The measurement of the running of α_{QED} at e+e- colliders in small angle Bhabha scattering

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We propose a method to determine the running of α_{QED} from the measurement of small-angle Bhabha scattering

The method is suited to high statistics experiments at e+e– colliders, which are equipped with luminometers in the appropriate angular region

We present a new simulation code predicting small-angle Bhabha scattering

The electroweak Standard Model SU(2) ⊗ U(1) contains Quantum Electrodynamics QED as a constitutive part.

The running of the electromagnetic coupling

$\alpha_{QED}(q^2)$

is determined by the theory as

$\alpha_{QED}(q^2) = \alpha_{QED}(0) / 1 - \Delta\alpha(q^2)$

$$\alpha_{QED}(0) = \alpha_0$$

$$\alpha_0 = \frac{e^2}{2\pi\varepsilon_0 hc}$$

is the Sommerfeld

"fine structure constant"

$\Delta\alpha(q^2)$ **Vacuum polarization** $\Delta\alpha = \Delta\alpha_{lept} + \Delta\alpha_{had}$

arises from quantum loop contribution to the photon propagator receiving contributions from quarks (hadrons), leptons and gauge bosons

The hadronic contribution is estimated in the s channel with a dispersion integral
from the cross-section e+e- to hadrons cross-section

S. Eidelman and F. Jegerlehner: Z. Phys. **C67** (1995) 602
F. Jegerlehner: hep-ph/0308117
M. Davier and A. Höcker: Phys. Lett. **B435** (1998) 427
M. Davier, S. Eidelman, A. Höcker and Z. Zhang: Eur. Phys. J. **C27** (2003) 497

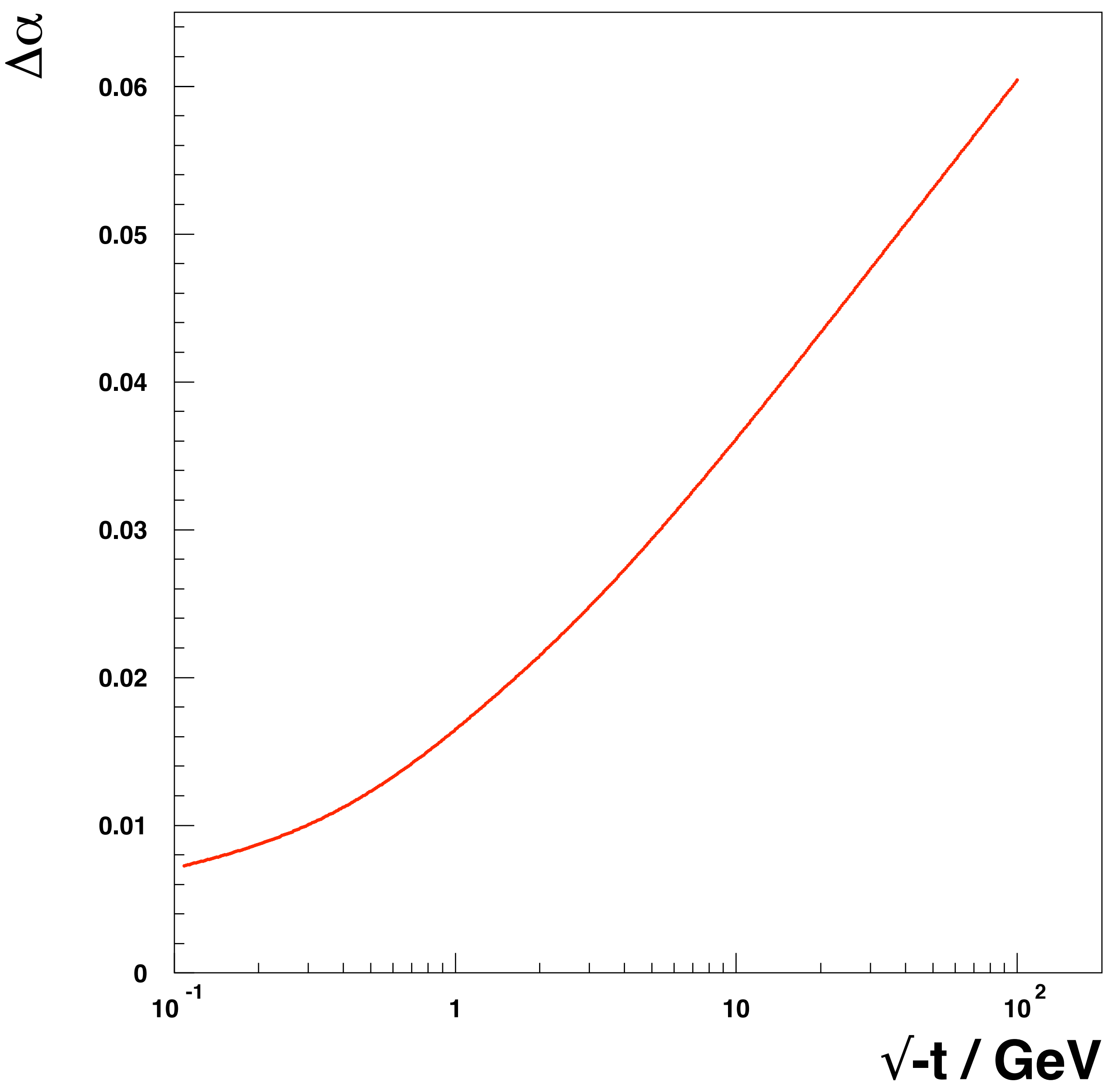
Here we follow an alternative approach:

- the running of α is studied using small-angle Bhabha scattering
- This process provides unique information on the QED coupling constant α at low space-like momentum transfer $t = -|q^2|$ in the t channel, with $t = -(1/2)s(1-\cos\theta)$ for example for

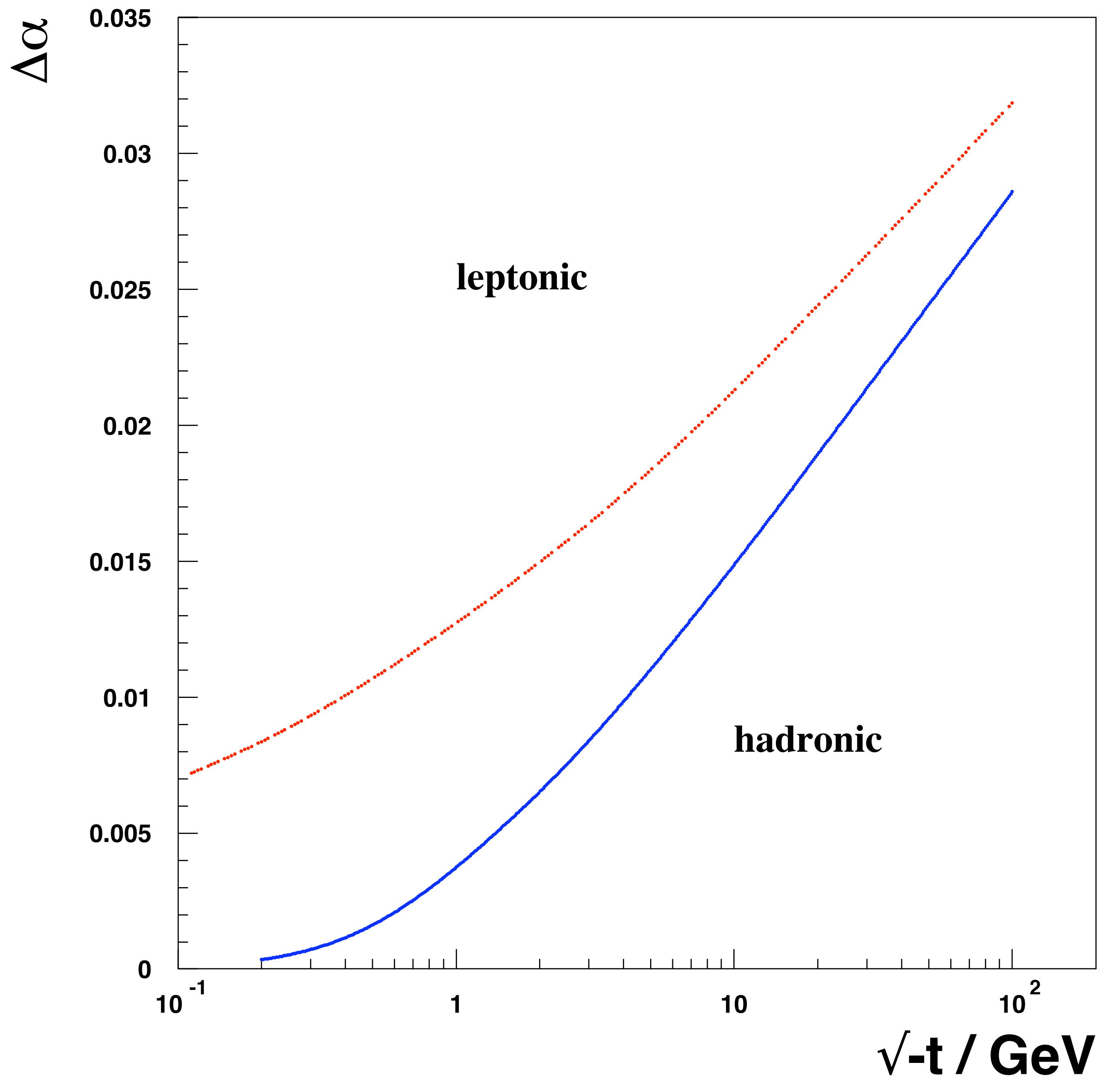
$$\sqrt{s} = 91.1 GeV$$

$$\theta = 30\ mrad \qquad t = 2.2\ GeV^2$$

$$\theta = 150\ mrad \qquad t = 30\ GeV^2$$



by using *alphaQED* F. Jegerlehner



by using *alphaQED* F. Jegerlehner

The method to measure the running of α

exploits the fact that the cross section for the process e+e–→e+e– can be conveniently decomposed into three factors:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \frac{\mathrm{d}\sigma^0}{\mathrm{d}t} \left(\frac{\alpha(t)}{\alpha(0)} \right)^2 (1 + \Delta r(t))$$

- The Bhabha Born cross section, including soft and virtual photons is precisely known and accounts for the strongest dependence on t

- The vacuum-polarization effect in the leading photon t-channel exchange is incorporated in the running of α and gives rise to the squared factor

- Δr(t) collects all the remaining real (in particular collinear) and virtual radiative effects not incorporated in the running of α

- The experimental data (after correction for detector effects) have to be compared with this distribution

The Luminosity measurement

The precise determination of the luminosity at e+e- colliders is a crucial ingredient to obtain an accurate evaluation of all the physically relevant cross sections.

They necessarily have to rely on some reference process, which is usually taken to be the small-angle Bhabha scattering.
Given the high statistical precision provided by the LEP collider, an equally precise knowledge of the theoretical small-angle Bhabha cross section is mandatory. In the 1990's the substantial progress in measuring the luminosity reached by the LEP machine has prompted several groups to make a theoretical effort aiming

at a **0.1%** accuracy.

An even better accuracy can be reached once the complete two-loop Bhabha (including α constants) cross-section will be computed

This goal has indeed been achieved by developing a dedicated strategy.
For the first time small-angle Bhabha scattering was evaluated analytically, following a new calculation technique that yields the required precision

Arbuzov,Fadin,Lipatov,Merenkov,Kuraev,T.(1995)
Nucl.Phys.B485(1997)457

Analytical calculations have been combined with Monte Carlo programs in order to simulate realistically the conditions of the LEP experiments

LABSMC NLLBHA SAMBHA

BHLUMI

The cross section $\mathrm{d}\sigma^0/\mathrm{d}t$

$$\frac{\mathrm{d}\sigma^0}{\mathrm{d}t} = \frac{\mathrm{d}\sigma^B}{\mathrm{d}t}\left(\frac{\alpha(0)}{\alpha(t)}\right)^2$$

$$\frac{\mathrm{d}\sigma^B}{\mathrm{d}t} = \frac{\pi\alpha_0^2}{2s^2}\mathrm{Re}\{B_t + B_s + B_i\},$$

$$\begin{aligned}
B_t &= \left(\frac{s}{t}\right)^2\Bigg\{\frac{5+2c+c^2}{(1-\Pi(t))^2} + \xi\frac{2(g_v^2+g_a^2)(5+2c+c^2)}{(1-\Pi(t))} \\
&+ \xi^2\left(4(g_v^2+g_a^2)^2 + (1+c)^2(g_v^4+g_a^4+6g_v^2g_a^2)\right)\Bigg\} \\
B_s &= \frac{2(1+c^2)}{|1-\Pi(s)|^2} + 2\chi\frac{(1-c)^2(g_v^2-g_a^2)+(1+c)^2(g_v^2+g_a^2)}{1-\Pi(s)} \\
&+ \chi^2\left[(1-c)^2(g_v^2-g_a^2)^2 + (1+c)^2(g_v^4+g_a^4+6g_v^2g_a^2)\right] \\
B_i &= 2\frac{s}{t}(1+c)^2\Bigg\{\frac{1}{(1-\Pi(t))(1-\Pi(s))} \\
&+ (g_v^2+g_a^2)\left(\frac{\xi}{1-\Pi(s)} + \frac{\chi}{1-\Pi(t)}\right) \\
&+ (g_v^4+6g_v^2g_a^2+g_a^4)\xi\chi\Bigg\}
\end{aligned}$$

$$\begin{aligned}
\chi &= \frac{s}{s-m_z^2+im_Z\Gamma_Z}\cdot\frac{1}{\sin 2\theta_w} \\
\xi &= \frac{t}{t-m_Z^2}\cdot\frac{1}{\sin 2\theta_w}, \\
g_a &= -\frac{1}{2}, \quad g_v = -\frac{1}{2} + 2\sin^2\theta_w), \\
t &= (p_1-q_1)^2 = -\frac{1}{2}\,s\,(1-c), \\
c &= \cos\theta, \qquad \theta = \widehat{\boldsymbol{p}_1\boldsymbol{q}_1}.
\end{aligned}$$

The running of α

$$\boxed{\Pi(t) = \Delta\alpha(t)}$$

$$\begin{aligned}
\Pi(t) &= \frac{\alpha_0}{\pi}\left(\delta_t + \frac{1}{3}L - \frac{5}{9}\right) \\
&+ \left(\frac{\alpha_0}{\pi}\right)^2 \left(\frac{1}{4}\,L + \zeta(3) - \frac{5}{24}\right) \\
&+ \left(\frac{\alpha_0}{\pi}\right)^3 \Pi^{(3)}(t) + \mathcal{O}\left(\frac{m_e^2}{t}\right),
\end{aligned}$$

$$L = \ln\frac{Q^2}{m_e^2}, \qquad Q^2 = -t, \qquad \zeta(3) = 1.202$$

The radiative factor $1 + \Delta r(t)$ and neglected terms

For the present investigation of the small-angle Bhabha cross section only the correction consistently needed to maintain the required accuracy are kept

All these corrections are included in the new code SAMBHA

All the following contributions have been proved to be negligible and are dropped:

- Any electroweak effect beyond the tree level, for instance appearing in boxes or vertices with Z^0 and W bosons, running weak coupling, etc.

- Box diagrams at order α^2 and larger

- Contributions of order α^2 without large logarithms, leading from order α^4 (i.e. $\alpha^4 L^4$) and subleading higher order ($\alpha^3 L^2$, $\alpha^4 L^3$, ...)

- Contributions from pair-produced hadrons, muons, taus and the corresponding virtual pair corrections to the vertices (estimated to be of the order of 0.5×10^{-4})

$\sqrt{s}$ (GeV)	91.187	91.2	189	206	500	1000	3000
	45 mrad $<\ \theta\ <$ 110 mrad						
$\sqrt{\langle -t \rangle}$ (GeV)	3.4	3.4	7.1	7.7	18.8	37.5	112.6
QED	51.428	51.413	11.971	10.077	1.7105	0.42763	0.047514
QED$_t$	51.484	51.469	11.984	10.088	1.7124	0.42809	0.047566
EW	51.436	51.413	11.965	10.072	1.7105	0.42871	0.049507
EW+VP$_t$	54.041	54.018	12.743	10.745	1.8590	0.47303	0.055748
EW+VP	54.036	54.013	12.742	10.744	1.8588	0.47296	0.055742
	5 mrad $<\ \theta\ <$ 50 mrad						
$\sqrt{\langle -t \rangle}$ (GeV)	1.1	1.1	2.2	2.4	5.8	11.6	34.8
QED	4963.4	4962.0	1155.4	972.54	165.08	41.271	4.5857
QED$_t$	4963.5	4962.1	1155.4	972.57	165.09	41.272	4.5858
EW	4963.4	4962.0	1155.4	972.53	165.08	41.272	4.5885
EW+VP$_t$	5075.0	5073.5	1190.6	1003.3	172.51	43.647	4.9603
EW+VP	5075.0	5073.5	1190.6	1003.3	172.51	43.646	4.9605

Table 1: Various cross sections in nb as a function of the centre-of- mass energy in GeV integrated over the two angular ranges 45–110 mrad and 5–50 mrad. The index t denotes the co[illegible]tion of the corresponding t channel Feynman diagrams alone. The last columns are of interest for [illegible]ture Linear Colliders.

Future Linear Collider

Monte Carlo codes and comparison

The new code SAMBA

The program LABSMC, which was intended to describe large-angle Bhabha scattering at high energies, has been complemented with a set of routines from NLLBHA so as to be applicable to small-angle Bhabha scattering. This implied the insertion of the relevant second-order next–to–leading radiative corrections ($\mathcal{O}(\alpha^2 L)$) in the Monte Carlo code[1], which are crucial to achieve the per mille accuracy. The extension to cover small angles resulted in the new code SAMBHA containing the previously existing features together with the following new characteristics :

- the complete electroweak matrix element at the Born level;
- the complete set of $\mathcal{O}(\alpha)$ QED radiative corrections (including radiation from amplitudes with Z-boson exchange);
- vacuum-polarization corrections by leptons, hadrons [19], and W-bosons;
- 1–loop electroweak radiative corrections and effective EW couplings by means of the DIZET v.6.30 [24] package;
- higher-order leading-logarithm photonic corrections by means of the electron structure functions [25, 26, 27, 28];
- light pair corrections in the $\mathcal{O}(\alpha^2 L^2)$ leading-logarithm approximation including (optionally) the two-photon and singlet mechanisms.

The code is applicable with the following restrictions:

a) $E_{\mathrm{beam}} \gg m_e$: the energy has to be much larger than the electron mass;

b) $m_e/E_{\mathrm{beam}} \ll \theta$: extemely small angles are not described well, but the condition is fulfilled in practice for both small- and large-angle Bhabha measurements in the experiments at LEP, SLC and NLC;

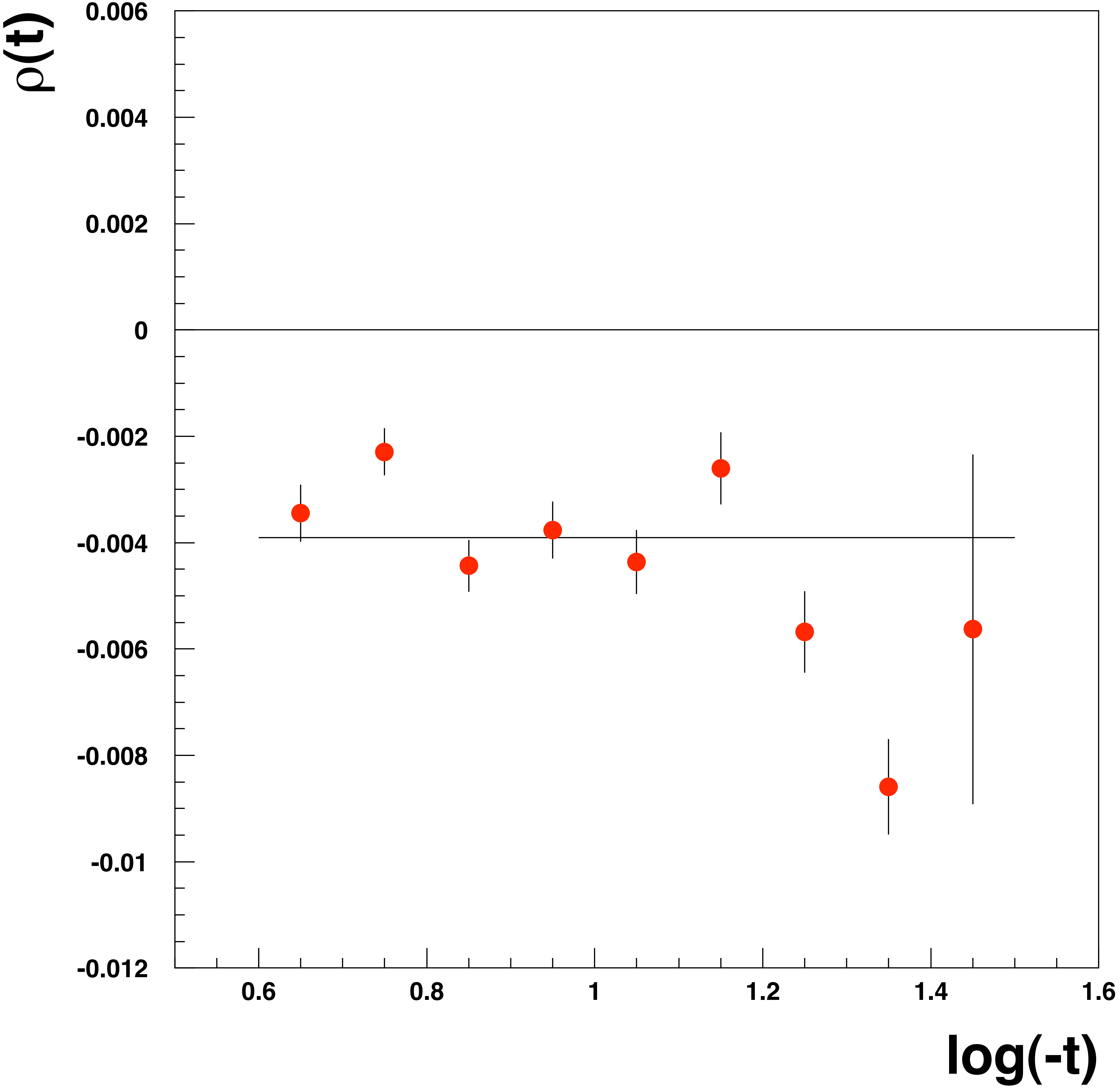
c) starting from the second order in α, real photon emission is integrated over, i.e. events with two photons separated from electrons are not generated.

Comparison of SAMBHA with BHLUMI

BHLUMI is compared with SAMBHA for integral and,
for the first time
also
differential distributions

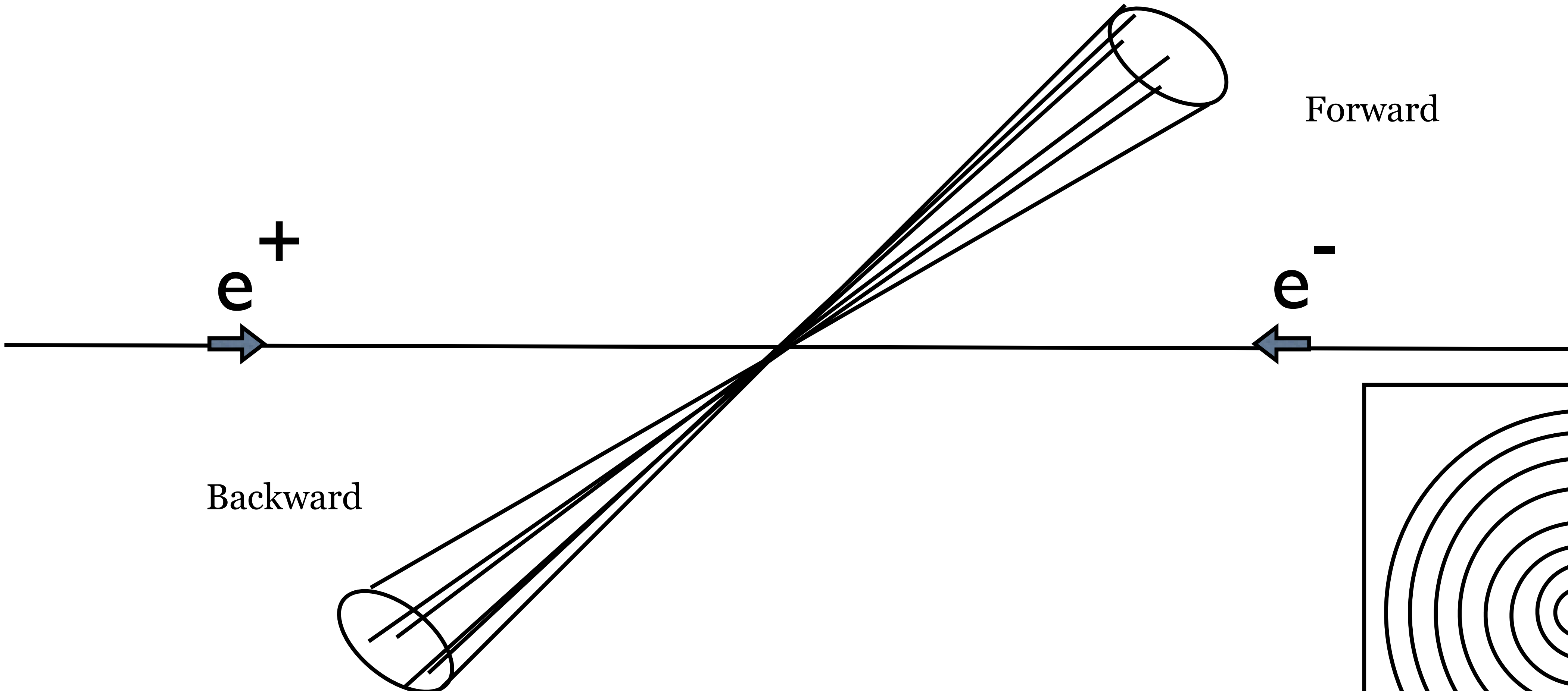
The actual measurements are of calorimetric type
Therefore, event samples are generated with both programs, subjecting each event to a common set of calorimeter-like criteria (CALO)

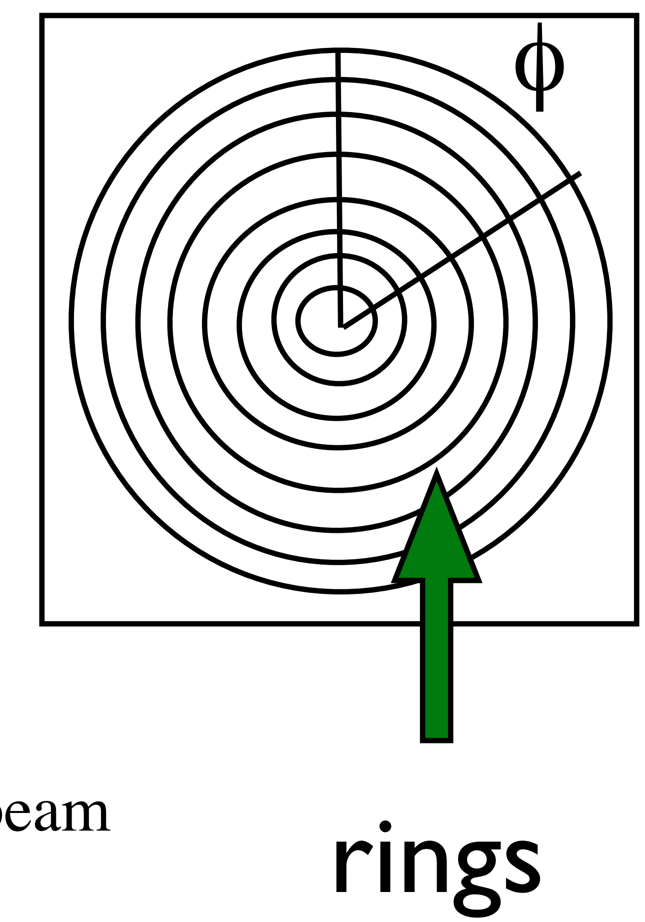
$$\rho(t) = (d\sigma_{sambha}/dt - d\sigma_{bhlumi}/dt) / d\sigma_{bhlumi}/dt$$



ρ(t)
0.006
0.004
0.002
0
-0.002
-0.004
-0.006
-0.008
-0.01
-0.012
0.6
0.8
1
1.2
1.4
1.6
log(-t)

Calorimetric type measurement





1.The cluster is reconstructed

2. Find the cluster center

3. The highest energy cluster in each emisphere F, B

4. Energy Cuts (suppresses the Initial State Radiation) $\min(E_F,E_B)>0.65\ E_{beam}$

$\mathrm{Max}(E_F,E_B)>0.95\ E_{beam}$

5.Take the angle of the center of the most energetic cluster: this defines $\Rightarrow$ θ

6.Repeat this procedure for several "rings" segmented detector to reproduce the differential distribution

Comparison and evaluation

Table 3: Numbers of events generated with BHLUMI

$\sqrt{s}$(GeV)	91.2	189	200
$\int \mathcal{L}\mathrm{d}t$ (pb^{-1})	75	150	200
Ring 2	1844850	863571	1028210
Ring 3	907754	425586	506131
Ring 4	513696	240550	286994
Ring 5	313218	146731	174740
Ring 6	201893	94033	112168

$$\sigma_i = \sigma_i^0 \left(\frac{\alpha(t_i)}{\alpha(0)} \right)^2 (1 + \Delta r_i),$$

$$\begin{aligned}
\sigma_i &= \int^{R_i} \mathrm{d}t \frac{\mathrm{d}\sigma}{\mathrm{d}t} \\
\sigma_i^0 &= \int^{R_i} \mathrm{d}t \frac{\mathrm{d}\sigma^0}{\mathrm{d}t} \\
\left(\frac{\alpha(t_i)}{\alpha(0)} \right)^2 &= \int^{R_i} \frac{\mathrm{d}t}{t_{\mathrm{max}} - t_{\mathrm{min}}} \left(\frac{\alpha(t)}{\alpha(0)} \right)^2, \\
1 + \Delta r_i &= \left(\frac{\alpha(0)}{\alpha(t_i)} \right)^2 \frac{\sigma_i}{\sigma_i^0}
\end{aligned}$$

$$\left(\frac{\alpha(t_i)}{\alpha(0)} \right)^2 = \frac{N_i}{\sigma_i^0 \int \mathcal{L}\mathrm{d}t} \frac{1}{1 + \Delta r_i},$$

$$\left(\frac{\alpha(t)}{\alpha(0)} \right)^2 = (u_0 \pm \delta u_0) + (u_1 \pm \delta u_1) \cdot \log \frac{-t}{\langle -t \rangle}$$

Table 4: Theoretical predictions for each ring of the three factors of eq. 7. For the conditions defined in sect. 5.1 the angular boundary of ring i is θ_i=arctan (7+3(i-1))/220).

No. of ring	1	2	3	4	5	6	7
	$\sqrt{s} = 91.2$ GeV						
σ_i^0	63.077	24.728	12.170	6.8694	4.2517	2.8120	1.9552
$\left(\alpha(t_i)/\alpha(0)\right)^2$	1.0425	1.0475	1.0516	1.0551	1.0582	1.0609	1.0634
$1+\Delta r_i$	0.9426	0.9440	0.9412	0.9395	0.9240	0.8915	0.7982
	$\sqrt{s} = 189$ GeV						
σ_i^0	14.685	5.7563	2.8324	1.5984	0.9889	0.6537	0.4542
$\left(\alpha(t_i)/\alpha(0)\right)^2$	1.0554	1.0613	1.0661	1.0702	1.0736	1.0767	1.0794
$1+\Delta r_i$	0.9377	0.9390	0.9360	0.9329	0.9165	0.8858	0.7898
	$\sqrt{s} = 200$ GeV						
σ_i^0	13.115	5.1406	2.5295	1.4274	0.8831	0.5838	0.4057
$\left(\alpha(t_i)/\alpha(0)\right)^2$	1.0565	1.0625	1.0673	1.0714	1.0749	1.0780	1.0807
$1+\Delta r_i$	0.9376	0.9387	0.9352	0.9330	0.9158	0.8847	0.7896
	$\sqrt{s} = 1000$ GeV						
σ_i^0	0.5248	0.2059	0.1014	0.0573	0.0356	0.0236	0.0165
$\left(\alpha(t_i)/\alpha(0)\right)^2$	1.0921	1.0994	1.1050	1.1096	1.1135	1.1169	1.1199
$1+\Delta r_i$	0.8622	0.8620	0.8590	0.8545	0.8398	0.8084	0.7205
	$\sqrt{s} = 3000$ GeV						
σ_i^0	0.0590	0.0234	0.0117	0.0067	0.0042	0.0028	0.0020
$\left(\alpha(t_i)/\alpha(0)\right)^2$	1.1192	1.1267	1.1325	1.1373	1.1414	1.1448	1.1479
$1+\Delta r_i$	0.8467	0.8457	0.8422	0.8381	0.8253	0.7956	0.6975

Final formula:

$$\left(\frac{\alpha(t_i)}{\alpha(0)}\right)^2 = \frac{N_i}{\sigma_i^0 \int \mathcal{L}\mathrm{d}t} \frac{1}{1+\Delta r_i},$$

Which can be transformed in a linear fit defining the t dependence of α:

$$\boxed{\left(\frac{\alpha(t)}{\alpha(0)}\right)^2 = (u_0 \pm \delta u_0) + (u_1 \pm \delta u_1) \cdot \log \frac{-t}{\langle -t \rangle}}$$

Table 5: Table of fit results; the uncertainties δu_0 and δu_1 are uncorrelated.

$\sqrt{s}$	91.2 GeV	189 GeV	200 GeV
u_0	1.0573±0.0005	1.0698±0.0008	1.0703±0.0007
u_1	0.0242±0.0028	0.0284±0.0041	0.0318±0.0038
$\langle -t \rangle$	8.5 GeV^2	36.6 GeV^2	40.9 GeV^2

$$\int \mathcal{L}\mathrm{d}t = \frac{n_0}{1+2\Delta\alpha(\langle t \rangle)}$$

$$\frac{\delta n_0}{n_0} = 10^{-3}$$

statistical precision

More formulae:

$$
\begin{aligned}
n_0 &= \int \mathcal{L}\mathrm{d}t \cdot \left(1 + 2\Delta\alpha(\langle t \rangle)\right) \\
n_1 &= \int \mathcal{L}\mathrm{d}t \cdot \left(\frac{\mathrm{d}}{\mathrm{d}\log(-t)} 2\Delta\alpha(t)\right).
\end{aligned}
$$

$$n_i = u_i \cdot \int \mathcal{L}\mathrm{d}t$$

$$\frac{\mathrm{d}}{\mathrm{d}\log(-t)}\Delta\alpha = \frac{n_1}{2n_0}\left(1 + 2\Delta\alpha(\langle t \rangle)\right)$$

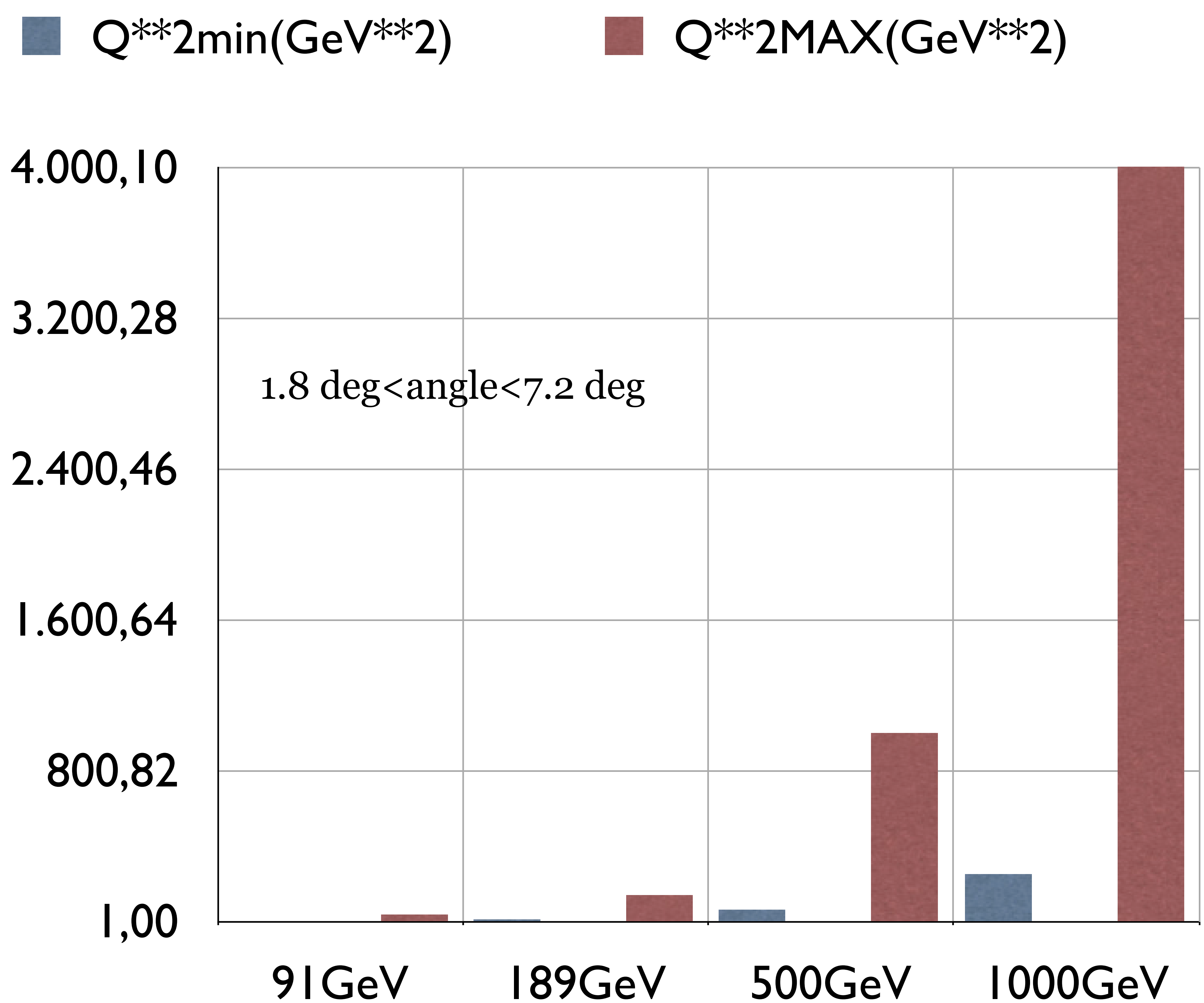
Linear Collider

Let us consider the case of a e+e- collider with $E_{c.m.}$ from 500 to 1000 GeV

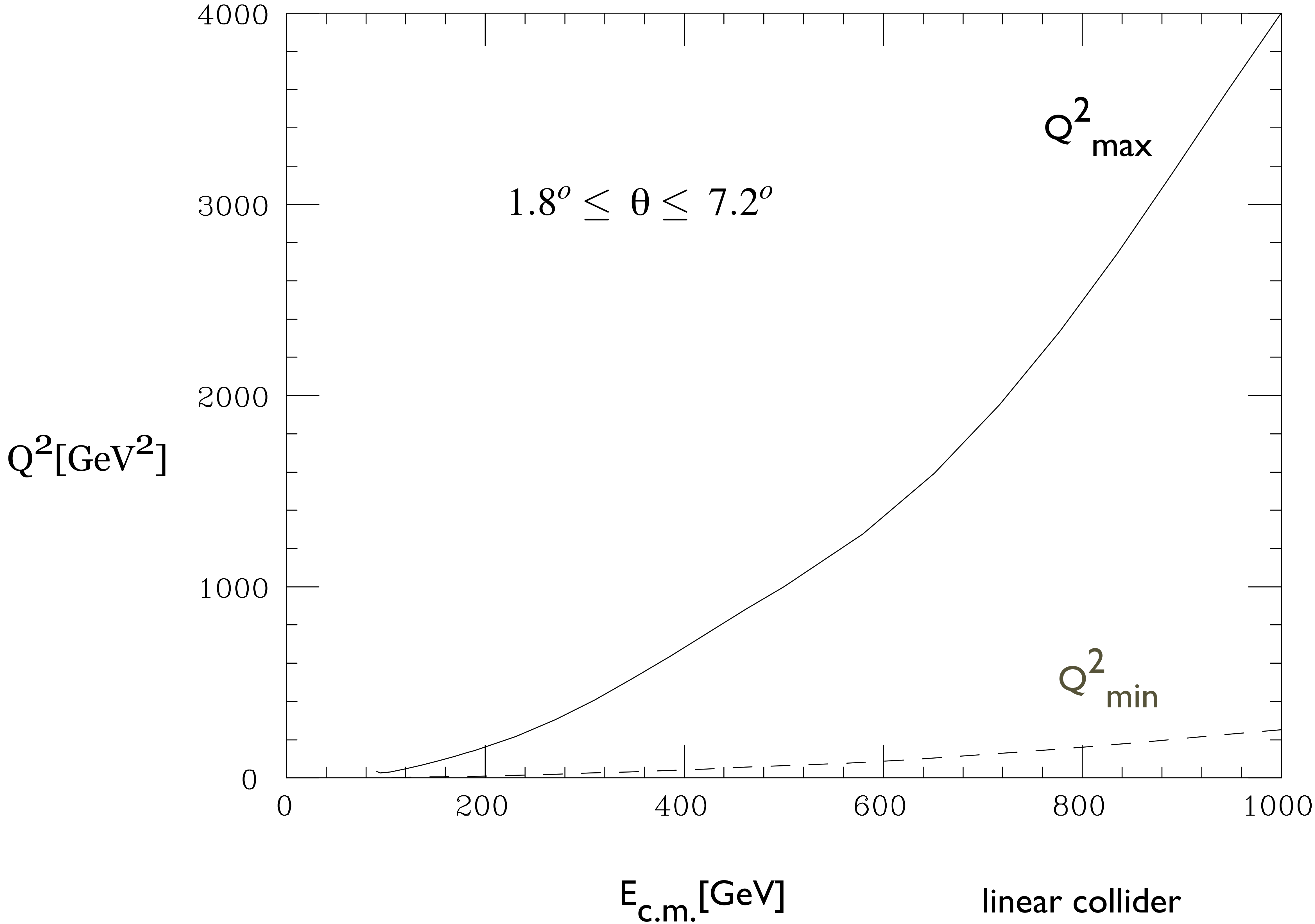
The acceptance angles for the Luminosity determination are:

$$1.8^0 < \theta < 7.2^0$$

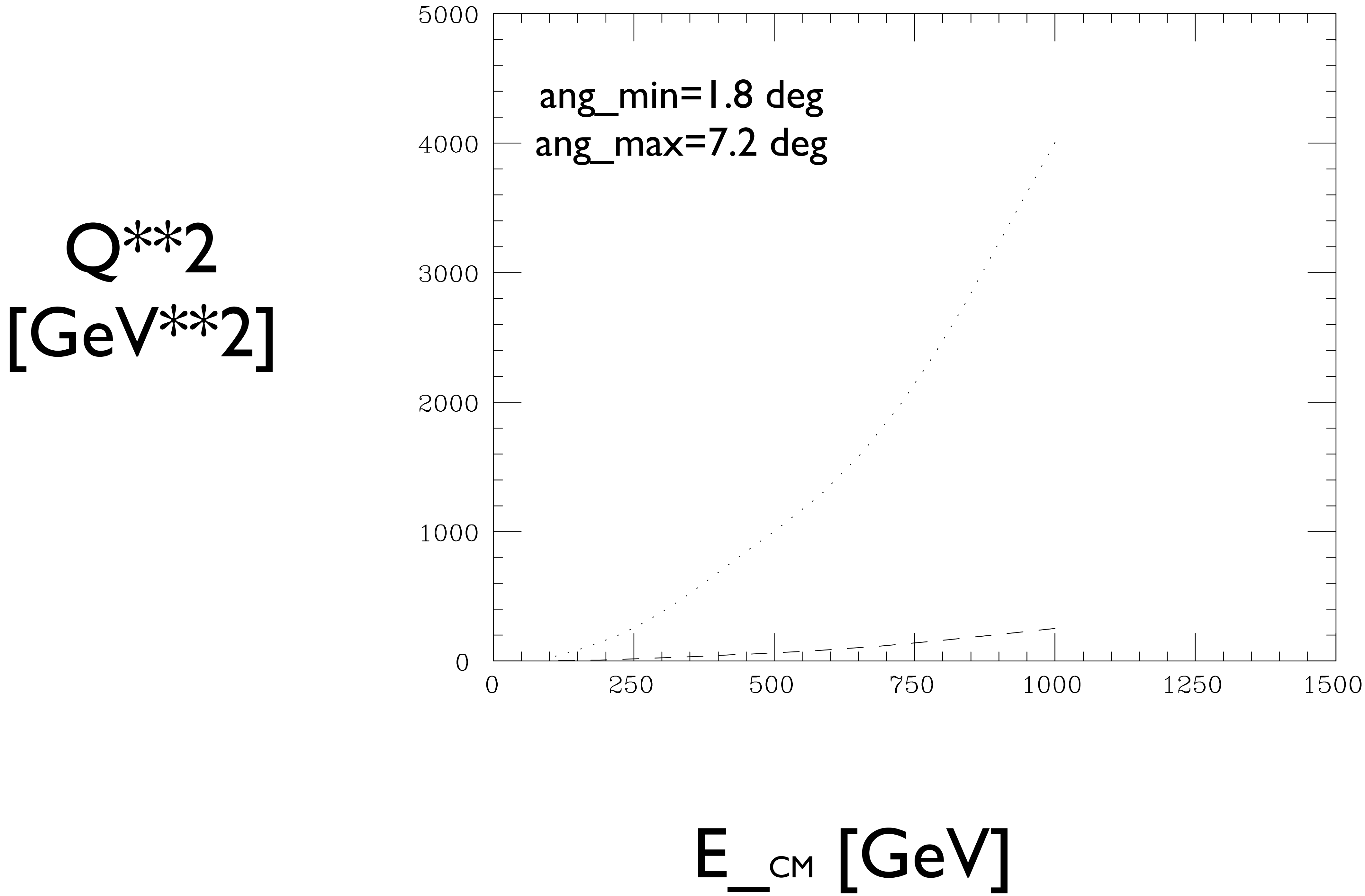
The min and Max values for $t= -Q^2$ can be readily estimated



Q**2min(GeV**2)
Q**2MAX(GeV**2)
4.000,10
3.200,28
2.400,46
1.600,64
800,82
1,00
1.8 deg<angle<7.2 deg
91GeV
189GeV
500GeV
1000GeV



4000
3000
2000
1000
0
Q^2[GeV^2]
$1.8^o \leq \theta \leq 7.2^o$
Q^2_{max}
Q^2_{min}
0
200
400
600
800
1000
$E_{c.m.}$[GeV]
linear collider



Q**2 [GeV**2]
5000
4000
3000
2000
1000
0
0
250
500
750
1000
1250
1500
ang_min=1.8 deg
ang_max=7.2 deg
E_CM [GeV]

Table 3: Numbers of events generated with BHLUMI

$\sqrt{s}$(GeV)	91.2	189	200
$\int \mathcal{L}\mathrm{d}t$ (pb^{-1})	75	150	200
Ring 2	1844850	863571	1028210
Ring 3	907754	425586	506131
Ring 4	513696	240550	286994
Ring 5	313218	146731	174740
Ring 6	201893	94033	112168

SAMBHA. In order to extract the t dependence of $\alpha(t)$, eq. 3 is evaluated for each ring R_i defined by the geometry of the DELPHI luminometer. Equation 3 then reads, for ring i :

$$\sigma_i = \sigma_i^0 \left(\frac{\alpha(t_i)}{\alpha(0)}\right)^2 (1 + \Delta r_i), \tag{7}$$

with the following definitions :

$$\begin{aligned}
\sigma_i &= \int^{R_i} \mathrm{d}t \frac{\mathrm{d}\sigma}{\mathrm{d}t} \\
\sigma_i^0 &= \int^{R_i} \mathrm{d}t \frac{\mathrm{d}\sigma^0}{\mathrm{d}t} \\
\left(\frac{\alpha(t_i)}{\alpha(0)}\right)^2 &= \int^{R_i} \frac{\mathrm{d}t}{t_{\max} - t_{\min}} \left(\frac{\alpha(t)}{\alpha(0)}\right)^2, \\
1 + \Delta r_i &= \left(\frac{\alpha(0)}{\alpha(t_i)}\right)^2 \frac{\sigma_i}{\sigma_i^0}
\end{aligned}$$

Table 4 contains the resulting theoretical values.

Putting together the experimental and theoretical ingredients, i.e. the observed number of events N_i in each ring, together with the relevant luminosities $\int \mathcal{L}\mathrm{d}t$ (from table 3) and σ_i^0, Δr_i (from table 4), we obtain the final formula:

$$\left(\frac{\alpha(t_i)}{\alpha(0)}\right)^2 = \frac{N_i}{\sigma_i^0 \int \mathcal{L}\mathrm{d}t} \frac{1}{1 + \Delta r_i}, \tag{8}$$

which can be exploited in a linear fit to access the parameters defining the t dependence of α :

$$\left(\frac{\alpha(t)}{\alpha(0)}\right)^2 = (u_0 \pm \delta u_0) + (u_1 \pm \delta u_1) \cdot \log \frac{-t}{\langle -t \rangle} \tag{9}$$

The parameters of the fit are listed in table 5.

6 Discussion

Table 5 demonstrates that for the case of the DELPHI setup (see sect. 5) and assuming typical integrated luminosities, the statistical accuracy is sufficient to verify the running of α for each of the three centre-of-mass energies.

Equation 8 can be expanded in terms of $\Delta\alpha$ (see eq. 1). It is convenient to consider

$$\frac{N_i}{\sigma_i^0} \frac{1}{1 + \Delta r_i} = n_0 + n_1 \log \frac{-t_i}{\langle -t \rangle} \tag{10}$$

Table 4: Theoretical predictions for each ring of the three factors of eq. 7. For the conditions defined in sect. 5.1 the angular boundary of ring i is θ_i=arctan (7+3(i-1))/220).

No. of ring	1	2	3	4	5	6	7
	$\sqrt{s} = 91.2$ GeV						
σ_i^0	63.077	24.728	12.170	6.8694	4.2517	2.8120	1.9552
$\left(\alpha(t_i)/\alpha(0)\right)^2$	1.0425	1.0475	1.0516	1.0551	1.0582	1.0609	1.0634
$1+\Delta r_i$	0.9426	0.9440	0.9412	0.9395	0.9240	0.8915	0.7982
	$\sqrt{s} = 189$ GeV						
σ_i^0	14.685	5.7563	2.8324	1.5984	0.9889	0.6537	0.4542
$\left(\alpha(t_i)/\alpha(0)\right)^2$	1.0554	1.0613	1.0661	1.0702	1.0736	1.0767	1.0794
$1+\Delta r_i$	0.9377	0.9390	0.9360	0.9329	0.9165	0.8858	0.7898
	$\sqrt{s} = 200$ GeV						
σ_i^0	13.115	5.1406	2.5295	1.4274	0.8831	0.5838	0.4057
$\left(\alpha(t_i)/\alpha(0)\right)^2$	1.0565	1.0625	1.0673	1.0714	1.0749	1.0780	1.0807
$1+\Delta r_i$	0.9376	0.9387	0.9352	0.9330	0.9158	0.8847	0.7896
	$\sqrt{s} = 1000$ GeV						
σ_i^0	0.5248	0.2059	0.1014	0.0573	0.0356	0.0236	0.0165
$\left(\alpha(t_i)/\alpha(0)\right)^2$	1.0921	1.0994	1.1050	1.1096	1.1135	1.1169	1.1199
$1+\Delta r_i$	0.8622	0.8620	0.8590	0.8545	0.8398	0.8084	0.7205
	$\sqrt{s} = 3000$ GeV						
σ_i^0	0.0590	0.0234	0.0117	0.0067	0.0042	0.0028	0.0020
$\left(\alpha(t_i)/\alpha(0)\right)^2$	1.1192	1.1267	1.1325	1.1373	1.1414	1.1448	1.1479
$1+\Delta r_i$	0.8467	0.8457	0.8422	0.8381	0.8253	0.7956	0.6975

rather than eq. 8, since in practice the integrated luminosity $\int \mathcal{L}dt$ is not known. The two coefficients n_0 and n_1 are obtained from a linear fit and contain the information on both the data and theory. Their interpretation is :

$$\begin{aligned} n_0 &= \int \mathcal{L}\mathrm{d}t \cdot \left(1 + 2\Delta\alpha(\langle t\rangle)\right) \\ n_1 &= \int \mathcal{L}\mathrm{d}t \cdot \left(\frac{\mathrm{d}}{\mathrm{d}\log(-t)} 2\Delta\alpha(t)\right). \end{aligned}$$

The dependence on the integrated luminosity is given explicitly: obviously, one has $n_i = u_i \cdot \int \mathcal{L}\mathrm{d}t$ by comparing eqs. 8, 9,10.

In the ratio n_1/n_0 the dependence of the integrated luminosity drops out :

$$\frac{\mathrm{d}}{\mathrm{d}\log(-t)}\Delta\alpha = \frac{n_1}{2n_0}\left(1 + 2\Delta\alpha(\langle t\rangle)\right)$$

The slope $\mathrm{d}\Delta\alpha/\mathrm{d}\log(-t)$, the quantity of interest, is then directly given by the ratio of the two experimentally measured quantities n_0 and n_1, namely $n_1/2n_0$. The contribution of $2\Delta\alpha(\langle t\rangle)$ is small with respect to 1 and can be neglected. The accuracy of the slope is determined by $\delta n_1/2n_0$, i.e. about 10% (see table 5), which is far smaller than the absolute value of $n_1/2n_0$.

Table 5: Table of fit results; the uncertainties δu_0 and δu_1 are uncorrelated.

$\sqrt{s}$	91.2 GeV	189 GeV	200 GeV
u_0	1.0573±0.0005	1.0698±0.0008	1.0703±0.0007
u_1	0.0242±0.0028	0.0284±0.0041	0.0318±0.0038
$\langle -t \rangle$	8.5 GeV^2	36.6 GeV^2	40.9 GeV^2

On the other hand, n_0 relates the integrated luminosity to $\Delta\alpha$ at the average value of t

$$\int \mathcal{L}\mathrm{d}t = \frac{n_0}{1 + 2\Delta\alpha(\langle t \rangle)}$$

Making use of $\Delta\alpha(\langle t \rangle)$ as a priori knowledge the fitted n_0 can be used to derive the integrated luminosity, which is the standard procedure. The statistical precision is given by $\delta n_0/n_0$, which is of the order of 10^{-3}.

In addition, the hadronic contribution to $\Delta\alpha(t)$ (see fig. 2) may be deduced by subtracting the leptonic contribution, which is theoretically known precisely. The extraction of the hadronic contribution is only limited by the experimental precision.

7 Conclusions

A novel approach to access directly and to measure the running of α in the space-like region is proposed. It consists in analysing small-angle Bhabha scattering. Depending on the particular angular detector coverage and on the energy of the beams, it allows a sizeable range of the t variable to be covered.

The feasibility of the method has been put in evidence by the use of a new tool, `SAMBHA` , to calculate the small-angle Bhabha differential cross section with a theoretical accuracy of better than 0.1%.

The information obtained in the t channel can be compared with the existing results of the s channel measurements. This represents a complementary approach, which is direct, transparent and based only on QED interactions and furthermore free of some of the drawbacks inherent in the s channel methods.

The method outlined can be readily applied to the experiments at LEP and SLC. It can also be exploited by future e^+e^- colliders as well as by existing lower energy machines.

An extremely precise measurement of the QED running coupling $\Delta\alpha(t)$ for small values of t may be envisaged with a dedicated luminometer even at low machine energies.

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Table 3: Numbers of events generated with BHLUMI

$\sqrt{s}$(GeV)	91.2	189	200
$\int \mathcal{L}dt$ (pb^{-1})	75	150	200
Ring 2	1844850	863571	1028210
Ring 3	907754	425586	506131
Ring 4	513696	240550	286994
Ring 5	313218	146731	174740
Ring 6	201893	94033	112168

SAMBHA. In order to extract the t dependence of $\alpha(t)$, eq. 3 is evaluated for each ring R_i defined by the geometry of the DELPHI luminometer. Equation 3 then reads, for ring i :

$$\sigma_i = \sigma_i^0 \left(\frac{\alpha(t_i)}{\alpha(0)}\right)^2 (1 + \Delta r_i), \tag{7}$$

with the following definitions :

$$\begin{aligned}
\sigma_i &= \int^{R_i} \mathrm{d}t \frac{\mathrm{d}\sigma}{\mathrm{d}t} \\
\sigma_i^0 &= \int^{R_i} \mathrm{d}t \frac{\mathrm{d}\sigma^0}{\mathrm{d}t} \\
\left(\frac{\alpha(t_i)}{\alpha(0)}\right)^2 &= \int^{R_i} \frac{\mathrm{d}t}{t_{\max} - t_{\min}} \left(\frac{\alpha(t)}{\alpha(0)}\right)^2, \\
1 + \Delta r_i &= \left(\frac{\alpha(0)}{\alpha(t_i)}\right)^2 \frac{\sigma_i}{\sigma_i^0}
\end{aligned}$$

Table 4 contains the resulting theoretical values.

Putting together the experimental and theoretical ingredients, i.e. the observed number of events N_i in each ring, together with the relevant luminosities $\int \mathcal{L}\mathrm{d}t$ (from table 3) and σ_i^0, Δr_i (from table 4), we obtain the final formula:

$$\left(\frac{\alpha(t_i)}{\alpha(0)}\right)^2 = \frac{N_i}{\sigma_i^0 \int \mathcal{L}\mathrm{d}t} \frac{1}{1 + \Delta r_i}, \tag{8}$$

which can be exploited in a linear fit to access the parameters defining the t dependence of α :

$$\left(\frac{\alpha(t)}{\alpha(0)}\right)^2 = (u_0 \pm \delta u_0) + (u_1 \pm \delta u_1) \cdot \log \frac{-t}{\langle -t \rangle} \tag{9}$$

The parameters of the fit are listed in table 5.

6 Discussion

Table 5 demonstrates that for the case of the DELPHI setup (see sect. 5) and assuming typical integrated luminosities, the statistical accuracy is sufficient to verify the running of α for each of the three centre-of-mass energies.

Equation 8 can be expanded in terms of $\Delta\alpha$ (see eq. 1). It is convenient to consider

$$\frac{N_i}{\sigma_i^0} \frac{1}{1 + \Delta r_i} = n_0 + n_1 \log \frac{-t_i}{\langle -t \rangle} \tag{10}$$

Conclusions

- We propose a novel approach to access directly and to measure the running of α_{QED} in the space-like region .

- It consists in analysing small-angle Bhabha scattering. Depending on the particular angular detector coverage and on the energy of the beams, it allows a sizeable range of the t variable to be covered.

- The feasibility of the method has been put in evidence by the use of a new tool, SAMBHA , to calculate the small-angle Bhabha differential cross section with a theoretical accuracy of better than 0.1%.

- The information obtained in the t channel can be compared with the existing results of the s channel measurements. This represents a complementary approach, which is direct, transparent and based only on QED interactions and furthermore free of some of the drawbacks inherent in the s channel methods.

- The method outlined can be readily applied to the experiments at LEP and SLC. It can also be exploited by future e+e– colliders as well as by existing lower energy machines.

- An extremely precise measurement of the QED running coupling $\Delta\alpha(t)$ for small values of t may be envisaged with a dedicated luminometer even at low machine energies.

$\sqrt{s}$ (GeV)	91.187	91.2	189	206	500	1000	3000
	45 mrad $<\ \theta\ <$ 110 mrad						
$\sqrt{\langle -t\rangle}$ (GeV)	3.4	3.4	7.1	7.7	18.8	37.5	112.6
QED	51.428	51.413	11.971	10.077	1.7105	0.42763	0.047514
QED$_t$	51.484	51.469	11.984	10.088	1.7124	0.42809	0.047566
EW	51.436	51.413	11.965	10.072	1.7105	0.42871	0.049507
EW+VP$_t$	54.041	54.018	12.743	10.745	1.8590	0.47303	0.055748
EW+VP	54.036	54.013	12.742	10.744	1.8588	0.47296	0.055742
	5 mrad $<\ \theta\ <$ 50 mrad						
$\sqrt{\langle -t\rangle}$ (GeV)	1.1	1.1	2.2	2.4	5.8	11.6	34.8
QED	4963.4	4962.0	1155.4	972.54	165.08	41.271	4.5857
QED$_t$	4963.5	4962.1	1155.4	972.57	165.09	41.272	4.5858
EW	4963.4	4962.0	1155.4	972.53	165.08	41.272	4.5885
EW+VP$_t$	5075.0	5073.5	1190.6	1003.3	172.51	43.647	4.9603
EW+VP	5075.0	5073.5	1190.6	1003.3	172.51	43.646	4.9605

fermion-loop insertions into the virtual photon lines:

$$\begin{aligned}\Pi(t) &= \frac{\alpha_0}{\pi}\left(\delta_t + \frac{1}{3}L - \frac{5}{9}\right) \\ &+ \left(\frac{\alpha_0}{\pi}\right)^2\left(\frac{1}{4}\,L + \zeta(3) - \frac{5}{24}\right) \\ &+ \left(\frac{\alpha_0}{\pi}\right)^3 \Pi^{(3)}(t) + \mathcal{O}\left(\frac{m_e^2}{t}\right),\end{aligned}$$

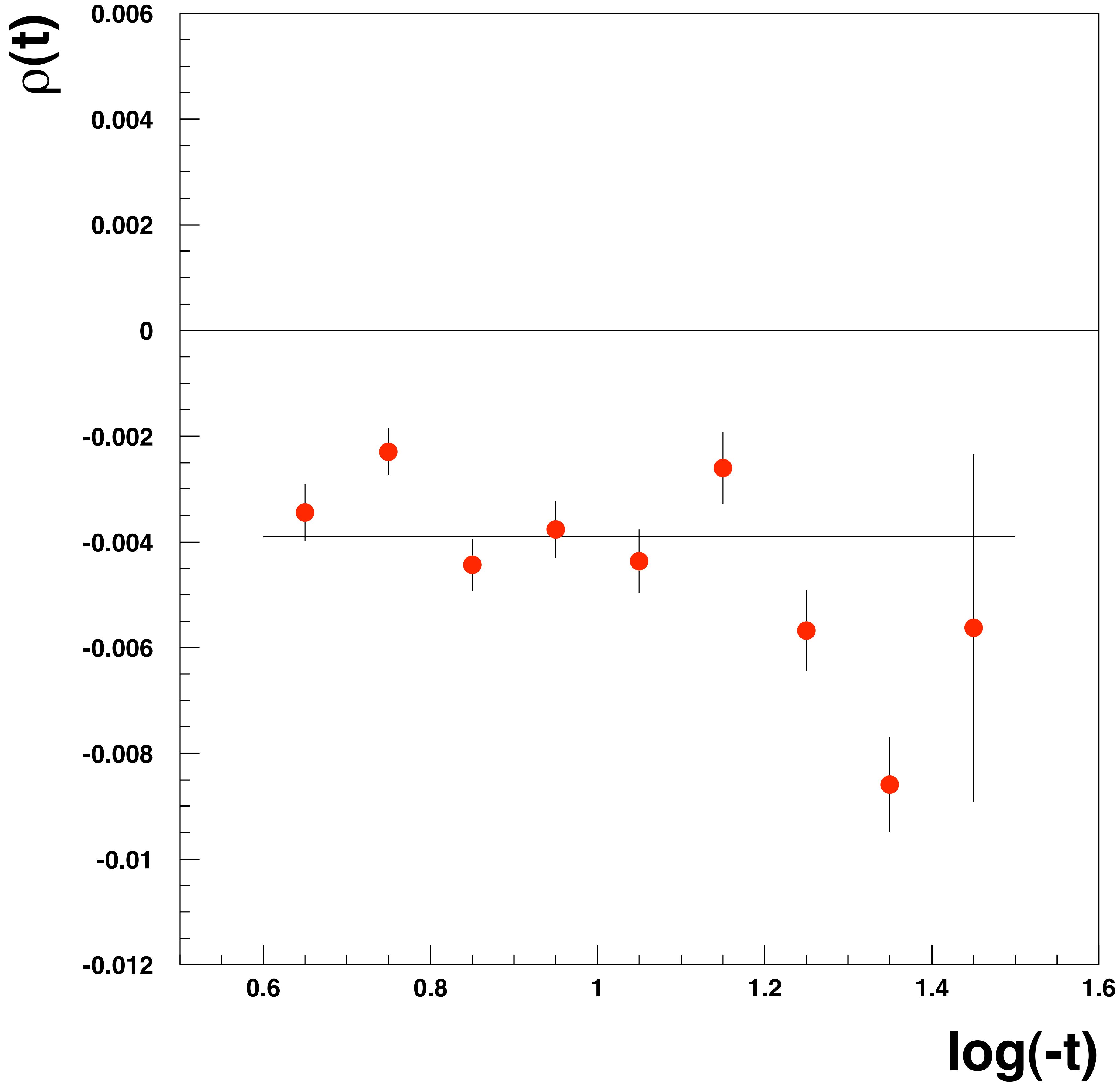
where

$$L = \ln\frac{Q^2}{m_e^2}, \qquad Q^2 = -t, \qquad \zeta(3) = 1.202$$

and where the leading part of the two-loop contribution to the polarization operator is taken into account. The most significant part arises from the electrons and is $L/3 - 5/9$.

The $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha^2)$ leptonic vacuum polarization has been known for many years [17]. The third-order (three–loop) leptonic contributions $\Pi^{(3)}(t)$ have recently been calculated [18]. In the Standard Model, δ_t contains contributions from muons, τ-leptons, W-bosons and hadrons :

$$\begin{aligned}\delta_t &= \delta_t^{\mu} + \delta_t^{\tau} + \delta_t^{W} + \delta_t^{H}, \\ \delta_s &= \delta_t\ (t \to s),\end{aligned}$$



ρ(t)
0.006
0.004
0.002
0
-0.002
-0.004
-0.006
-0.008
-0.01
-0.012
0.6
0.8
1
1.2
1.4
1.6
log(-t)

$$\frac{\mathrm{d}\sigma^B}{\mathrm{d}t} = \frac{\pi\alpha_0^2}{2s^2}\mathrm{Re}\{B_t + B_s + B_i\},$$

$$\begin{aligned}
B_t &= \left(\frac{s}{t}\right)^2 \Bigg\{ \frac{5+2c+c^2}{(1-\Pi(t))^2} + \xi \frac{2(g_v^2+g_a^2)(5+2c+c^2)}{(1-\Pi(t))} \\
&+ \xi^2 \Big(4(g_v^2+g_a^2)^2 + (1+c)^2(g_v^4+g_a^4+6g_v^2g_a^2) \Big) \Bigg\} \\
B_s &= \frac{2(1+c^2)}{|1-\Pi(s)|^2} + 2\chi \frac{(1-c)^2(g_v^2-g_a^2)+(1+c)^2(g_v^2+g_a^2)}{1-\Pi(s)} \\
&+ \chi^2\left[(1-c)^2(g_v^2-g_a^2)^2 + (1+c)^2(g_v^4+g_a^4+6g_v^2g_a^2)\right] \\
B_i &= 2\frac{s}{t}(1+c)^2 \Bigg\{ \frac{1}{(1-\Pi(t))(1-\Pi(s))} \\
&+ (g_v^2+g_a^2)\left(\frac{\xi}{1-\Pi(s)} + \frac{\chi}{1-\Pi(t)}\right) \\
&+ (g_v^4+6g_v^2g_a^2+g_a^4)\xi\chi \Bigg\}
\end{aligned}$$