Lattice Results and HERA Physics

Hartmut Wittig
Theory Group

DESY, 21 May 2002

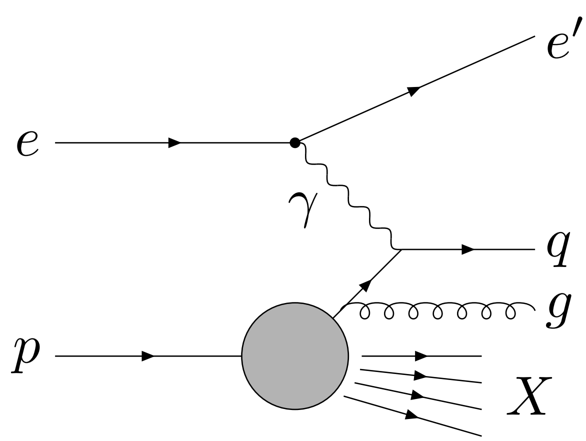
Introduction

- Does QCD correctly describe the strong interactions?
- QCD: formulated as a theory of quarks and gluons

  coupling: $\overline{g}^2$

  quark masses: $\overline{m}_{\mathrm{f}}, \quad \mathrm{f} = u, d, s, \ldots$
- High energy: weakly coupled quarks and gluons:

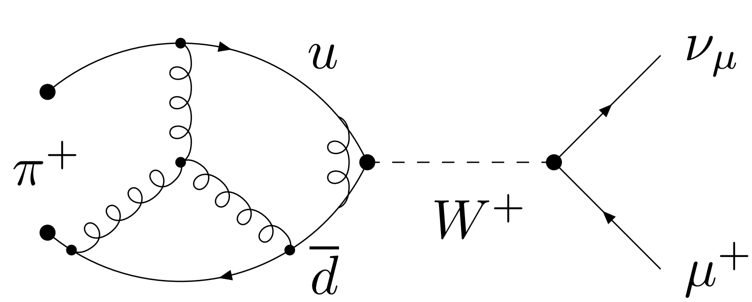
$e + p \to e' + X$



$$\alpha_s(M_Z) = 0.1171 \pm 0.0017(\text{expt}) \pm 0.0017(\text{syst})$$

- Low energy: bound states of quarks and gluons:
  $\pi, K, \ldots, \rho, K^*, \ldots, P, \Sigma, \ldots, \Delta, \Sigma^*, \ldots, G(?)$

$\pi^+ \to \mu^+ \nu_\mu$



$$f_{\pi^+} = 130.7 \pm 0.1 \pm 0.36\,\mathrm{MeV}$$

- Can we connect the low- and high-energy regimes of QCD?

$$\left.\begin{array}{l} f_\pi \\ m_{K^+} \end{array}\right\} \quad \mapsto \quad \left\{\begin{array}{l} \alpha_s \\ m_{\rm u}+m_{\rm s} \end{array}\right.$$

$$f_\pi \quad \mapsto \quad \langle x^n \rangle$$

$\longrightarrow$ stringent tests of QCD

$\longrightarrow$ predict parameters of SM

- Relating low- and high-energy regimes must go beyond perturbation theory

$\longrightarrow$ Lattice QCD

- Problem: large scale differences are involved

Outline:

(1) **Lattice Primer**

- Basic concepts & definitions
- Why realistic lattice calculations are hard

(2) **Scale dependence in QCD**

- Renormalisation Group
- QCD in a (small) box

(3) **Results**

- Running coupling
- Strange quark mass
- Moments of quark distribution functions

(4) **Summary & outlook**

(See also: Rainer Sommer, HW, `physics/0204015`)

I. Lattice Primer

Lattice formulation: replace continuous Minkowskian space-time with discretised, Euclidean version:

lattice spacing	$a, \quad a^{-1} \sim \Lambda_{\mathsf{UV}}$
finite volume	$L^3 \cdot T$

Lattice action:

$$S[U,\overline{\psi},\psi] = S_{\rm G}[U] + S_{\rm F}[U,\overline{\psi},\psi]$$

Expectation value:

$$\begin{aligned} Z &= \int D[U]D[\overline{\psi}]D[\psi]\,{\rm e}^{-S_{\rm G}[U]-S_{\rm F}[U,\overline{\psi},\psi]} \\ &= \int D[U]\,\prod_f \det\left(\not{D}+m_f\right)\,{\rm e}^{-S_{\rm G}[U]} \\ \langle\Omega\rangle &= \frac{1}{Z}\int\prod_{x,\mu} dU_\mu(x)\,\Omega\,\prod_f \det\left(\not{D}+m_f\right)\,{\rm e}^{-S_{\rm G}[U]} \end{aligned}$$

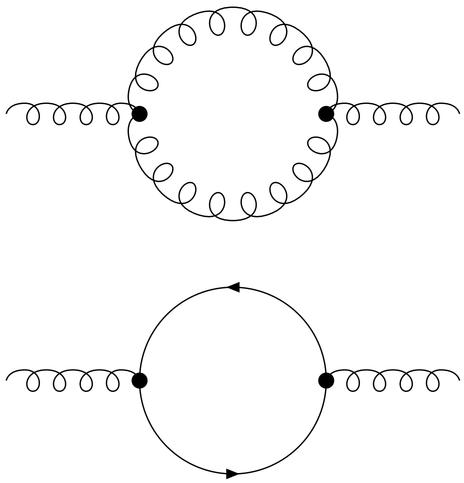
$\longrightarrow$ allows for stochastic evaluation of $\langle\Omega\rangle$

$\longrightarrow$ gauge invariant regularisation procedure

$\longrightarrow$ definition of observables does not rely on perturbation theory

Why realistic QCD simulations are hard

(1) **Inclusion of dynamical quark effects:**



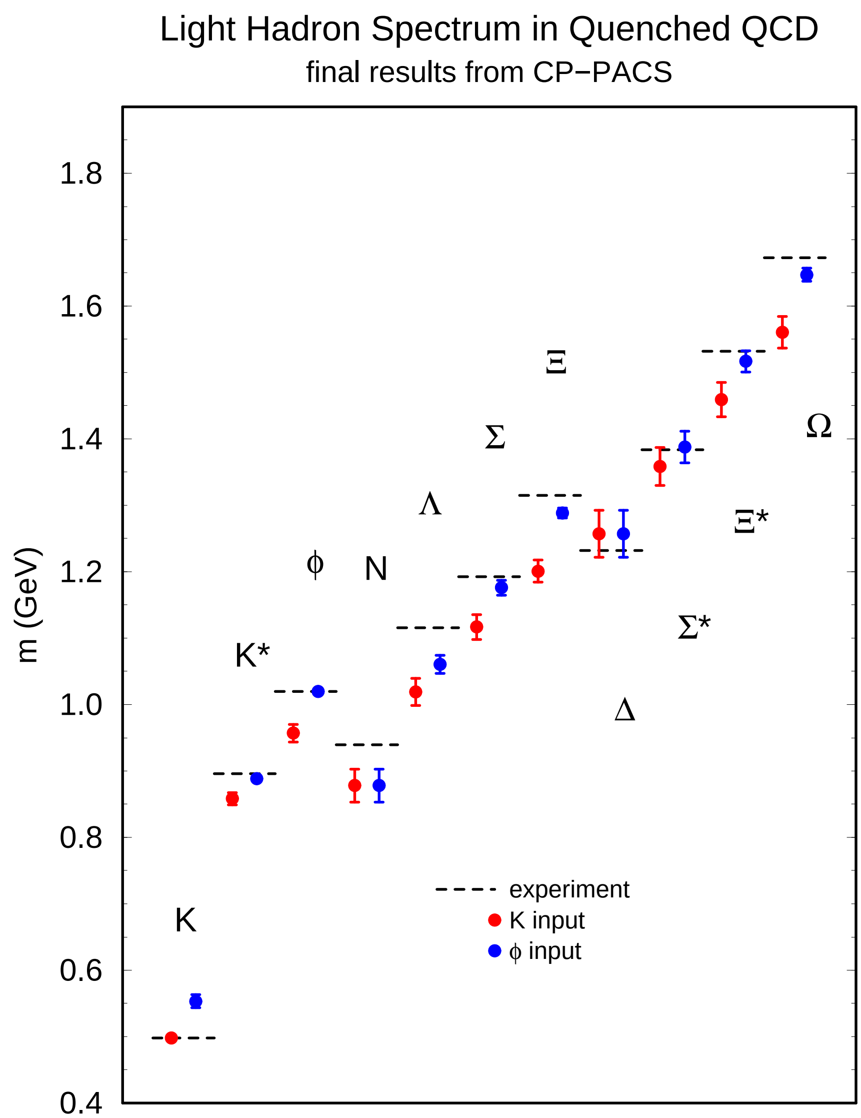
$\det(\not{D} + m_f) = 1$: Quenched Approximation

$\longrightarrow$ neglect quark loops in the evaluation of $\langle\Omega\rangle$.

Cost: $$\frac{\text{“full” QCD}}{\text{quenched QCD}} \gtrsim 100 - 1000$$

- Light hadron spectrum in quenched QCD:

CP-PACS Collaboration, Phys. Rev. Lett.84 (2000) 238



→ Quenched QCD describes the light hadron spectrum at the 10% level.

(2) **Restrictions on quark masses:**

$$a \ll m_{\rm q}^{-1} \ll L$$

$\longrightarrow$ must extrapolate to $m_{\rm u}$, $m_{\rm d}$ and $m_{\rm b}$
$\longrightarrow$ guidance from ChPT, HQET

(3) **Lattice artefacts** (cutoff effects)

$$\langle\Omega\rangle^{\rm lat} = \langle\Omega\rangle^{\rm cont} + O(a^p)$$

$\longrightarrow$ need to extrapolate to continuum limit: $a \to 0$
$\longrightarrow$ fast convergence to c.l. for large p

(4) **Limitations on energy range:**

Typical number of sites:	$L/a \leq 32$
Required volume:	$L \gtrsim 1.5\,\rm fm$

This implies

$$a \gtrsim 0.05\,\rm fm, \quad \Rightarrow \quad \mu < a^{-1} \lesssim 4\,\rm GeV$$

II. Scale dependence in QCD

Renormalisation Group:

- Scale dependence of masses and couplings described by RG equations:

$$\mu\frac{\partial\overline{g}}{\partial\mu} = \beta(\overline{g}), \qquad \beta(\overline{g}) = -\overline{g}^3 b_0 - \overline{g}^5 b_1 \ldots$$
$$\mu\frac{\partial\overline{m}}{\partial\mu} = \tau(\overline{g})\,\overline{m}, \quad \tau(\overline{g}) = -\overline{g}^2 d_0 - \overline{g}^4 d_1 \ldots$$

- From the solution in the asymptotic regime one can define

$$\Lambda = \lim_{\mu\to\infty}\left\{\mu(b_0\overline{g}^2)^{-b_1/2b_0^2}\,\mathrm{e}^{-1/2b_0\overline{g}^2}\right\},$$
$$M = \lim_{\mu\to\infty}\left\{\overline{m}(2b_0\overline{g}^2)^{-d_0/2b_0}\right\}$$

M : RG-invariant quark mass

- Simple relation between different schemes at high energies:

$$\Lambda' = k\,\Lambda, \qquad M' = M$$

$\longrightarrow$ can use any scheme to compute Λ and M.

- At low energy scales the running of $\overline{g}$, $\overline{m}$ needs to be determined non-perturbatively

- Lattice: $\mu < a^{-1} \lesssim 4\,\mathrm{GeV}$
  $\Rightarrow$ hard to make contact with Perturbation Theory

Solution:

- Formulate QCD in a finite volume of size L^4;
  masses and couplings run with box size L:

$$\mu = 1/L \ll a^{-1}$$

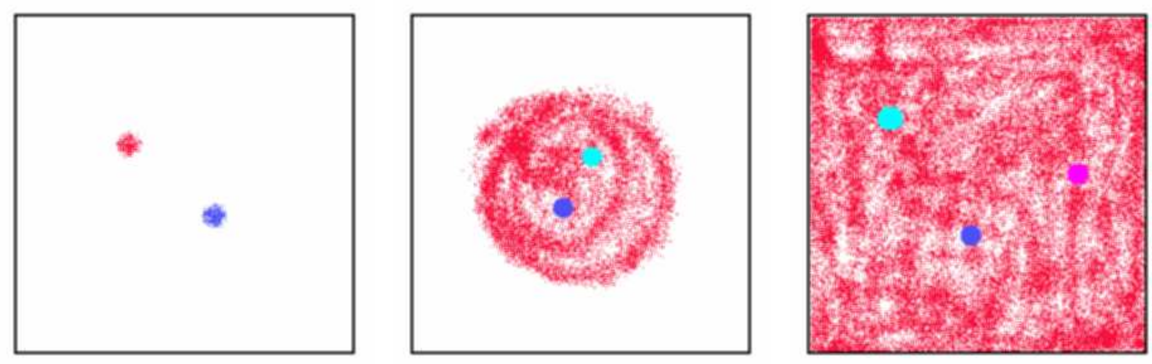
- Can arrange calculation such that all reference to finite volume drops out in the final result.

  Approach applied by the

  ALPHA Collaboration

  (HU Berlin, CERN, DESY, Milan, MPI Munich, NIC/DESY, Rome II)

QCD in finite volume



(a) $L \gg 2\,\mathrm{fm}$

Single hadrons unaffected by volume effects

(b) $L \lesssim 1.5\,\mathrm{fm}$

Hadrons "feel" finite extent;
masses are shifted by terms $\propto \mathrm{e}^{-m_\pi L}$

(c) $L \lesssim 0.5\,\mathrm{fm}$

– Hadrons cease to exist;
  $\longrightarrow$ quarks and gluons in "femto universe"
– As L becomes arbitrarily small one enters the short distance regime of QCD:

$$L \to 0 \quad \text{implies} \quad \overline{g}^2 \to 0$$

$\longrightarrow$ regard $\mu = 1/L$ as renormalisation scale

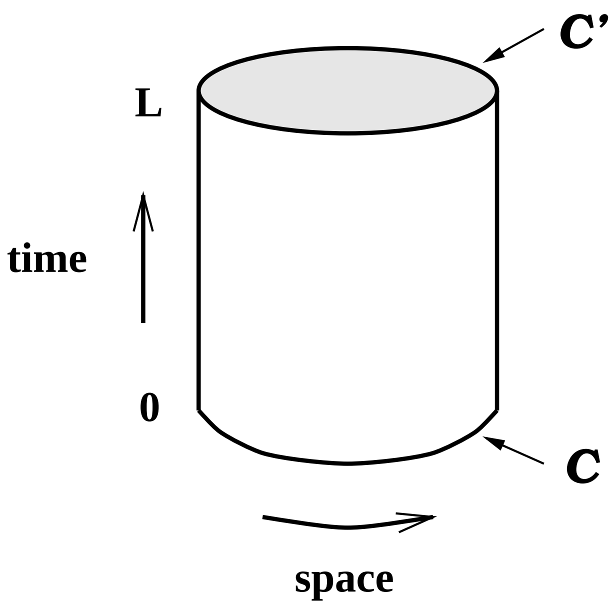
– QCD is well defined in femto universe;
  specify suitable boundary conditions

The Schrödinger Functional:

- Realisation of QCD in a finite volume;
  box size L^4.
  - periodic boundary conditions in spatial directions
  - Dirichlet boundary conditions in time direction:

$$A_k(x) = \begin{cases} C_k(\mathbf{x}), & x_0 = 0 \\ C'_k(\mathbf{x}), & x_0 = L \end{cases}$$

$C,\ C'$: classical colour-electric field



(LxLxL box with periodic b.c.)

- Definition of the running coupling:

$$\bar{g}^2_{\mathsf{SF}}(L) = \left\langle \frac{\partial \Gamma_0}{\partial \eta} \right\rangle \Big/ \left\langle \frac{\partial \Gamma}{\partial \eta} \right\rangle = \frac{F^{\mathsf{class}}_{0k}}{\langle F_{0k} \rangle}$$

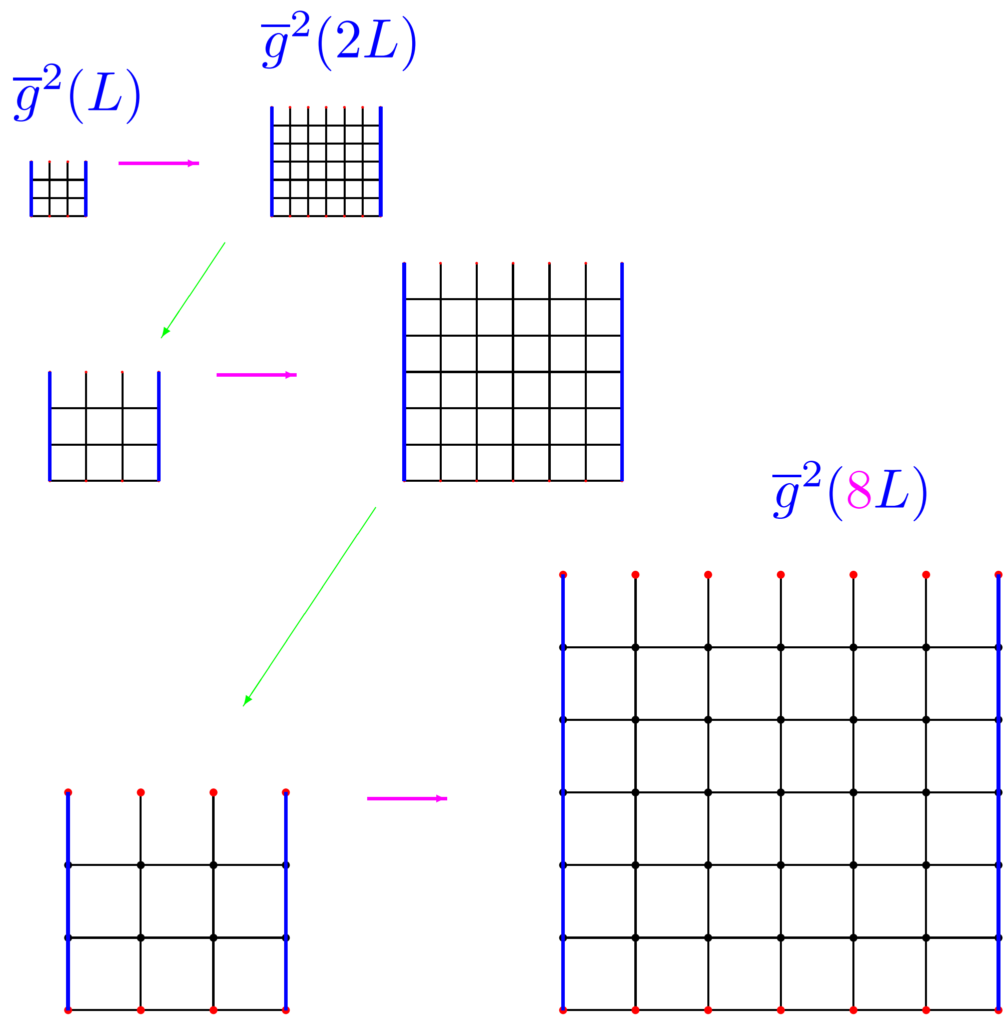
III. Results: The running coupling

- Define "step scaling function" σ

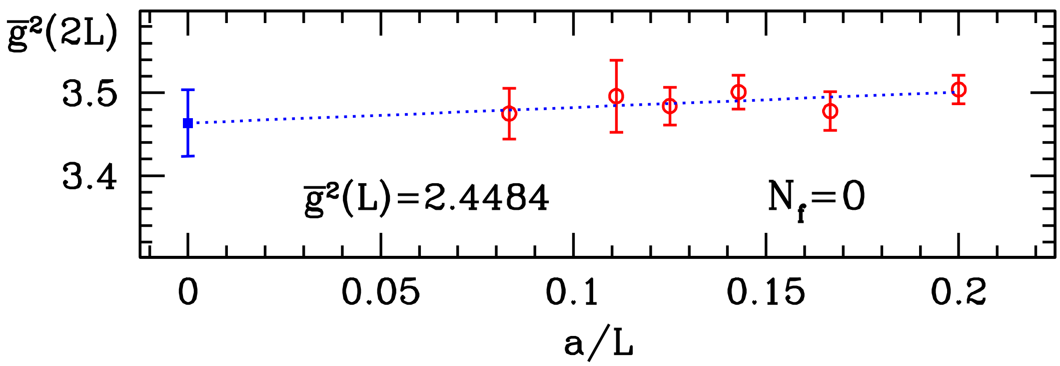
$$\overline{g}^2_{\mathrm{SF}}(2L) = \sigma(2, \overline{g}^2_{\mathrm{SF}}(L))$$

- σ describes the change in $\overline{g}^2_{\mathrm{SF}}$ when the renormalisation scale is changed by a factor 2.

- Can compute σ recursively over several orders of magnitude

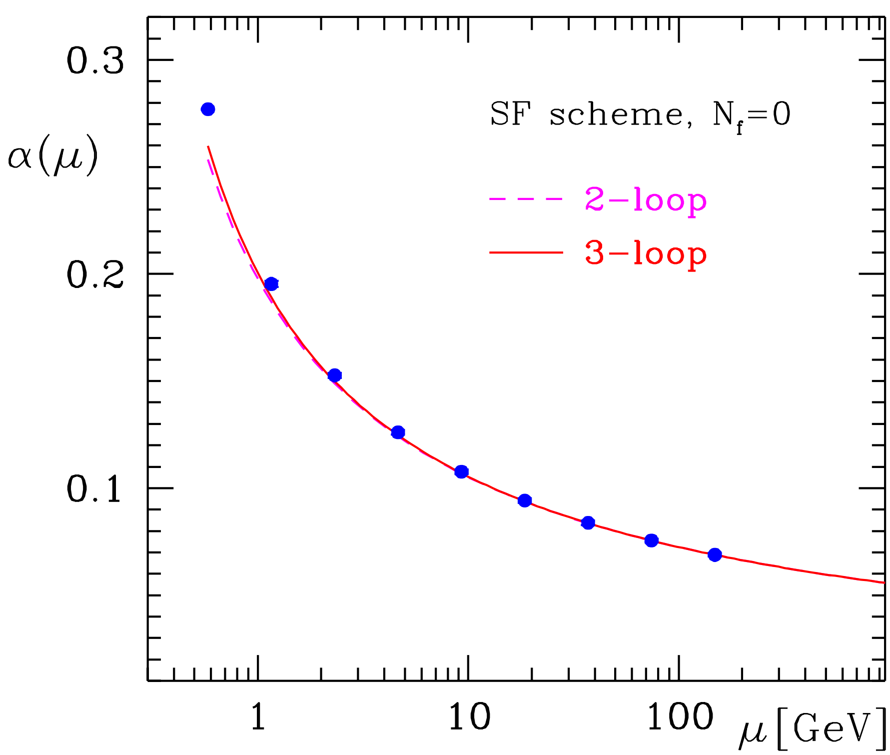


- Continuum limit is reached by repeating each horizontal step for different lattice resolutions



- Successive application of σ yields non-perturbative scale evolution of $\overline{g}^2_{\mathrm{SF}}$;

  Quenched approximation:



- Extract Λ_{SF} using perturbative RG functions for $\mu \gtrsim 100\,\mathrm{GeV}$

- At $L = L_{\mathsf{max}}$ the matching with infinite-volume physics is performed:

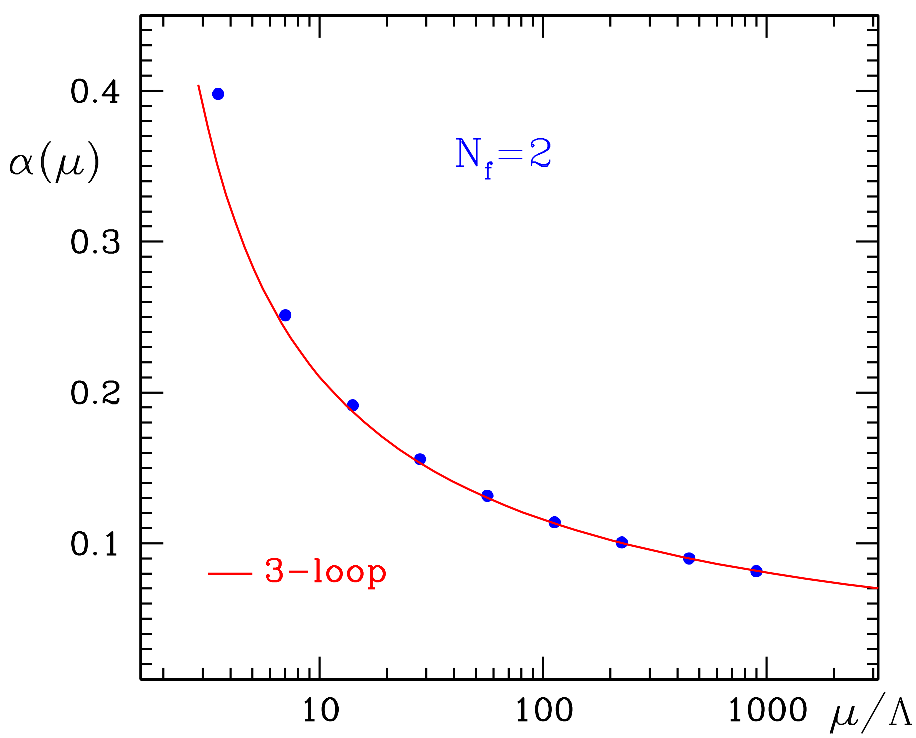
$$\Lambda_{\mathsf{SF}} L_{\mathsf{max}} = 0.211(16), \quad L_{\mathsf{max}} f_{\mathrm{K}} = 0.291(6)$$

$$\Lambda_{\mathsf{SF}} = 0.48811(1)\,\Lambda_{\overline{\mathrm{MS}}}$$

$$\Rightarrow \quad \boxed{\Lambda_{\overline{\mathrm{MS}}}^{(0)} = 238 \pm 19\,\mathrm{MeV}}$$

(ALPHA Collab, Nucl. Phys. B544 (1999) 669)

- Including dynamical quark effects: $N_{\mathsf{f}} = 2$:



- Relation of L_{max} to f_{K} not yet computed for $N_{\mathsf{f}} = 2$ (ALPHA Collab, Phys. Lett. B515 (2001) 49), and in preparation)

Results (cntd.): The strange quark mass

- PCAC relation:

$$f_{\rm K} m_{\rm K}^2 = (\overline{m}_{\rm u} + \overline{m}_{\rm s}) \langle 0 | \overline{u}\gamma_5 s | K^+ \rangle$$

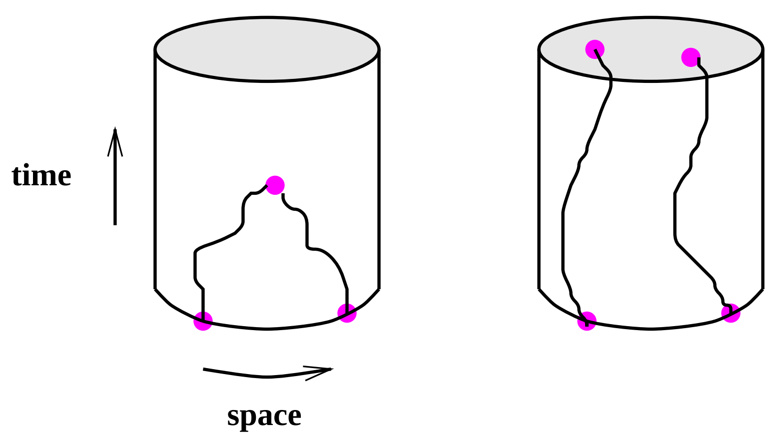
- Must relate the lattice density $(\overline{u}\gamma_5 s)_{\rm lat}$ to the renormalised density in the $\overline{\rm MS}$ scheme:

$$\begin{aligned} (\overline{u}\gamma_5 s)_{\overline{\rm MS}} &= Z_P(g_0, a\mu)\,(\overline{u}\gamma_5 s)_{\rm lat} \\ Z_P(g_0, a\mu) &= 1 + \frac{g_0^2}{4\pi}\left\{\frac{2}{\pi}\ln(a\mu) + C\right\} + O(g_0^4) \end{aligned}$$

- Normalisation condition for the pseudoscalar density (SF $\leftrightarrow$ lat):

$$Z_{\rm P}(g_0, a/L) = c\sqrt{f_1}/f_{\rm P}(z)\Big|_{z_0=L/2}$$

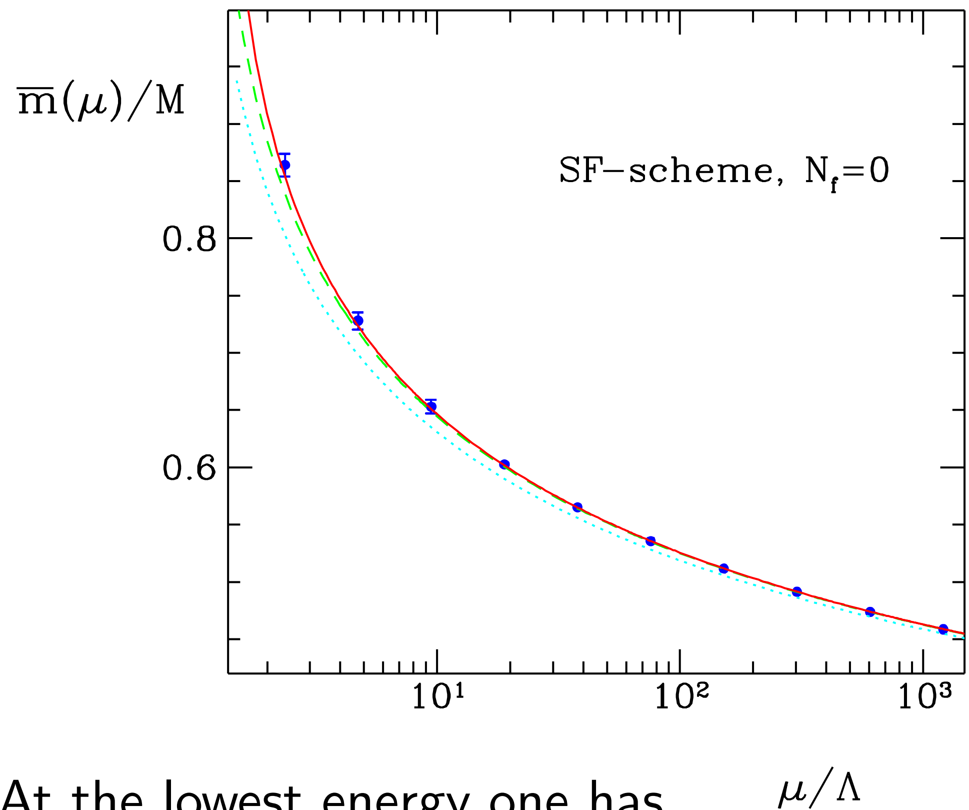
$f_{\rm P}, f_1$: correlation functions in Schrödinger functional:



- Step scaling function:

$$\sigma_{\mathrm{P}}(2, \overline{g}^2_{\mathsf{SF}}(L)) = \frac{Z_{\mathrm{P}}(2L)}{Z_{\mathrm{P}}(L)}$$

- Evolution of the running quark mass (quenched!)



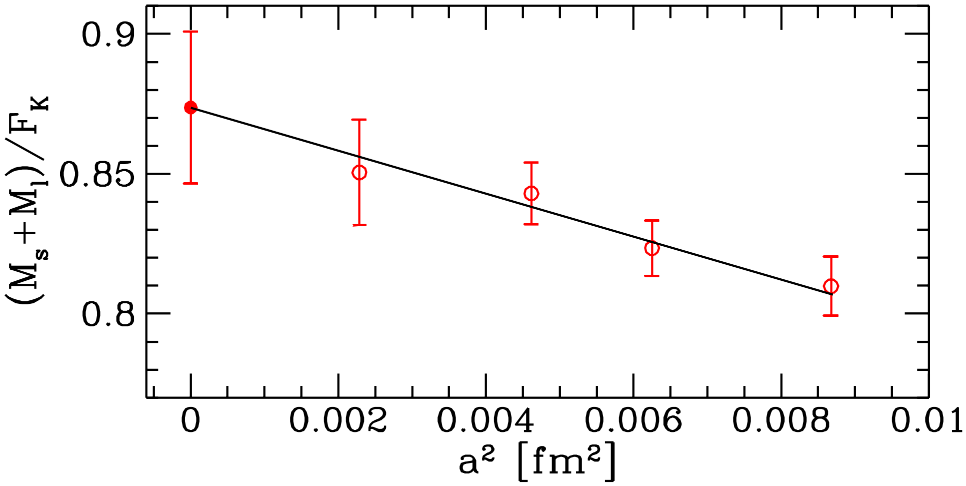
- At the lowest energy one has

$$\frac{M}{\overline{m}_{\mathrm{SF}}}(2L_{\mathrm{max}}) = 1.157(15)$$

- Renormalised PCAC relation:

$$\frac{\widehat{M} + M_{\mathrm{s}}}{f_{\mathrm{K}}} = \frac{M}{\overline{m}_{\mathrm{SF}}} \cdot \frac{1}{Z_{\mathrm{P}}} \times \left. \frac{m_{\mathrm{P}}^2}{\langle 0|\overline{\ell}\gamma_5 s|P\rangle} \right|_{m_{\mathrm{P}}=m_{\mathrm{K}}} + O(a^2)$$

- Continuum extrapolation of $(\widehat{M} + M_{\rm s})/f_{\rm K}$:



- Convert into $\overline{m}_{\rm s}^{\overline{\rm MS}}(2\,{\rm GeV})$ using

$$f_{\rm K} = 160(2)\,{\rm MeV},$$

$$M_{\rm s}/\widehat{M} = 24.4 \pm 1.5, \quad \text{(ChPT)}$$

$$\overline{m}^{\overline{\rm MS}}(2\,{\rm GeV})/M = 0.7208, \quad \text{(4-loop)}$$

$$\Rightarrow \quad \boxed{\overline{m}_{\rm s}^{\overline{\rm MS}}(2\,{\rm GeV}) = 97 \pm 4\,{\rm MeV}}$$

(Capitani, Lüscher, Sommer, HW, Nucl.Phys.B544 (1999) 669)
(Garden, Heitger, Sommer, HW, Nucl. Phys. B571 (2000) 237)

$\longrightarrow$ $N_{\rm f} = 2$: in preparation $\longleftarrow$

Results (ctnd.): Moments of PDF's

- Study more complicated cases:

  Moments of Parton Distribution Functions

  $\updownarrow$

  Matrix elements of local operators between hadronic states

- Focus on the simplest case:

  - PDF for the pion
  - twist-2, non-singlet operator

$$O_{\mu_1\mu_2} = \tfrac{1}{4}\,\overline{\psi}(x)\gamma_{\{\mu_1}\,\overleftrightarrow{D}_{\mu_2\}}\,\tfrac{\tau^3}{2}\psi(x)$$

- Renormalised average momentum:

$$\langle x\rangle_R(\mu) = \langle \pi;\vec{p}\,|O^R_{\mu_1\mu_2}|\,\pi;\vec{p}\rangle$$

$$O^R_{\mu_1\mu_2}(\mu) = \frac{1}{Z_{\rm O}(g_0, a\mu)}\,O^{\rm lat}_{\mu_1\mu_2}$$

- Schrödinger functional set-up:

$$f_{\rm O}(x_0) = \sum_{\vec{x},\vec{y},\vec{z}} \langle O^{\rm lat}_{\mu_1\mu_2}(x)\,\overline{\zeta}(\vec{y})\Gamma\tau^3\zeta(\vec{z})\rangle$$

$$Z_{\rm O}(L) = k\,f_{\rm O}(x_0)/\sqrt{f_1}$$

$$\sigma_{\rm O}(2,\overline{g}^2) = \frac{Z_{\rm O}(2L)}{Z_{\rm O}(L)}$$

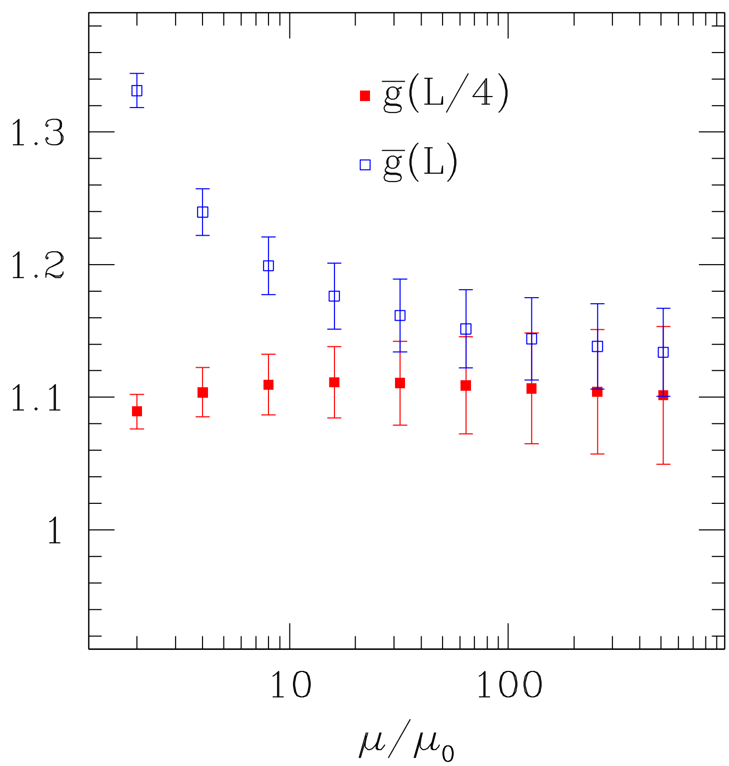
- RGI matrix element:

$$\langle O^{\rm RGI}_{\mu_1\mu_2}\rangle = \langle O^{\rm R}_{\mu_1\mu_2}(\mu)\rangle \cdot f^R(\bar{g}^2(\mu))$$

$$f^R(\bar{g}^2(\mu)) = (\bar{g}^2(\mu))^{-\gamma_0/2b_0} \times \exp\left\{-\int_0^{\bar{g}(\mu)} dg \left[\frac{\gamma(g)}{\beta(g)} - \frac{\gamma_0}{b_0 g}\right]\right\}$$

- For large $\mu = 1/L$ the r.h.s. is scale-independent:

$$\frac{\langle O^{\rm SF}_{\mu_1\mu_2}(L)\rangle\, f^{\rm SF}(\bar{g}^2_{\sf SF}(L))}{\langle O^{\rm SF}_{\mu_1\mu_2}(L_{\rm max})\rangle} \xrightarrow{L\to 0} \frac{\langle O^{\rm RGI}_{\mu_1\mu_2}\rangle}{\langle O^{\rm SF}_{\mu_1\mu_2}(L_{\rm max})\rangle}$$



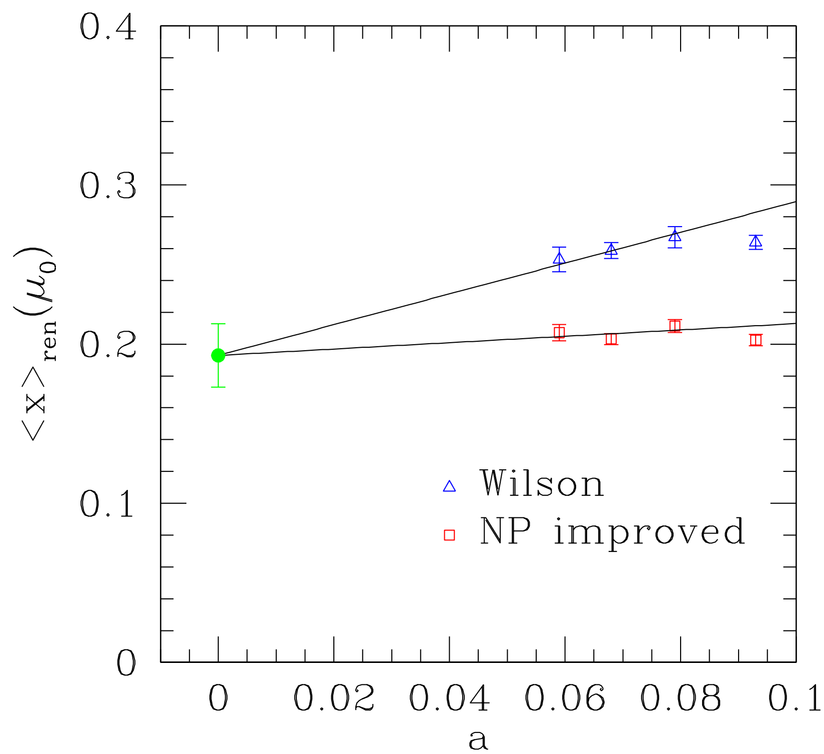
(Guagnelli, Jansen, Petronzio, Phys.Lett. B459 (1999) 594)

$\Rightarrow$ $\langle O^{\rm RGI}_{\mu_1\mu_2}\rangle / \langle O^{\rm SF}_{\mu_1\mu_2}(L_{\rm max})\rangle = 1.11(4)$

$$\langle x \rangle_{\mathrm{SF}}(L) = \frac{1}{Z_{\mathrm{O}}(L)} \left\langle \mathrm{PS} \left| O^{\mathrm{lat}}_{\mu_1\mu_2} \right| \mathrm{PS} \right\rangle \Big|_{m_{\mathrm{PS}}=m_\pi} + O(a)$$

→ involves chiral extrapolation to physical pion

- Continuum extrapolation



(Guagnelli, Jansen, Petronzio, Phys.Lett. B493 (2000) 77; K Jansen, `hep-lat/0010038`)

$$\langle x \rangle_{\mathrm{SF}}(L_{\mathrm{max}}) = 0.20(2)$$

$$\langle x \rangle_{\mathrm{RGI}} = 1.11(4) \cdot \langle x \rangle_{\mathrm{SF}}(L_{\mathrm{max}}) = 0.22(2)$$

$$\langle x \rangle_{\overline{\mathrm{MS}}}(\mu = 2.4\,\mathrm{GeV}) = 0.30(3)$$

$$\langle x \rangle^{\mathrm{exp}}_{\overline{\mathrm{MS}}}(\mu = 2.4\,\mathrm{GeV}) = 0.23(2)$$

- Quenching effects?, chiral extrapolations?

VI. Summary & Outlook

- Gap between low- and high-enery regimes in QCD can be bridged with controlled errors:

  Lattice simulations + Finite-size scaling

- Accuracy in the continuum limit (quenched approximation):

  8% for Λ-parameter

  4% for $m_{\rm s}$

  10% for $\langle x \rangle$

- Remaining systematics are due to
  - Quenched approximation
  - Quark mass dependence

- In future we can expect:
  - determinations of other quantities
  - investigation of quark mass dependence (smaller masses, combine with ChPT)
  - extension to simulations with dynamical quarks