Deep-inelastic scattering
— from HERA to LHC —

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Proton colliders

- HERA: deep structure of proton at highest $Q^2$ and smallest $x$
- LHC: Higgs boson search at highest energies $\sqrt{S} = 14\text{TeV}$

Quantum Chromodynamics (QCD) ubiquitous at proton colliders
  - reliable understanding essential for precision and discovery physics
Perturbative QCD at colliders

- Hard scattering at hadron colliders
  - constituent partons from incoming hadrons interact at short distance (large momentum transfer $Q^2$)

\[ Q^2 = -q^2, \quad x = Q^2 / (2p \cdot q) \]

- QCD factorization
  - separate sensitivity to dynamics from different scales

**DIS**: photon momentum $Q^2 = -q^2$, Bjorken’s $x = Q^2 / (2p \cdot q)$

**pp**: energy $Q^2 = 2p_1 \cdot p_2$, ratio to $W/Z$-boson mass $x = M_{W/Z}^2 / Q^2$
Hard scattering cross section

- QCD factorization

$$\sigma_{\text{had}} = \left\{ \begin{array}{ll}
\sum_i f_i \otimes \hat{\sigma}_{\gamma i \rightarrow X} \left( \alpha_s(\mu^2), Q^2, \mu^2 \right) & \text{DIS} \\
\sum_{ij} f_i \otimes f_j \otimes \hat{\sigma}_{ij \rightarrow X} \left( \alpha_s(\mu^2), Q^2, \mu^2 \right) & \text{pp}
\end{array} \right.$$  

- Parton cross sections $\hat{\sigma}_{\gamma i \rightarrow X}$ and $\hat{\sigma}_{ij \rightarrow X}$ calculable pertubatively in powers of $\alpha_s$
  - short distance interaction of constituent partons

- Parton distributions $f_i \rightarrow$ parton luminosity
  - proton: very complicated multi-particle bound state
  - colliders: wide-band beams of quarks and gluons

- Standard approach to uncertainties in theoretical predictions
  - variation of factorization scale $\mu$: $\frac{d}{d \ln \mu^2} \sigma_{\text{had}} = O(\alpha_s^{l+1})$
Asymptotic freedom of QCD

- Effective coupling constant $\alpha_s$ depends on resolution, momentum scale $Q$
  - screening (like in QED)
  - anti-screening (color charge of $g$)

- At large scales: application of perturbation theory (but $\alpha_s \gg \alpha_{\text{QED}}$)

\[
\alpha_s(Q^2) = \begin{cases} 
0.115 & \text{1-loop} \\
0.1 & \text{2-loop} \\
0.05 & \text{3-loop} \\
0.025 & \text{4-loop}
\end{cases}
\]

$Q^2 (\text{GeV}^2)$

$\alpha_s(M_Z^2) = 0.115$
Approaches to the calculation of $\sigma_{\text{had}}$

- **LO (leading order)**
  - Automated tree level calculations in Standard Model, MSSM, ... (Madgraph, Alpgen, CompHEP, ...)
  - LO + parton shower
  - String inspired techniques

- **NLO (next-to-leading order)**
  - Analytical (or numerical) calculations of diagrams yield parton level Monte Carlos (NLOJET++, MCFM, ...)
  - NLO + parton shower (MC@NLO, VINCIA)

- **NNLO (next-to-next-to-leading order)**
  - selected results known (mostly inclusive kinematics)

- **$N^3$LO (next-to-next-to-next-to-leading order)**
  - very few ...
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Parton luminosity

Proton in resolution $1/Q$ → sensitive to lower momentum partons

Feynman diagrams in leading order:

$P_{qq}(x)$

$p \ x p$

$P_{gg}(x)$

$p \ x p$

$P_{gg}(x)$

$p \ x p$
Parton luminosity

Feynman diagrams in leading order

Proton in resolution $1/Q \rightarrow$ sensitive to lower momentum partons

Evolution equations for parton distributions $f_i$
  - predictions from fits to reference processes (universality)

$$\frac{d}{d \ln \mu^2} f_i(x, \mu^2) = \sum_k \left[ P_{ik}(\alpha_s(\mu^2)) \otimes f_k(\mu^2) \right](x)$$

Splitting functions $P$

$$P = \alpha_s P^{(0)} + \alpha_s^2 P^{(1)} + \alpha_s^3 P^{(2)} + \ldots$$

NLO: standard approximation (large uncertainties)
Parton distributions in proton

- Valence $q - \bar{q}$ (additive quantum numbers) sea (part with $q + \bar{q}$)

Parameterization (bulk of data from deep-inelastic scattering)

- structure function $F_2$ $\rightarrow$ quark distribution
- scale evolution (perturbative QCD) $\rightarrow$ gluon distribution
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**PDFs from HERA to LHC**

HERA $F_2$

```
\begin{align*}
  x &= 0.000102 \\
  x &= 0.000161 \\
  x &= 0.000253 \\
  x &= 0.0004 \\
  x &= 0.000632 \\
  x &= 0.0008 \\
  x &= 0.0013 \\
  x &= 0.0021 \\
  x &= 0.0032 \\
  x &= 0.005 \\
  x &= 0.008 \\
  x &= 0.013 \\
  x &= 0.021 \\
  x &= 0.032 \\
  x &= 0.05 \\
  x &= 0.08 \\
  x &= 0.13 \\
  x &= 0.18 \\
  x &= 0.25 \\
  x &= 0.4 \\
  x &= 0.65
\end{align*}
```

LHC parton kinematics

$x_{1,2} = (M/14\text{ TeV}) \exp(\pm y)$

$Q = M$

- $M = 10\text{ TeV}$
- $M = 1\text{ TeV}$
- $M = 100\text{ GeV}$
- $M = 10\text{ GeV}$

**HERA → LHC:** scale evolution in $Q^2$ over three orders of magnitude

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Higgs boson production at LHC (I)

- Dominant channel $gg \to H + X$ via top-quark loop

- Estimate of uncertainty: apparent convergence, variation of scale $\mu$

- NLO approximation insufficient for reliable predictions
Our calculation in deep-inelastic scattering

"Loop technology": optical theorem

total cross section $\mapsto$ imaginary part of Compton amplitude

$$V^*(q) f(p) \quad 2 \quad f(p) V^*$$

$$V^* f \quad V^* f$$
Our calculation in deep-inelastic scattering

- "Loop technology": optical theorem
  - total cross section $\leftrightarrow$ imaginary part of Compton amplitude

![Diagram of loop technology]

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<th>2-loop</th>
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</table>

- more than 10 FTE years and a few CPU years
  - computer algebra updates: \textsc{form} $\rightarrow$ \textsc{form 3.1} $\rightarrow$ \textsc{form 3.2} $\rightarrow$ ...
  - $>10^5$ tabulated symbolic integrals ($>3\text{GB}$)
Splitting functions for a quarter of a century

\[
P^{(0)}_m(x) = C_F \left( 2p_{qg}(x) + 3\delta(1-x) \right)
\]

\[
P^{(0)}_p(x) = 0
\]

\[
P^{(0)}_g(x) = 2n_f p_{qg}(x)
\]

\[
P^{(0)}_q(x) = 2C_F p_{qg}(x)
\]

\[
P^{(0)}_v(x) = C_A \left( 4p_{qg}(x) + \frac{11}{3} \delta(1-x) \right) - \frac{2}{3} n_f \delta(1-x)
\]

\[
P^{(1)}_m(x) = 4C_F C_F \left( p_{qq}(x) \left[ \frac{67}{18} - \frac{11}{6} H_0 + H_{0,0} \right] + p_{qg}(x) \left[ \frac{1}{2} + 2H_{-1,0} - H_{0,0} \right] \right)
\]

\[
+ \frac{14}{3} \left[ (1-x) + 3\delta(1-x) \right] \left[ \frac{17}{25} + \frac{11}{3} \delta_2 - 3\delta_1 \right]
\]

\[
- 4C_F n_f \left( p_{qq}(x) \left[ \frac{5}{9} + \frac{1}{3} H_0 \right] + \frac{2}{3} \delta_1 \right)
\]

\[
+ \delta(1-x) \left[ \frac{1}{12} + \frac{2}{3} \delta_1 \right] + 4C_F^2 \left( 2p_{qq}(x) \left[ H_{1,0} - \frac{3}{4} H_0 + H_2 \right] - 2p_{qg}(x) \right) \left[ \frac{3}{16} + 2H_{1,0} - H_{0,0} \right] - \delta_1 \left[ \frac{3}{16} - 3\delta_2 + 6\delta_1 \right]
\]

\[
P^{(1)}_p(x) = P^{(1)}_m(x) + 16C_F \left( C_F - \frac{C_A}{2} \right) \left( p_{qq}(x) \left[ \frac{1}{2} + 2H_{-1,0} - H_{0,0} \right] - 2(1-x) \right)
\]

\[
- (1+x)H_0 \right)
\]

\[
P^{(1)}_v(x) = 4C_F n_f \left( \frac{20}{9} - 2 + 6x - 4H_0 + 4x^2 \left[ \frac{9}{3} H_0 - \frac{56}{9} \right] + (1+x) \left[ 5H_0 - 2H_{0,0} \right] \right)
\]

\[
P^{(1)}_q(x) = 4C_F n_f \left( \frac{20}{9} - 2 + 25x - 2p_{qg}(x) \left[ H_{-1,0} - 2p_{qg}(x)H_{1,1} + \delta(1-x) \right] \left[ \frac{29}{3} - H_0 - \frac{1}{3} H_0 \right] \right)
\]

\[
+ 4(1-x) \left[ H_{0,0} - 2H_0 + xH_2 \right] - 4\delta_2 - 2H_0 + 2H_{0,0} - 2H_{1,1} + \frac{37}{7} H_0 + \frac{7}{9} \delta_2 \left[ 2p_{qg}(x)H_{-1,0} - 4C_F n_f \left( \frac{2}{3} \delta_1 \right) - p_{qg}(x) \left[ \frac{2}{3} - \frac{10}{9} \right] + 4C_F^2 \left( p_{qg}(x) \left[ 3H_1 - 2H_{1,1} \right] + (1+x) \left[ H_{0,0} - \frac{7}{3} + \frac{7}{9} H_0 \right] - 3H_{0,0} \right)
\]

\[
+ 1 - \frac{2}{3} H_0 + 2H_{1,0} \right) \right)
\]

\[
P^{(1)}_g(x) = 4C_A C_F \left( \frac{1}{4} (H_{1,1} - H_{1,1} - H_2) + \frac{11}{6} H_0 \right) - \delta_2 \left[ \frac{1}{5} - \frac{44}{3} H_0 - \frac{218}{9} \right] + 4C_A^2 \left( \frac{1}{3} - x \right) \left[ \frac{8}{3} H_0 - \frac{44}{9} \right] + 4C_A \left[ 7H_0 + 2H_{0,0} - 2H_0 + \frac{37}{9} \right] - 2p_{gq}(x) \left[ H_{1,0} \right] - 4C_F n_f \left( \frac{2}{3} \delta_1 \right) - p_{qg}(x) \left[ \frac{2}{3} H_1 - \frac{10}{9} \right] + 4C_F^2 \left( p_{qg}(x) \left[ 3H_1 - 2H_{1,1} \right] + (1+x) \left[ H_{0,0} - \frac{7}{3} + \frac{7}{9} H_0 \right] - 3H_{0,0} \right)
\]

\[
+ 1 - \frac{2}{3} H_0 + 2H_{1,0} \right) \right)
\]

\[
P^{(1)}_v(x) = 4C_A n_f \left( 1 - x + \frac{10}{9} p_{qg}(x) - \frac{13}{9} \left( \frac{1}{3} - x \right)^2 - \frac{2}{3} (1+x) H_0 - \frac{2}{3} \delta(1-x) \right) + 4C_A^2 \left( \frac{1}{3} + x \right) \left[ \frac{11}{6} H_0 + H_{0,0} - \frac{27}{9} \right] + 2p_{qg}(x) \left[ H_{0,0} - 2H_{-1,0} - \delta_2 \right] - 2H_0 - \frac{44}{9} x^2 H_0 + 2p_{gq}(x) \left[ \frac{67}{18} - \delta_2 + H_0 + 2H_{1,1} + H_2 \right] + \delta(1-x) \left[ \frac{8}{3} + 3\delta_1 \right] + 4C_F n_f \left( 2H_0 \right)
\]

\[
+ \frac{21}{3} x + \frac{10}{3} x^2 - 12 + (1+x) \left[ 4 - 5H_0 - 2H_{0,0} - \frac{1}{2} \delta(1-x) \right]
\]
Splitting functions at NNLO

\[ p^{(2)} \approx 3 |n| \left( \frac{2}{3} \right)^{n+1} \left( 1 + \frac{2}{3} \right) \]

\[ p^{(1)} \approx 3 |n| \left( \frac{2}{3} \right)^{n+1} \left( 1 + \frac{2}{3} \right) \]

\[ p^{(0)} \approx 3 |n| \left( \frac{2}{3} \right)^{n+1} \left( 1 + \frac{2}{3} \right) \]
Perturbative stability of evolution

- Scale derivatives of quark and gluon distributions at \( Q^2 \approx 30 \text{ GeV}^2 \)

\[
\frac{d \ln q}{d \ln Q^2}, \quad \frac{d \ln g}{d \ln Q^2}
\]

\( \alpha_s = 0.2, \ N_f = 4 \)
Perturbative stability of evolution

- Scale derivatives of quark and gluon distributions at $Q^2 \approx 30 \text{ GeV}^2$

- Expansion very stable except for very small momenta $x \lesssim 10^{-4}$
Impact on precision of LHC predictions

- $W^{\pm}, Z$-boson rapidity distribution (scale variation $\frac{m_{W,Z}}{2} \leq \mu \leq 2m_{W,Z}$)

Anastasiou, Petriello, Melnikov '05

- NNLO QCD theoretical uncertainties (renormalization / factorization scale) at level of 1% Dissertori et al. '05
  - one of the few cross sections known to NNLO in pQCD
  - "Standard candle" process for parton luminosity
Updates of PDFs (exp)

- New experimental data
  - results from neutrino-nucleon DIS for strange quark PDFs ($s \neq \bar{s}$)
- Uncertainty on $\bar{u}$, $\bar{d}$ doubles from 1.5% to 3% at $Q^2 \simeq M_W^2$ MSTW '07
- $s$, $\bar{s}$ feed into $F_2$ NC DIS constraint $4/9(u + \bar{u}) + 1/9(d + \bar{d} + s + \bar{s})$
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Updates of PDFs (th)

- Improved heavy quark (charm) threshold
  - matching consistent with QCD factorization CTEQ '08
- Significant changes due to larger light flavor PDFs
Predictions for $W^\pm/Z$ cross sections at LHC shift by 8% between PDF sets CTEQ6.6 and CTEQ6.1 (improved theory!)

- sensitivity to PDFs in the $x \sim 10^{-3}$ range
- $W^\pm/Z$-ratio golden calibration measurement
Large extra dimensions

- Sensitivity of LHC dijet cross section to large extra dimensions Ferrag ‘04
  - large extra dimensions accelerate running of $\alpha_s$ as compactification scale $M_c$ is approached

- PDF uncertainties
  - potential sensitivity to $M_c$ reduced from 6 TeV to 2 TeV

\[ M_c = 2 \text{ TeV no PDF error} \quad M_c = 2 \text{ TeV with PDF error} \]
Drell-Yan process and Higgs production

- Mapping of DIS to Drell-Yan lepton-pair production (or to Higgs production in gluon fusion)
  - re-engineering the soft and collinear limit
  - threshold enhanced terms at N^3LO (numerically most important)
  
  S.M., Vogt '05; Laenen, Magnea '05; Idilbi et al. '05

\[ \text{DIS} \quad \text{from} \quad P_{gg}^{(2)} \quad \text{to} \quad \text{Higgs} \quad \text{DY} \]
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Recycling of theory

\[ P_{gg}^{(2)} \]
Anatomy of DIS result (1 loop)

\[ \mathcal{T}_1^b = 2 \text{Re} \mathcal{F}_1 \delta(1 - x) + S_1 \]

- Forward Compton amplitude $\mathcal{T}_n$ in $D = 4 - 2\epsilon$-dimensions combines
  - virtual corrections $\mathcal{F}_n$ (dependent on $\delta(1 - x)$)
  - pure real-emission contributions $S_n$ (dependent on $D$-dimensional +-distributions)
- Infrared finiteness implies cancellation of poles between $\mathcal{F}_n$ and $S_n$
  
  Kinoshita '62; Lee, Nauenberg '64
- Constructive approach to form factor $\mathcal{F}_n$ and $S_n$
Anatomy of DIS result (2 loops)

\[
T_2^b = (2 \text{ Re } F_2 + |F_1|^2) \delta(1 - x) + 2 \text{ Re } F_1 S_1 + S_2
\]
Anatomy of DIS result (3 loops)

\[ T^b_3 = (2 \text{Re } F_3 + 2 |F_1 F_2|) \delta(1 - x) + (2 \text{Re } F_2 + |F_1|^2) S_1 + 2 \text{Re } F_1 S_2 + S_3 \]
Drell-Yan (1 loop)

- Construction of cross sections for hadron-hadron scattering
  - form factor with time-like kinematics $Q^2 > 0$
  - soft emission with $D$-dimensional +-distributions
- Drell-Yan lepton-pair production in $qq$-annihilation
- Higgs production from gluon fusion
Drell-Yan (2 loops)

\[ \int d\text{LIPS}(1) \]

\[ \int d\text{LIPS}(2) \]

\[ F^{(2)} \]

\[ F^{(1)} S^{(1)} \]

\[ (F^{(1)})^2 \]

\[ S^{(2)} \]

- Checks at two loops
  - Drell-Yan
    Hamberg, van Neerven, Matsuura ‘91; Harlander, Kilgore ’02
  - Higgs production
    Harlander, Kilgore ‘02; Anastasiou, Melnikov ‘02; Ravindran, Smith, van Neerven ‘03
Drell-Yan (3 loops)
Higgs boson production at LHC (II)

\[ \mathcal{O}(\mu_0, \mu_r) = \mathcal{O}(M_H) \]

- \( N^3\text{LO}_{\text{approx.}} \) increase at \( \mu_r = M_H \) 5\% (NNLO PDF’s) S.M., Vogt ’05
- \( \mu_r \) variation 4\%
- Overall accuracy of 5\% reached with approx. \( N^3\text{LO} \) prediction

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Instanton induced processes at LHC

- Multi-gluon production in instanton background
- Transfer of DIS phenomenology to Drell-Yan lepton-pair production
  Brandenburg, Ringwald, Utermann ’06; Petermann, Schrempp ’08
- Amplitudes related by crossing

**DIS**: $I$-induced photon-gluon scattering

- **pp**: $I$-induced $q\bar{q}$-annihilation

  Characteristic final state signature
  - Isotropic multi-particle production, very high multiplicity
Summary

HERA

- Deep-inelastic scattering (electron-proton collision)
  - wealth of experimental information on proton structure
- QCD precision predictions
  - radiative corrections for parton evolution
- HERA PDFs have strong impact on measurements at LHC
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Outlook
- If past performance is an indicator, we are well prepared for future challenges...