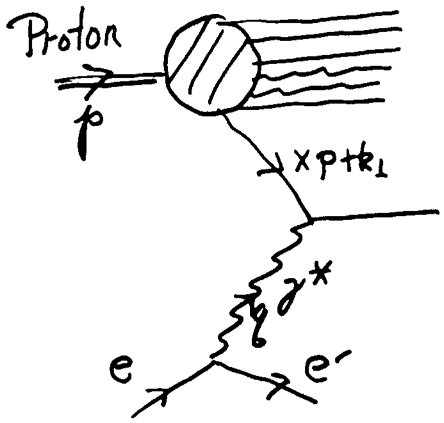
Diffraction, Low-x, and Parton Saturation at HERA

1. What is Parton Saturation?

A. Recall Parton Picture of Deep Inelastic Scattering



Bjorken Frame

$$p_\mu = \left(p + \frac{M^2}{2p}, 0, 0, p\right) \quad (t, x, y, z)$$

$$q_\mu = (q_0, q_\perp, q_3 = 0)$$

$$q_0 = \frac{P\cdot q}{p} \xrightarrow[p\to\infty]{} 0$$

$$\left.\begin{aligned} Q^2 &= -q_\perp^2 \\ x &= \frac{Q^2}{2p\cdot q} \end{aligned}\right\} \text{Variables (invariant) of Structure Functions}$$

γ^* absorbed by quark over short time $\Rightarrow$ γ^* makes instantaneous measurement of Proton wave Functio

γ^* resolves transverse coordinates at level $\Delta x_\perp \sim 1/Q$ $\Rightarrow$ γ^* absorbed by an individual charged parton (quark

$$F_2(x, Q^2) = \sum_f e_f^2 \left[x\, q_f(x, Q^2) + x\, \bar{q}_f(x, Q^2)\right]$$

Picture is unambiguous in leading order DELAP formalism. (Leading logs)

B. Saturation

Give a more refined description of quarks and gluons in the proton wave function.

$$\frac{d\, x(q_F+\bar{q}_F)}{d^2b\, d^2k_\perp}$$ is the quark density in transverse phase space in the proton.

b: impact parameter; $k_\perp$: transverse momentum

Then

$$x\, q_F(x,Q^2) + x\,\bar{q}_F(x,Q^2) = \int d^2b \int_{k_\perp \le Q} d^2k_\perp\, \frac{d\, x(q_F+\bar{q}_F)}{d^2b\, d^2k_\perp}$$

in leading order formalism. (Such a description is valid so long as the important values of $k_\perp$ obey $k_\perp \gg 1/\mathrm{fm}$.)

Quark saturation

$$\frac{d\, x(q_F+\bar{q}_F)}{d^2b\, d^2k_\perp} \quad \begin{cases} \text{const}\ \dfrac{N_c}{2\pi^4} & k_\perp^2/Q_s^2 \ll 1 \\ \text{const}'\ N_c\, \dfrac{Q_s^2}{k_\perp^2} & k_\perp^2/Q_s^2 \gg 1 \end{cases}$$

← perturbation theory

Gluon saturation

$$\frac{d\, xG}{d^2b\, d^2k_\perp} = \begin{cases} \text{"const"}\ \dfrac{N_c^2-1}{4\pi^3 \alpha N_c} & k_\perp^2/Q_s^2 \ll 1 \\ \text{const}'\ (N_c^2-1)\, Q_s^2/k_\perp^2 & k_\perp^2/Q_s^2 \gg 1 \end{cases}$$

In the saturation regime quarks are packed in two-dimensional phase space much like a Fermi gas with $T \sim Q_s$. In the gluon case there are $1/\alpha$ gluons "sitting one on top of the other" in each cell in transverse phase space leading to

$$A_\mu^i \sim \frac{1}{\sqrt{\alpha N_c}}$$

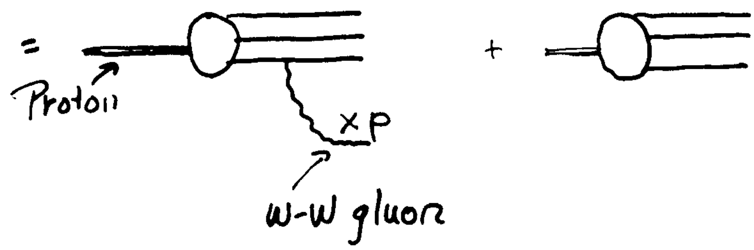
which is likely the maximum allowed value.

2. Simple Models of Saturation [McLerran, Venugopalan]

Saturation requires high gluon density. At HERA this can occur through the rapid growth of xG as x decreases. It is conceptually simpler to get high gluon densities from a simple kinematic argument using large nuclei.

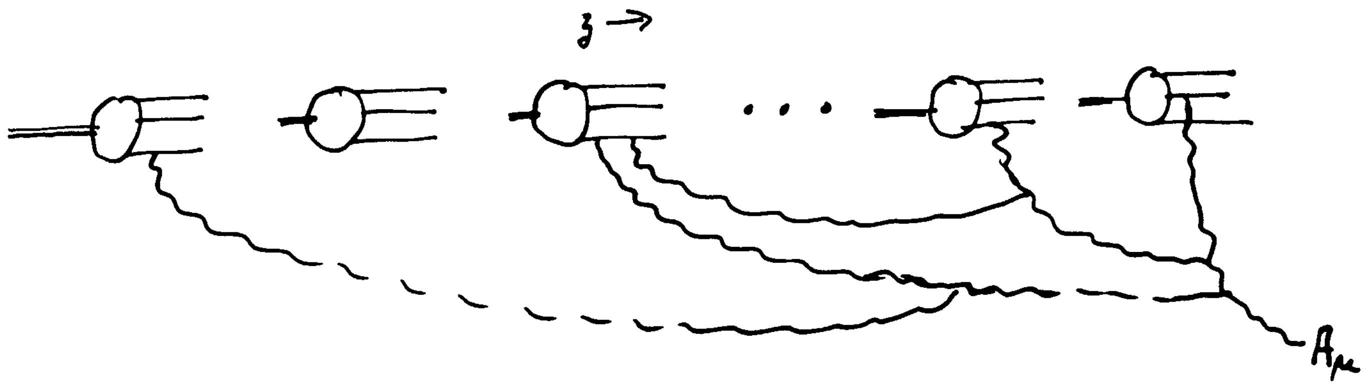
To that end view a high momentum proton as three valence quarks along with a (weak) Weizsäcker-Williams Field.

$|\psi_{Proton}\rangle =$



For large nucleus, when $\frac{1}{xP} \gg 2R\frac{M}{P}$ or $x \ll \frac{1}{2RM}$

all gluons at a given impact parameter will interact.



Solving for A is a "semi-classical" problem and can be done exactly McLerran et al. Kovchegov

$$\frac{d\,xG_A}{d^2b\,d^2k_\perp} = \int \frac{d^2x}{4\pi^2}\cdot\frac{2}{\pi}\,\mathrm{tr}\langle A_i^a(\underline{b}+\underline{x})A_i^a(\underline{b})\rangle$$

$$\mathrm{tr}\langle A_i^a(\underline{b}+\underline{x})A_i^a(\underline{b})\rangle = \frac{N_c^2-1}{2\pi\alpha N_c\,\underline{x}^2}\left(1-e^{-\underline{x}^2Q_s^2/4}\right)$$

Nucleon density

$$Q_s^2 = \frac{8\pi^2\alpha N_c}{N_c^2-1}\sqrt{R^2-b^2}\,\rho\, xG_p(x, Q_s^2)$$

Nuclear radius

IF one calculates the $q\bar{q}$ sea in this background field

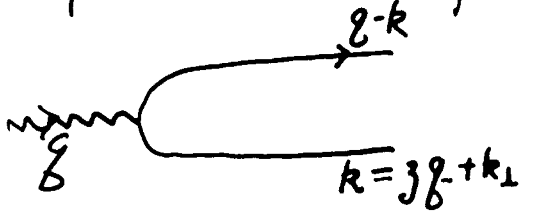
a saturated quark distribution emerges

3. Is Saturation a General Low-x Phenomenon?

Yes, it is equivalent (dual) to unitarity!

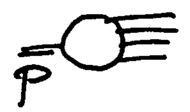
A. Dipole Frame in Deep Inelastic Scattering

In order to make DIS look like the scattering of a dipole on the proton choose Frame



$$\overset{}{q} = \left(\overset{t}{q - \frac{Q^2}{2q}}, \overset{x}{0}, \overset{y}{0}, \overset{z}{-q}\right)$$

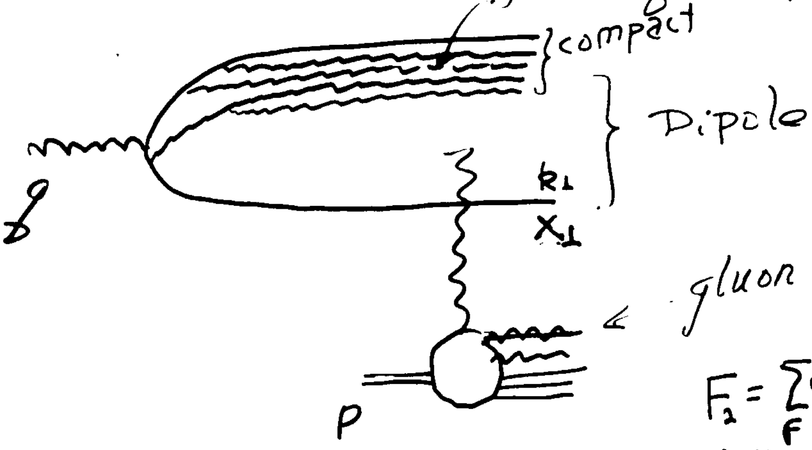
$$p = \left(p + \frac{M^2}{2p}, 0, 0, p\right)$$



q/Q is large, but not ~~too large~~ and is x-independent

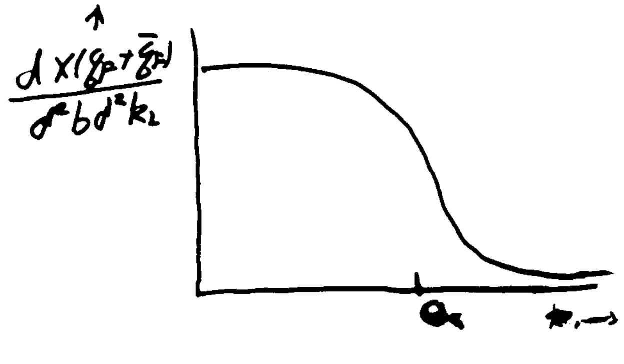
need $\frac{k_-}{k_\perp} \gg 1$. (Recall $\frac{k_-}{k_\perp^2} \sim \frac{q_-}{Q^2} \Rightarrow \frac{q}{Q} = \frac{k}{k_\perp} \cdot \frac{Q}{k_\perp} \gg 1$

B. Quark Saturation (First order DGLAP)



$$F_2 = \sum_F e_F^2 (q_F + \bar{q}_F) = \frac{Q^2}{4\pi^2 \alpha_{em}} [\sigma_T + \sigma_L]$$

$$\frac{dx(q_F + \bar{q}_F)}{d^2b\, d^2k_\perp} \sim Q^2 \int d^2x_\perp |\psi_{\gamma}(x_\perp, Q)|^2 e^{ik_\perp x_\perp} (1 - S(x_\perp, b, x))$$

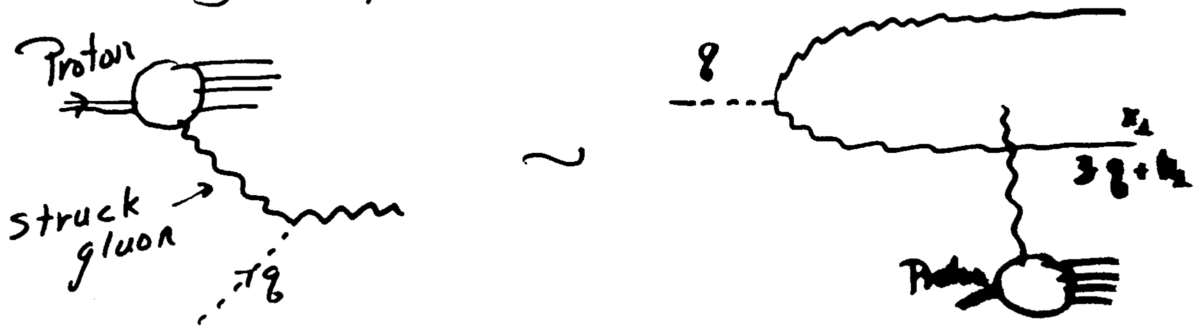


C. Gluon Saturation

Deeply inelastic lepton proton scattering does not directly measure gluons. Gluon distributions are determined from evolution, heavy quark production or from "direct" two jet production. Theoretically, one can introduce a "current" which couples directly to gluons,

$$j = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu}.$$

Then



Bj frame

Dipole Frame

and

$$\frac{dxG}{d^2b\, d^2k_\perp} \sim \int dz \int d^2x_\perp\, e^{ik_\perp \cdot x_\perp} |\psi_{ij}|^2 (1 - S(x_\perp, b, x))$$

Only difference from quark case is dz integration, proportional to dz/z. Model calculations give

$$\int dz/z \to 1/\alpha .$$

4. Saturation at HERA?

Two directions to pursue

(i) Understand better the high density gluon system

(ii) Find evidence for (or against) saturation in HERA data

Presently the major effort is on (ii).

Not so easy because it is only the central impact parameters for which one can expect saturation. One can try (A) models which try to incorporate most data or (B) specific observables which relate to saturation.

A. Golec-Biernat Wüsthoff model

Also related models by Levin et al. and by Frankfurt, Strikman et al. which try to correct some of the weak points of G-B W.

Golec-Biernat Wüsthoff say

$$\sigma_{\gamma^* p} = \int d^2x_\perp \, |\psi_{\gamma^*}(x_\perp, x)|^2 \, \sigma_0 \left(1 - S(x_\perp, x)\right)$$

$$\left.\frac{d\sigma^{DD}_{\gamma^* p}}{dt}\right|_{t=0} = \frac{1}{16\pi} \int d^2x_\perp \, |\psi_{\gamma^*}|^2 \, |\sigma_0 (1 - S(x_\perp, x))|^2 + q\bar{q}g \text{ term}$$

$$S(x_\perp, x) = e^{-x_\perp^2 Q_s^2/4}$$

3 parameters: $\sigma_0 = 23\ \text{mb}$ $\quad Q_s^2 = 1\,\text{GeV}^2 \cdot (x/x_0)^{-\lambda}$

$x_0 = 3 \times 10^{-4}$ $\quad \lambda = 0.3$

Get surprisingly good fits to F_2 and F_2^D as well as to vector meson production. Caldwell, Soares

But 2^{nd} order DGLAP, an orthogonal picture, also gives good fits, but with a surprising gluon distribution at $Q^2 \sim 1\text{-}2\ GeV^2$.

B. Impact Parameter Dependent Cross Sections

A lesson from soft physics

Total hadronic cross sections, the hadronic analogies of F_2, are well fit by

$$\sigma_{AB} = a_{AB}(s/s_0)^{0.08} + b_{AB}\frac{1}{\sqrt{s}}$$

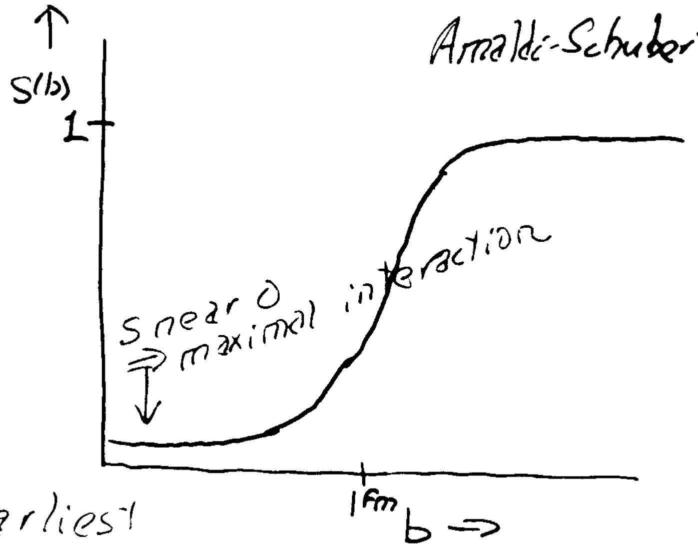
over a wide range of energies. Suggests a simple Pomeron pole interpretation and weak coupling. However, this is not the case.

Amaldi-Schubert

$$\sigma_{tot} = 2\int d^2b\,(1-S(b))$$

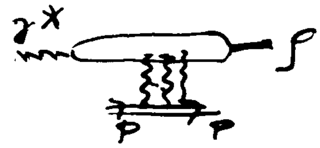
$$\sigma_{el} = \int d^2b\,(1-S(b))^2 \quad \text{with}$$

$$\sigma_{in} = \int d^2b\,(1-S^2(b))$$



Unitarity limits are felt earliest at small impact parameters

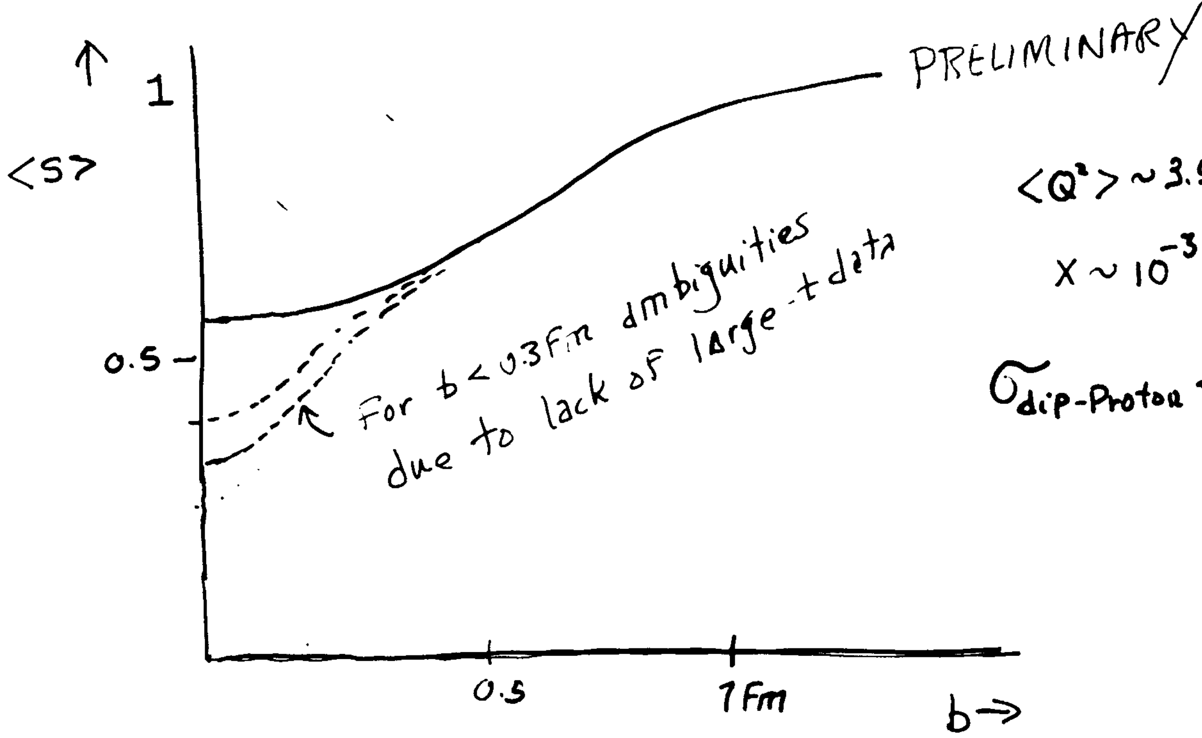
Best place to do similar analysis at HERA is in vector meson production Munier, Stasto, A.M.



$$\frac{d\sigma^{\gamma^*_L \to \rho}}{d\Delta^2} = \frac{1}{16\pi}\left[\int d^2b\, (\psi_\rho, (1 - S(\underline{b},x))\psi_{\gamma^*})\, e^{i\underline{b}\cdot\underline{\Delta}}\right]^2$$

$$\Downarrow$$

$$\langle S(\underline{b},x)\rangle = 1 - \frac{1}{\pi^{3/2}(\psi_\rho,\psi_{\gamma^*})}\int d^2\Delta\, e^{-i\underline{b}\cdot\underline{\Delta}}\sqrt{\frac{d\sigma}{d\Delta^2}}$$



$\langle Q^2\rangle \sim 3.5\,\text{GeV}^2$

$x \sim 10^{-3}$

$\sigma_{\text{dip-Proton}} \sim 10\,\text{mb}$

5. Nuclei and Heavy Ion Collisions

A. Protons vs Nuclei; Q_s-values

Key parameter in saturation is Q_s. Recall in McLerran-Venugopalan model

$$Q_s^2 = \frac{8\pi^2 \alpha N_c}{N_c^2-1}\sqrt{R^2-b^2}\;\rho\, x G_p(x, Q_s^2)$$

In comparing Nuclei with Protons (at $b=0$) the relevant parameter is $R\rho$.

Roughly: $\frac{(\rho R)_A}{(\rho R)_p} = \frac{7\,\text{Fm}\,\frac{1}{7}\frac{1}{\text{Fm}^3}}{1\,\text{Fm}\cdot\frac{1}{\frac{4\pi}{3}\text{Fm}^3}} \approx 4$

$R \approx 7\,\text{Fm}$

IF $Q_s^2 \sim x^{-0.3}$ (as in Golec-Biernat Wüsthoff) the factor of 4 is as good as a ratio of x_p/x_A for equal Q_s^2

$$4 = \left(\frac{x_p}{x_A}\right)^{-0.3} \Rightarrow x_p/x_A \approx e^4 \approx 50$$

B. A_μ vs $F_{\mu\nu}$

The gluons in a saturated wave function are mainly pure gauge fields. This is not bad but saturated gluons only become fully interacting when they are freed in a collision. ($A_\mu \to F_{\mu\nu}$ after freeing.) DIS only frees a few gluons while a heavy ion collision frees all saturated gluons.

Indeed,

$$\frac{dN_{glue}}{dy\,d^2b} \sim \int_{k_\perp \le Q_s} \frac{dxG}{d^2b\,d^2k_t}\, d^2k_t$$

gives a reasonable estimate for the initial density in the central region of a head-on high energy heavy ion collision. The very early stages of the collision, $\tau \sim 1/Q_s$, involve highly nonperturbative (but probably classical) high field strength QCD. (Only study so far is numerical by Krasnitz and Venugopalan.) Later system $\rightarrow$ equilibrium. Estimates are $Q_s \sim 1\,GeV$ at RHIC and $Q_s \sim 3\,GeV$ at LHC.