# Strings for Everybody

DESY, Hamburg

November 05

# Strings for Every Body

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# Introduction: S/W-coupling dualities

Dualities (strong-weak coupling) have been main theme of string theory in last decade.

Generalization of Kramers-Wannier duality in 2D Ising model

non-perturbatively defined !

$$E = -\varepsilon \sum s_i s_j$$

$$Z(\beta) \sim \Sigma_{\{s\}} e^{-\beta E} \sim \Sigma_{\{s\}} e^{-\beta^* E} \sim Z(\beta^*)$$

$$\beta^* = -\tfrac{1}{2} \log \tanh \beta$$

**β small**

**↔ β* large**

# Introduction: S/W-coupling dualities

2D Sine-Gordon model  Massive Thirring model

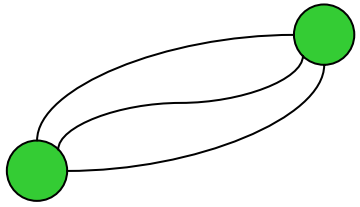
**Boson Φ;  V(Φ) ~ cos b Φ**  **Fermion Ψ;  V(Ψ) ~ g Ψ⁴**

$b^2 \sim 1/(1+g)$

*Is there dual to strongly coupled gauge theory ?*

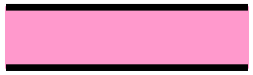
Proposal: Strongly coupled gauge theory is dual to weakly coupled strings.

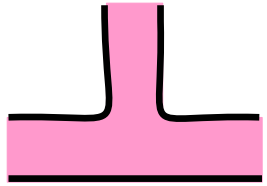
**Recall: QCD flux tubes ~ strings**



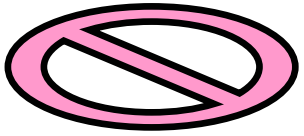
# Intro: ‘t Hooft’s analysis of large N

$$S(A) \sim \frac{1}{g^2} * \int \mathrm{Tr}\,(\partial A\,\partial A + A^3 + \ldots)$$

propagator ~ $g^2$ ~ 

3-vertex ~ $1/g^2$ ~ 

$$Z^{YM} \sim \Sigma_{\text{double line networks } \Gamma} N^L\, g^{2E-2V}\, f(\Gamma) \sim \Sigma\, N^{V-E+L}\, \lambda^{E-V}\, f(\Gamma)$$



~ $g^2\, N^3 \sim \lambda\, N^2$

$\lambda = g^2 N$

`t Hooft coupling

$$\sim \Sigma\, N^{2-2h}\, \lambda^{E-V}\, f(\Gamma) \sim \Sigma_h\, (g_s)^{2h-2}\, F(X_\lambda) \sim Z^{\text{string}}$$

Network $\Gamma \rightarrow$ 2D surface $S_\Gamma$

h = # handles of $S_\Gamma$

$g_s = 1/N$

String coupl.

# Plan of talk

Part 0: Introduction **(done)**

Part I: Elements of modern string theory

**closed strings & SUGRA, p-branes,**

**branes & gauge theories, dualities.**

Part II: AdS/CFT duality – claim & tests

**N=4 SYM: scaling dimension & dilaton**

**operator, string in AdS, test of duality.**

Part III: Conclusions and Outlook

# Part I: Closed strings and gravity

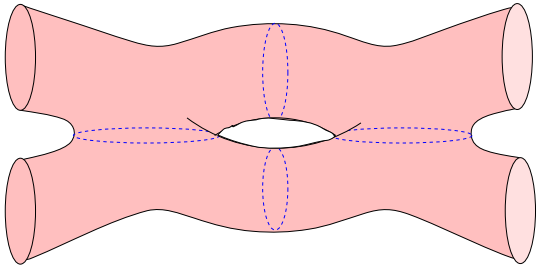
Closed string theory in background X

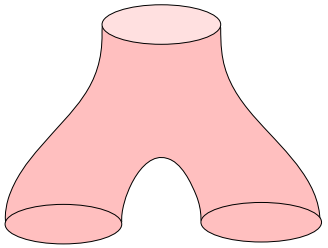
Infinite tower of vibrational modes $M \sim n/\ell_s$

String length

String interaction through 3-vertex $g_s$

String coupling





At <u>low energies</u> ($E << \ell_s$): massless closed str.

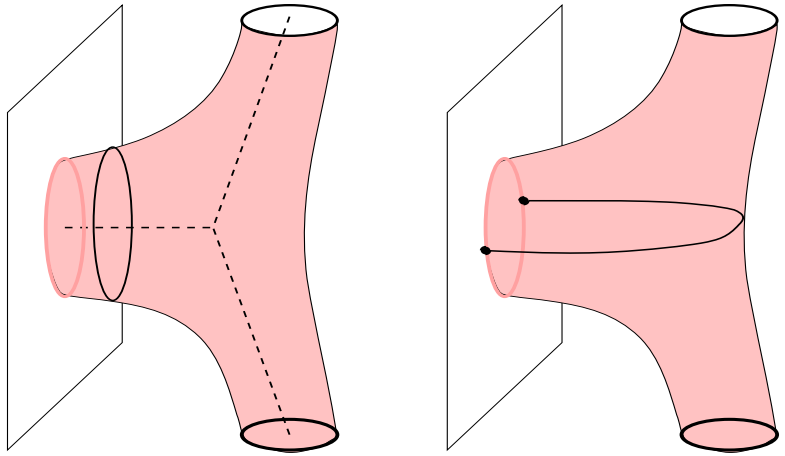
modes behave like gravitons → 10D SUGRA

# Part I: solitonic p- and D-branes

10-dim SUGRA has solutions describing massive & charged objects localized on p+1 - dim. surfaces.

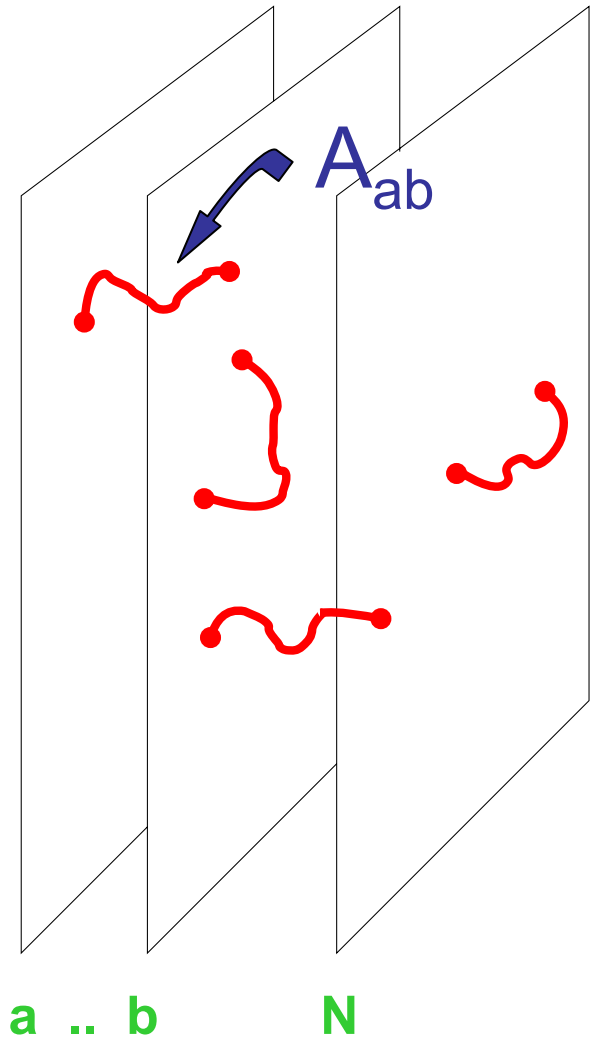
[black branes]

*Branes in string theory ?*



D-branes are objects on which open strings can end [Polchinski]

# Part I: Open strings & gauge theory

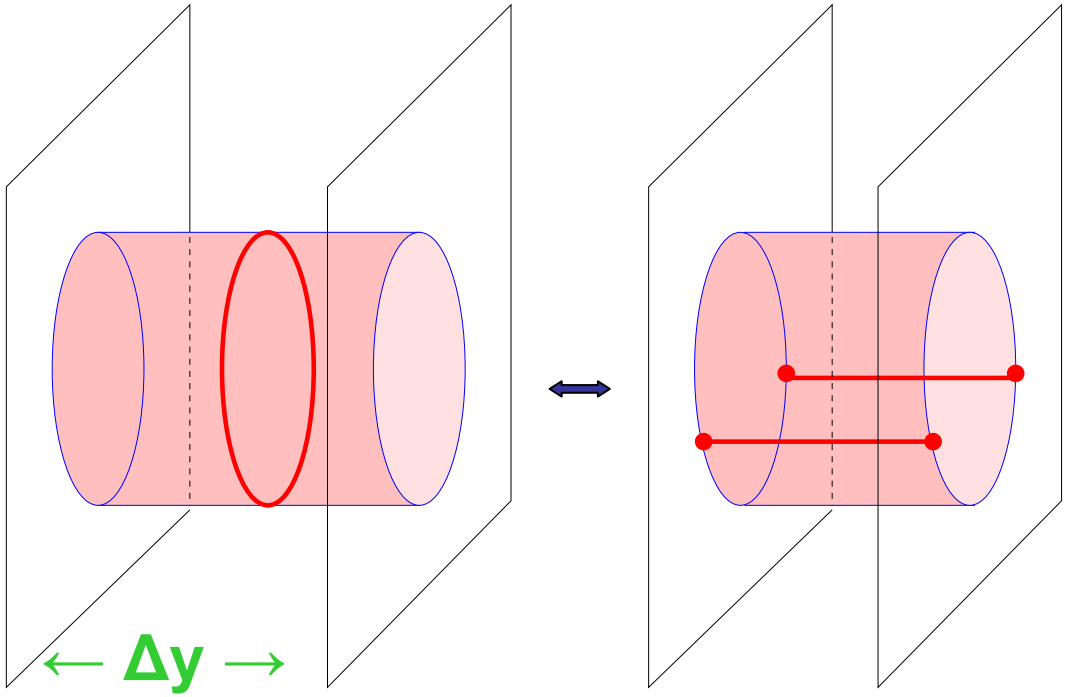



At low energies ($E<<\ell_s$):

massless modes of the open string behave like gauge bosons + matter on the world-volume of D-branes.

**p+1- dimensional gauge theory**

# Part I: Gauge-String theory dualities




Depending on Δy

one side simpler

$\Delta y << \ell_s$: gauge th

$\Delta y >> \ell_s$: SUGRA

- Loop order is not preserved

**e.g. classical string → quantum gauge theory**

- Closed string propagates in different dim. !

# Part II: The $AdS_5 / CFT_4$ duality

**Gauge theory on a stack of D3-branes** ↔ **Strings in the near horizon geometry**

$\mathcal{N}$=4 4D S Yang-Mills — Closed str. in $AdS_5 x S^5$

**U(N) gauge field A; 6 scalars Φ**

**& their fermionic superpartners**

$$x_0^2 + x_1^2 - x_2^2 - x_3^2 - x_4^2 - x_5^2 = R^2$$

4-dim conformal QFT — Curved backgrnd $X_R$

$\lambda = g^2_{YM} N \quad \sim \quad R^4 / \ell_s^4$

# Part II: Scaling dimension in CFT

Scaling dimension $\Delta$ characterizes behavior of fields under scale trafos: $\Psi(ax) \rightarrow a^{-\Delta}\Psi(x)$

Class. theory: $A(x), \Phi(x), \ldots Tr(\Phi_1(x)\Phi_2(x)\partial\Phi_1(x))$

$\Delta = 1 \quad \ldots \quad \Delta = 1+1+1+1 = 4$

<u>Quantum theory</u>: Scaling dimension encoded in

$$D = D_0 + \lambda D_1 + \lambda^2 D_2 + \ldots.$$

Degenerate Perturbation theory

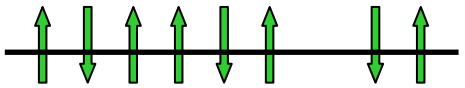
**Dilaton operator** — **diagonal** $D_0$ **+** $\lambda D_1 + \lambda^2 D_2$ **perturbative corrections**

**diag($\Delta$)** — **Feynman diagrams**

# Part II: Dilaton operator of SYM

Space of (single trace) composite fields ~ state space of one-dimensional spin chain :

e.g. $\mathrm{Tr}\,(Z\,\bar{Z}\,Z\,Z\,\bar{Z}\,Z\,\ldots\,\bar{Z}\,Z)$



$Z = \Phi_1 + i\Phi_2\,,\ \bar{Z} = \Phi_1 - i\Phi_2$

n = # of spins

$D_1$ acts like Hamiltonian of spin chain model (e.g. of Heisenberg model) on fields in SYM

$D_1 \sim H \sim \Sigma\, \sigma_i\, \sigma_{i+1}$

Diagonalizable for small n & in the limit n >> 1 (TBA)

# Part II: Spectrum for string in AdS

For a particle moving freely in some background X

$\rightarrow$ spectrum of Laplace operator $H = \Delta_X$

e.g. real line: $H = -\partial^2 \Rightarrow E = p^2$ **AdS harder (bound states)**

Analogous infinite dimensional problem for a string in AdS not solved (spectrum unknown)

String spectrum known • $R/\ell_s >> 1$ (flat space)

• semi-classical limit (large quantum numbers)

e.g. rotating strings => $E = E_J(R)$ $J >> 1$ **spin J**

# Part II: Tests of AdS/CFT duality

**Spectrum in AdS ↔ Scaling dim. in SYM**

Semi-classical limit: String states with large J

↔ long composite operators in gauge theory.

$$E_J(\rho) \sim E_J^{(0)} + E_J^{(1)}\,\rho + .. \leftrightarrow\ D \sim D_0 + D_1\,\lambda + ...$$

$\rho = R/\ell_s$

$D_1$ acts like Hamiltonian of spin chain model

(e.g. of Heisenberg model) on fields in SYM

**classical spinning strings ↔ diag. of spin chain H (TBA)**

# Part III: Outlook and conclusions

AdS/CFT dualities generalize to field theories

- in other dimensions (e.g. $AdS_3/CFT_2$) **solvable ?**
- with less symmetry (less SUSY; w/o scale inv.)

**$S^5 \to X^5$ ; confining theories**

<u>Open problem:</u> solution of perturbative string theory in AdS spaces (spectrum, interaction).

$\leftrightarrow$ 2D σ-model on super spaces $\leftrightarrow$ **cond mat.**

**$S_{Pol}(X) \sim \int dxdt\, G_{ab}(X)\, \partial X^a\, \partial X^b$** **$AdS_3 \sim PSU(1,1|2)$ …**

# Part III: Further aspects of duality

Tempting to read duality in opposite direction:

Gauge theory $\rightarrow$ stringy extension of gravity

**cosmological back-grounds, black holes**

Every Body for Strings

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