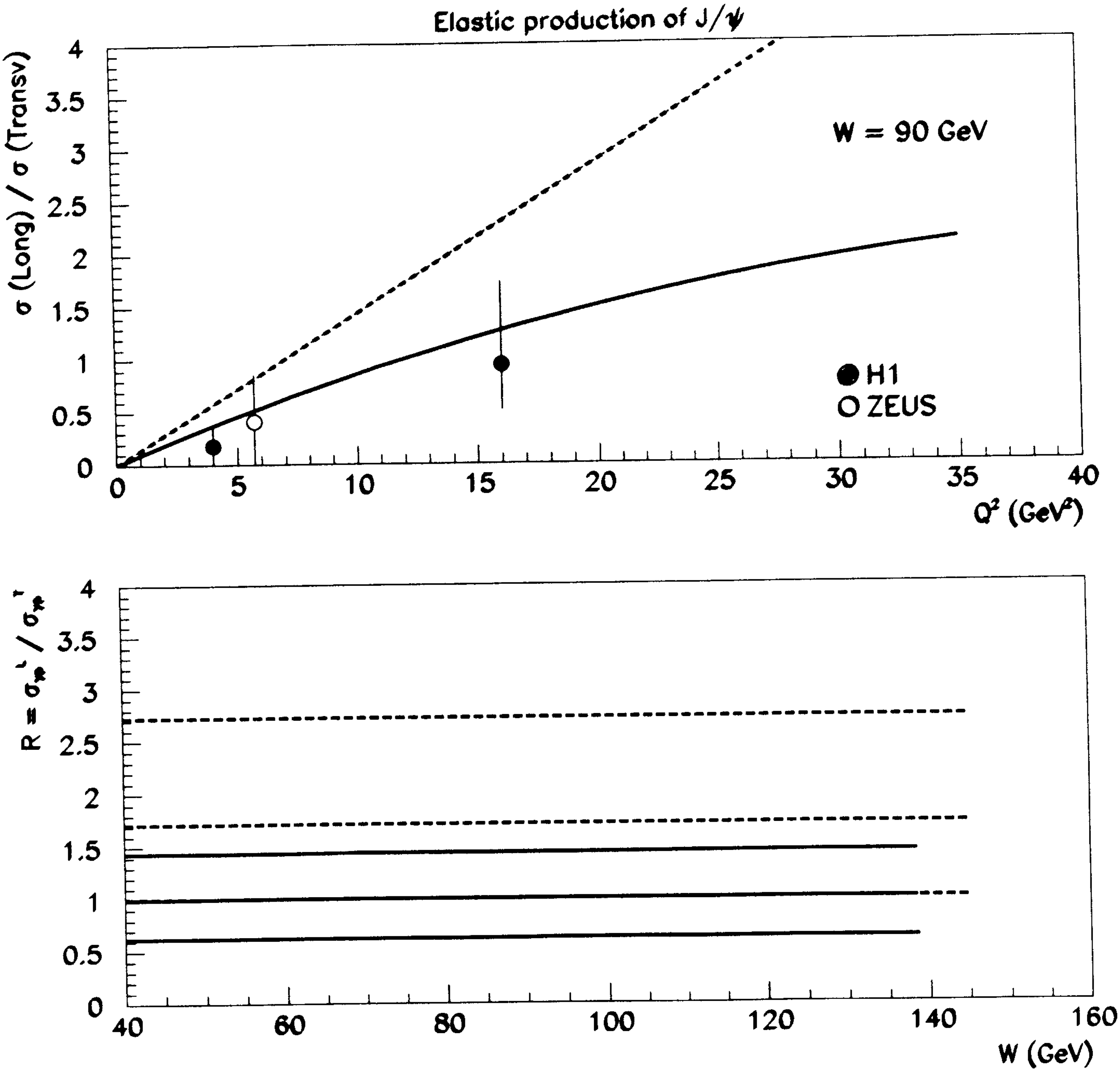
δ increases with Q^2

At Q^2 of 27 GeV^2,

$\sigma \sim W^{0.88}$!




ρ electroproduction
R = σ_γp^L / σ_γp^T
W=75 GeV
Q² (GeV²)
Q²=19 GeV²
Q²=12 GeV²
Q²=7 GeV²
W (GeV)



Elastic production of J/ψ
σ (Long) / σ (Transv)
W = 90 GeV
H1
ZEUS
Q² (GeV²)
R = σ_γp^L / σ_γp^T
W (GeV)