

# QCD & small $x$ & factorization

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Areas:

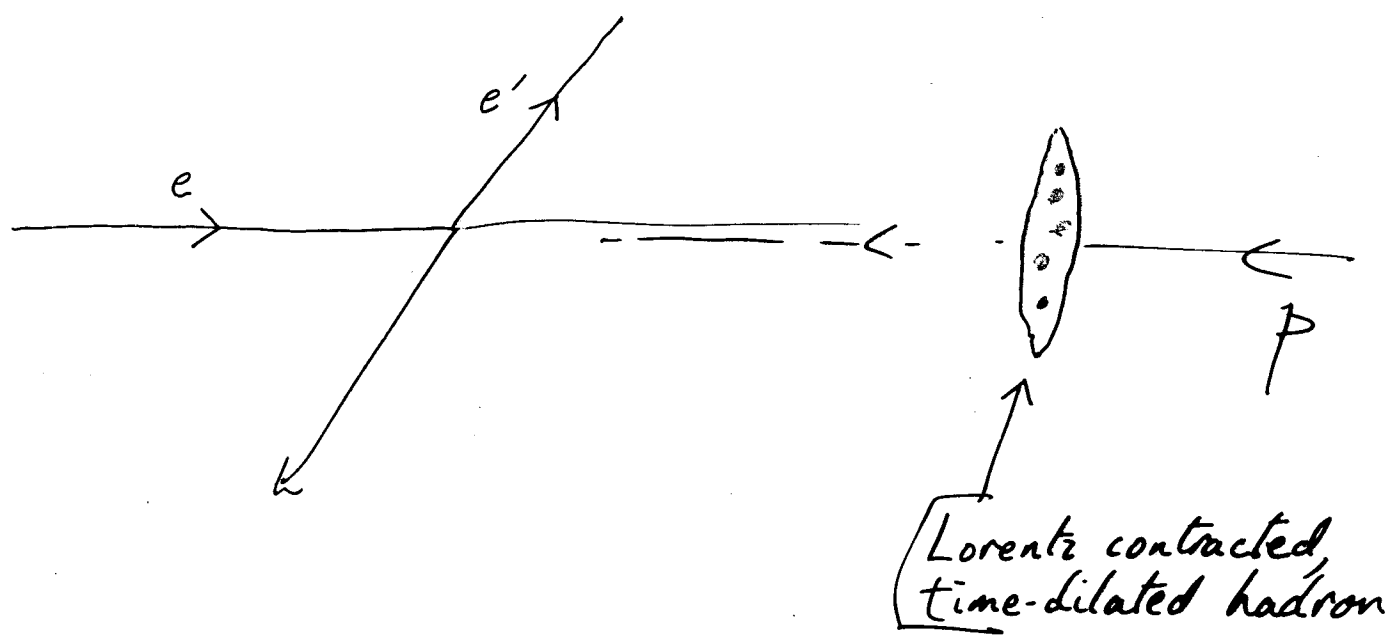
Factorization

Regge theory

Dipole model & c.

BFKL

# Hard scattering factorization (QCD-improved parton model)



Large  $Q \Rightarrow$  short-distance hard scattering of approximately free quarks & gluons

$$d\sigma = \hat{\sigma} \otimes f + \text{power law correction}$$

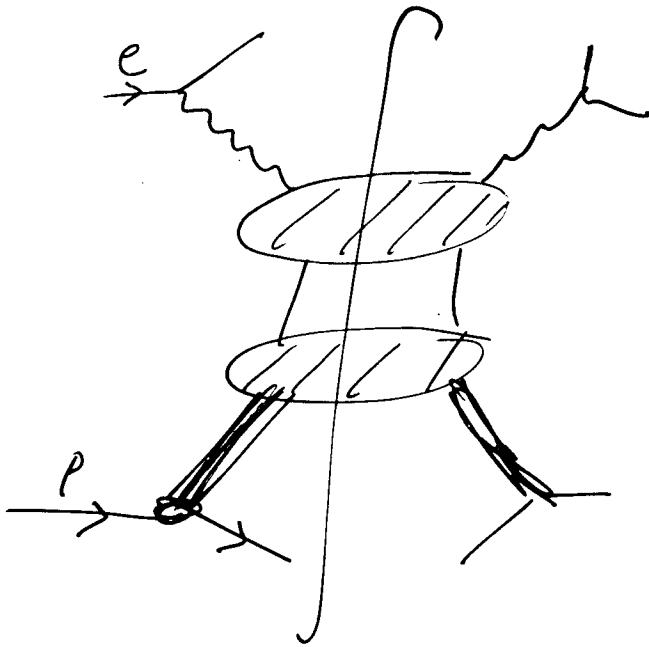
& DGLAP evolution of pdfs.

Predictions - same pdfs in many x-sects.

Calculations of  $\hat{\sigma}$  & DGLAP to arbitrarily high order (in principle!).  
(Includes multi-jet final states!)

Diffractive DIS — same with diffractive pdfs

∠ i.e. number densities of partons conditional on final state proton



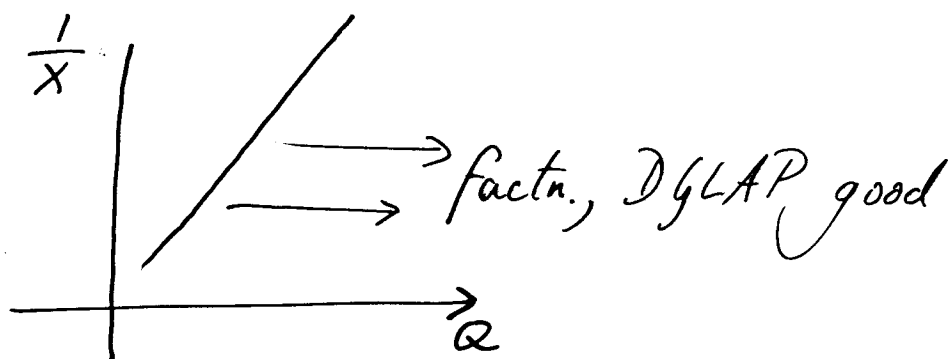
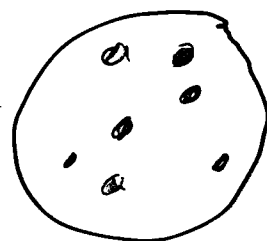
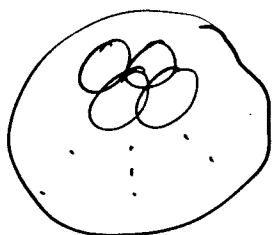
Measured — large gluon density

∅ corresponding verified predictions  
— dijets, heavy quarks.

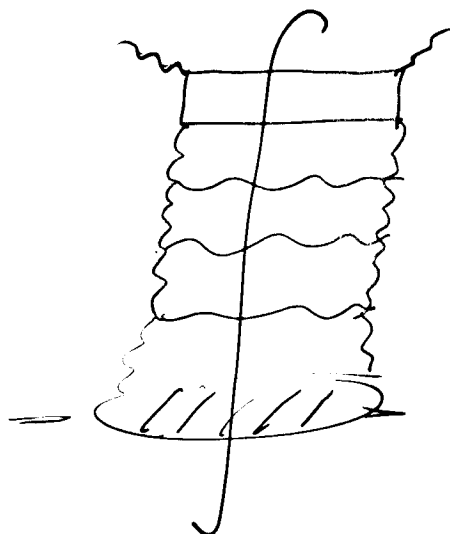
# Breakdown

1. Small  $Q$  : impulse approx. no good
2. Saturation :  $\frac{\# \text{partons} \propto \frac{1}{Q^2}}{\text{proton area}}$  too large.

$\Rightarrow$  overlap



Fixed order worsens in accuracy as  $x \rightarrow 0$



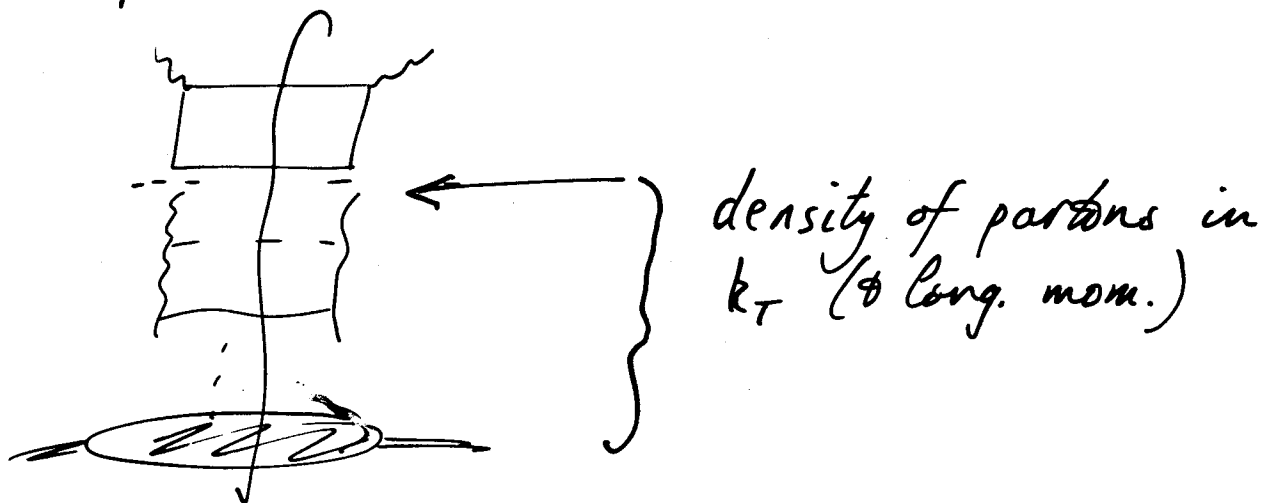
} several gluons o.c.  
w/  $k_T \sim Q$

BUT: part of this is in DGLAP evol<sup>n</sup>.

## $k_T$ -factorization

Use BFKL to handle  $\ln \frac{1}{x}$ 's

Rewrite factor. as



New evolution eq. includes BFKL & DGLAP.

Details?

BFKL  $\implies$  strong rise w/  $1/x$ .

(DGLAP gives a lot of rise, in practice)

What gluon density is to be used in saturation criterion?

$$\left[ \frac{\text{"# gluons"}}{Q^2 R^2} \right]$$

a. NOT  $\overline{MS}$

b.  $\int_{k_T \leq Q} d^2 k_T \frac{f(x, k_T)}{k_T^2}$

$k_T \leq Q$   
↑

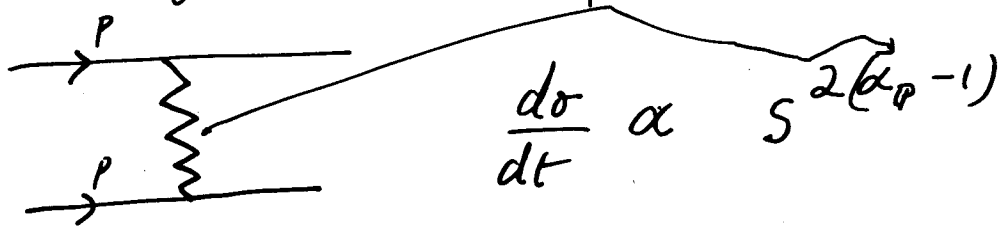
Exactly what's the defn.?  
Cut off on virtual gluon rapidity?  
Light-cone gauge not suitable!

NLO &c. corrections are generally significant, especially at small  $x$  — evolution gives large anomalous dimension

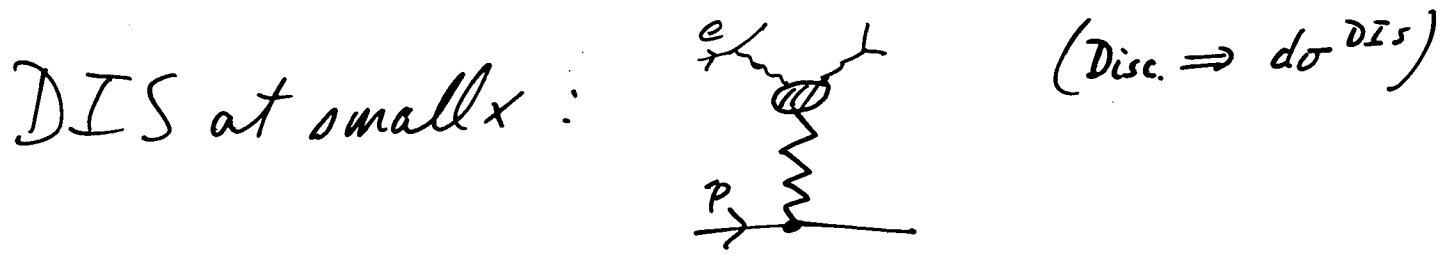
This is a question of scheme. Scheme-dependence is important — fixed order hard scattering coeff. or resummed ~~it~~.

# Regge

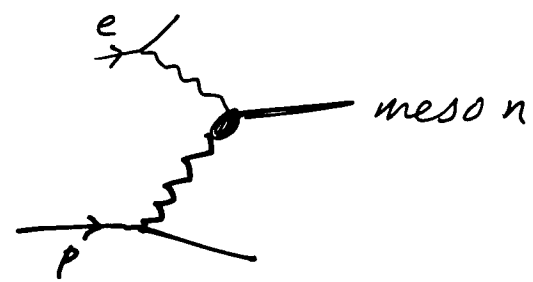
Elastic scattering at small angle. Text books say there is a particle-like exchange



Obvious generalization



Diff. DIS:

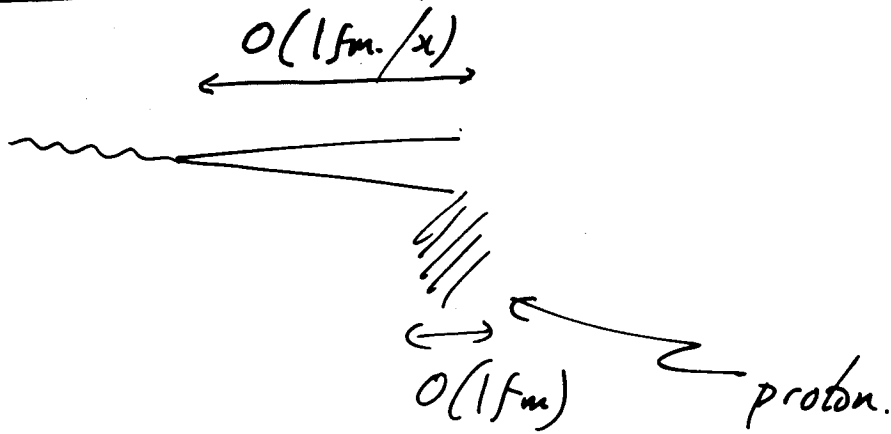


But, e.g.  $F_2$  is not  $(\frac{1}{x})^{0.08}$

$\frac{d\sigma}{dt}(\gamma^* p \rightarrow \nu p)$  is not  $s^{0.16}$

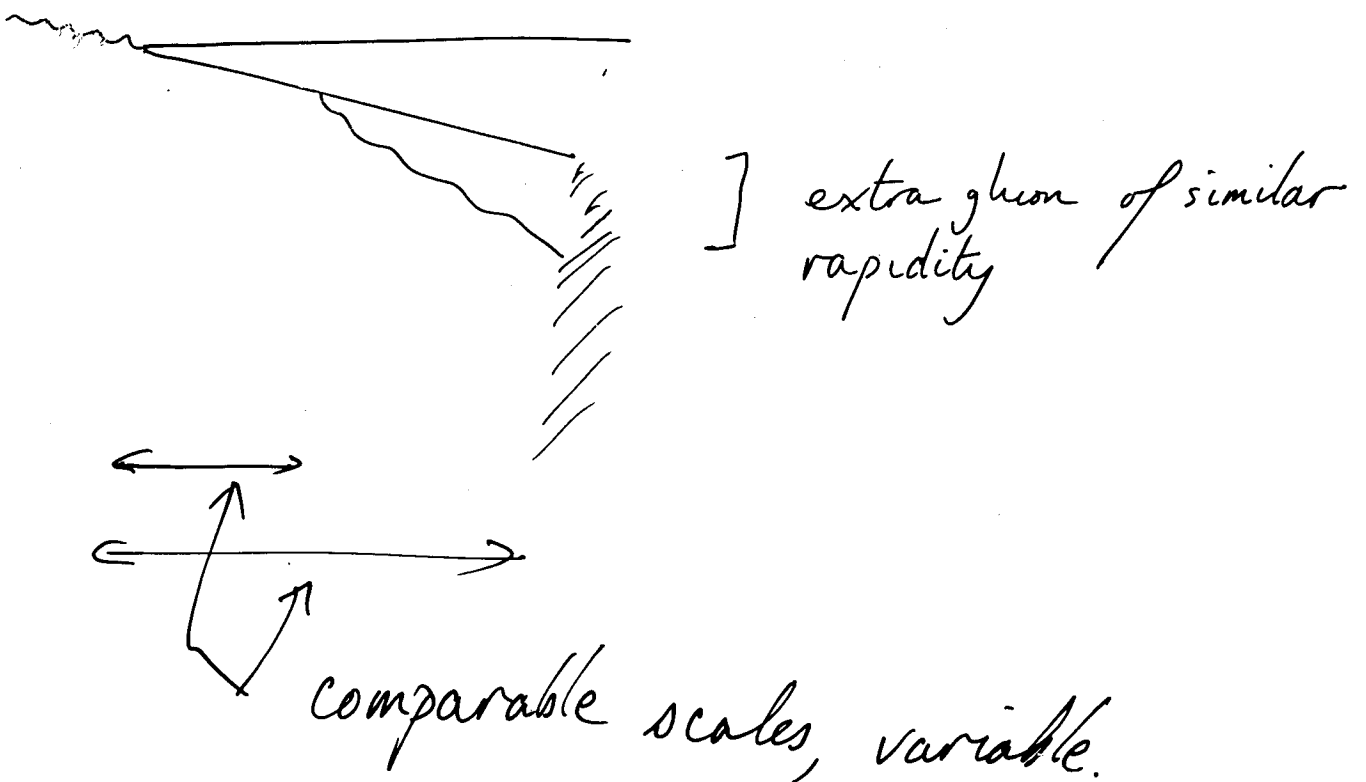
QCD  $\rightarrow$  BFKL  $\oplus$  DGLAP  $\rightarrow$  agree!

# Dipole model — proton rest frame



Transverse position of quarks doesn't change during scattering by short proton.

But non-leading logarithm corrections are different





# Status

Factn.	All orders & logarithms	Fixed order problematic at sm. $x$ . Higher twist?	Saturation — goodness of fit
Diffractive Factn	<del>—</del>	<del>—</del>	??
BFKL, CCFM & c.	LLA, NLLA	proof?	Saturation??
Dipole model	LLx	parameterization?	
Regge	??	??	