Saturation model

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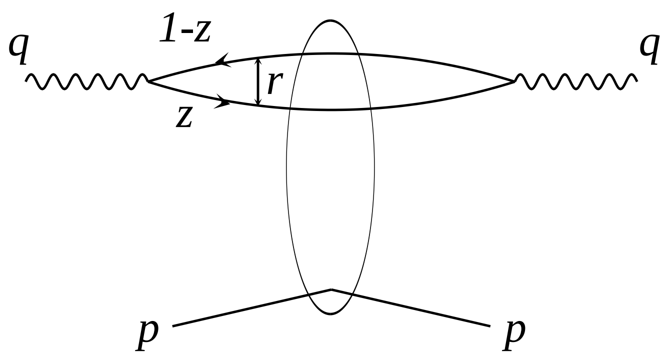
PRD 59 (1999) 014017
PRD 60 (1999) 114023

DESY
30 November 2000

Model formulation



k_T-factorization formula in the dipole representation for $\gamma^* p$ cross sections at small x

$$\sigma_{T,L}(x,Q^2) = \int d^2\mathbf{r}\, dz\, |\Psi_{T,L}(r,z,Q^2)|^2\; \hat{\sigma}(x,r)$$

where the dipole cross section is modelled

$$\hat{\sigma}(x,r) = \hat{\sigma}(r/R_0(x)) = \sigma_0\, \left\{1 - \exp\left(-r^2/R_0^2(x)\right)\right\}$$

and the saturation radius

$$R_0^2(x) \;=\; \mathcal{N}\, x^{\lambda}$$

Three parameters: $\sigma_0, \mathcal{N}, \lambda > 0$ are fitted to all available DIS data at $x < 0.01$ (including $Q^2 < 1\ \mathrm{GeV}^2$ points).

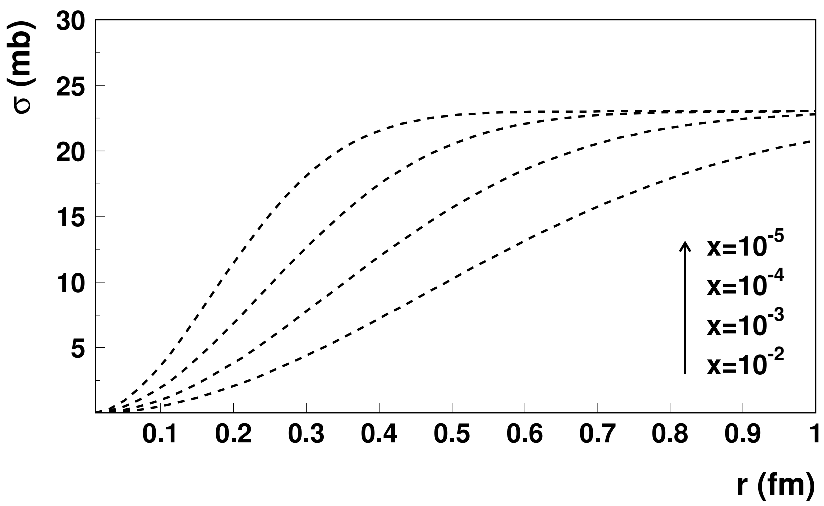
Remember: $\quad F_2 \;=\; Q^2\,(\sigma_T + \sigma_L)\,/\,4\pi^2\alpha_{em}$

Motivation

The photon wave function $\Psi_{T,L}$ is known from QED but the dipole cross section $\hat{\sigma}$ is postulated incorporating important features of QCD:

$$\hat{\sigma}(x,r) \;\simeq\; \begin{cases} \sigma_0\, r^2/R_0^2(x) & \text{for} \quad r \ll R_0(x) \\ \\ \sigma_0 & \text{for} \quad r \gg R_0(x)\,. \end{cases}$$

Transition from color transparency form for small r to geometric cross section for large r occurs for smaller r when $x \to 0$.



In this way the structure function is unitarized:

$$F_2 \;\sim\; Q^2\,\sigma_0\,\log(1/x) \qquad \text{as} \qquad x \to 0$$

Remember: $\quad x \;\simeq\; Q^2/W^2$

Transition to low Q^2

The form of the dipole cross section $\hat{\sigma}$ allows to describe the transition to low Q^2 in DIS at small x.

The relation between the characteristic $q\bar{q}$ size $1/Q$ and the saturation radius $R_0(x)$ is crucial for that.

Large Q^2: $\quad 1/Q \ll R_0(x)$

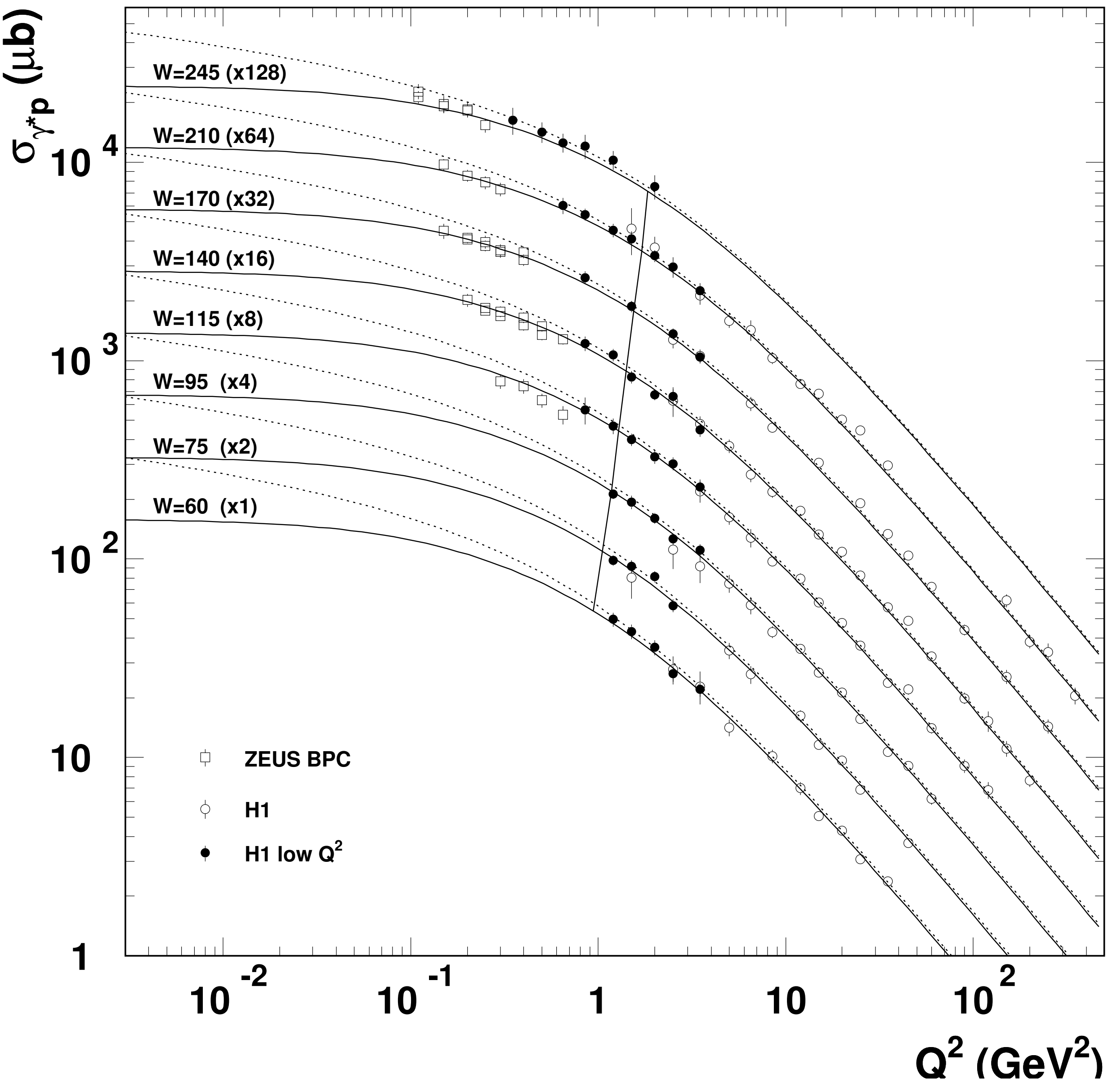
$$\sigma^{tot} \sim \frac{\sigma_0}{Q^2 R_0^2(x)} \log(Q^2 R_0^2(x)) \quad \Rightarrow \quad F_2 \sim x^{-\lambda} \log(Q^2)$$

Small Q^2: $\quad 1/Q \gg R_0(x)$

$$\sigma^{tot} \sim \sigma_0 \log(1/Q^2 R_0^2(x)) \quad \Rightarrow \quad F_2 \sim Q^2\, \sigma_0 \log(1/x)$$

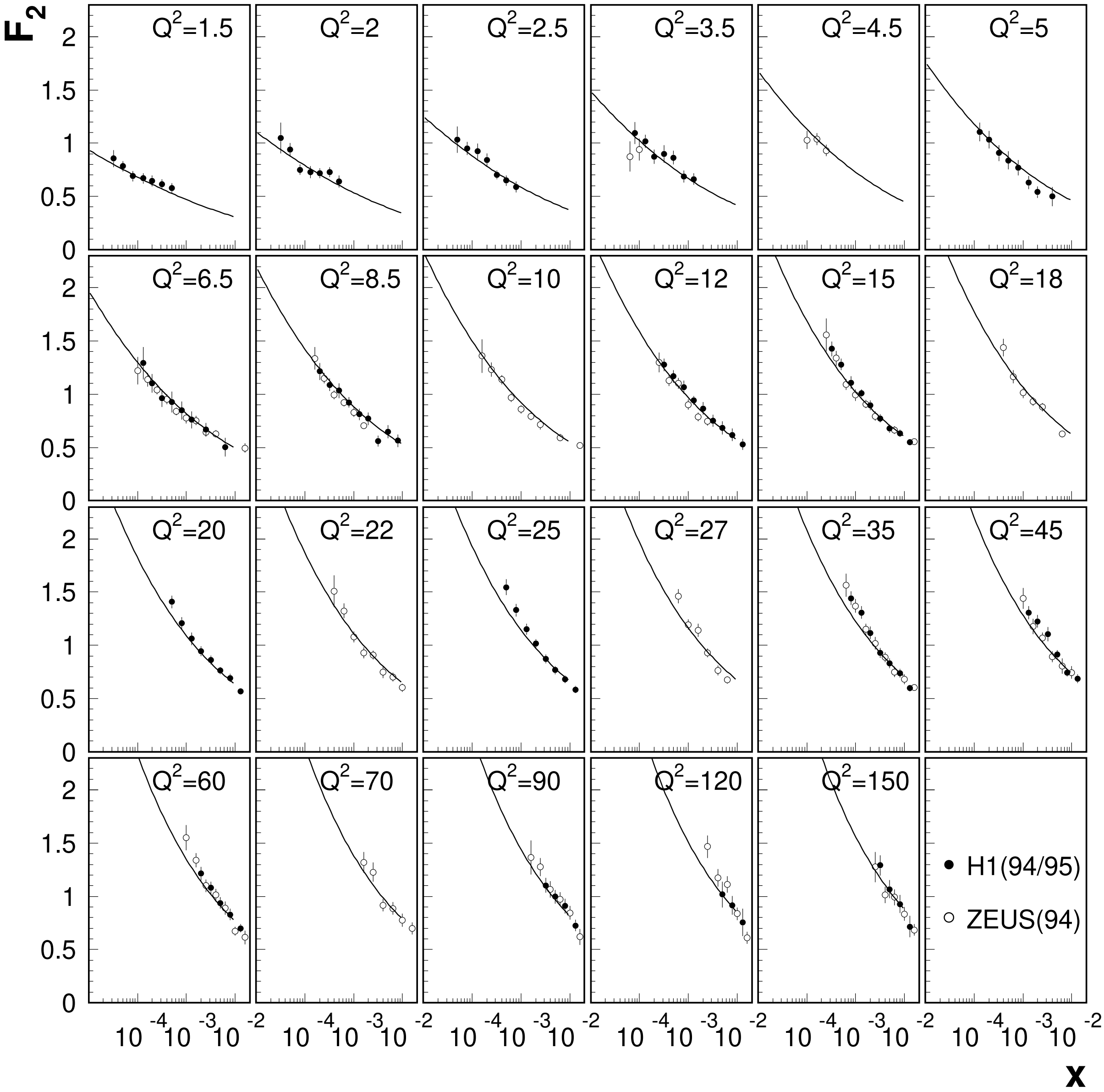
The relation $1/Q \approx R_0(x)$ defines transition region in the (x, Q^2)-plane.

$\sigma^{\gamma^* p}$ as a function of Q^2

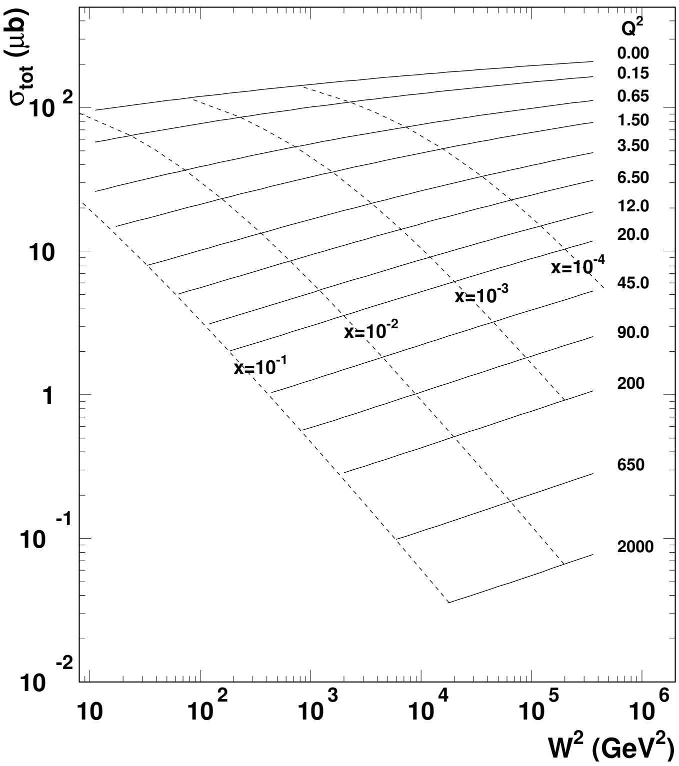


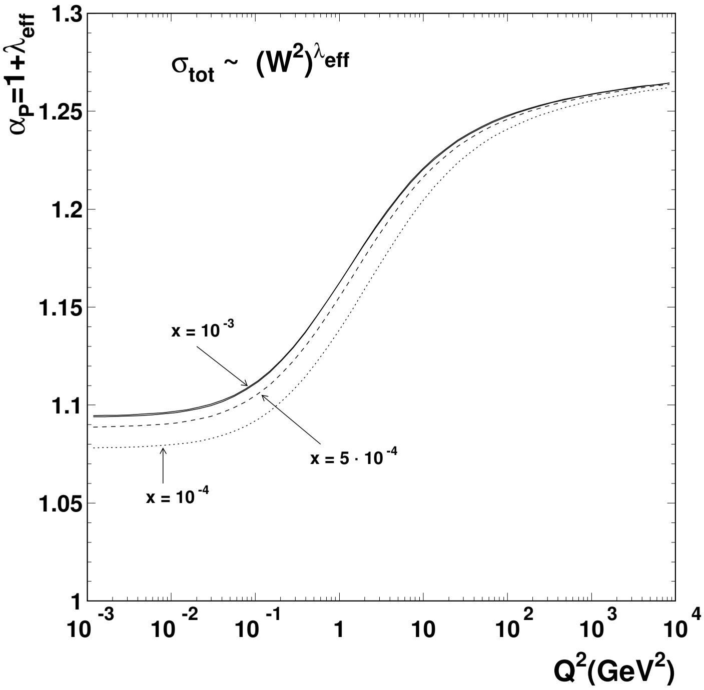
solid lines : $m_{u,d,s} = 140$ MeV
dotted lines: $m_{u,d,s} = 0$

F_2 as a function of x

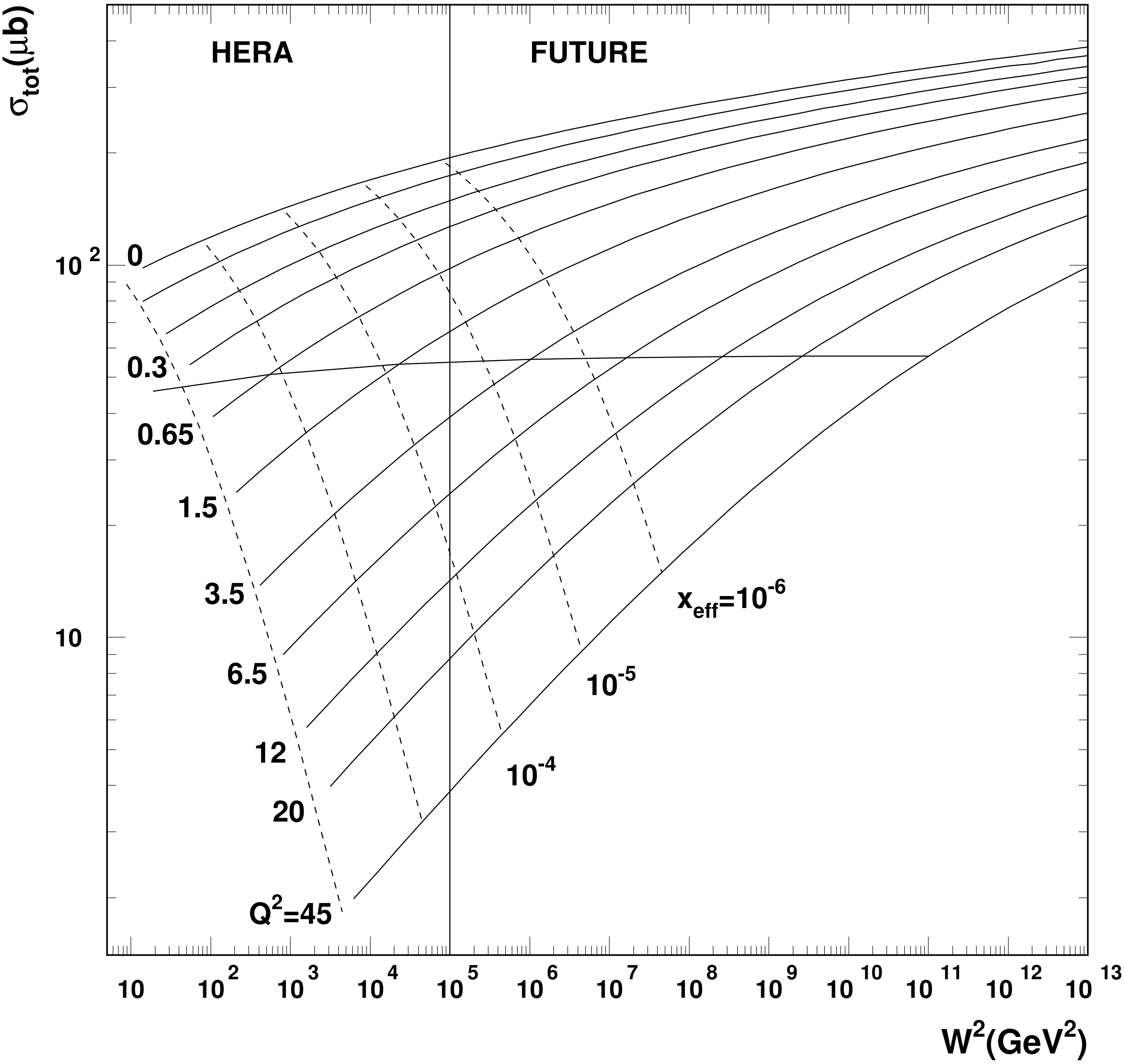


Effective Pomeron intercept





σ^{tot} as a function of energy W^2



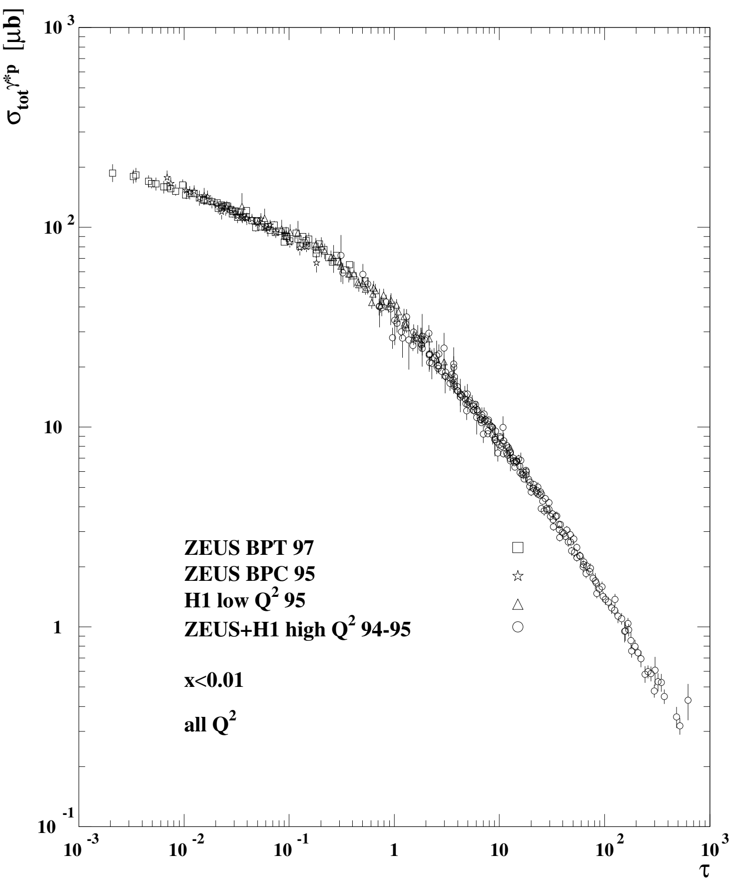
Geometric scaling

The assumption that the dipole cross section

$$\hat{\sigma}(x, r) = \hat{\sigma}(r/R_0(x))$$

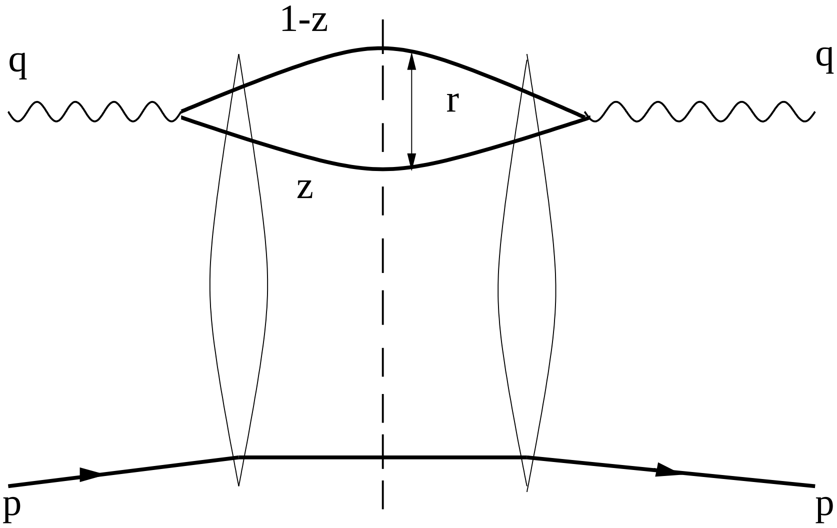
leads to a new scaling at small x

$$\sigma^{tot}(x, Q^2) = \sigma^{tot}\Big(\tau = Q^2 R_0^2(x)\Big)$$



(Staśto, G-B, Kwieciński, hep-ph/0007192)

DIS diffraction



Optical theorem relates total and difractive cross section

$$\frac{d\sigma^{diff}}{dt}_{|\,t=0} = \frac{1}{16\pi}\int d^2\mathbf{r}\,dz\,|\Psi_{T,L}(r,z,Q^2)|^2\,\hat{\sigma}^2(r/R_0(x))$$

x-dependent saturation of $\hat{\sigma}$ alows to find the leading behaviour for large Q^2

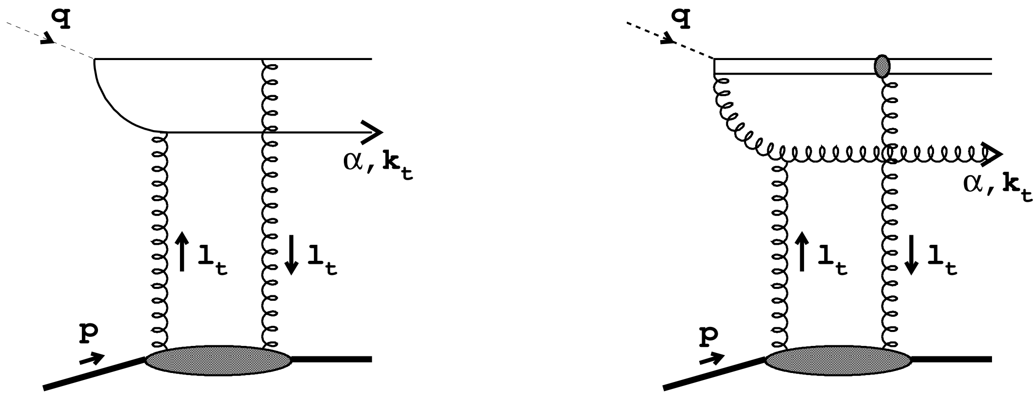
$$\sigma^{diff} \sim \frac{\sigma_0^2}{Q^2 R_0^2(x)}$$

and the constant ratio

$$\frac{\sigma^{diff}}{\sigma^{tot}} \sim \frac{1}{\log(Q^2 R_0^2(x))}$$

Saturation model in diffraction

Diffractive state is formed by $q\bar{q}$ and $q\bar{q}g = gg$ dipoles which interact with the proton with the found dipole cross section $\hat{\sigma}$.

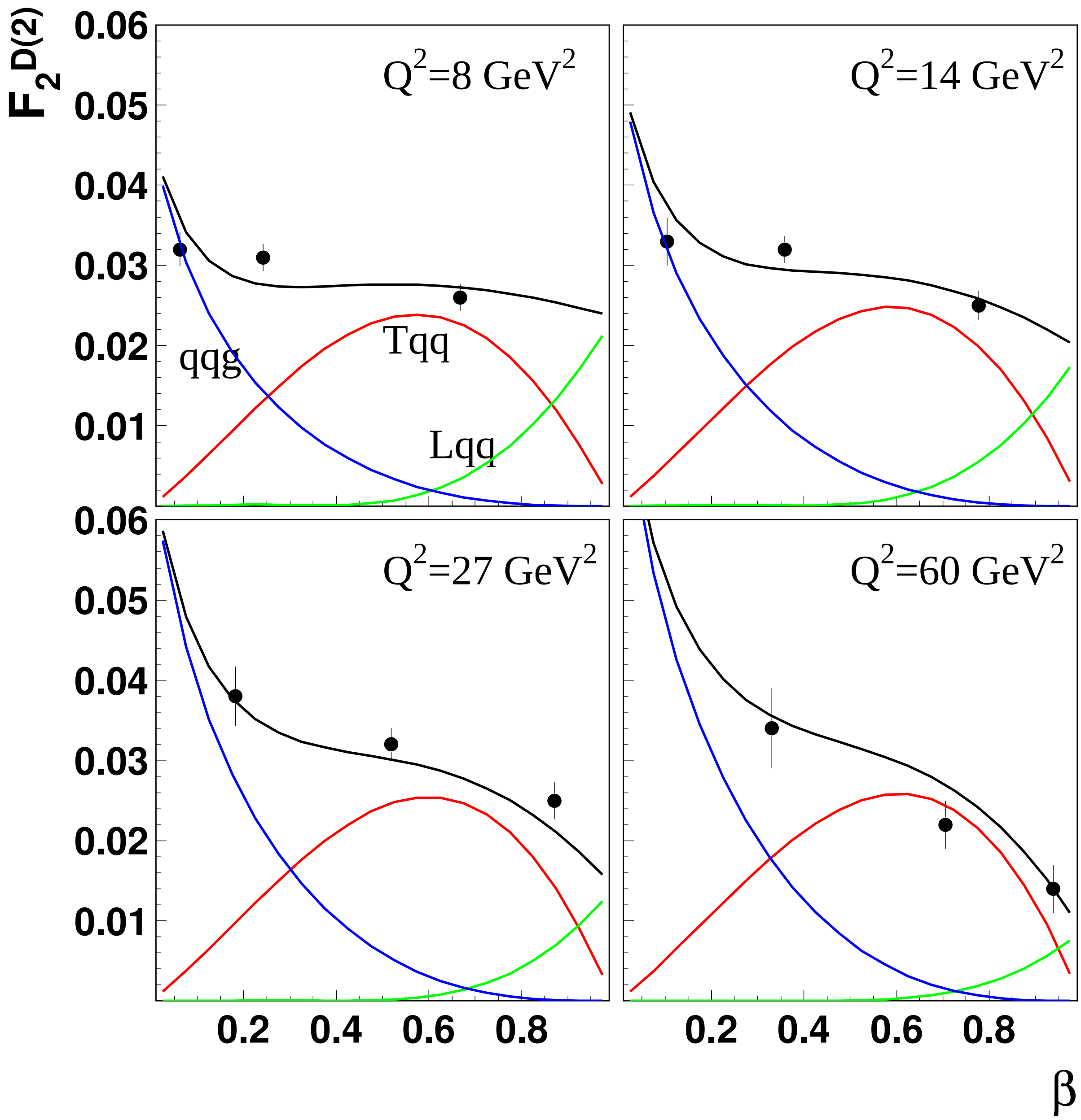


Thus

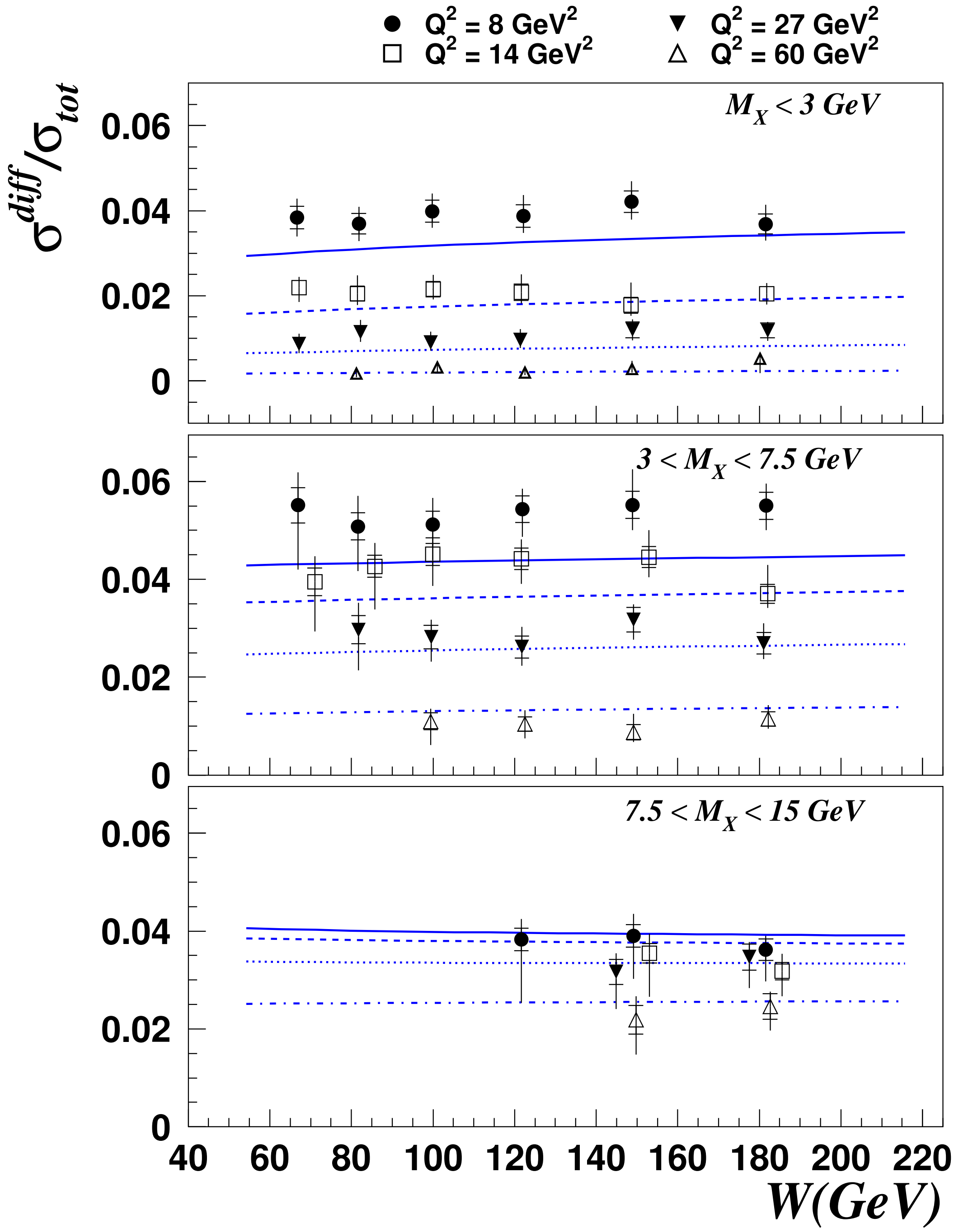
$$F_2^{D(3)}(\beta, Q^2, x_{I\!P}) = F_{qq}^T + F_{qq}^L + F_{qqg}^T$$

- The shape in $\beta = Q^2/(Q^2+M^2)$ determined by the dipole wave functions.
- Longitudinal $q\bar{q}$-contribution is higher twist ($\sim 1/Q^2$), important at $\beta \to 1$.
- The found dipole cross section provides energy dependence ($x_{I\!P}$) and partially normalization (up to diffractive slope B_D, taken from experiment).

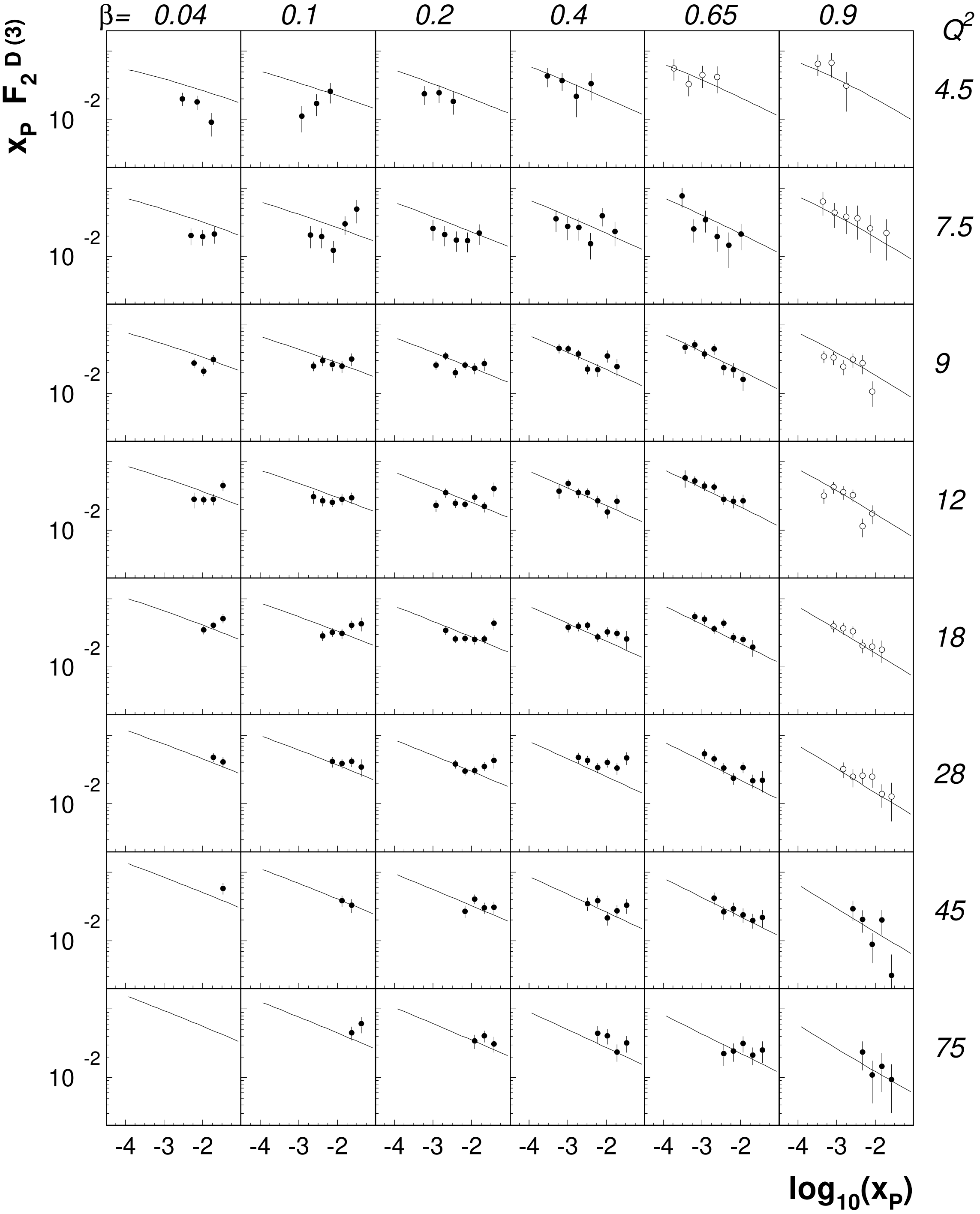
β-shape vs ZEUS 94 data at $x_{I\!P} = 0.0042$



$\sigma^{diff}/\sigma^{tot}$ ratio and ZEUS 94 data



xp-dependence vs H1 94 data



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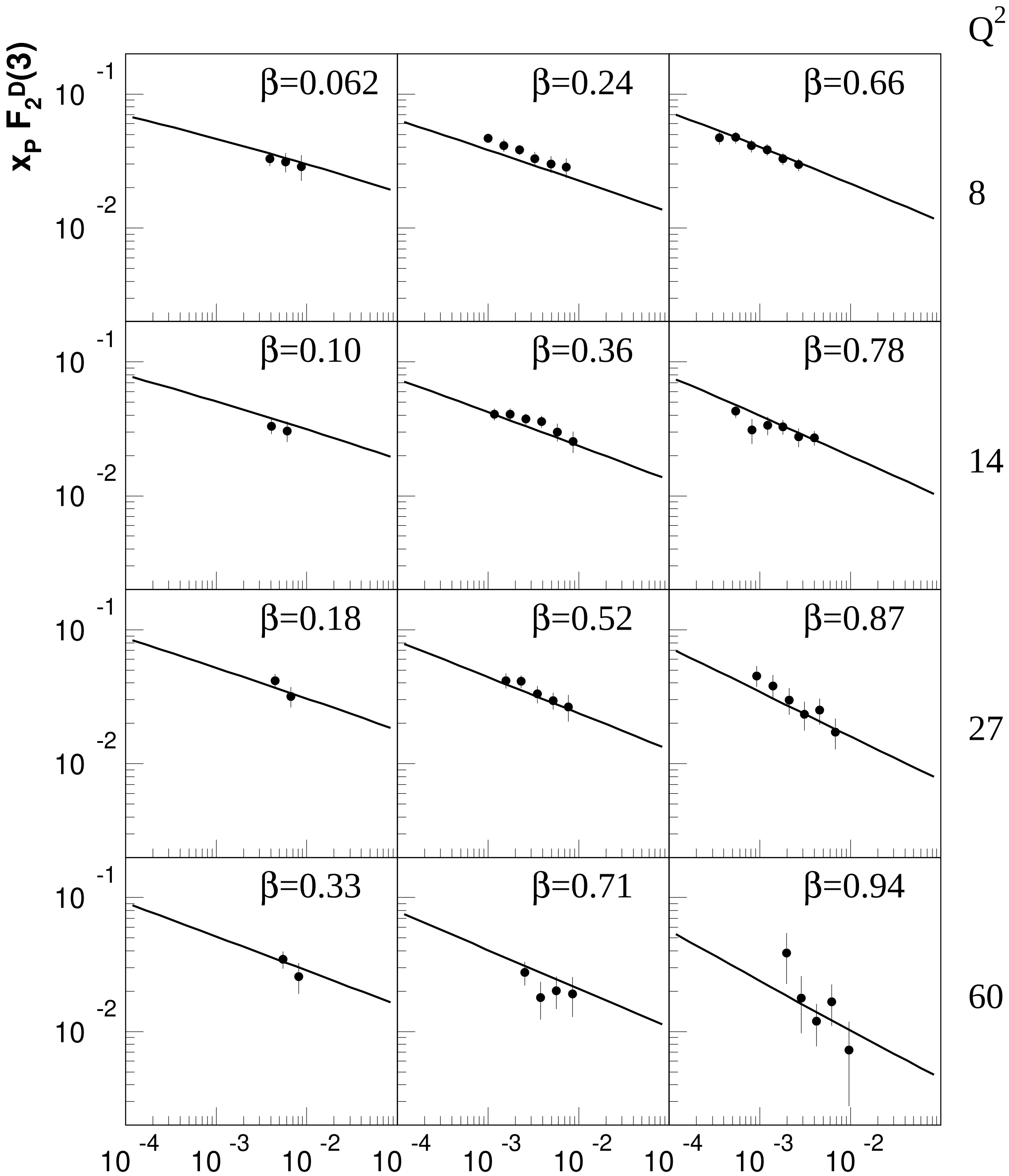

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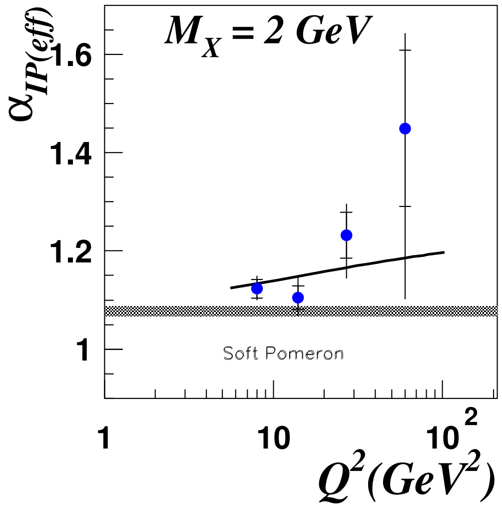

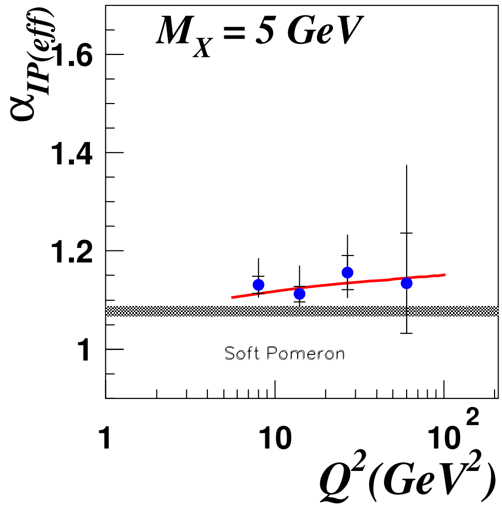

$$x_{I\!P}\, F_2^{D(3)} \;\sim\; x_{I\!P}^{\,1-2\,\alpha_{I\!P(eff)}}$$

The energy ($x_{I\!P}$) dependence of DIS diffraction is correctly predicted from the inclusive DIS analysis.

Conclusions

- $k_\perp$-factorization formula in the dipole representation provides a framework for a successful description of DIS at small x.

- The dipole cross section with the x-dependent saturation incorporates unitarity and gives a good description of the transition from high to low Q^2 in DIS.

- The saturation model applied to DIS diffraction explains constant ratio $\sigma^{diff}/\sigma^{tot}$, and predicts correct energy dependence of diffractive cross section.

- DIS diffraction is described with the help of tha additional qqg component. Taken from the experiment diffractive slope B_D is the only new parameter.