



Deep-inelastic scattering — *from HERA to LHC* —

Sven-Olaf Moch

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– Festkolloquium für Fridger Schrempf, Hamburg, February 19, 2007 –

Proton colliders

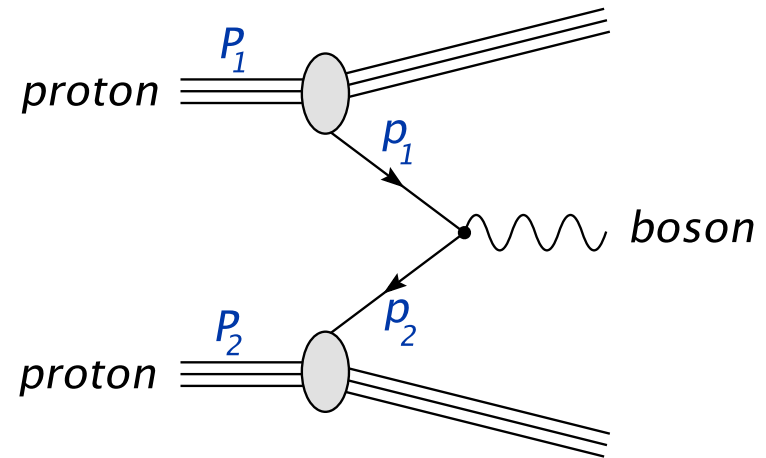
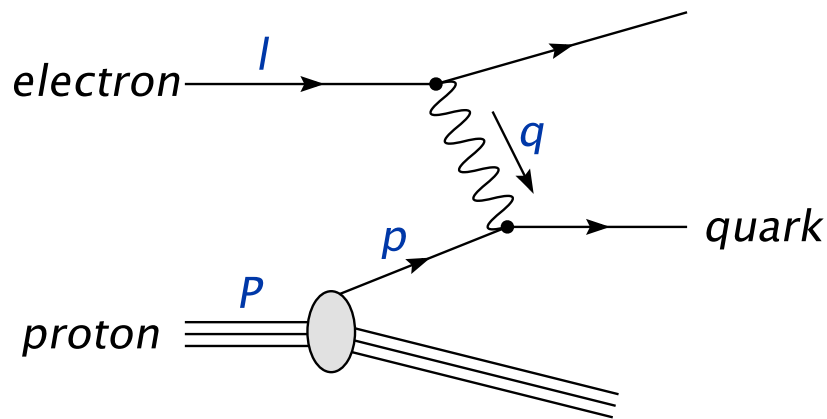
- HERA: deep structure of proton at highest Q^2 and smallest x
- LHC: Higgs boson search at highest energies $\sqrt{S} = 14\text{TeV}$



- Quantum Chromodynamics (QCD) ubiquitous at proton colliders
 - reliable understanding essential for precision and discovery physics

Perturbative QCD at colliders

- Hard scattering at hadron colliders
 - constituent partons from incoming hadrons interact at short distance (large momentum transfer Q^2)



- QCD factorization
 - separate sensitivity to dynamics from different scales
- **DIS**: photon momentum $Q^2 = -q^2$, Bjorken's $x = Q^2 / (2p \cdot q)$
- **pp**: energy $Q^2 = 2p_1 \cdot p_2$, ratio to W/Z -boson mass $x = M_{W/Z}^2 / Q^2$

Hard scattering cross section

- QCD factorization

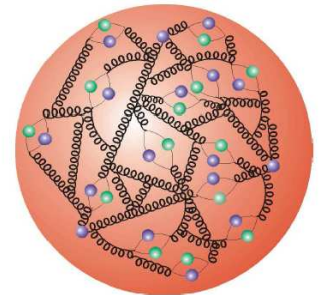
$$\sigma_{had} = \begin{cases} \sum_i f_i \otimes \hat{\sigma}_{\gamma i \rightarrow X}(\alpha_s(\mu^2), Q^2, \mu^2) & \text{DIS} \\ \sum_{ij} f_i \otimes f_j \otimes \hat{\sigma}_{ij \rightarrow X}(\alpha_s(\mu^2), Q^2, \mu^2) & \text{pp} \end{cases}$$

- Parton cross sections $\hat{\sigma}_{\gamma i \rightarrow X}$ and $\hat{\sigma}_{ij \rightarrow X}$ calculable perturbatively in powers of α_s
 - short distance interaction of constituent partons

- Parton distributions $f_i \rightarrow$ parton luminosity
 - proton: very complicated multi-particle bound state
 - colliders: wide-band beams of quarks and gluons

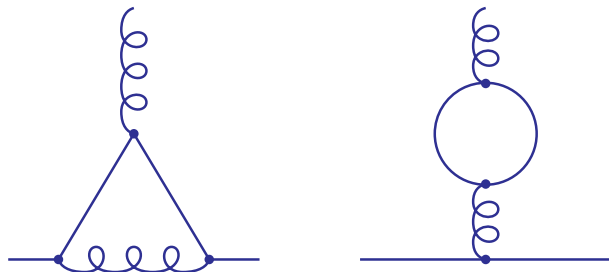
- Standard approach to uncertainties in theoretical predictions

- variation of factorization scale μ : $\frac{d}{d \ln \mu^2} \sigma_{had} = \mathcal{O}(\alpha_s^{l+1})$

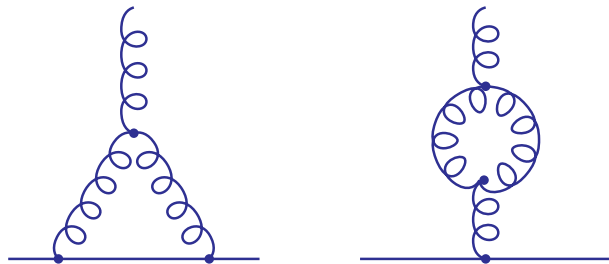


Asymptotic freedom of QCD

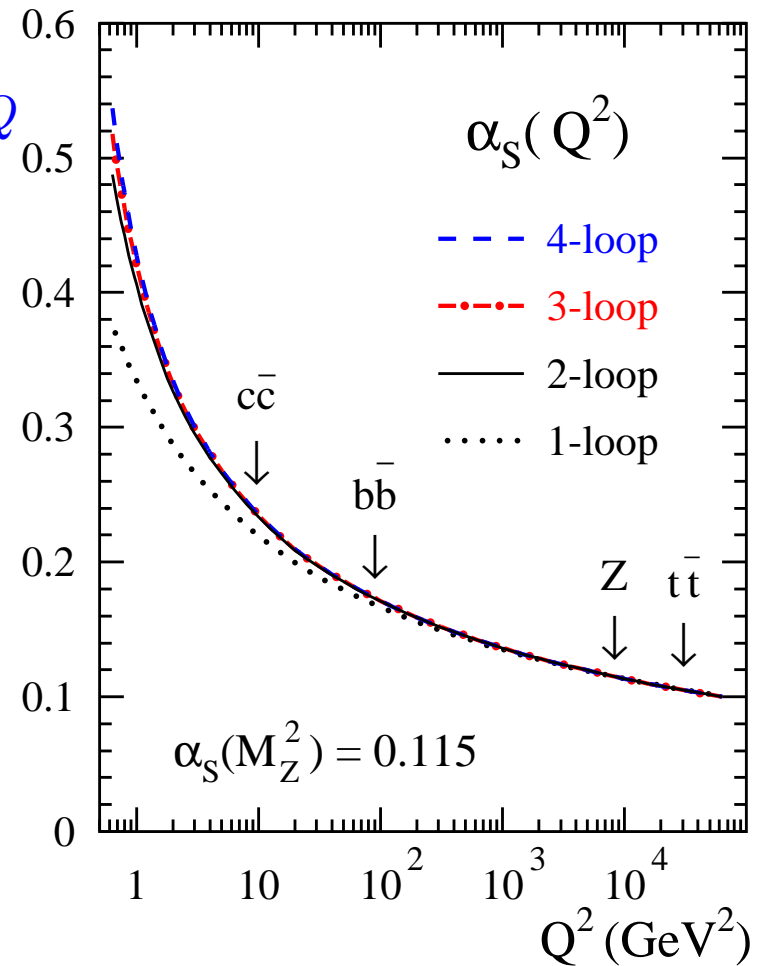
- Effective coupling constant α_s depends on resolution, momentum scale Q



– screening (like in QED)



– anti-screening (color charge of g)



- At large scales: application of perturbation theory (but $\alpha_s \gg \alpha_{\text{QED}}$)

Approaches to the calculation of σ_{had}

- LO (leading order)
 - Automated tree level calculations in Standard Model, MSSM, ... (Madgraph,Alpgen, CompHEP, ...)
 - LO + parton shower
 - String inspired techniques
- NLO (next-to-leading order)
 - Analytical (or numerical) calculations of diagrams yield parton level Monte Carlos (NLOJET++, MCFM, ...)
 - NLO + parton shower (MC@NLO, VINCIA)
- NNLO (next-to-next-to-leading order)
 - selected results known (mostly inclusive kinematics)
- N³LO (next-to-next-to-next-to-leading order)
 - very few ...

Approaches to the calculation of σ_{had}

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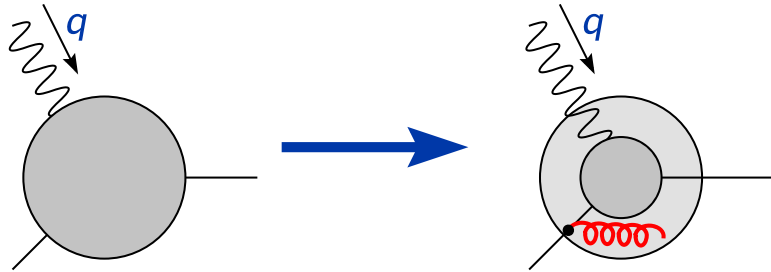
← level of talk here

● N³LO (next-to-next-to-next-to-leading order)

- very few ...

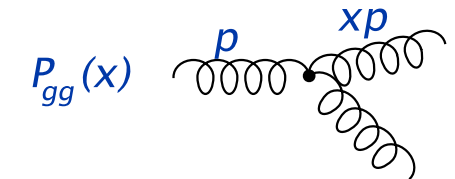
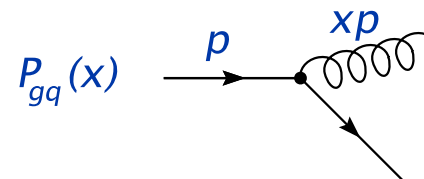
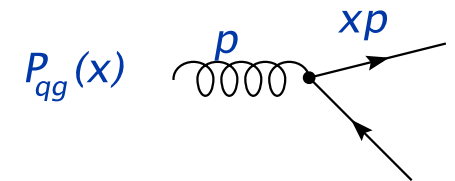
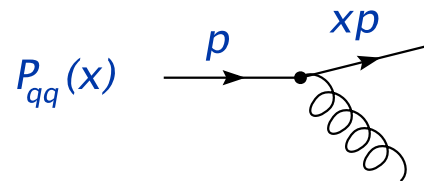
difficult

Parton luminosity

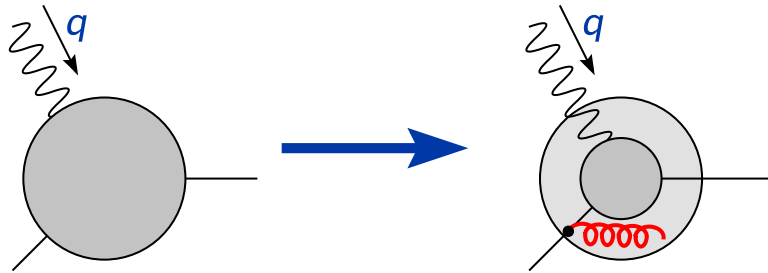


- Proton in resolution $1/Q \rightarrow$ sensitive to lower momentum partons

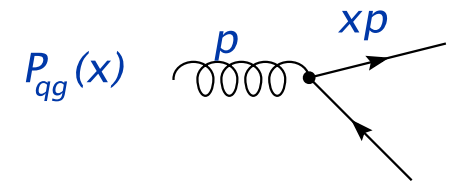
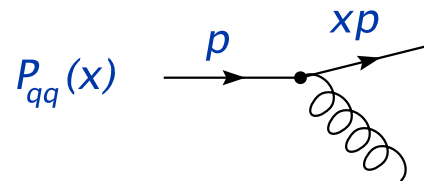
- Feynman diagrams in leading order



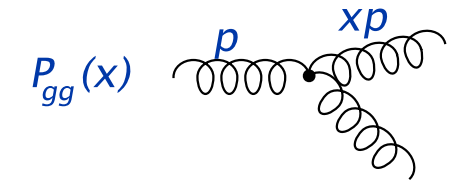
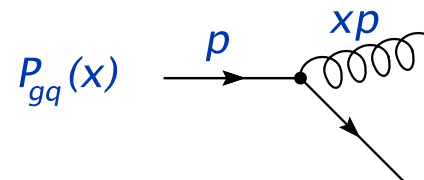
Parton luminosity



● Feynman diagrams in leading order



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● Evolution equations for parton distributions f_i

● predictions from fits to reference processes (universality)

$$\frac{d}{d \ln \mu^2} f_i(x, \mu^2) = \sum_k \left[P_{ik}(\alpha_s(\mu^2)) \otimes f_k(\mu^2) \right] (x)$$

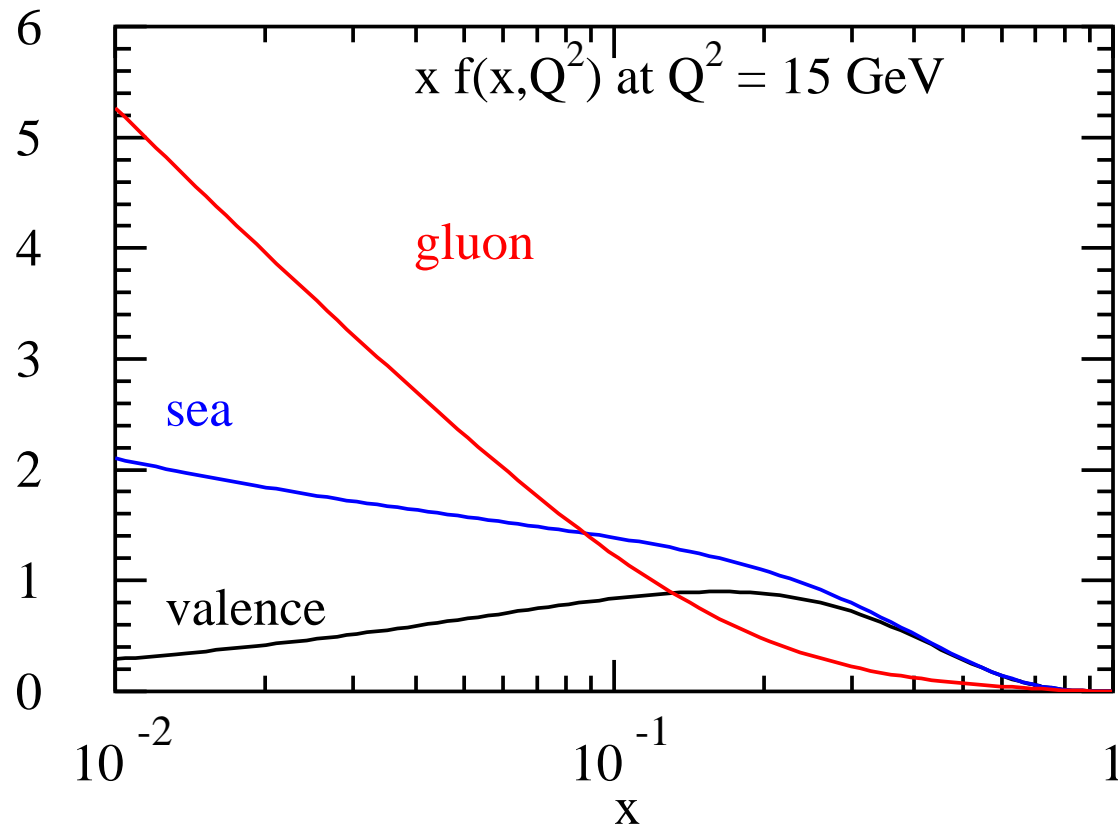
● Splitting functions P

$$P = \underbrace{\alpha_s P^{(0)} + \alpha_s^2 P^{(1)}} + \alpha_s^3 P^{(2)} + \dots$$

NLO: standard approximation (large uncertainties)

Parton distributions in proton

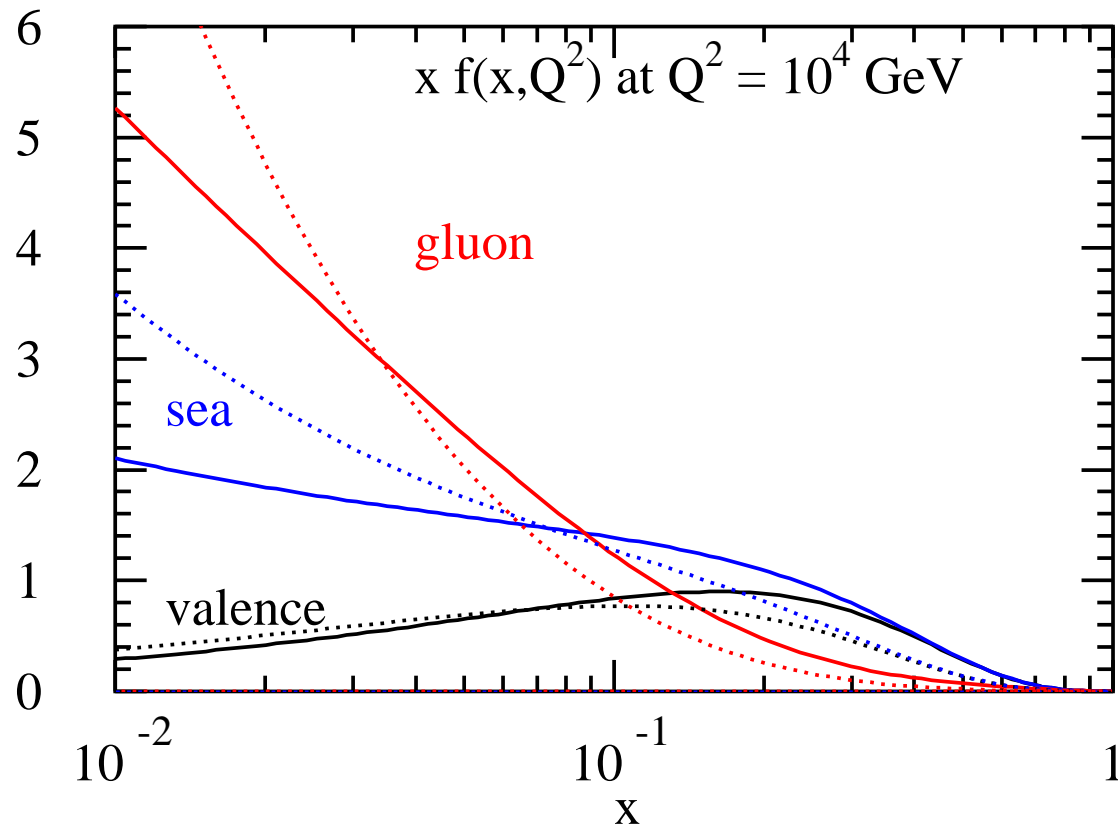
- Valence $q - \bar{q}$ (additive quantum numbers) sea (part with $q + \bar{q}$)



- Parameterization (bulk of data from deep-inelastic scattering)
 - structure function F_2 \longrightarrow quark distribution
 - scale evolution (perturbative QCD) \longrightarrow gluon distribution

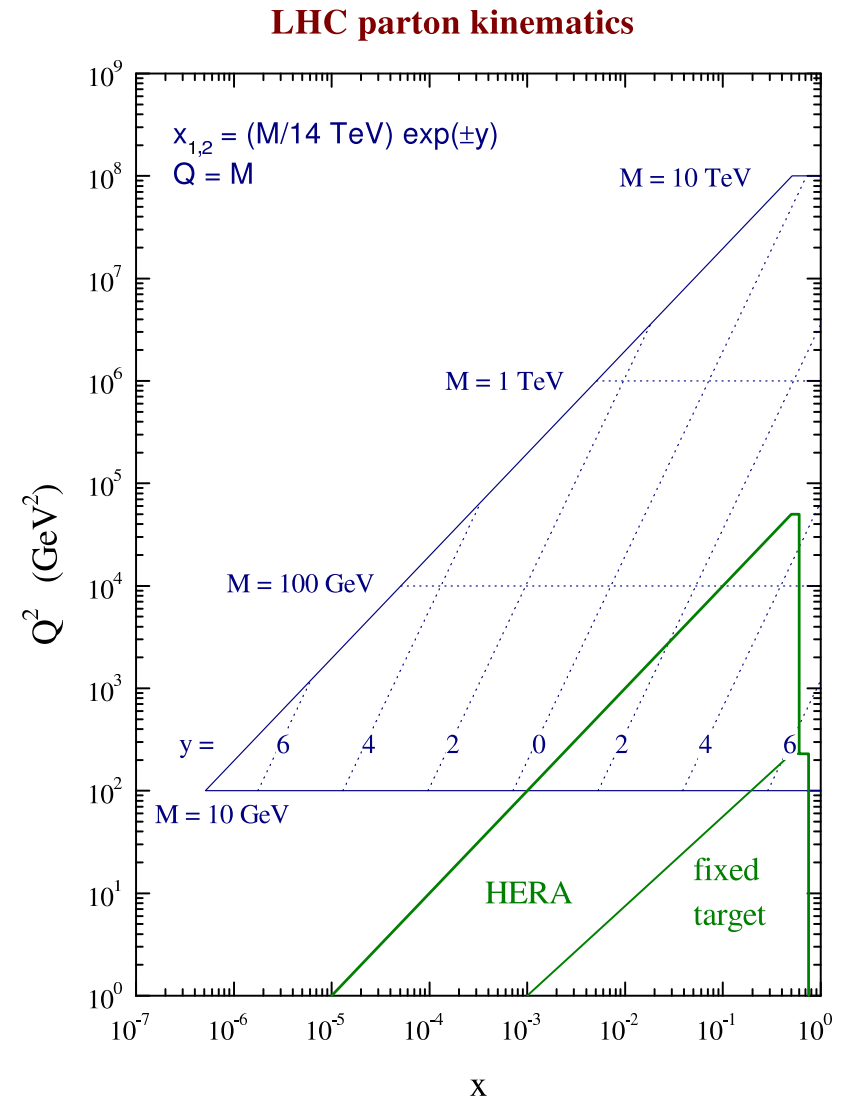
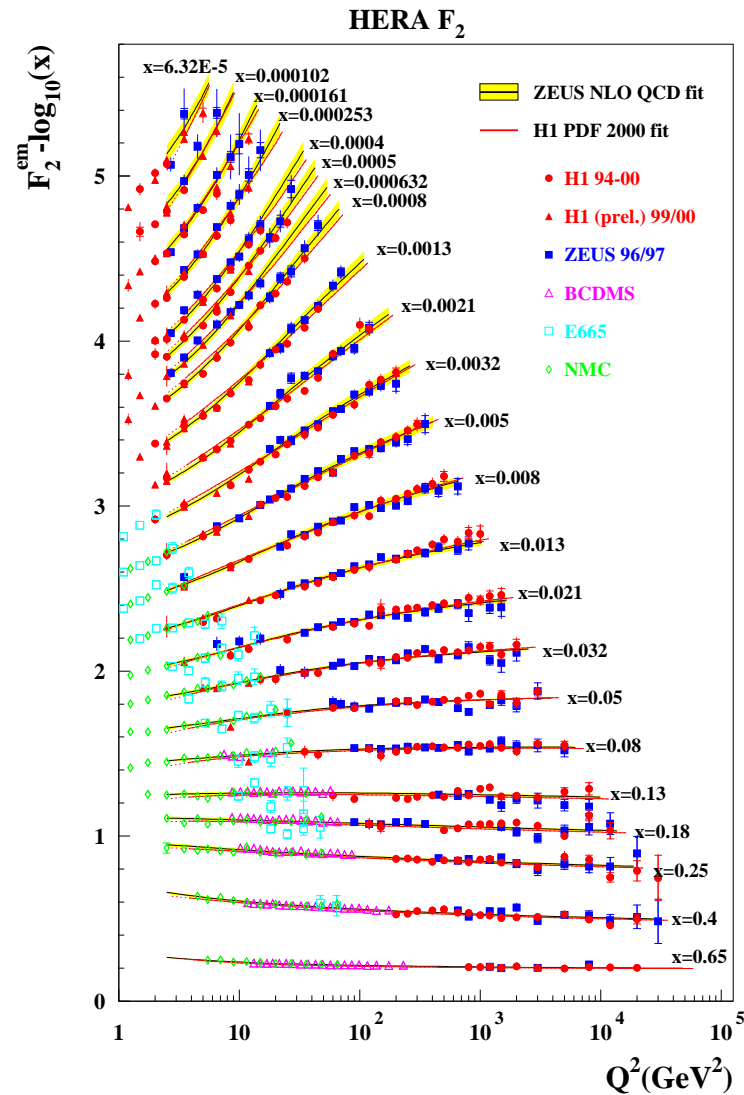
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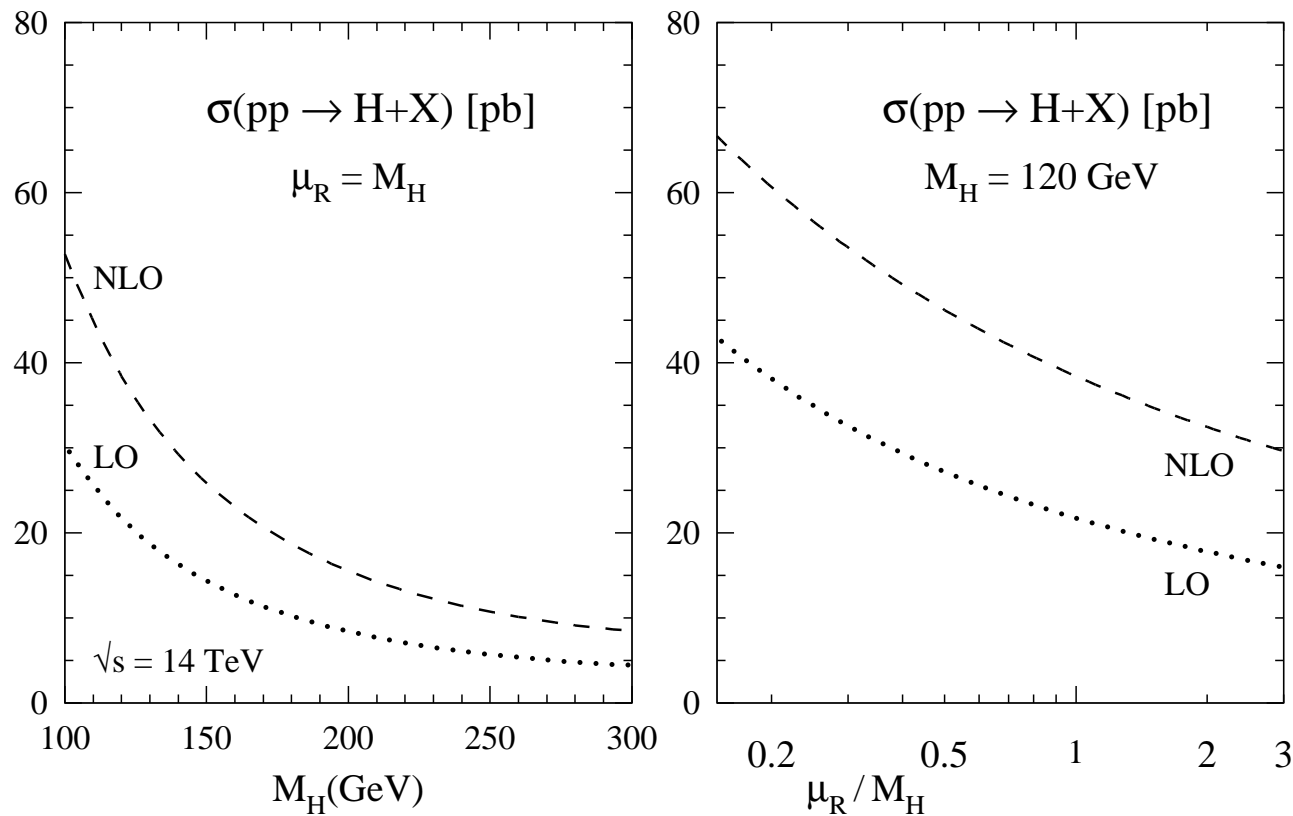
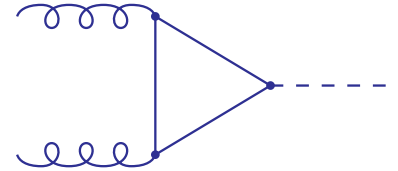
PDFs from HERA to LHC



HERA → LHC: scale evolution in Q^2 over three orders of magnitude

Higgs boson production at LHC (I)

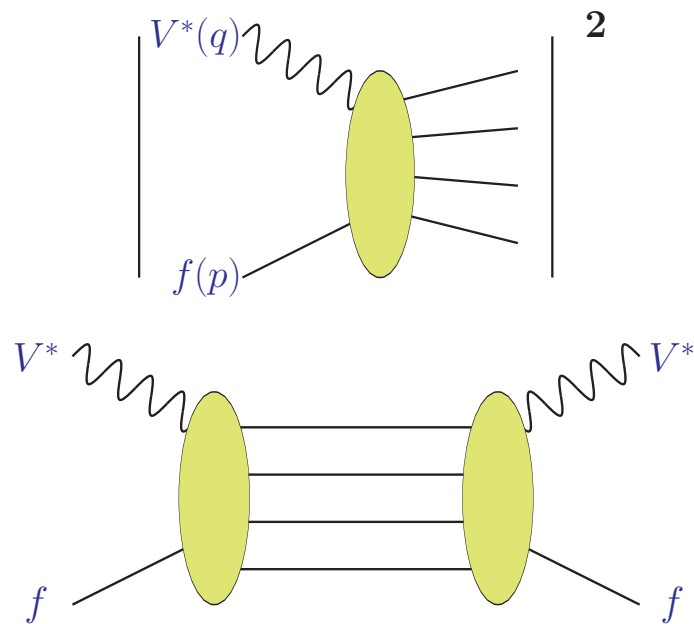
- Dominant channel $gg \rightarrow H + X$ via top-quark loop



- Estimate of uncertainty: apparent convergence, variation of scale μ
- NLO approximation insufficient for reliable predictions

Our calculation in deep-inelastic scattering

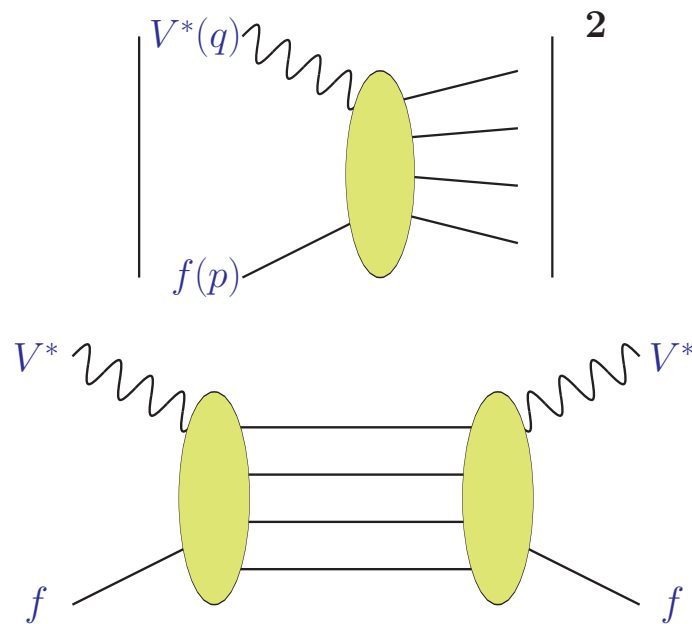
- **"Loop technology"**: optical theorem
total cross section \longleftrightarrow imaginary part of Compton amplitude



Our calculation in deep-inelastic scattering

- **"Loop technology"**: optical theorem

total cross section \longleftrightarrow imaginary part of Compton amplitude



	tree	1-loop	2-loop	3-loop
$q\gamma$	1	3	25	359
$g\gamma$		2	17	345
$h\gamma$			2	56
qW	1	3	32	589
$q\phi$		1	23	696
$g\phi$	1	8	218	6378
$h\phi$		1	33	1184
sum	3	18	350	9607


- more than 10 FTE years and a few CPU years

- computer algebra updates: FORM \rightarrow FORM 3.1 \rightarrow FORM 3.2 \rightarrow ...
- $> 10^5$ tabulated symbolic integrals ($> 3\text{GB}$)

Splitting functions for a quarter of a century


$$\begin{aligned}
 P_{\text{ns}}^{(0)}(x) &= C_F(2p_{\text{qq}}(x) + 3\delta(1-x)) \\
 P_{\text{ps}}^{(0)}(x) &= 0 \\
 P_{\text{qg}}^{(0)}(x) &= 2n_f p_{\text{qg}}(x) \\
 P_{\text{gq}}^{(0)}(x) &= 2C_F p_{\text{gq}}(x) \\
 P_{\text{gg}}^{(0)}(x) &= C_A\left(4p_{\text{gg}}(x) + \frac{11}{3}\delta(1-x)\right) - \frac{2}{3}n_f\delta(1-x)
 \end{aligned}$$

1973




The Nobel Prize in Physics 2004


for the discovery of asymptotic freedom in the theory of the strong interaction



David J. Gross



H. David Politzer



Frank Wilczek

$$\begin{aligned}
 P_{\text{ns}}^{(1)+}(x) &= 4C_A C_F \left(p_{\text{qq}}(x) \left[\frac{67}{18} - \zeta_2 + \frac{11}{6}H_0 + H_{0,0} \right] + p_{\text{qq}}(-x) \left[\zeta_2 + 2H_{-1,0} - H_{0,0} \right] \right. \\
 &\quad \left. + \frac{14}{3}(1-x) + \delta(1-x) \left[\frac{17}{24} + \frac{11}{3}\zeta_2 - 3\zeta_3 \right] \right) - 4C_F n_f \left(p_{\text{qq}}(x) \left[\frac{5}{9} + \frac{1}{3}H_0 \right] + \frac{2}{3}(1-x) \right. \\
 &\quad \left. + \delta(1-x) \left[\frac{1}{12} + \frac{2}{3}\zeta_2 \right] \right) + 4C_F^2 \left(2p_{\text{qq}}(x) \left[H_{1,0} - \frac{3}{4}H_0 + H_2 \right] - 2p_{\text{qq}}(-x) \left[\zeta_2 + 2H_{-1,0} \right. \right. \\
 &\quad \left. \left. - H_{0,0} \right] - (1-x) \left[1 - \frac{3}{2}H_0 \right] - H_0 - (1+x)H_{0,0} + \delta(1-x) \left[\frac{3}{8} - 3\zeta_2 + 6\zeta_3 \right] \right)
 \end{aligned}$$

$$\begin{aligned}
 P_{\text{ns}}^{(1)-}(x) &= P_{\text{ns}}^{(1)+}(x) + 16C_F \left(C_F - \frac{C_A}{2} \right) \left(p_{\text{qq}}(-x) \left[\zeta_2 + 2H_{-1,0} - H_{0,0} \right] - 2(1-x) \right. \\
 &\quad \left. - (1+x)H_0 \right)
 \end{aligned}$$

$$P_{\text{ps}}^{(1)}(x) = 4C_F n_f \left(\frac{20}{9} \frac{1}{x} - 2 + 6x - 4H_0 + x^2 \left[\frac{8}{3}H_0 - \frac{56}{9} \right] + (1+x) \left[5H_0 - 2H_{0,0} \right] \right)$$

$$\begin{aligned}
 P_{\text{qg}}^{(1)}(x) &= 4C_A n_f \left(\frac{20}{9} \frac{1}{x} - 2 + 25x - 2p_{\text{qg}}(-x)H_{-1,0} - 2p_{\text{qg}}(x)H_{1,1} + x^2 \left[\frac{44}{3}H_0 - \frac{218}{9} \right] \right. \\
 &\quad \left. + 4(1-x) \left[H_{0,0} - 2H_0 + xH_1 \right] - 4\zeta_2 x - 6H_{0,0} + 9H_0 \right) + 4C_F n_f \left(2p_{\text{qg}}(x) \left[H_{1,0} + H_{1,1} + H_2 \right. \right. \\
 &\quad \left. \left. - \zeta_2 \right] + 4x^2 \left[H_0 + H_{0,0} + \frac{5}{2} \right] + 2(1-x) \left[H_0 + H_{0,0} - 2xH_1 + \frac{29}{4} \right] - \frac{15}{2} - H_{0,0} - \frac{1}{2}H_0 \right)
 \end{aligned}$$

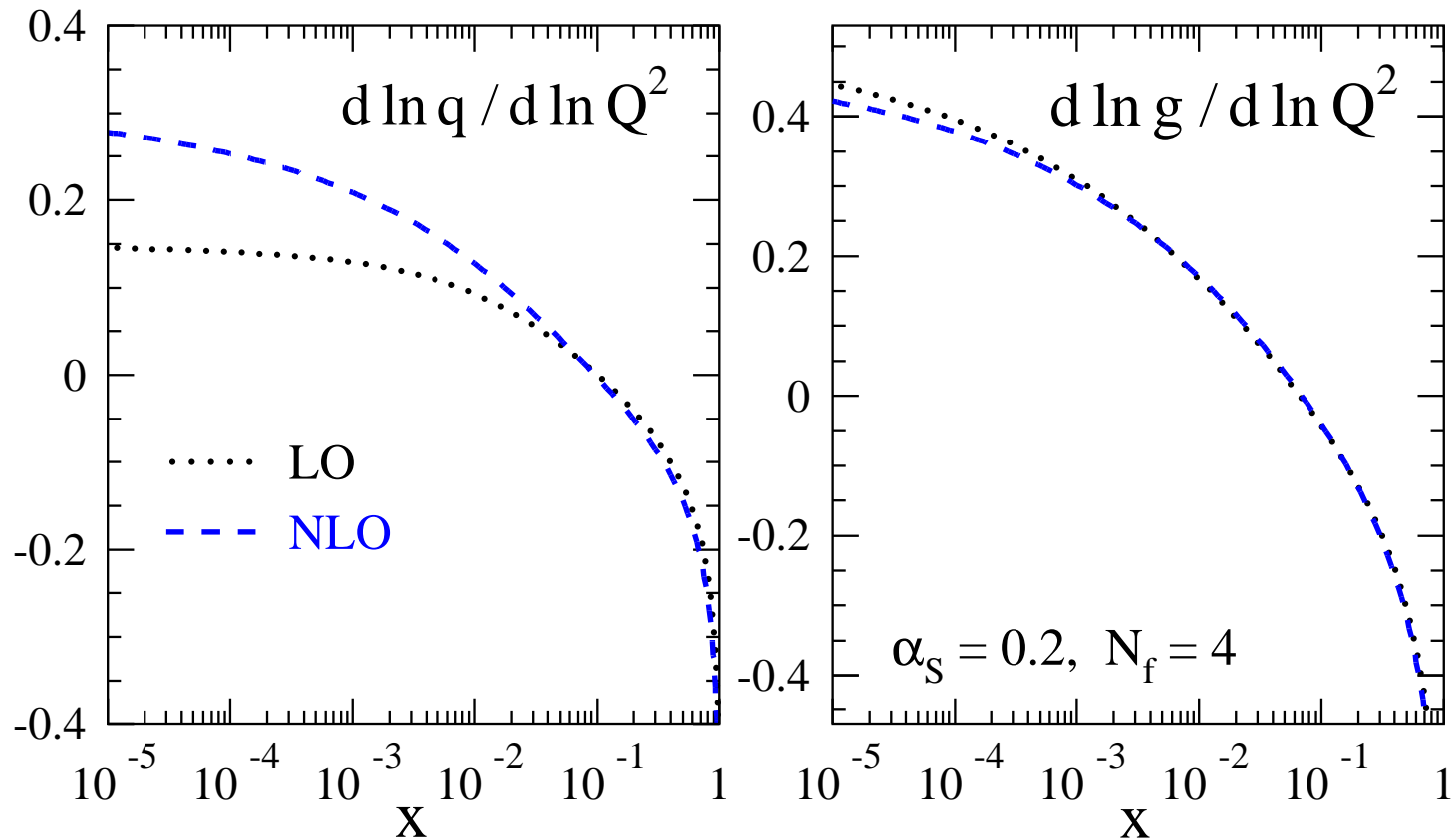
$$\begin{aligned}
 P_{\text{gq}}^{(1)}(x) &= 4C_A C_F \left(\frac{1}{x} + 2p_{\text{gq}}(x) \left[H_{1,0} + H_{1,1} + H_2 - \frac{11}{6}H_1 \right] - x^2 \left[\frac{8}{3}H_0 - \frac{44}{9} \right] + 4\zeta_2 - 2 \right. \\
 &\quad \left. - 7H_0 + 2H_{0,0} - 2H_1 x + (1+x) \left[2H_{0,0} - 5H_0 + \frac{37}{9} \right] - 2p_{\text{gq}}(-x)H_{-1,0} \right) - 4C_F n_f \left(\frac{2}{3}x \right. \\
 &\quad \left. - p_{\text{gq}}(x) \left[\frac{2}{3}H_1 - \frac{10}{9} \right] \right) + 4C_F^2 \left(p_{\text{gq}}(x) \left[3H_1 - 2H_{1,1} \right] + (1+x) \left[H_{0,0} - \frac{7}{2} + \frac{7}{2}H_0 \right] - 3H_{0,0} \right. \\
 &\quad \left. + 1 - \frac{3}{2}H_0 + 2H_1 x \right)
 \end{aligned}$$

$$\begin{aligned}
 P_{\text{gg}}^{(1)}(x) &= 4C_A n_f \left(1-x - \frac{10}{9}p_{\text{gg}}(x) - \frac{13}{9} \left(\frac{1}{x} - x^2 \right) - \frac{2}{3}(1+x)H_0 - \frac{2}{3}\delta(1-x) \right) + 4C_A^2 \left(27 \right. \\
 &\quad \left. + (1+x) \left[\frac{11}{3}H_0 + 8H_{0,0} - \frac{27}{2} \right] + 2p_{\text{gg}}(-x) \left[H_{0,0} - 2H_{-1,0} - \zeta_2 \right] - \frac{67}{9} \left(\frac{1}{x} - x^2 \right) - 12H_0 \right. \\
 &\quad \left. - \frac{44}{3}x^2 H_0 + 2p_{\text{gg}}(x) \left[\frac{67}{18} - \zeta_2 + H_{0,0} + 2H_{1,0} + 2H_2 \right] + \delta(1-x) \left[\frac{8}{3} + 3\zeta_3 \right] \right) + 4C_F n_f \left(2H_0 \right. \\
 &\quad \left. + \frac{2}{3} \frac{1}{x} + \frac{10}{3}x^2 - 12 + (1+x) \left[4 - 5H_0 - 2H_{0,0} \right] - \frac{1}{2}\delta(1-x) \right) .
 \end{aligned}$$

1980

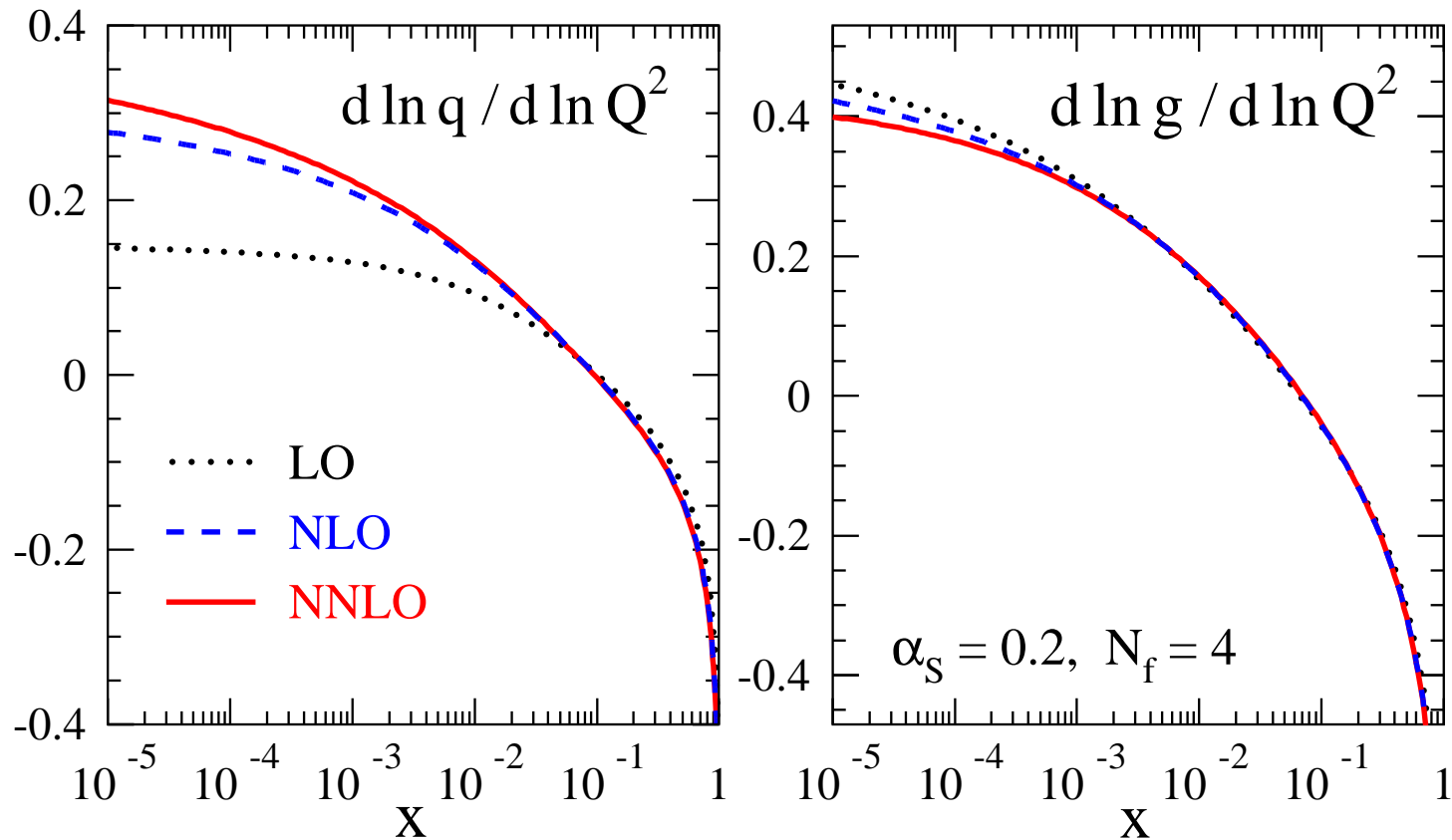
Perturbative stability of evolution

- Scale derivatives of quark and gluon distributions at $Q^2 \approx 30 \text{ GeV}^2$



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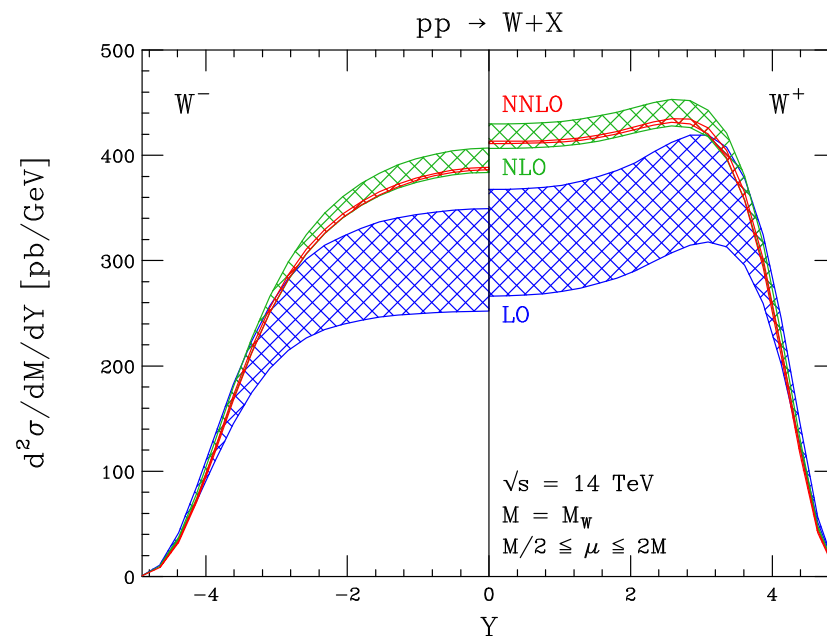
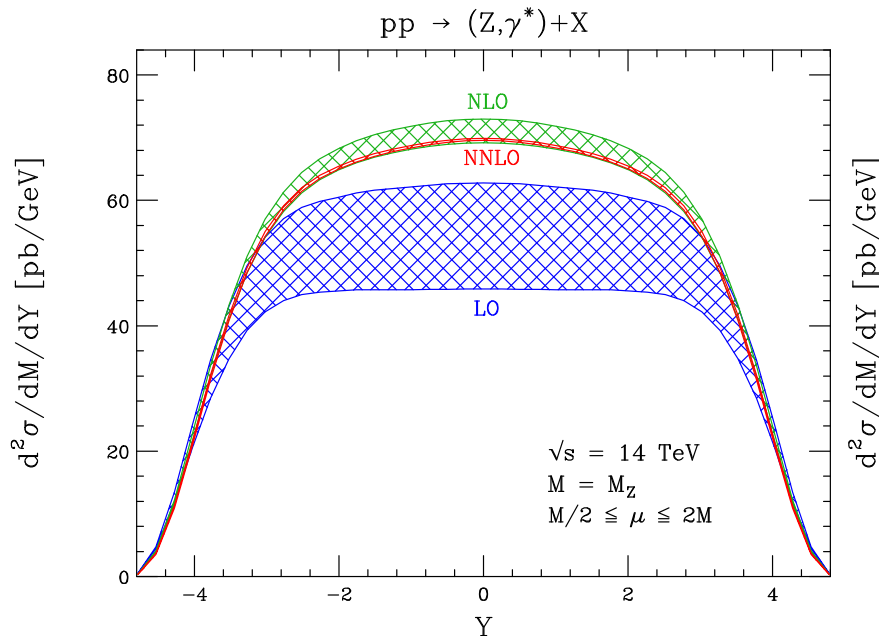


- Expansion very stable except for very small momenta $x \lesssim 10^{-4}$

Impact on precision of LHC predictions

- W^\pm, Z -boson rapidity distribution (scale variation $\frac{m_{W,Z}}{2} \leq \mu \leq 2m_{W,Z}$)

Anastasiou, Petriello, Melnikov '05



- NNLO QCD theoretical uncertainties (renormalization / factorization scale) at level of 1% Dissertori et al. '05
 - one of the few cross sections known to NNLO in pQCD
- "Standard candle" process for parton luminosity

Updates of PDFs (exp)

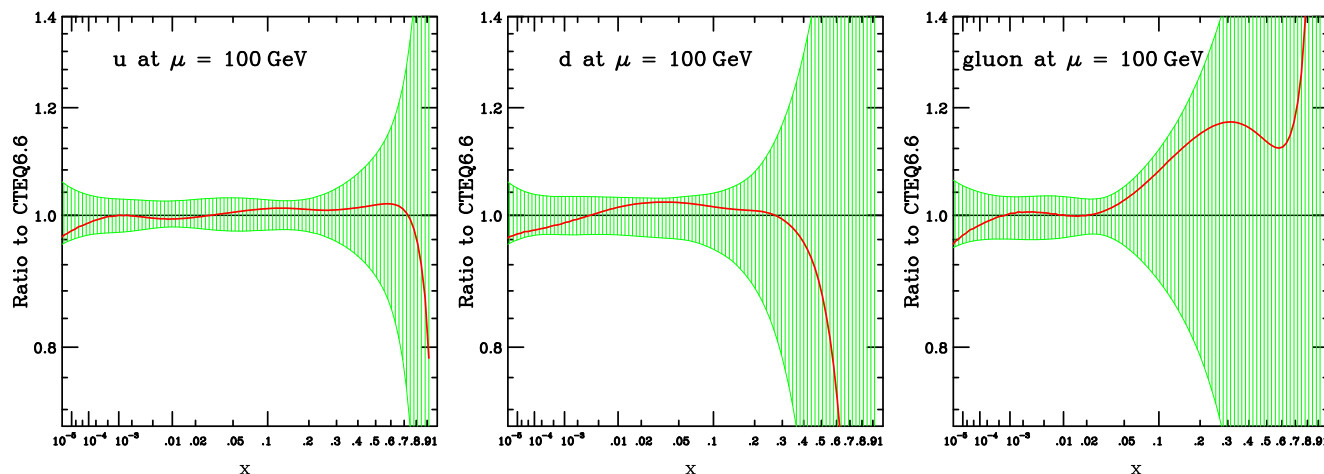
- New experimental data
 - results from neutrino-nucleon DIS for strange quark PDFs ($s \neq \bar{s}$)
- Uncertainty on \bar{u} , \bar{d} doubles from 1.5% to 3% at $Q^2 \simeq M_W^2$ MSTW '07
 - s , \bar{s} feed into F_2 NC DIS constraint $4/9(u + \bar{u}) + 1/9(d + \bar{d} + s + \bar{s})$

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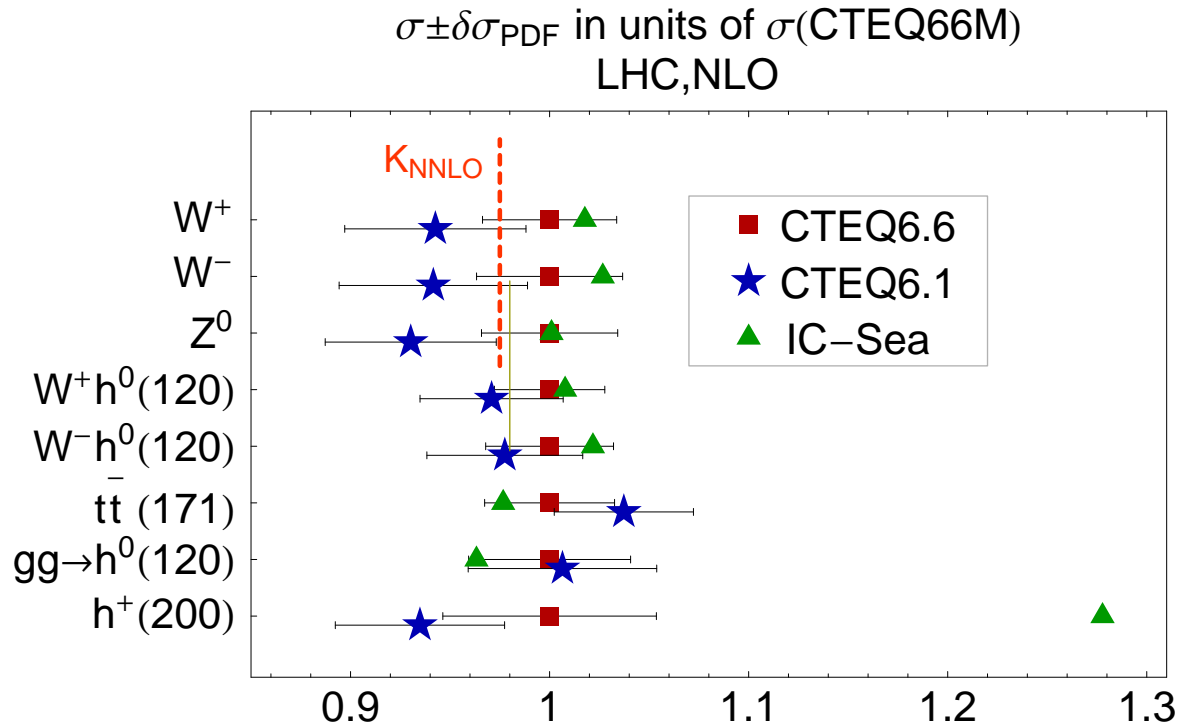
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Updates of PDFs (th)

- Improved heavy quark (charm) threshold
 - matching consistent with QCD factorization CTEQ '08
- Significant changes due to larger light flavor PDFs



Cross sections at LHC



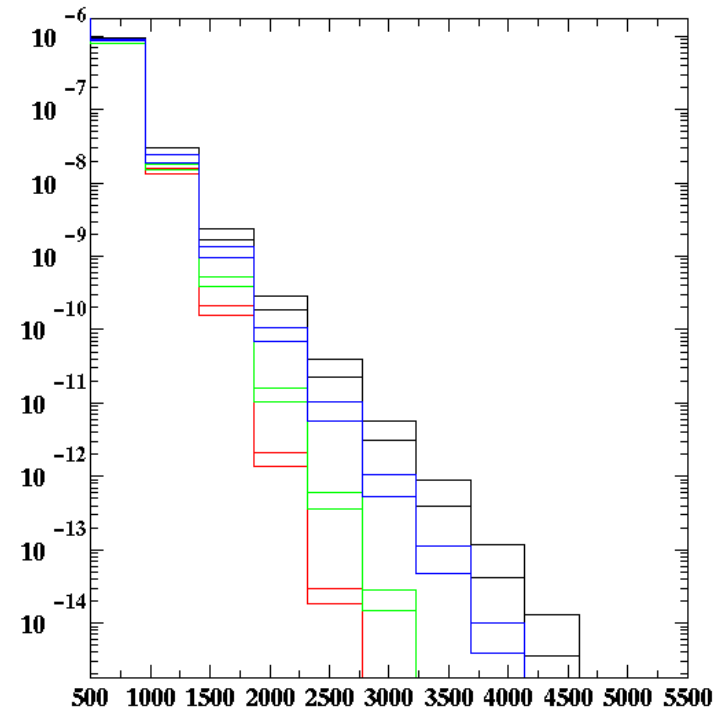
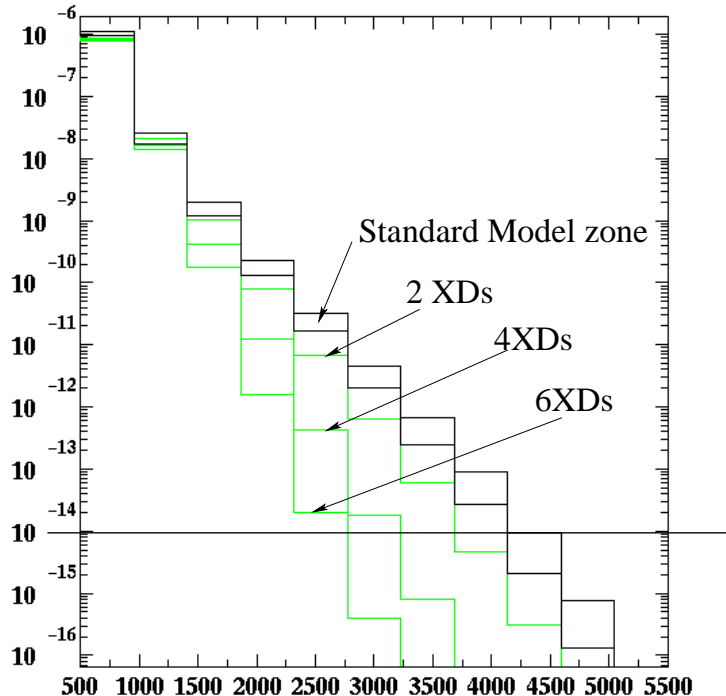
- Predictions for W^\pm/Z cross sections at LHC shift by 8% between PDF sets CTEQ6.6 and CTEQ6.1 (improved theory!)
 - sensitivity to PDFs in the $x \sim 10^{-3}$ range
- W^\pm/Z -ratio golden calibration measurement

Large extra dimensions

- Sensitivity of LHC dijet cross section to large extra dimensions Ferrag '04
 - large extra dimensions accelerate running of α_s as compactification scale M_c is approached
- PDF uncertainties
 - potential sensitivity to M_c reduced from 6 TeV to 2 TeV

$M_c = 2 \text{ TeV}$ no PDF error

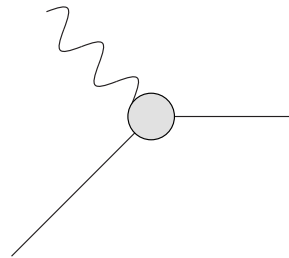
$M_c = 2 \text{ TeV}$ with PDF error



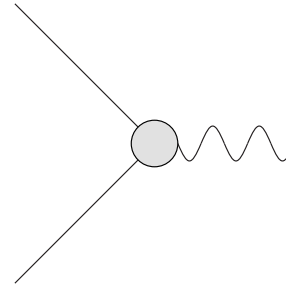
Drell-Yan process and Higgs production

- Mapping of DIS to Drell-Yan lepton-pair production (or to Higgs production in gluon fusion)
 - re-engineering the soft and collinear limit
 - threshold enhanced terms at N³LO (numerically most important)
S.M., Vogt '05; Laenen, Magnea '05; Idilbi et al. '05

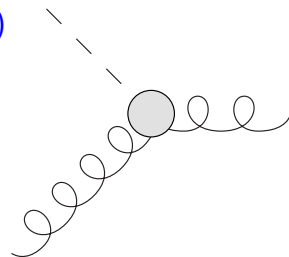
DIS



DY



from $P_{gg}^{(2)}$



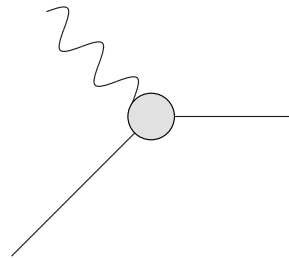
Higgs



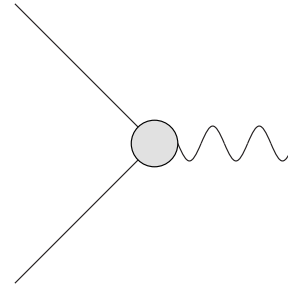
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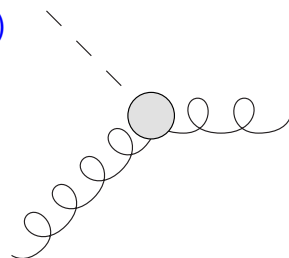
DIS



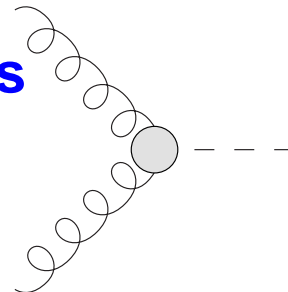
DY



from $P_{gg}^{(2)}$

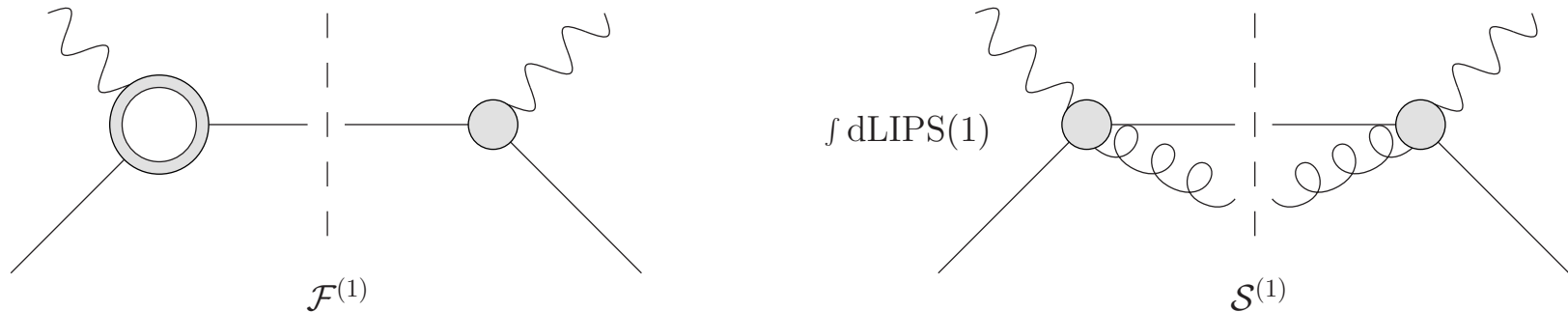


Higgs



Recycling of
theory 

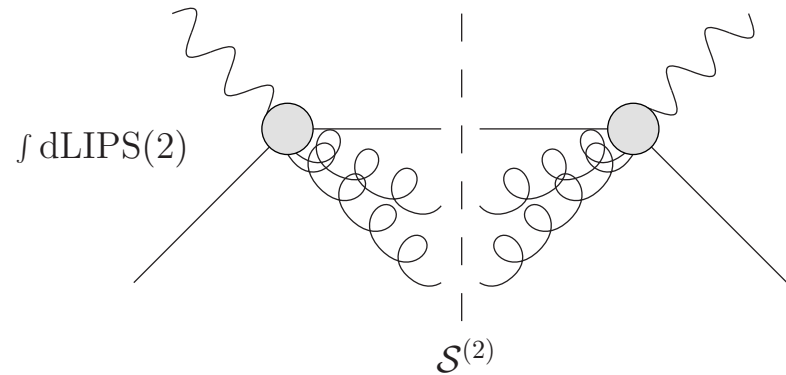
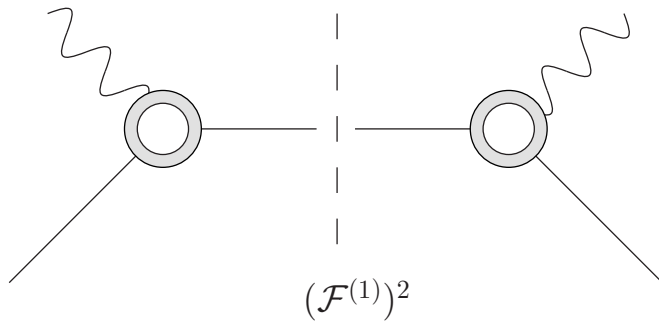
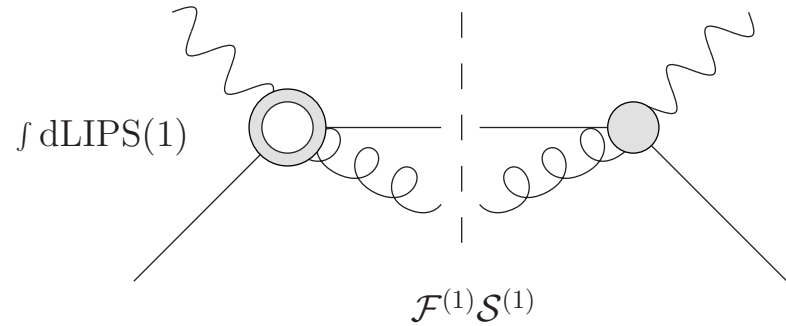
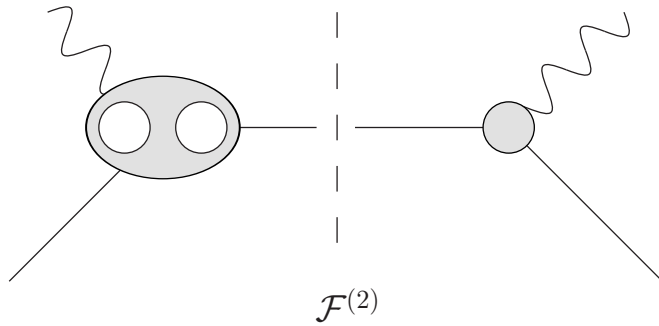
Anatomy of DIS result (1 loop)



$$\mathcal{T}_1^b = 2 \operatorname{Re} \mathcal{F}_1 \delta(1-x) + \mathcal{S}_1$$

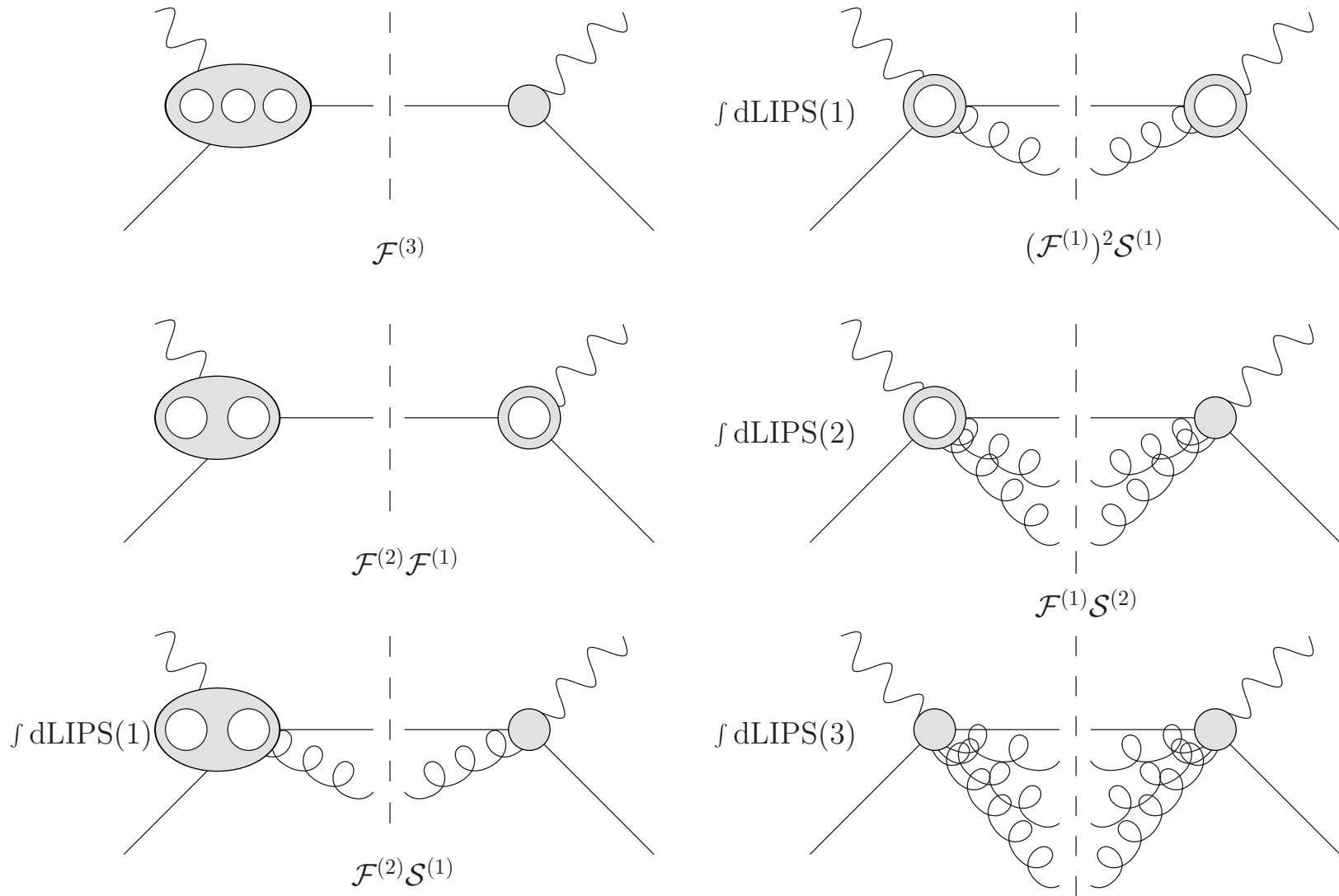
- Forward Compton amplitude \mathcal{T}_n in $D = 4 - 2\epsilon$ -dimensions combines
 - virtual corrections \mathcal{F}_n (dependent on $\delta(1-x)$)
 - pure real-emission contributions \mathcal{S}_n
(dependent on D -dimensional +-distributions)
- Infrared finiteness implies cancellation of poles between \mathcal{F}_n and \mathcal{S}_n
Kinoshita '62; Lee, Nauenberg '64
- Constructive approach to form factor \mathcal{F}_n and \mathcal{S}_n

Anatomy of DIS result (2 loops)



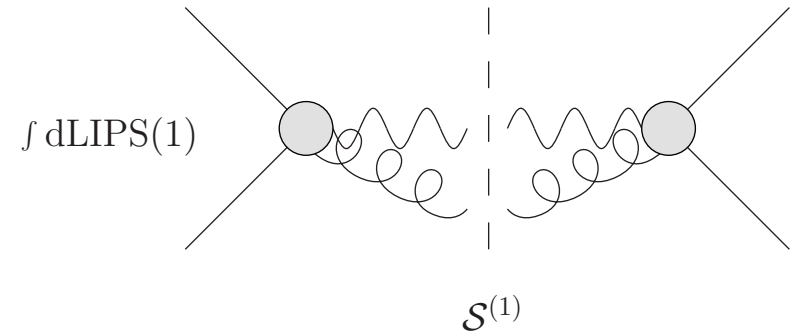
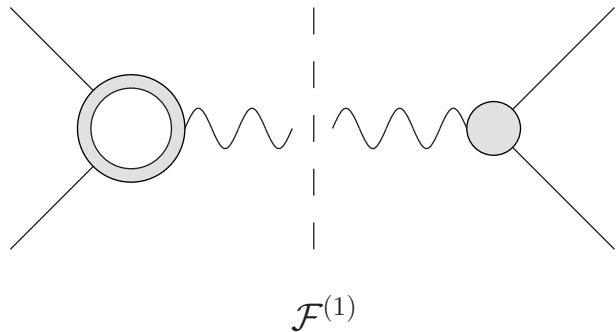
$$\mathcal{T}_2^b = (2 \text{Re } \mathcal{F}_2 + |\mathcal{F}_1|^2) \delta(1-x) + 2 \text{Re } \mathcal{F}_1 \mathcal{S}_1 + \mathcal{S}_2$$

Anatomy of DIS result (3 loops)



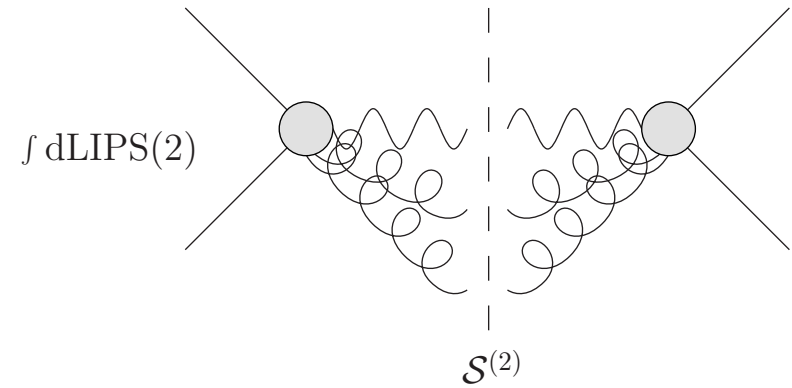
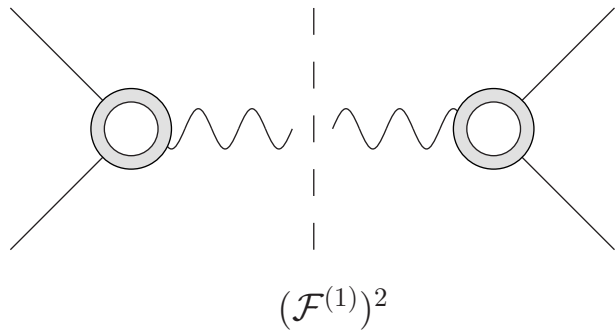
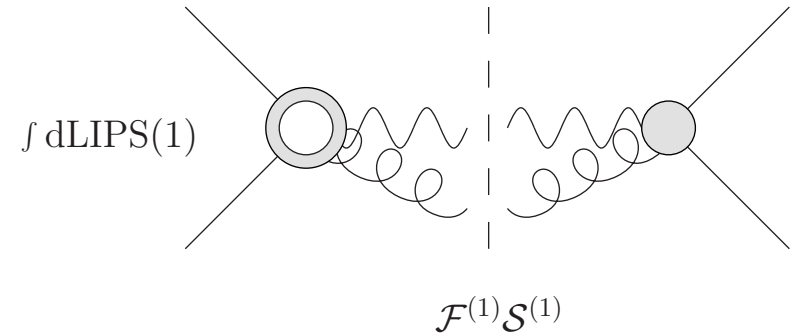
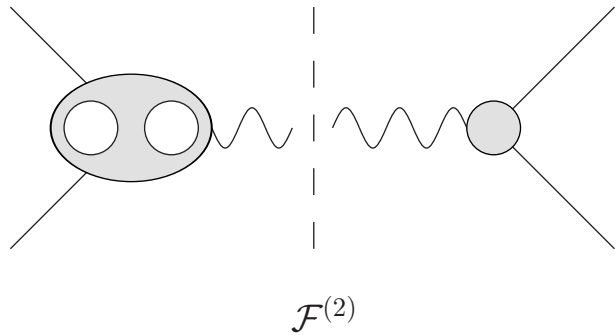
$$\mathcal{T}_3^b = (2 \operatorname{Re} \mathcal{F}_3 + 2 |\mathcal{F}_1 \mathcal{F}_2|) \delta(1-x) + (2 \operatorname{Re} \mathcal{F}_2 + |\mathcal{F}_1|^2) \mathcal{S}_1 + 2 \operatorname{Re} \mathcal{F}_1 \mathcal{S}_2 + \mathcal{S}_3$$

Drell-Yan (1 loop)



- Construction of cross sections for hadron-hadron scattering
 - form factor with time-like kinematics $Q^2 > 0$
 - soft emission with D -dimensional +-distributions
- Drell-Yan lepton-pair production in $q\bar{q}$ -annihilation
- Higgs production from gluon fusion

Drell-Yan (2 loops)



- Checks at two loops

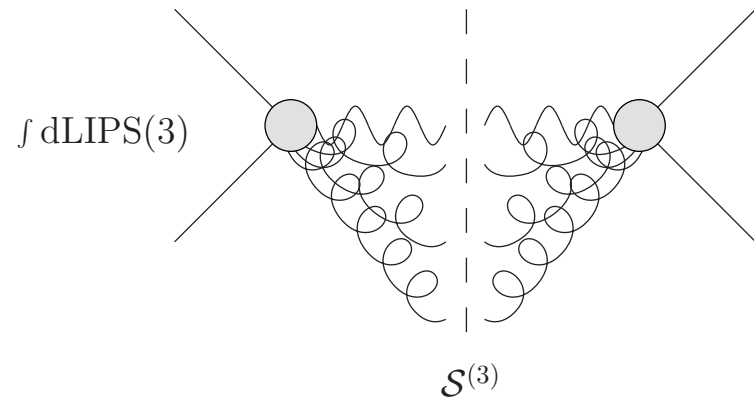
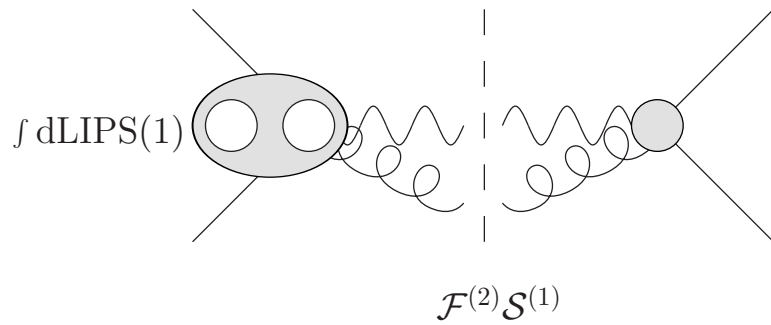
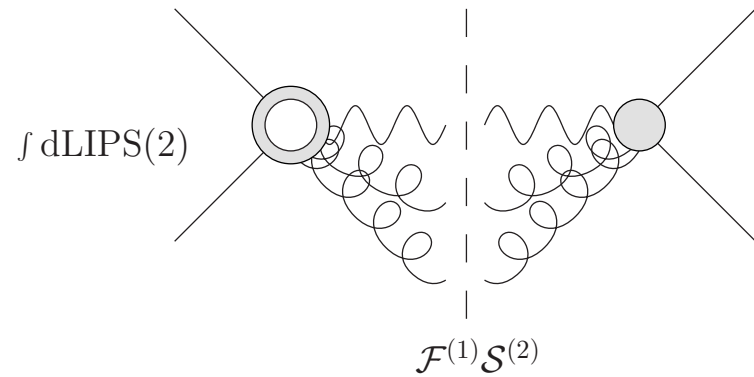
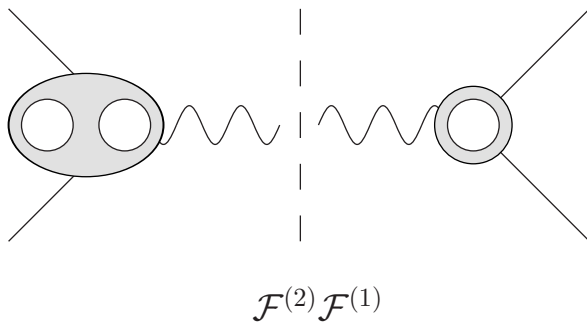
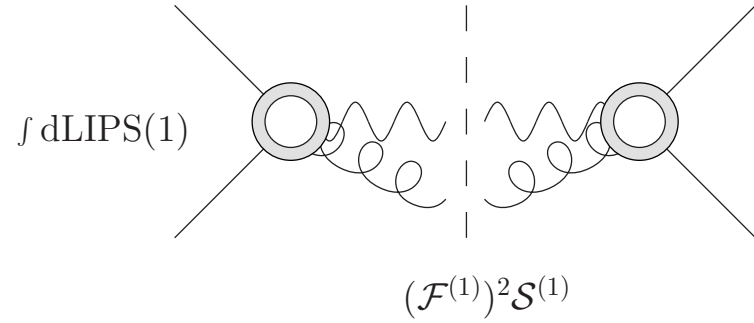
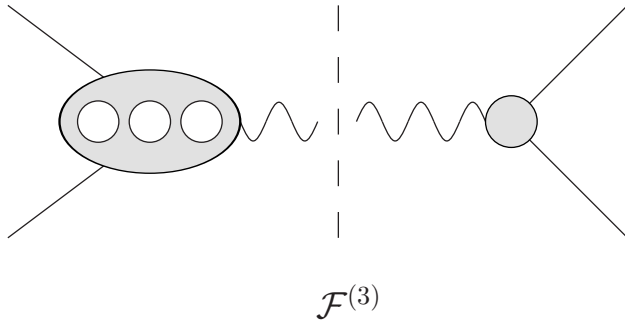
- Drell-Yan

Hamberg, van Neerven, Matsuura '91; Harlander, Kilgore '02

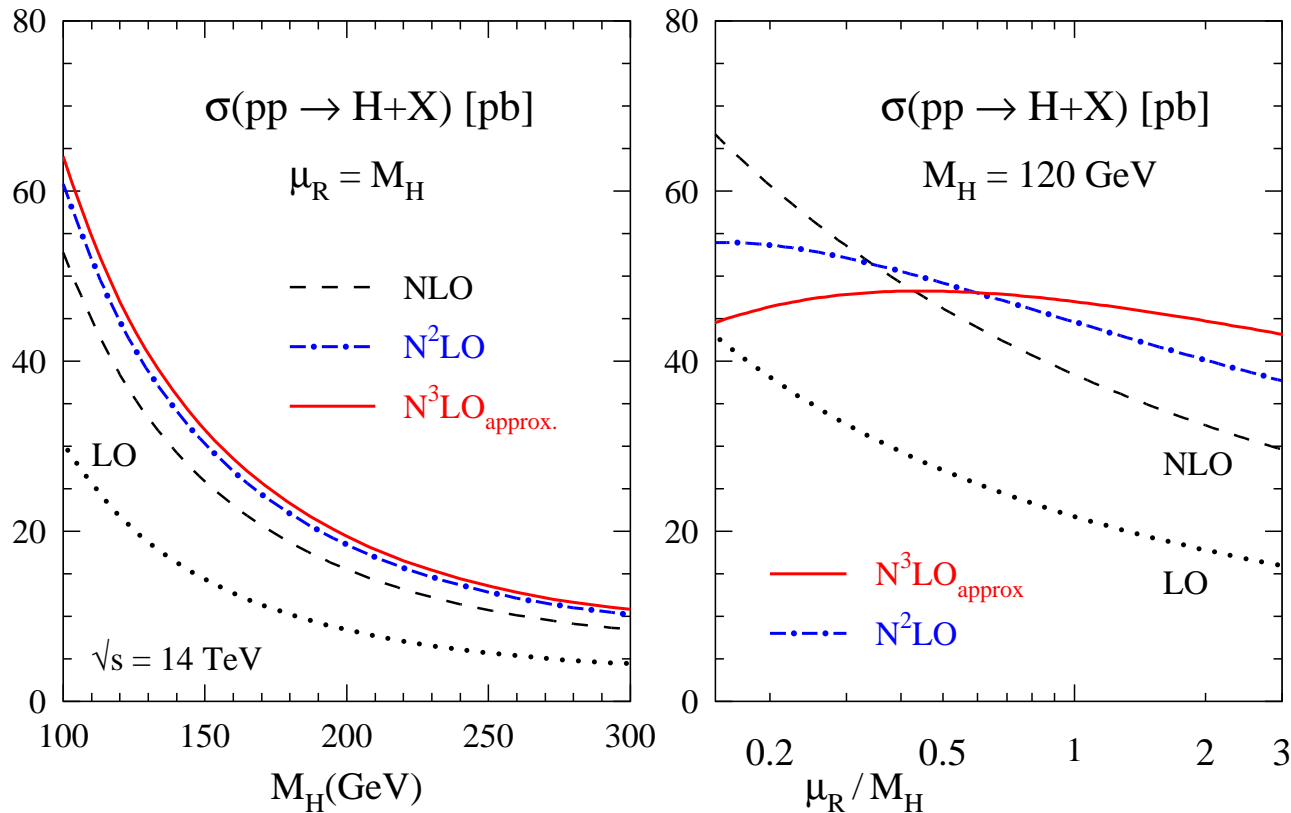
- Higgs production

Harlander, Kilgore '02; Anastasiou, Melnikov '02; Ravindran, Smith, van Neerven '03

Drell-Yan (3 loops)



Higgs boson production at LHC (II)

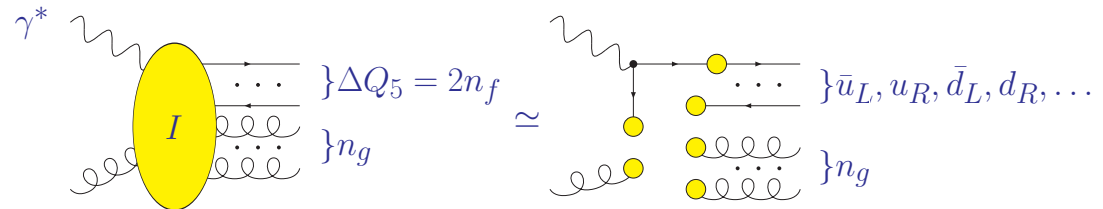


- N^3 LO_{approx.} increase at $\mu_r = M_H$ 5% (NNLO PDF's) S.M., Vogt '05
 - μ_r variation 4%
- Overall accuracy of 5% reached with approx. N^3 LO prediction

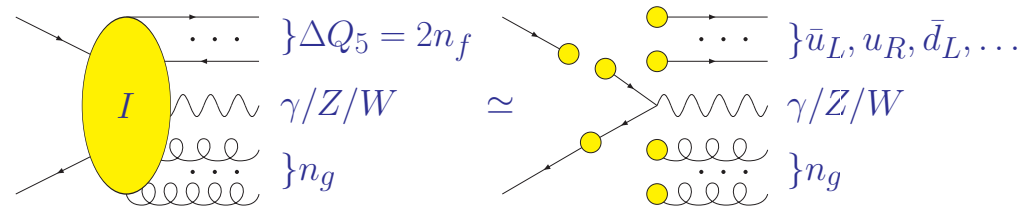
Instanton induced processes at LHC

- Multi-gluon production in instanton background
 - transfer of DIS phenomenology to Drell-Yan lepton-pair production
Brandenburg, Ringwald, Utermann '06; Petermann, Schrempp '08
 - amplitudes related by crossing

- **DIS:** I -induced photon-gluon scattering



- **pp:** I -induced $q\bar{q}$ -annihilation



- Characteristic final state signature
 - isotropic multi-particle production, very high multiplicity

Summary

HERA

- Deep-inelastic scattering (electron-proton collision)
 - wealth of experimental information on proton structure
- QCD precision predictions
 - radiative corrections for parton evolution
- HERA PDFs have strong impact on measurements at LHC

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Outlook

- If past performance is an indicator, we are well prepared for future challenges ...