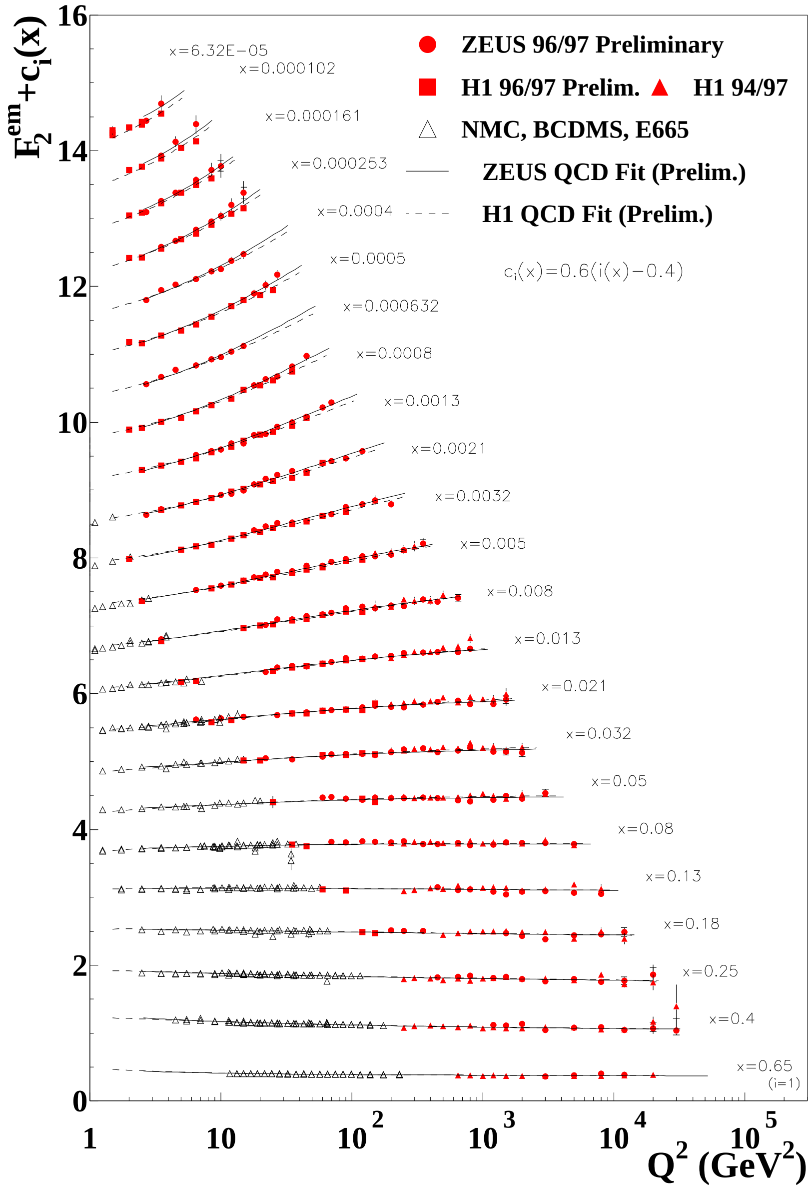
k_t - factorization and CCFM: the solution for describing the hadronic final states - everywhere ?

H. Jung, University of Lund

DESY - seminar 22.Oct 2002

- where is the problem ?
  hadronic final states - jets, heavy quarks - even at Tevatron
  approximations
- doing it better !
  CCFM equation in one - loop (DGLAP) and all - loop, solution
  implementation into new hadron level MC CASCADE
- solve problems, also for heavy quarks and even for Tevatron
- nobody is perfect
- conclusion

The structure function $F_2(x, Q^2)$: DGLAP



Scaling violations perfectly described with DGLAP:

$0.63 \cdot 10^{-5} < x < 0.65$

$1 < Q^2 < 25000$ GeV^2

- adjust input pdf to fit F_2 data

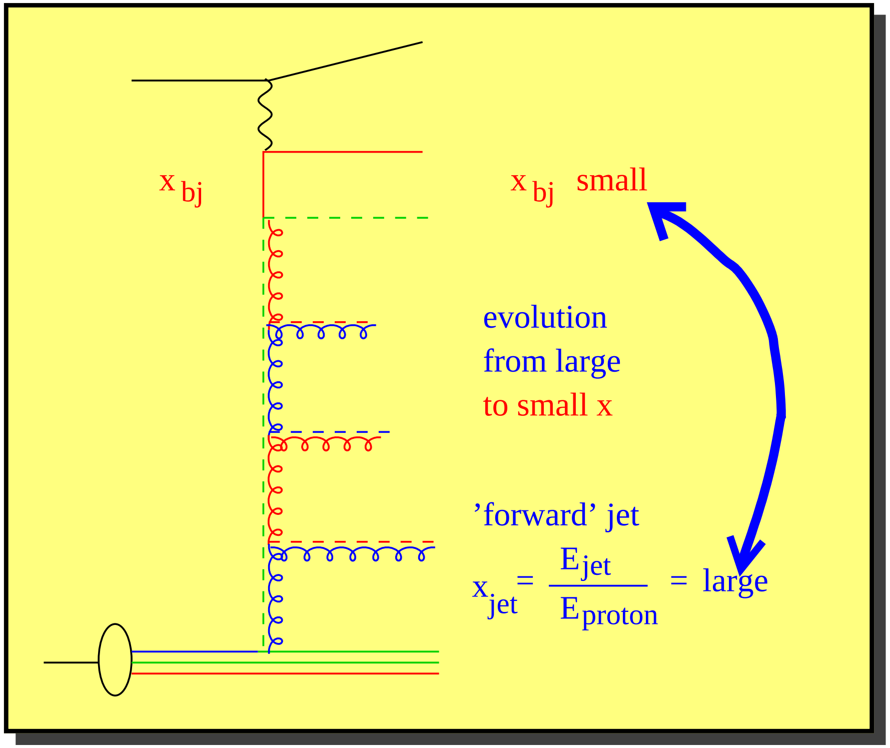
BUT different sets: MRS, GRV etc

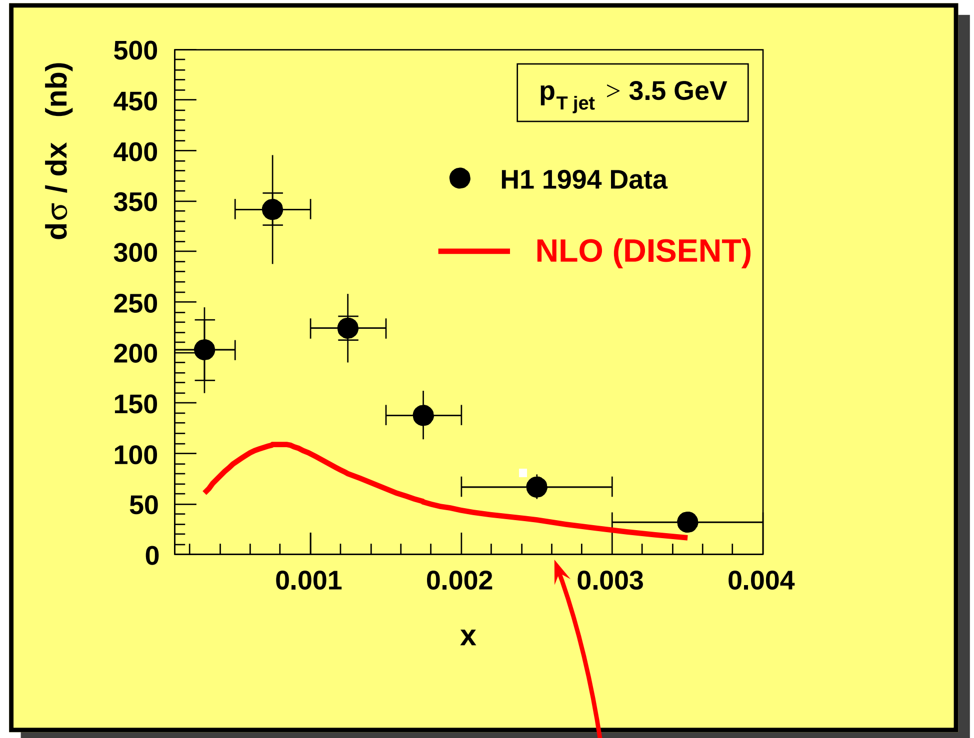
- use extracted pdf to predict x - sections
- even at $p\bar{p}$

BUT for reliable predictions at HERA II/III, THERA, LHC etc

- better understand pdf's

Where is the problem? Forward Jets

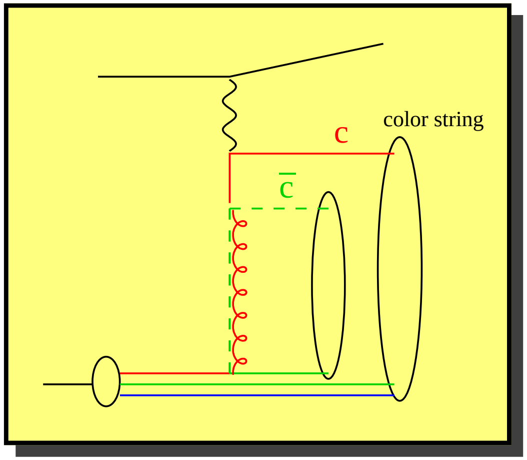




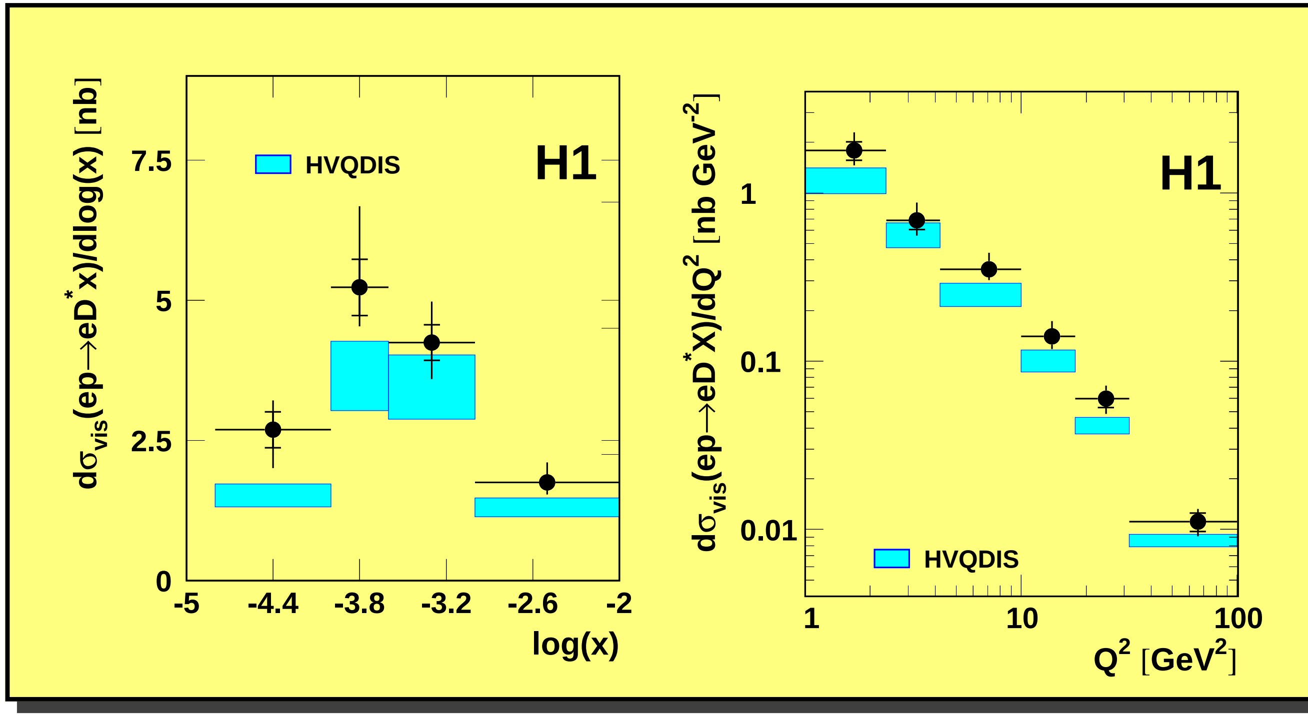
Mueller - Navelet jets in DIS: **Jet in p - direction with $p_t^2 \sim Q^2$, x_{jet} large, BUT small x_{bj}**

- ☞ **suppress DGLAP evolution allow evolution in x**
- ☞ **standard DGLAP $\sim$ factor 2 too small!**

Where is the problem? Charm in DIS



exp. cuts
$1 < Q^2 < 100$ GeV^2
$0.05 < y < 0.7$
$p_t^{D^*} > 1.5$ GeV $|\eta^{D^*}| > 1.5$



D^* production in DIS

- **standard DGLAP with NLO calculation too small**
- **Problem at small x and small Q^2 !**
- **How to determine F_2^c ?**

Where is the problem ?
Bottom at HERA and Tevatron

HERA

ZEUS (ZEUS Coll. *EPJC* (2001))

$Q^2 < 1$ **GeV**^{2}, $0.2 < y < 0.8$,

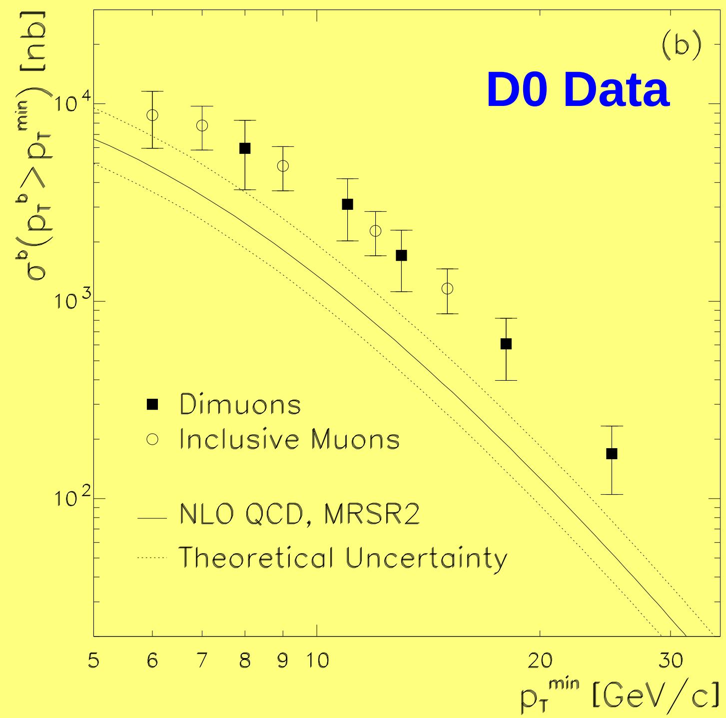
$p_t^b > 5$ **GeV**, $|\eta^b| < 2$

$\sigma = 1.6 \pm 0.4$**(stat.)**$^{+0.3}_{-0.5}$**(syst.)**$^{+0.2}_{-0.4}$**(ext.) nb**

NLO: $\sigma = 0.64^{+0.15}_{-0.1}$ **nb**

---

safe extrapolation from visible to total x-section ???



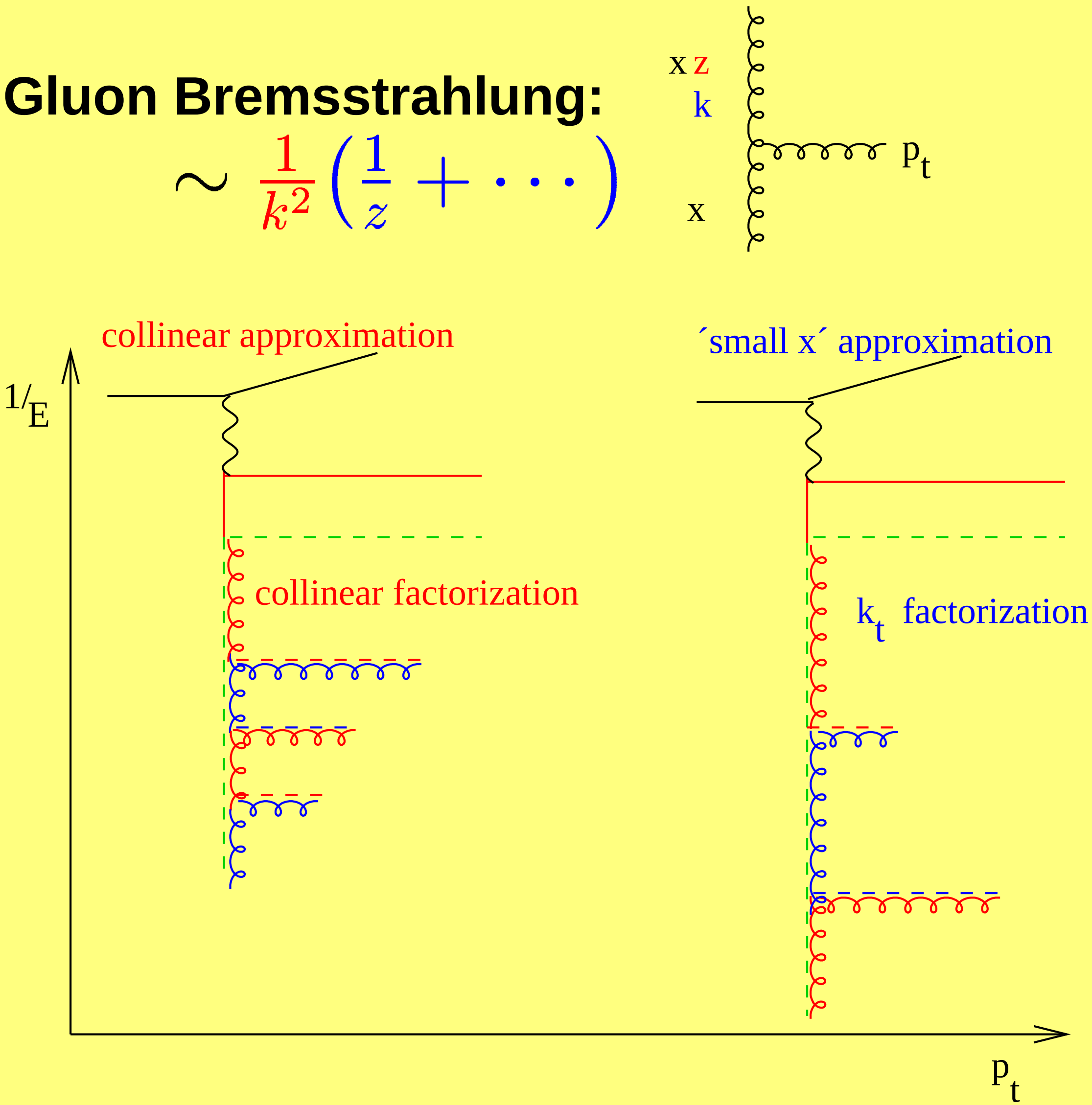
$b\bar{b}$ in photoproduction and hadroproduction:

- ☞ **standard DGLAP with NLO calculation**
- ☞ $\sim$ **factor 2 - 4 too small !**

Approximation in QCD cascade: Factorization

Gluon Bremsstrahlung:

$$\sim \frac{1}{k^2}\left(\frac{1}{z} + \cdots\right)$$



DGLAP:

- **collinear singularities factorized in pdf**
- **evolution in $Q^2 \sim k^2$, k_t^2 or p_t^2**
- $\sigma = \sigma_0 \int \frac{dz}{z} C^a\left(\frac{x}{z}\right) f_a(z, Q^2)$

BFKL:

- **k_t dependent pdf**
- **$\rightarrow$ unintegrated pdf**
- **evolution in x**
- $\sigma = \int \frac{dz}{z} d^2k_t \hat{\sigma}\left(\frac{x}{z}, k_t\right) \mathcal{F}(z, k_t)$

The problem of Asymptotia...

DGLAP is great

at large $Q^2 \to \infty$

But has problems:

- ➤ **Small x processes:**
- ☞ **heavy quarks**
- ☞ **particle spectra**
- ☞ **jets**

BFKL is great

at small $x \to 0$

But has problems:

- ➤ **at finite x:**
- ☞ **NLO corrections**
- ☞ **predictive power**
- ☞ **how to simulate?**

But asymptotia still far away

even for THERA or LHC ...

Attempts to survive in reality

- ☞ **hack DGLAP and BFKL**

  prediction ?
- ☞ **introduce new concepts: resolved (virtual) photons**

  ↪**evolve with DGLAP from proton and photon side**

  ↪**similar to $p\bar{p}$**

  ↪**works nicely ($\rightarrow$ RAPGAP MC generator)**

  BUT theoretical questions: which scale etc ???
- ☞ **CCFM - new investigation of color coherence**

Attempts to survive in reality

hacker: person paid to do hard and uninteresting work ...

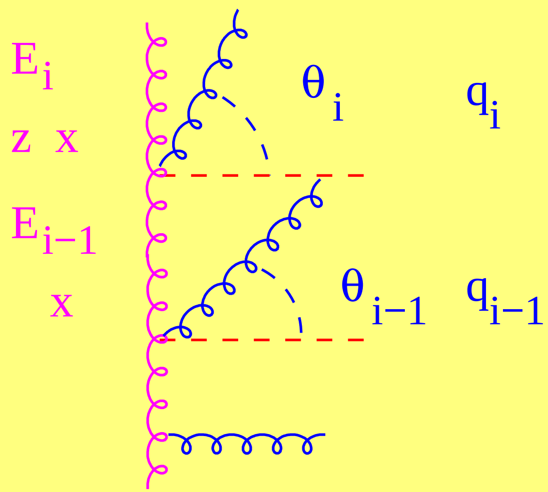
Oxford Advanced Dictionary

- hack DGLAP and BFKL prediction ?
- introduce new concepts: resolved (virtual) photons
  - ➥evolve with DGLAP from proton and photon side
  - ➥similar to $p\bar{p}$
  - ➥works nicely ($\rightarrow$ RAPGAP MC generator)

  BUT theoretical questions: which scale etc ???
- **CCFM - new investigation of color coherence**

C atani C iafaloni F iorani M archesini (1988 - 1990)

- including color coherence effects in multi-gluon emissions
- angular ordering of emission angles:



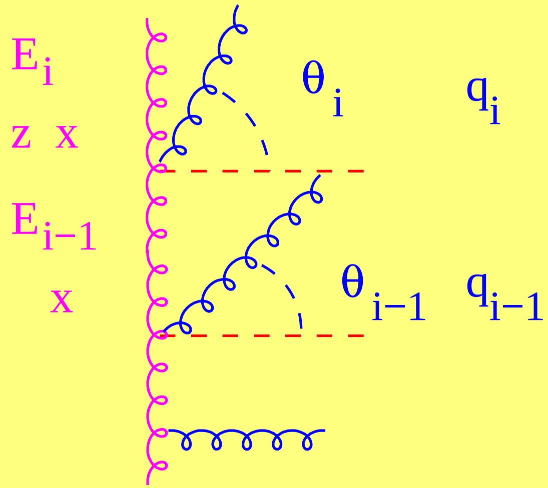
- ordering in q (DGLAP) implies also angular ordering
- unification of DGLAP and BFKL

☛ **WOW**

for small z no restriction in q: ☛ random walk in q

CCFM: Catani Ciafaloni Fiorani Marchesini (1988 - 1990)

- including color coherence effects in multi-gluon emissions
- angular ordering of emission angles:



$$p_{ti} = |q_i^0| \sin\Theta_i, \; z = \frac{E_i}{E_{i-1}}$$

$$E_{i-1} = E_i + q_i^0 = zE_{i-1} + q_i^0, \; ☛ \; q_i^0 = (1-z)E_{i-1}$$

$$p_{ti} = q_i^0 \sin\Theta_i \simeq (1-z)E_{i-1}\Theta_i$$

$$\frac{p_{ti}}{1-z} \simeq E_{i-1}\Theta_i$$

with: $q_i = \frac{p_{ti}}{1-z_i}$ ☛ $\Theta_i = \frac{q_i}{E_{i-1}}$ **and** $\Theta_{i+1} = \frac{q_{i+1}}{E_i}$

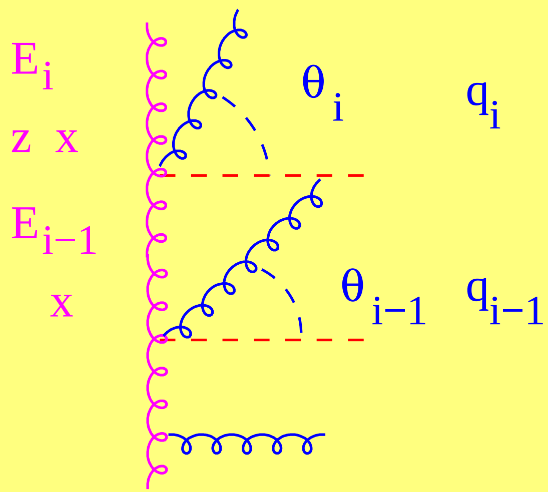
- ordering in q (DGLAP) implies also angular ordering
- unification of DGLAP and BFKL

☛ **WOW**

for small z no restriction in q: ☛ random walk in q

Catani **C**iafaloni **F**iorani **M**archesini (1988 - 1990)

- including color coherence effects in multi-gluon emissions
- angular ordering of emission angles:



in lab. frame

$$\Theta_{i+1} > \Theta_i$$

$$\boxed{q_{i+1} > z_i q_i}$$

with $q = \frac{p_t}{1-z}$

- ordering in q (DGLAP) implies also angular ordering
- unification of DGLAP and BFKL

WOW

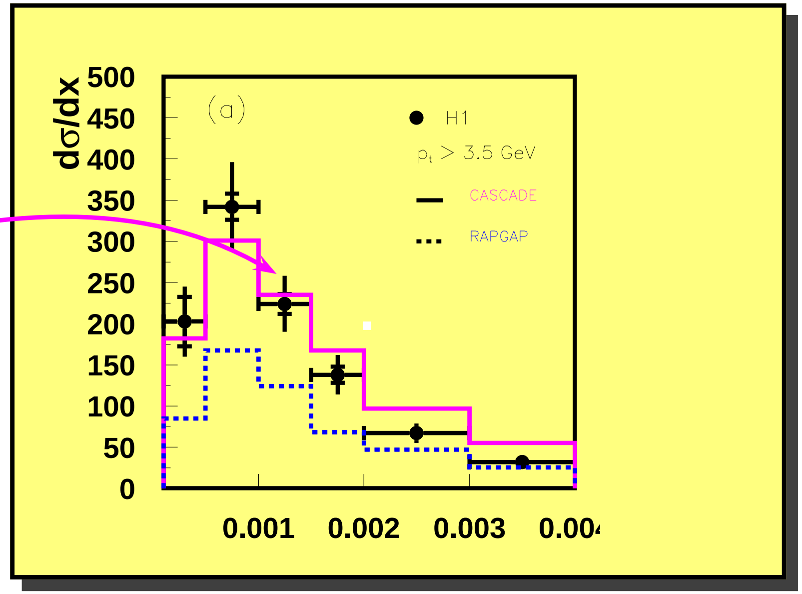
for small z no restriction in q: random walk in q

CASCADE with CCFM solves the problems

Solve CCFM equation
to fit F_2 data from HERA

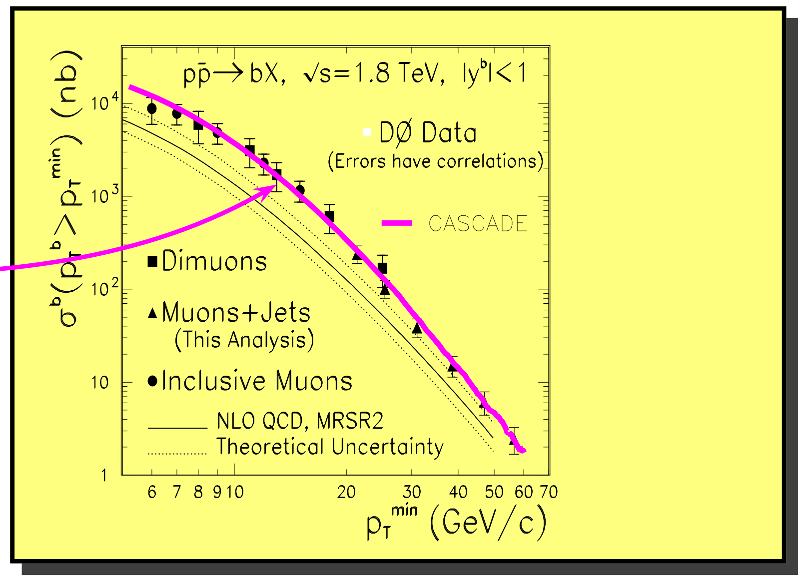
- obtain CCFM un-integrated gluon

  CASCADE MC implements CCFM:
- predict fwd jet x-section at HERA ✔
- predict charm at HERA ✔
- predict bottom at HERA ✔



- test universality of un-integrated gluon density from HERA
- predict bottom at Tevatron ✔
- w/o additional free parameters

WOW !!!



CCFM Monte Carlo - CASCADE

Why is CASCADE doing well

and

why did it take so long

to develop it ???

Or

Why did all the other brave approaches fail ???

Apologies to my friends in Lund ...

CCFM history

1988 - 1990	Formulation of CCFM	M. Ciafaloni, Nucl. Phys. B 296 (1988) 49 S. Catani, F. Fiorani, G. Marchesini, Phys. Lett B 234 (1990) 339 S. Catani, F. Fiorani, G. Marchesini, Nucl. Phys. B 336 (1990) 18 G. Marchesini, Nucl. Phys. B 445 (1995) 49
1992	SMALLX- MC program	G. Marchesini. B. Webber, Nucl. Phys. B 386 (1992) 215
1996 - 1998	Formulation of LDC	B. Andersson, G. Gustafson, J. Samuelsson, Nucl. Phys. B 467 (1996) 443 B. Andersson, G. Gustafson, H. Kharrazhia, J. Samuelsson, Z. Phys. C 71 (1996) 613 H. Kharrazhia, L.Lönnblad, JHEP 03 (1998) 006
1998	Had. final states in CCFM	G. Bottazzi, G. Marchesini, G.P. Salam, M. Scorletti, JHEP 9812:011 (1998)

CCFM history (cont'd)

status in 1999

CCFM is great !

can describe $\sigma(\gamma^* p)$ (F_2)

(hm..., DGLAP does it also...)

But fails for hadronic final states: Forward Jets ...

CCFM history (cont'd)

status in 1999

CCFM is great !

can describe $\sigma(\gamma^* p)$ (F_2)

(hm..., DGLAP does it also...)

But fails for hadronic final states: Forward Jets ...

The End of small x ?

CCFM history (cont'd)

status in 1999

CCFM is great !

can describe $\sigma(\gamma^* p)$ (F_2)

(hm..., DGLAP does it also...)

But fails for hadronic final states: Forward Jets ...

Much progress since then....

Small x workshops in Lund

(thanks to DESY for support)

Small x Collaboration

CCFM equation: small and large x

$$\mathcal{A}(x,k_t,\bar{q}) = \mathcal{A}_0(x,k_t)\Delta_s(\bar{q},Q_0) + \int \frac{dz}{z} \int \frac{d^2q}{\pi q^2} \Theta(\bar{q} - zq) \cdot \Delta_s(\bar{q},zq) \tilde{P}(z,q,k_t) \mathcal{A}\left(\frac{x}{z}, k_t', q\right)$$

CCFM Splitting fct: $\tilde{P}(z,q,k_t) = \frac{\bar{\alpha}_s(q(1-z))}{1-z} + \frac{\bar{\alpha}_s(k_t)}{z}\Delta_{\textbf{ns}}(z,q,k_t)$

Sudakov $\Delta_s(a,b)$: **probablility for no radiation in $[a,b]$**

angular ordering: $\bar{q} > z_n q_n, q_n > z_{n-1} q_{n_1}, \ldots, q_1 > Q_0$

small x

- **BFKL limit ($z \to 0$)**
- **angular ordering**

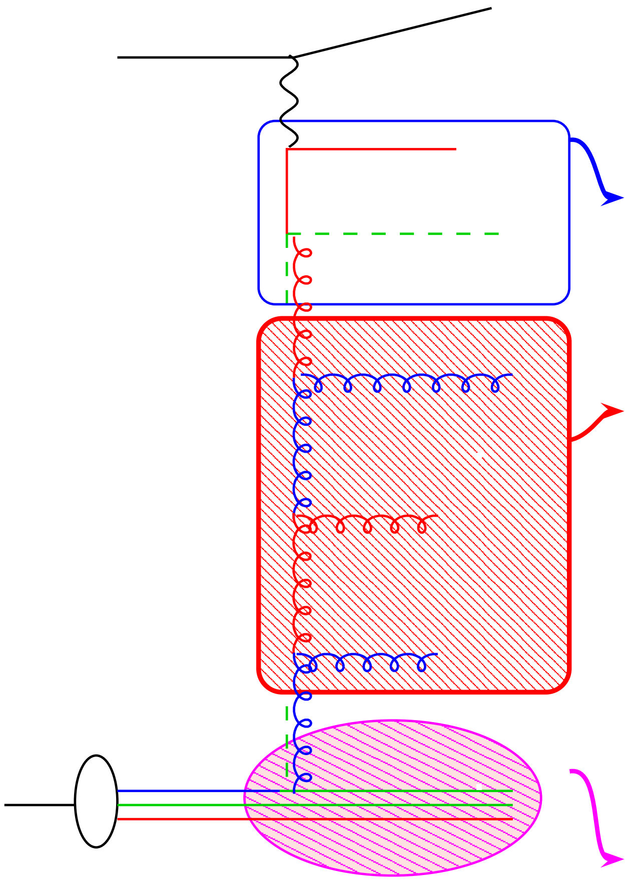
$\rightarrow$ **no restriction on q_i**

large x

- **DGLAP limit ($z \gg 0$)**
- **DGLAP splitting fct $\tilde{P}$ with $\Delta_{\textbf{ns}} = 1$**
- **angular ordering $\rightarrow q_i$ ordering**

Basic idea - Collinear factorisation

DGLAP

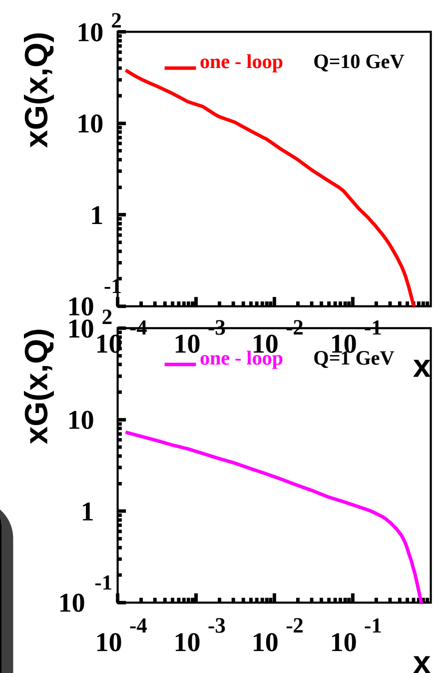


BGF matrix element
on mass shell

evolution of parton cascade
q - ordered
with DGLAP splitting fct.

$$\tilde{P}(z,q,k_t) = \bar{\alpha}_s \left(\frac{1}{1-z} + \frac{1}{z} \right)$$

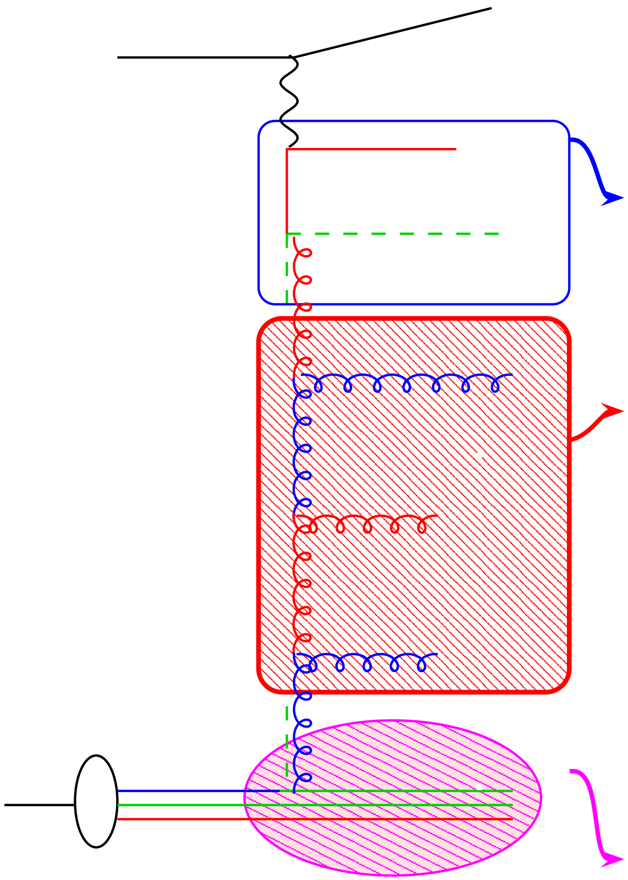
initial distribution: steep



$$\sigma(ep \to e' q\bar{q}) = \int \frac{dy}{y} d^2Q \frac{dx_g}{x_g} \hat{\sigma}(\hat{s}, 0, Q) \int d^2k_t x_g \mathcal{A}(x_g, k_t, \bar{q})$$

with $\int d^2k_t x_g \mathcal{A}(x_g, k_t, \bar{q}) = x_g G(x_g, Q^2)$

Basic idea - Collinear factorisation



DGLAP

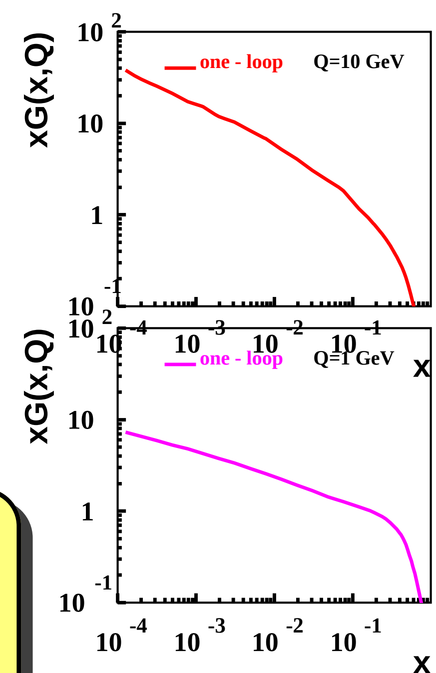
BGF matrix element
on mass shell

evolution of parton cascade
q - ordered
with DGLAP splitting fct.

$$\tilde{P}(z,q,k_t) = \bar{\alpha}_s \left(\frac{1}{1-z} + \frac{1}{z}\right)$$

initial distribution: steep

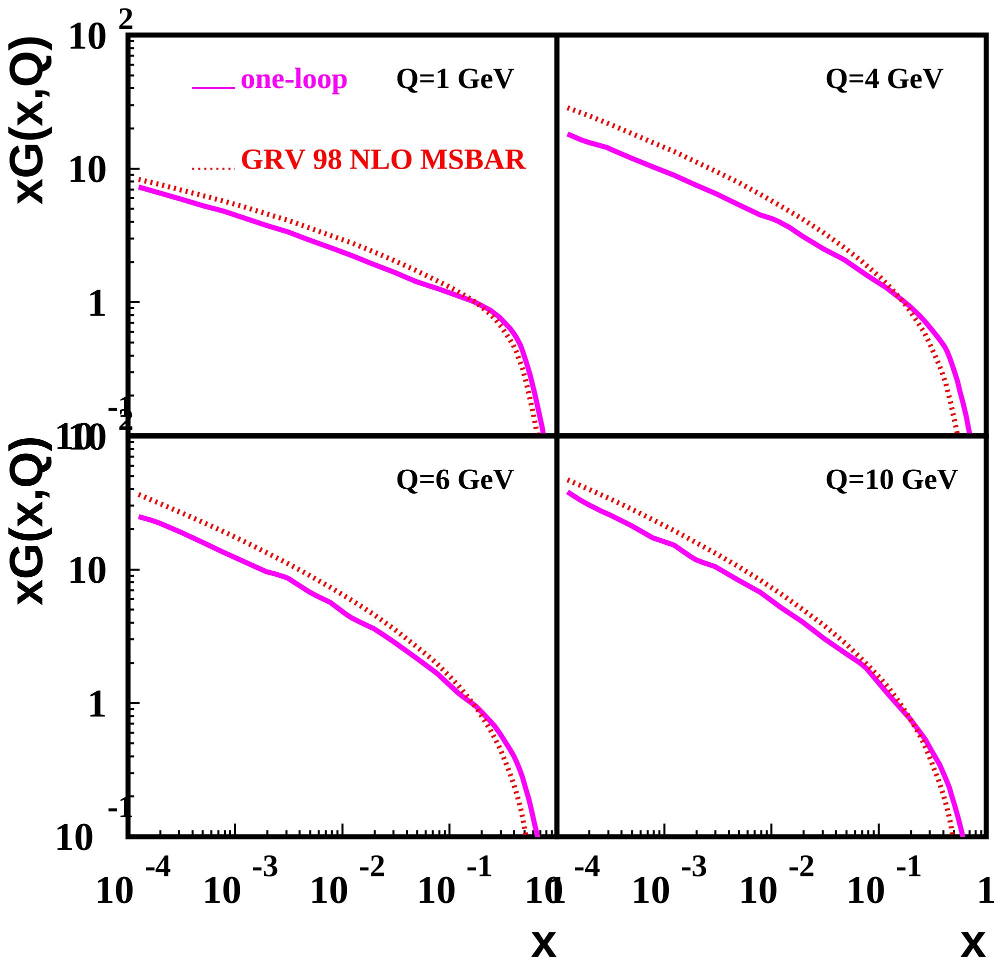
J.C. Collins, X. Zu JHEP 06 (2002) 018
on shell matrix element
is only assumption without proof
and is unphysical !!!
and not necessary for proof of factorization !!!



$$\sigma(ep \to e'q\bar{q}) = \int\int d^2k_t \frac{dy}{y} d^2Q \frac{dx_g}{x_g} \hat{\sigma}(\hat{s},k_t,Q) x_g \mathcal{A}(x_g,k_t,\bar{q})$$

with $\int d^2k_t x_g \mathcal{A}(x_g,k_t,\bar{q}) \stackrel{?}{=} x_g G(x_g,Q^2)$

DGLAP unintegrated gluon density - integrated -



one-loop gluon integrated over k_t

$$\int_0^{\bar{q}} dk_t^2 x\mathcal{A}(x,k_t,\bar{q}) = xG(x,\bar{q})$$

➤ **compare to evolved DGLAP gluon**

one-loop gluon:

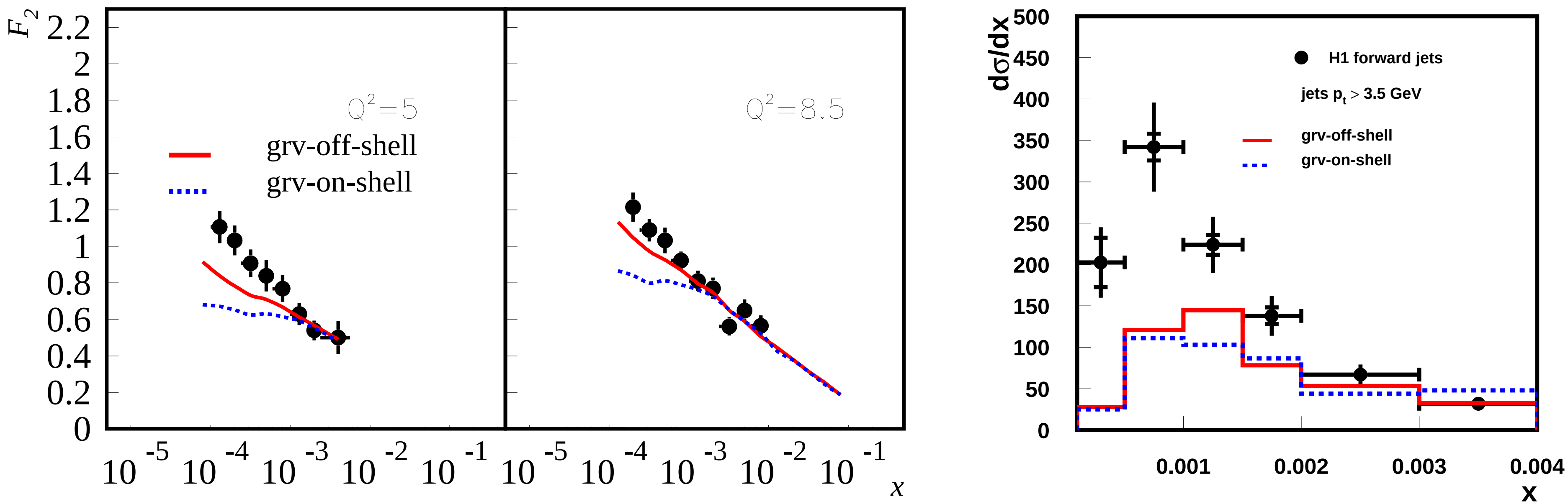
- **at starting scale use GRV**
- **test evolution machinery**
- **full treatment of kinematics**
- **more than standard DGLAP**
- **small differences from special form of** $\tilde{P} = \bar{\alpha}_s \left(\frac{1}{1-z} + \frac{1}{z}\right)$
- **small differences since only gluons**

$F_2(x, Q^2)$ and forward jets (in one-loop)

With $\sigma = \int dx_g \sigma(\gamma^* g \to q\bar{q}) \int dk_t^2 \mathcal{A}(x_g, k_t^2, \bar{q})$ fit $F_2(x, Q^2)$

(data from H1 Coll, NPB 470 (1996) 3.)

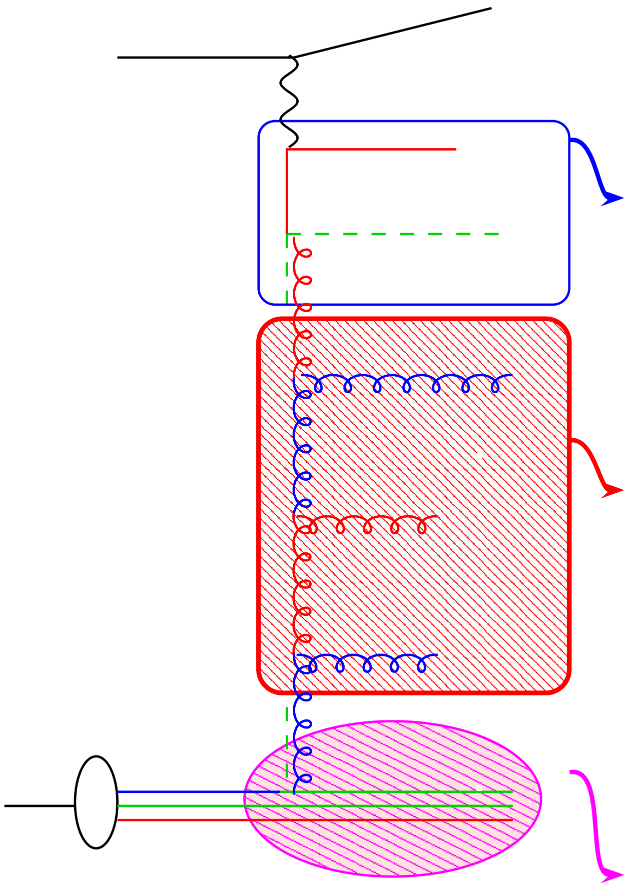
(fitted for $Q^2 > 5$ GeV^2, $x < 10^{-2}$)



- only $1/z$ and $1/(1-z)$ included in P_{gg}
- problem: small x region
- But also off-shell ME helps (with q-ordering and $k_t < p_t$) for rise at small x !!!
- similar findings with standard NLO splitting kernels
- still to small for forward jets... but the same as NLO

Basic idea - k_t factorisation

CCFM

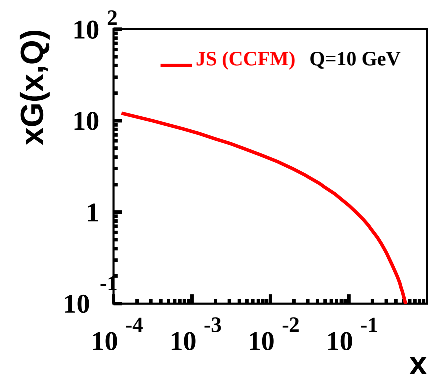


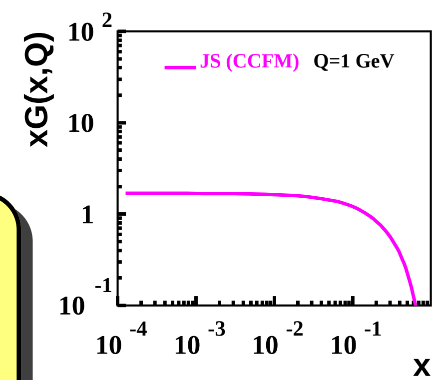
BGF matrix element
off mass shell

evolution of parton cascade
with CCFM splitting fct.

$$\tilde{P} = \bar{\alpha}_s \left(\frac{1}{1-z} + \frac{1}{z}\Delta_{ns} \right)$$

initial distribution: flat



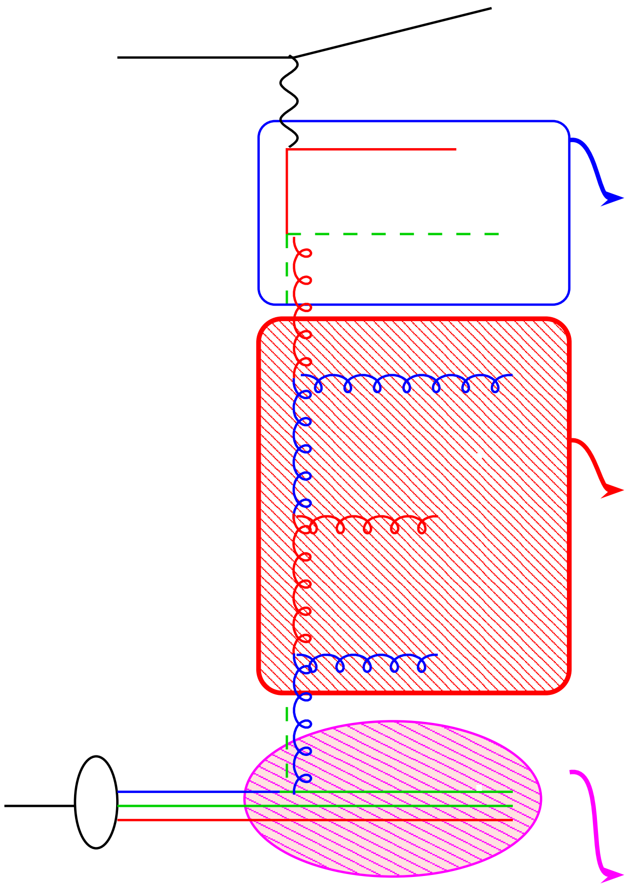


$$\sigma(ep \to e'q\bar{q}) = \int \frac{dy}{y} d^2Q \frac{dx_g}{x_g} \int d^2k_t \hat{\sigma}(\hat{s}, k_t, Q) x_g \mathcal{A}(x_g, k_t, \bar{q})$$

with $\int d^2k_t x_g \mathcal{A}(x_g, k_t, \bar{q}) \simeq x_g G(x_g, Q^2)$

Basic idea - k_t factorisation

CCFM



BGF matrix element
off mass shell

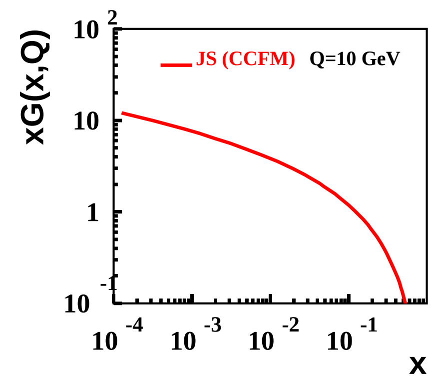
evolution of parton cascade
with CCFM splitting fct.

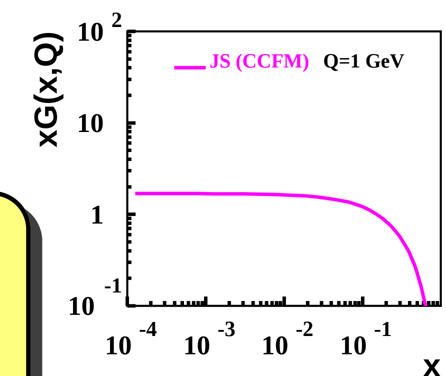
$$\tilde{P} = \bar{\alpha}_s \left(\frac{1}{1-z} + \frac{1}{z} \Delta_{ns} \right)$$

initial distribution: flat

NEW

- **angular ordering (instead of q_t ordering)**
- **Δ_{ns} (non - Sudakov)**





$$\sigma(ep \to e' q\bar{q}) = \int \frac{dy}{y} d^2Q \frac{dx_g}{x_g} \int d^2k_t \hat{\sigma}(\hat{s}, k_t, Q) x_g \mathcal{A}(x_g, k_t, \bar{q})$$

with $\int d^2k_t x_g \mathcal{A}(x_g, k_t, \bar{q}) \simeq x_g G(x_g, Q^2)$

Splitting Function,non-Sudakov and all that...

Splitting Fct: $\tilde{P} = \frac{\bar{\alpha}_s(q(1-z))}{1-z} + \frac{\bar{\alpha}_s(k_t)}{z} \Delta_{\textbf{ns}}(z, q, k_t)$

...contains only singular parts....

non-Sudakov $\log \Delta_{\textbf{ns}} = -\bar{\alpha}_s(k_t^2) \int_0^1 \frac{dz'}{z'} \int \frac{dq^2}{q^2} \Theta(k_t - q) \Theta(q - z' q_t)$

Splitting Function,non-Sudakov and all that...

Splitting Fct: $\tilde{P} = \frac{\bar{\alpha}_s(q(1-z))}{1-z} + \frac{\bar{\alpha}_s(k_t)}{z}\Delta_{\mathbf{ns}}(z, q, k_t)$

...contains only singular parts....

non-Sudakov $\log\Delta_{\mathbf{ns}} = -\bar{\alpha}_s(k_t^2)\int_0^1 \frac{dz'}{z'}\int \frac{dq^2}{q^2}\Theta(k_t - q)\Theta(q - z'q_t)$

reason why others fail:
- only 1/z terms
- non -singular terms

Splitting Function,non-Sudakov and all that...

Splitting Fct: $\tilde{P} = \frac{\bar{\alpha}_s(q(1-z))}{1-z} + \frac{\bar{\alpha}_s(k_t)}{z} \Delta_{\textbf{ns}}(z, q, k_t)$

...contains only singular parts....

non-Sudakov $\log\Delta_{\textbf{ns}} = -\bar{\alpha}_s(k_t^2) \int_0^1 \frac{dz'}{z'} \int \frac{dq^2}{q^2} \Theta(k_t - q)\Theta(q - z'q_t)$

reason why others fail:

- **only 1/z terms**
- **non -singular terms**

naive solution:

$\log\Delta_{\textbf{ns}} = -\bar{\alpha}_s(k_t^2)\log\left(\frac{1}{z}\right)\log\left(\frac{k_t^2}{zq^2}\right)$

consistency constraint (CC):

$k_t^2 > zq^2$

CC **removes part of phase space**

reason why others failed:

- **integrals not properly evaluated**
- *Consistency Constraint !!!*

Splitting Function,non-Sudakov and all that...

Splitting Fct: $\tilde{P} = \frac{\bar{\alpha}_s(q(1-z))}{1-z} + \frac{\bar{\alpha}_s(k_t)}{z} \Delta_{\text{ns}}(z, q, k_t)$

...contains only singular parts....

non-Sudakov $\log\Delta_{\text{ns}} = -\bar{\alpha}_s(k_t^2) \int_0^1 \frac{dz'}{z'} \int \frac{dq^2}{q^2} \Theta(k_t - q)\Theta(q - z' q_t)$

reason why others fail:

- only 1/z terms
- non -singular terms

solution for CCFM splitting fct:

(Kwiecinski, Martin, Sutton PRD 52 (1995) 1445)

$\log\Delta_{\text{ns}} = -\bar{\alpha}_{\text{s}}(k_t) \log\left(\frac{z_0}{z}\right) \log\left(\frac{k_t^2}{z_0 z q^2}\right)$

with $z_0 = f(z, k_t, q)$

valid in full phase space

non - Sudakov depends on non - local (history of cascade):

$\mathbf{k}_{ti} = \mathbf{k}_{ti-1} + \mathbf{p}_{ti} = \mathbf{p}_{t1} + \mathbf{p}_{t2} + ... + \mathbf{p}_{ti}$

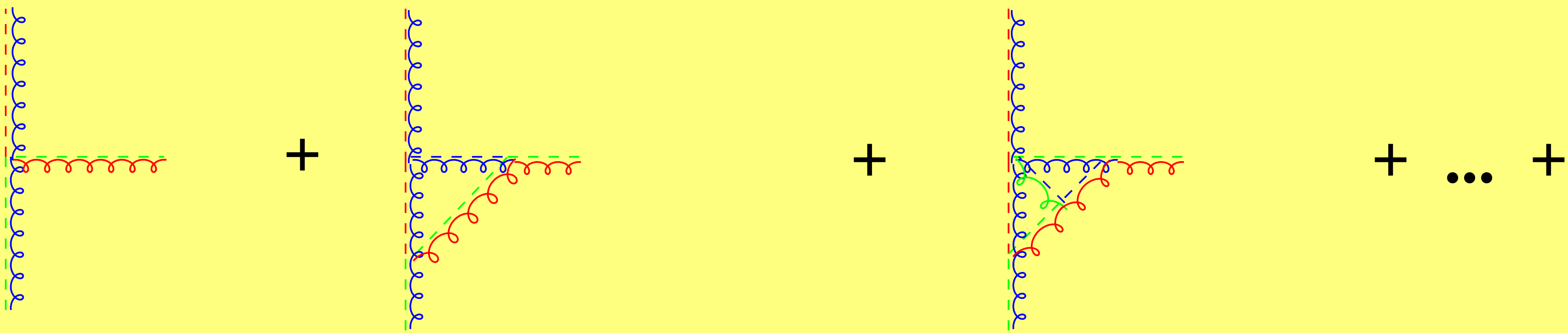
Non-Sudakov and all - loop resummation

Splitting Fct: $\tilde{P} = \frac{\bar{\alpha}_s(q(1-z))}{1-z} + \frac{\bar{\alpha}_s(k_t)}{z}\Delta_{\textbf{ns}}(z,q,k_t)$

Non - Sudakov form factor ➤ all loop resummation:

$$\Delta_{\textbf{ns}} = \exp\left[-\bar{\alpha}_s(k_t^2)\int_0^1 \frac{dz'}{z'}\int \frac{dq^2}{q^2}\Theta(k_t-q)\Theta(q-z'q_t)\right]$$

$$\Delta_{\textbf{ns}} = 1 + \left(-\bar{\alpha}_s(k_t^2)\int \frac{dz'}{z'}\int \frac{dq^2}{q^2}\right)^1 + \frac{1}{2!}\left(-\bar{\alpha}_s(k_t^2)\int \frac{dz'}{z'}\int \frac{dq^2}{q^2}\right)^2 \ldots$$



$$\bar{\alpha}_\mathrm{s}(k_t)\frac{1}{z}\left[\,1 \quad + \quad \bar{\alpha}_\mathrm{s}\log\left(\frac{z}{z_0}\right)\log\left(\frac{k_t^2}{z_0 z q^2}\right) \quad + \quad \frac{1}{2!}\left(\bar{\alpha}_\mathrm{s}\log\left(\frac{z}{z_0}\right)\log\left(\frac{k_t^2}{z_0 z q^2}\right)\right)^2 \ldots\right]$$

Structure Function $F_2(x, Q^2)$

together with G.P. Salam, EPJC 19, 351 (2001)

With $\sigma = \int dk_t^2 dx_g \mathcal{A}(x_g, k_t^2, \bar{q}) \sigma(\gamma^* g^* \to q\bar{q})$ fit $F_2(x, Q^2)$

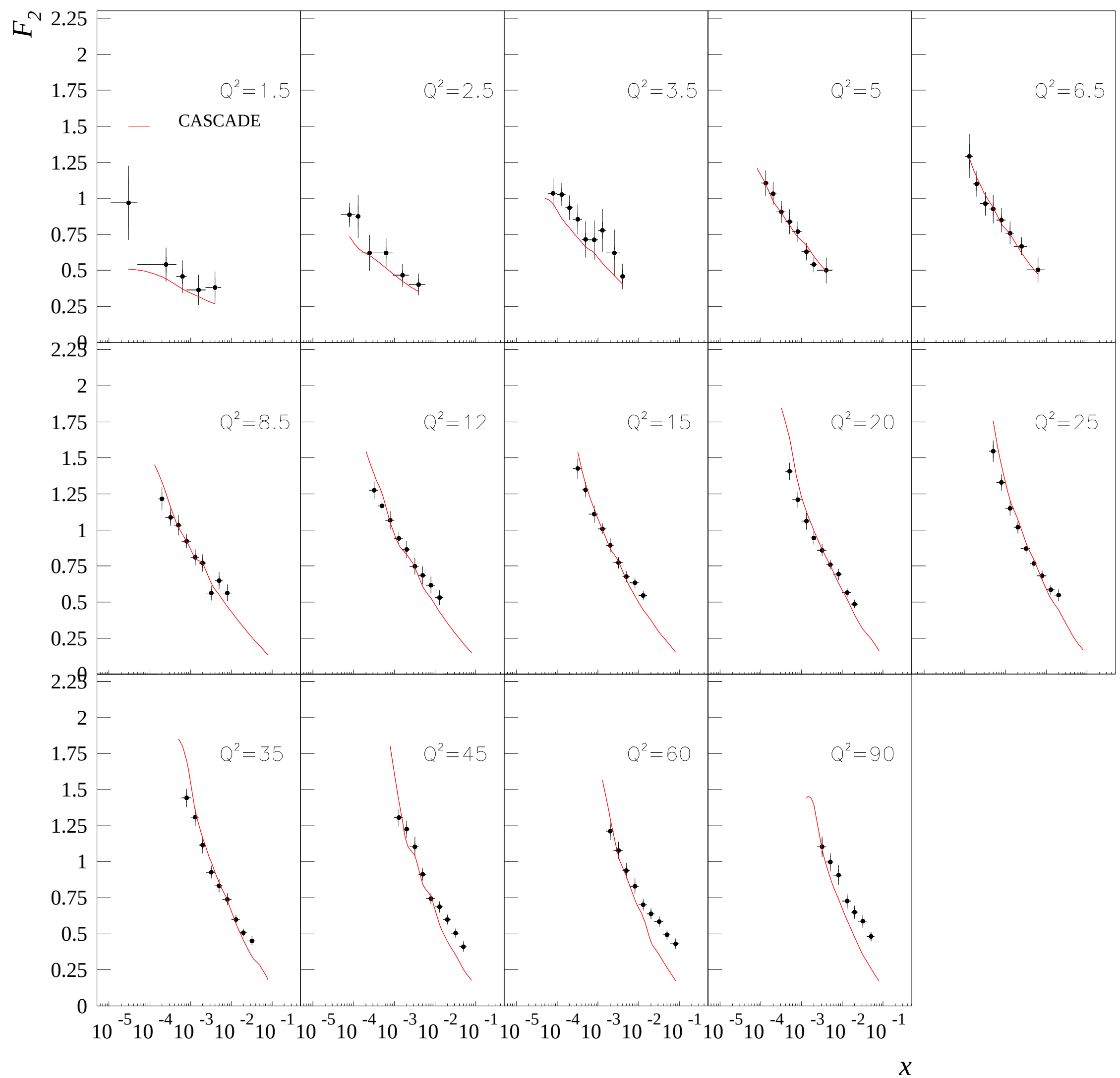
(data from H1 Coll, NPB 470 (1996) 3.)

Parameters in fit

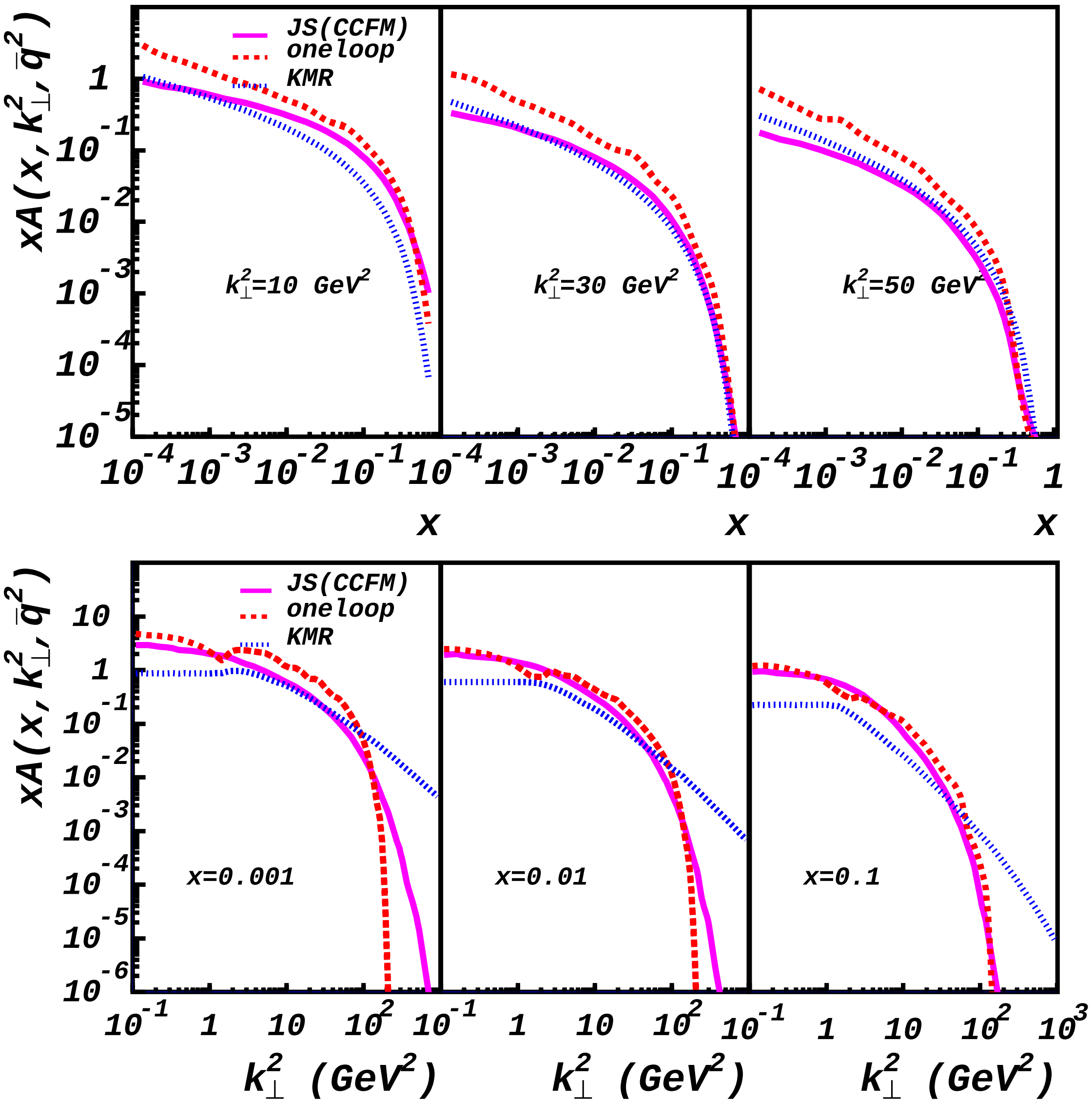
(fitted for $Q^2 > 5$ GeV^2, $x < 10^{-2}$)

- collinear cut-off $Q_0 = 1.4$ GeV
- initial gluon $x\mathcal{A}_0(x, k_{t0}^2)$
- freezing of $\alpha_s(k_t)$ for $k_t \to 0$
  k_t not constrained ...
- light quark masses: $m_q = 0.140$ GeV, $m_c = 1.5$ GeV

unintegrated gluon density
$x\mathcal{A}(x, k_t^2, \bar{q})$
obtained from fit to F_2



Unintegrated gluon density



JS (CCFM) gluon

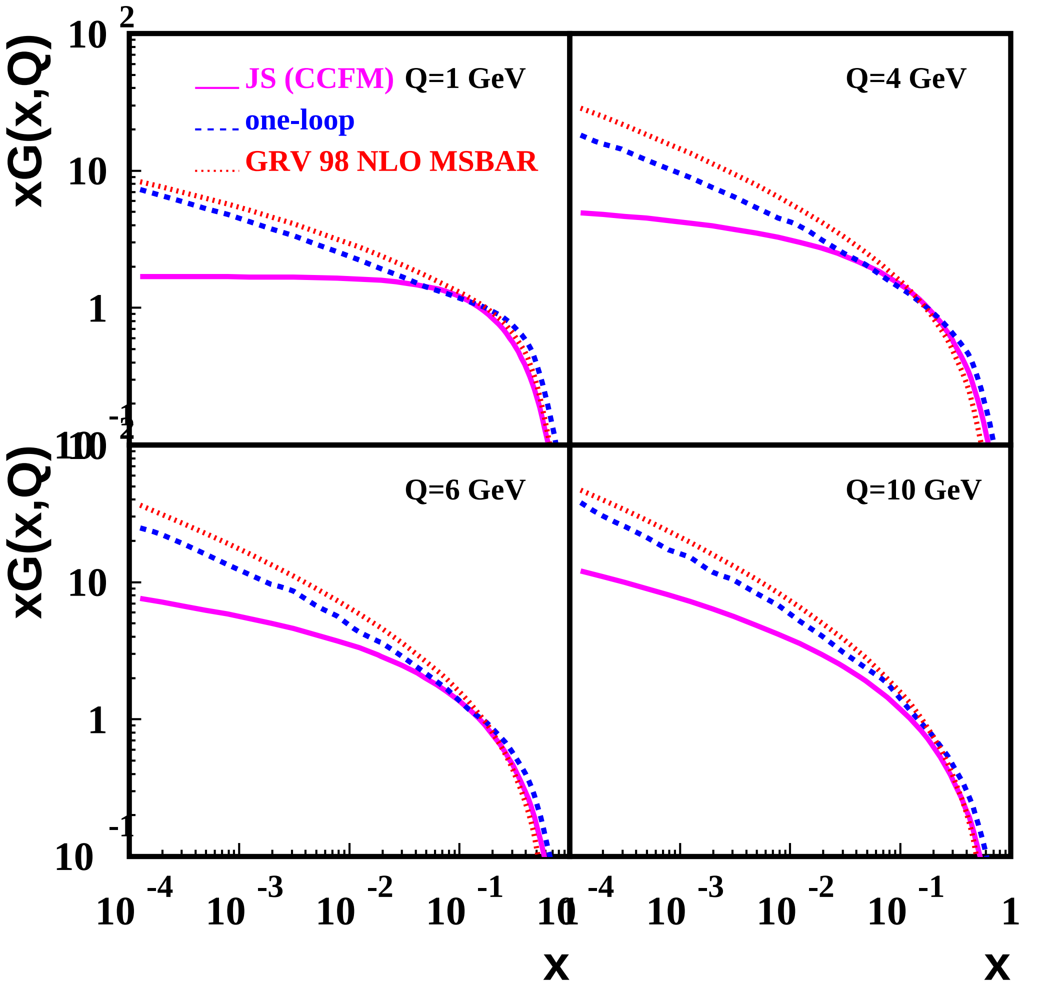
H. Jung, G.P. Salam, EPJC 19, 351 (2001)

constrained from F_2 fit only for $x_g < 0.03$

Compare JS (CCFM) to other unintegrated gluons at $\bar{q} = 10$ GeV:

- **DGLAP (one-loop) gluon with GRV at start (NEW !!!)**
- **BFKL+DGLAP gluon from KMR with MRST at start**
  M. Kimber, A. Martin, M. Ryskin
  PRD 63 (2001) 114027

CCFM unintegrated gluon density - integrated -



CCFM gluon integrated over k_t

$$\int_0^{\bar{q}} dk_t^2 x\mathcal{A}(x,k_t,\bar{q}) = xG(x,\bar{q})$$

➤ compare to DGLAP gluon

CCFM gluon:

- at starting scale $q = 1$ GeV **flat !!!**
- small x rise of gluon only for DGLAP needed
- in CCFM small x rise generated perturbatively

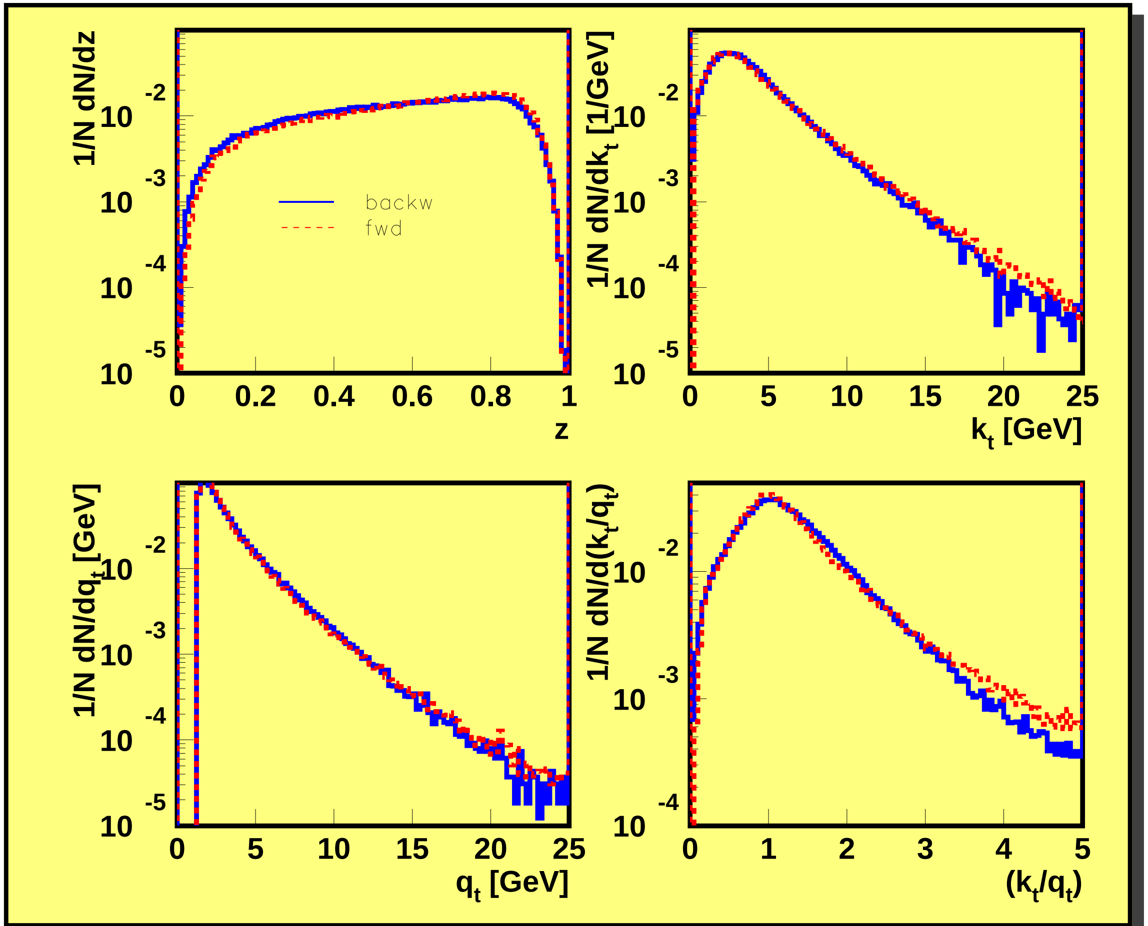
Remember: gluon density is no observable, only cross sections **!!!**

The Monte Carlo Generator CASCADE

- **Implement CCFM backward evolution into *NEW* MC generator CASCADE (http://www.quark.lu.se/ hannes/cascade)**
- **hard scattering processes included:**
  - $\gamma g^* \to q\bar{q}$, $\gamma^* g^* \to Q\bar{Q}$, $\gamma g^* \to J/\psi g$ **for** ep **scattering**
  - $g^* g^* \to q\bar{q}$, $g^* g^* \to Q\bar{Q}$ **for** $p\bar{p}$ **scattering**
- **initial state parton cascade acc. to CCFM with angular ordering**
- P**-remnant treatment like in PYTHIA (**q**-di-**q**, primordial** k_t**)**
- **final state parton showers added to quarks, hadronization via JETSET/PYTHIA**

CASCADE is MC implementation of CCFM for ep and also for $p\bar{p}$

Backward evolution of initial state radiation

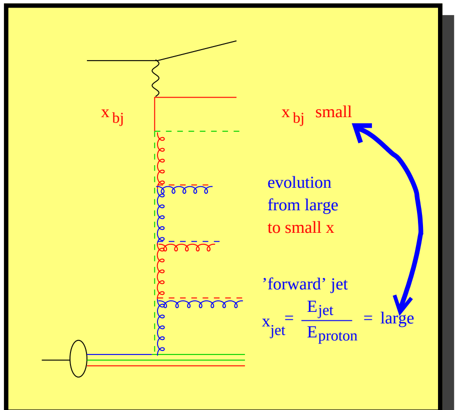


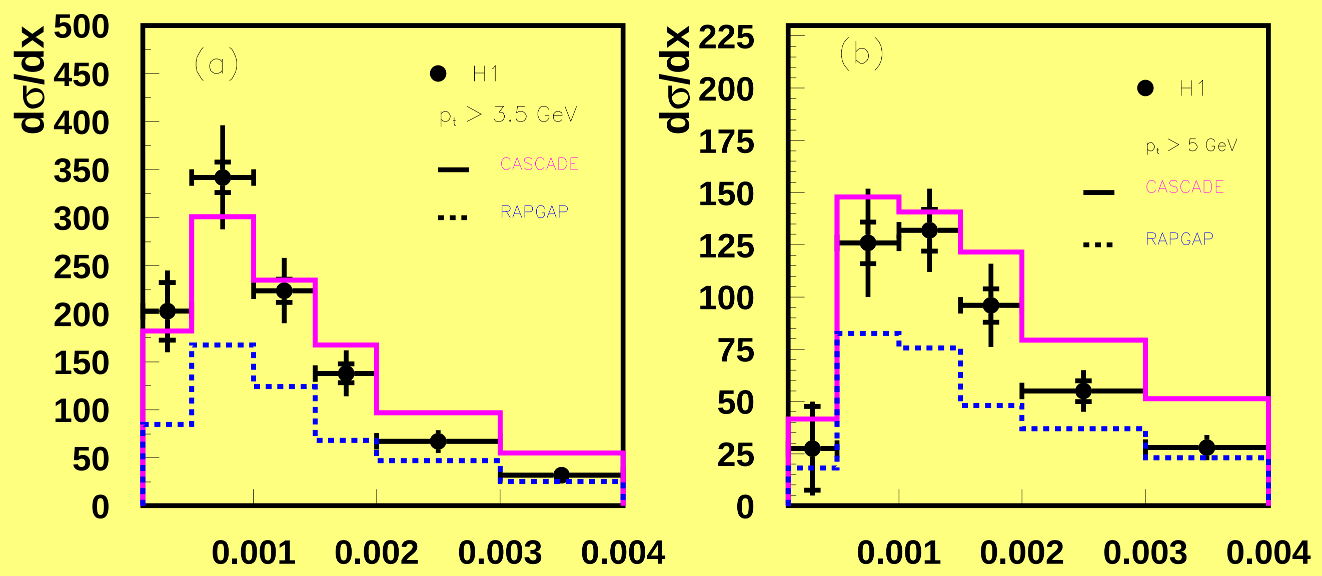
- Compare **forward** with **backward** evolution
- *NEW* **backward evolution** for small x processes possible
- **backward evolution:** parton kinematics of evolution equation with angular ordering reproduced **!!!**

forward - backward evolution comparison

never shown for DGLAP type MC's!!!

Solution to the problem: Forward Jets



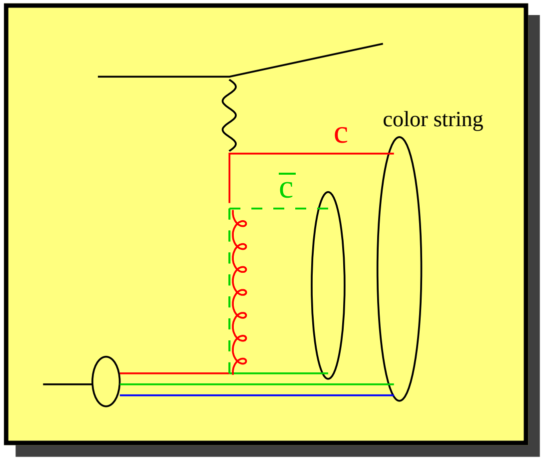


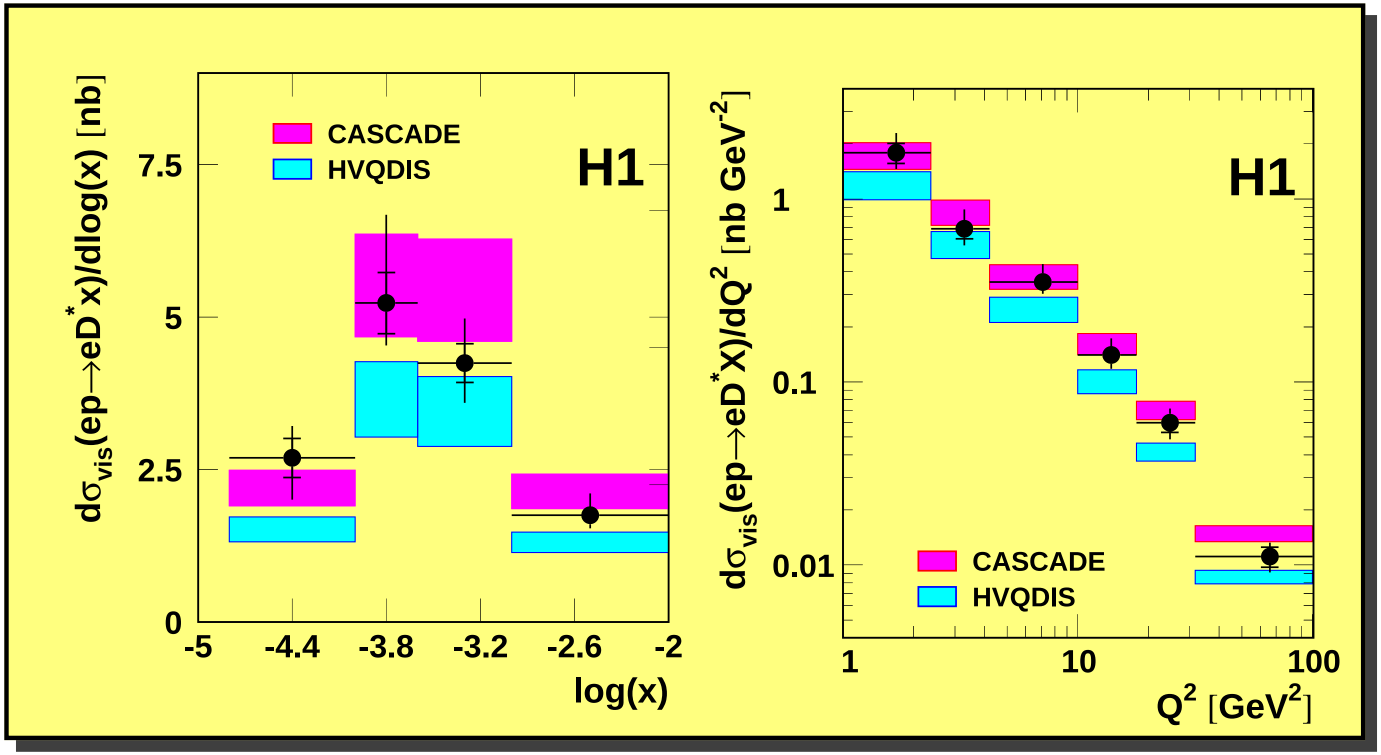
require jets with $p_t > 3.5(5.0)$ GeV and $0.5 < E_t^2/Q^2 < 2$

(H1 Coll. *NPB* 538 (1999) 3)

- CASCADE **well in shape and normalization !!!**
- RAPGAP **off as expected from DGLAP type evolution**

Solution to the problem: charm in DIS





D^* production in DIS

- **standard DGLAP with NLO calculation too small**
- CASCADE $\sim$ **perfect even at low x and Q^2**
- **free parameter: m_c**

$b\bar{b}$ production at HERA: H1 and ZEUS

H1 (H1 Coll. *PLB* 467 (1999) 156)

$Q^2 < 1$ **GeV**^{2}, $0.1 < y < 0.8$,

$p_t^\mu > 2$ **GeV**, $35^o < \theta^\mu < 130^o$

visible x-section $ep \to b\bar{b}X \to \mu X$:

$\sigma_{vis} = 176 \pm 16(stat.)^{+26}_{-17}(syst.)$ **pb**

NLO: $\sigma = 54 \pm 9$ **pb**

CASCADE $\sigma(ep \to e'b\bar{b}X \to \mu X) = 65$ pb

$R_{MC}(\textbf{H1}) = \frac{\sigma_{data}}{\sigma_{MC}} = 2.7 \pm 0.25^{+0.4}_{-0.26}$

ZEUS (ZEUS Coll. *EPJC* (2001))

$Q^2 < 1$ **GeV**^{2}, $0.2 < y < 0.8$,

$p_t^b > 5$ **GeV**, $|\eta^b| < 2$

$\sigma = 1.6 \pm 0.4(stat.)^{+0.3}_{-0.5}(syst.)^{+0.2}_{-0.4}(ext.)$ **nb**

NLO: $\sigma = 0.64^{+0.15}_{-0.1}$ **nb**

CASCADE $\sigma(ep \to e'b\bar{b}X) = 0.88 \pm 0.08$ nb

$R_{MC}(\textbf{ZEUS}) = \frac{\sigma_{data}}{\sigma_{MC}} = 1.25 \pm 0.32^{+0.21}_{-0.37}$

Measurements rely on large extrapolation from visible to total x-section

difference in visible x-sections !?!

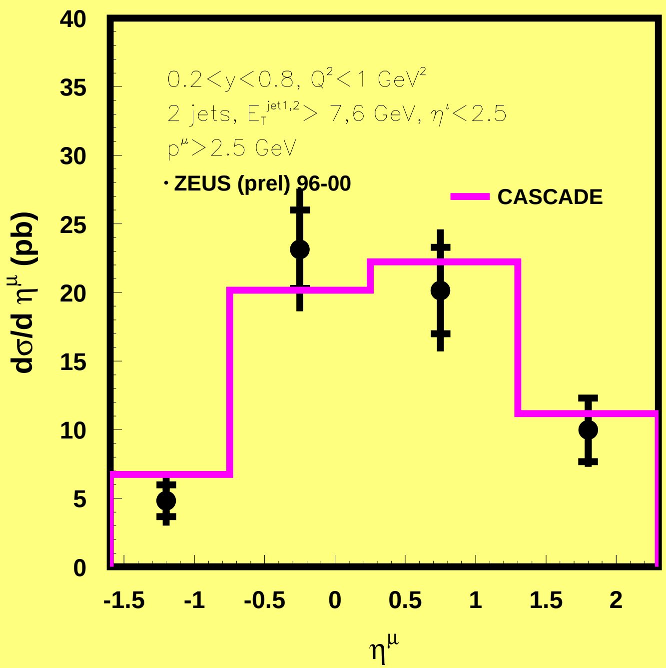
Solution to the problem at HERA: $b\bar{b}$

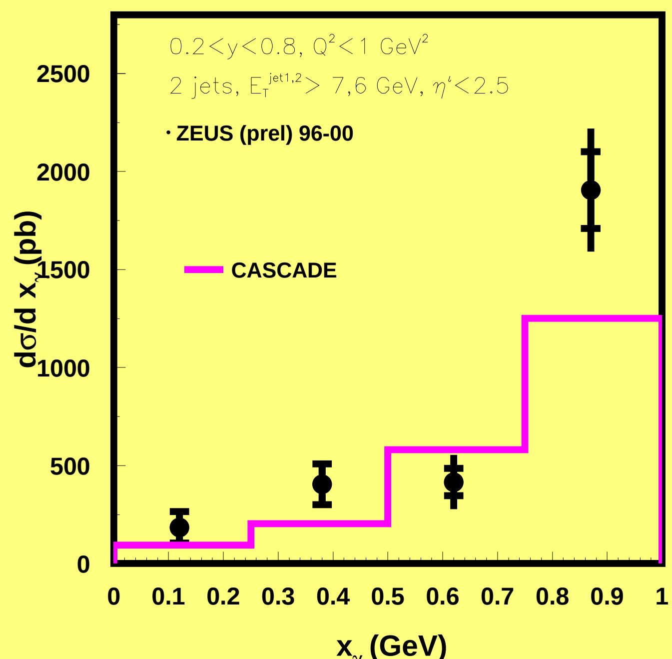
ZEUS (ICHEP (2002) Abstract 785) $Q^2 < 1$ **GeV**^{2}, $0.2 < y < 0.8$,

two jets + muon with $p^\mu > 3$ **GeV**, $-1.75 < \eta^\mu < 2.3$

ZEUS: $\sigma(e^+p \to b\bar{b} \to dijets + X) = 733 \pm 61 \pm 104$ **pb**

CASCADE: $\sigma(e^+p \to b\bar{b} \to dijets + X) = 533$ **pb** **NLO:** $\sigma = 381$ **pb**





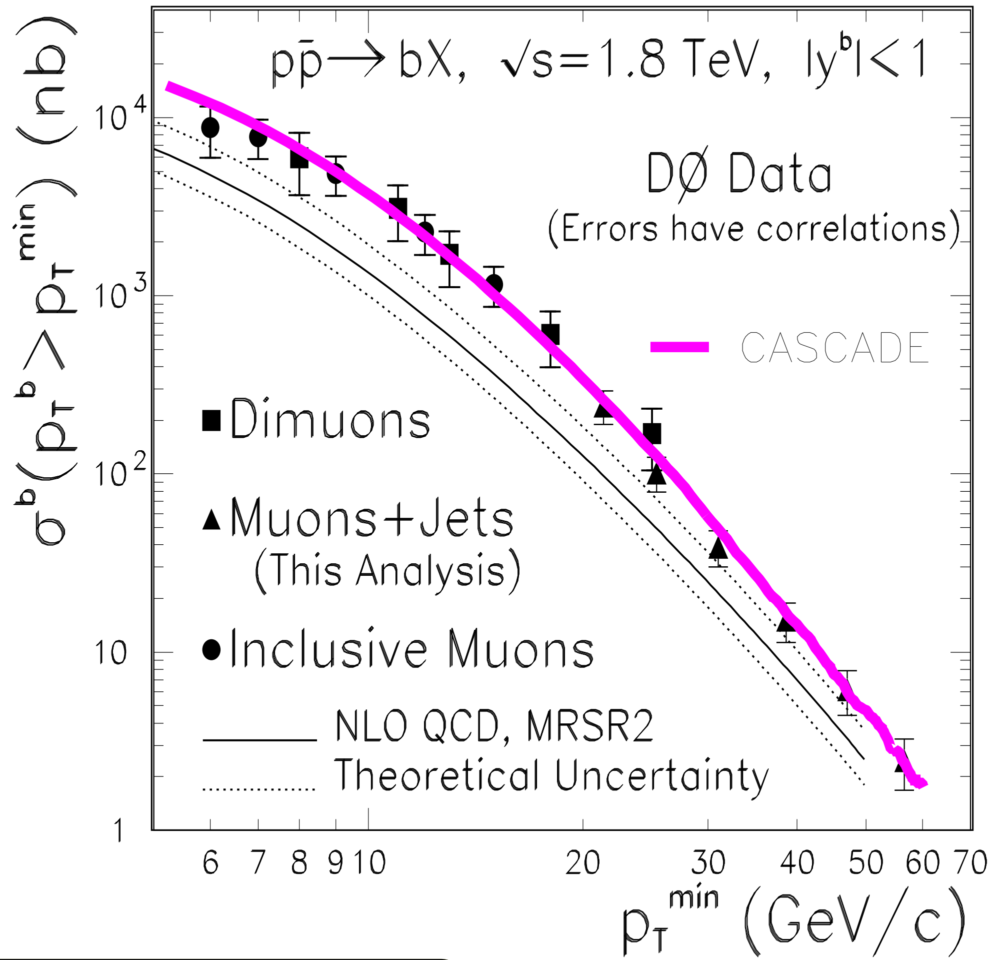
CASCADE **agrees for μ's, extrapolation to jets ???**

Solution to the problem: $b\bar{b}$ production at Tevatron

Test universality of unintegrated gluon density from HERA

- use unintegrated gluon as before (from F_2 fit at HERA)
- use of shell matrix element for $g^* g^* \to b\bar{b}$ with $m_b = 4.75$ GeV.

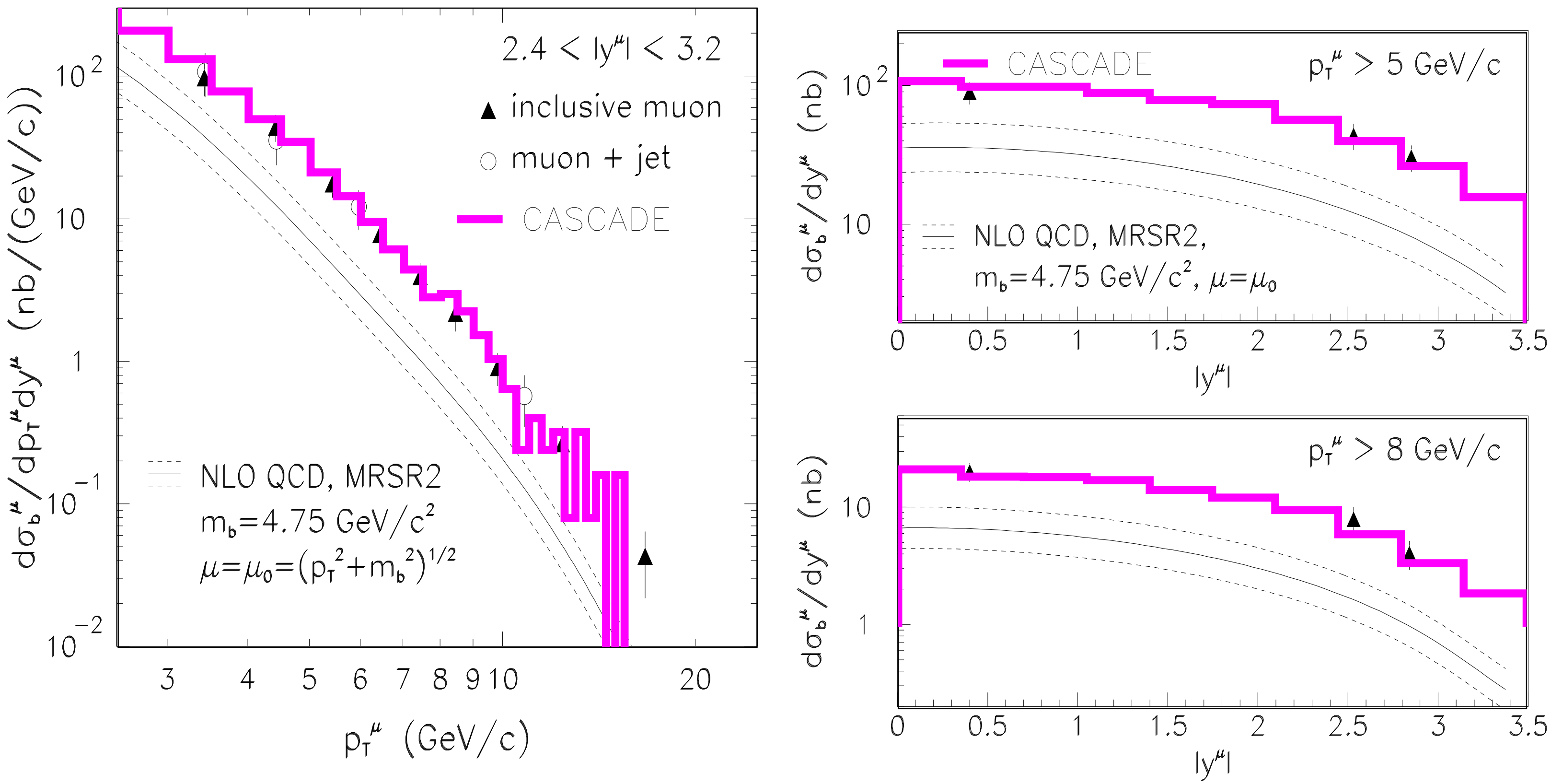
NOTE NLO off by factor 2



CASCADE w/o additional free parameters

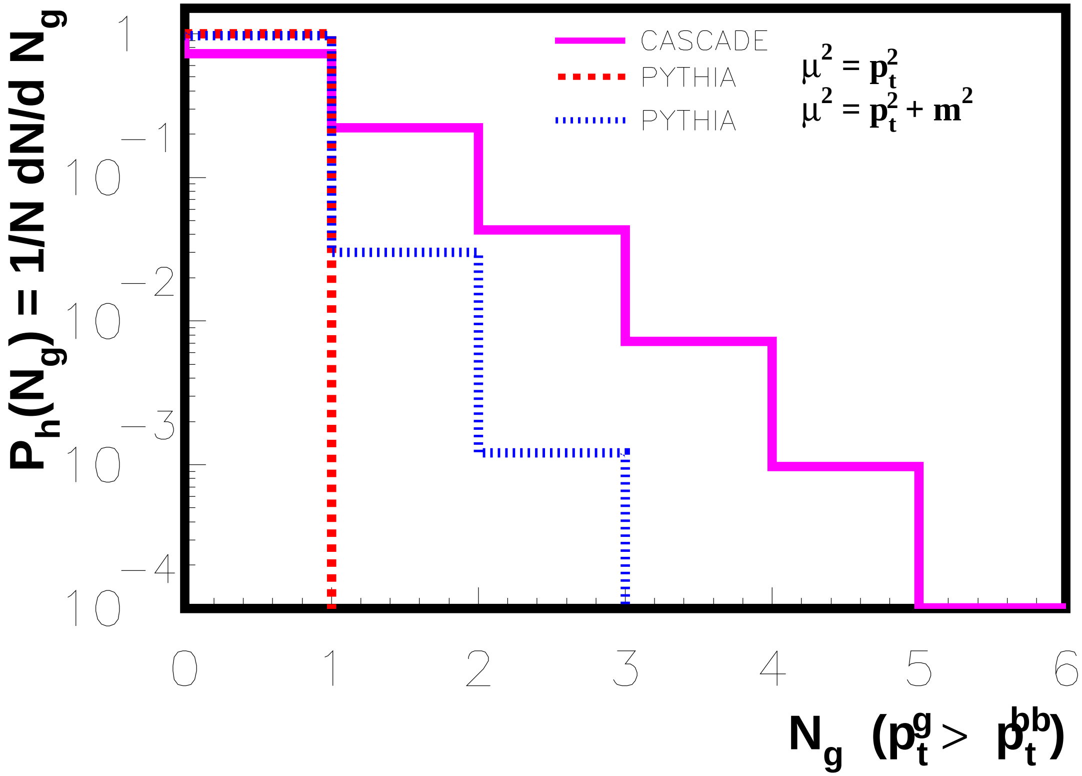
Solution to the problem: $b\bar{b}$ production at Tevatron

data from: D0 Collaboration B. Abbott et al., *Phy.Rev.Lett* 84 (2000) 5478



- CASCADE **describes μ spectrum over huge range well**
- **NLO fails by factor $\sim$ 2 (central) and $\sim$ 4 (forward)**

Why does k_t-factorization help for $b\bar{b}$ production at Tevatron



estimate higher order corrections

Nr of gluons with $p_t > p_t^{b\bar{b}}$

LO: $\mathcal{O}(\alpha_s^2) \rightarrow N_g = 0$

NLO: $\mathcal{O}(\alpha_s^3) \rightarrow N_g = 1$

NNLO: $\mathcal{O}(\alpha_s^4) \rightarrow N_g = 2$

.....

CASCADE $\rightarrow \mathcal{O}(\alpha_s^6)$

CASCADE with k_t factorization for estimation of higher order corrections

Nobody is perfect ...

- fwd π's and fwd jets with k_t - algo are less well described
  - only gluons included, need to worry about quarks also ?
- bottom production at HERA: what about difference between H1 and ZEUS ?
  - need to care about fragmentation issues ?
- jets in DIS and charm
  - x-section seems to be at high end....
  - think about fine tuning and re-fitting of CCFM unintegrated gluon ?

BUT also:

- improve CCFM splitting function P_{gg} to be valid for all z
  - non-singular terms
  - scale q_t^2 in α_s also for small z part

- CCFM unintegrated gluon density also for diffraction at HERA
  - better suited because of angular ordering - rapidity gap - angle
  - 1st attempts look promising
- unintegrated gluon density for photon (M. Hansson (Lund) started ...)
  - solve also $b\bar{b}$ in $\gamma\gamma$???

What has been achieved ???

- important to go beyond DGLAP: treat kinematics properly.....
- full machinery for MC evolution developed:
  suitable for DGLAP - reproduces standard evolution
  reliable for small x evolution with k_t factorization and CCFM
- MC solution of CCFM evolution equation obtained
- full hadron level CCFM MC generator developed: CASCADE
- resolves nearly all discrepancies with data at s mall x
  - forward jets
  - heavy quarks: charm (open charm and J/ψ) and bottom
- solves $b\bar{b}$ crisis at Tevatron with unintegrated gluon from HERA

The Beginning, Not the End

- k_t - factorization has reached new phase
  - precision tests
  - fine tuning possible and necessary
- k_t - factorization is now at comparable level with DGLAP
- many new items are opened up:
  - more studies with high statistics at HERA II
  - more studies in small angle region at HERA III
  - $\gamma\gamma$ - physics at TESLA
  - new perspectives for THERA
- new theoretical interest in k_t - factorization generated:
  - Lund small x workshops
  - Small x collaboration

The Beginning, Not the End

Even if there is still a bit to go for a

Theory **O**f **E**verything

we are facing the beginning of an

interesting, bright and challenging future

in small x physics

Problems in small x evolution: scale of α_s

together with G.P. Salam

Splitting Fct:

$$\tilde{P} = \frac{\bar{\alpha}_s(q(1-z))}{1-z} + \frac{\bar{\alpha}_s(k_t)}{z}\Delta_{ns}(z,q,k_t)$$

$$\log\Delta_{ns} = -\bar{\alpha}_s(k_t^2)$$
$$\int_0^1 \frac{dz'}{z'} \int \frac{dq^2}{q^2}\Theta(k_t - q)\Theta(q - z'q_t)$$

Splitting Fct:

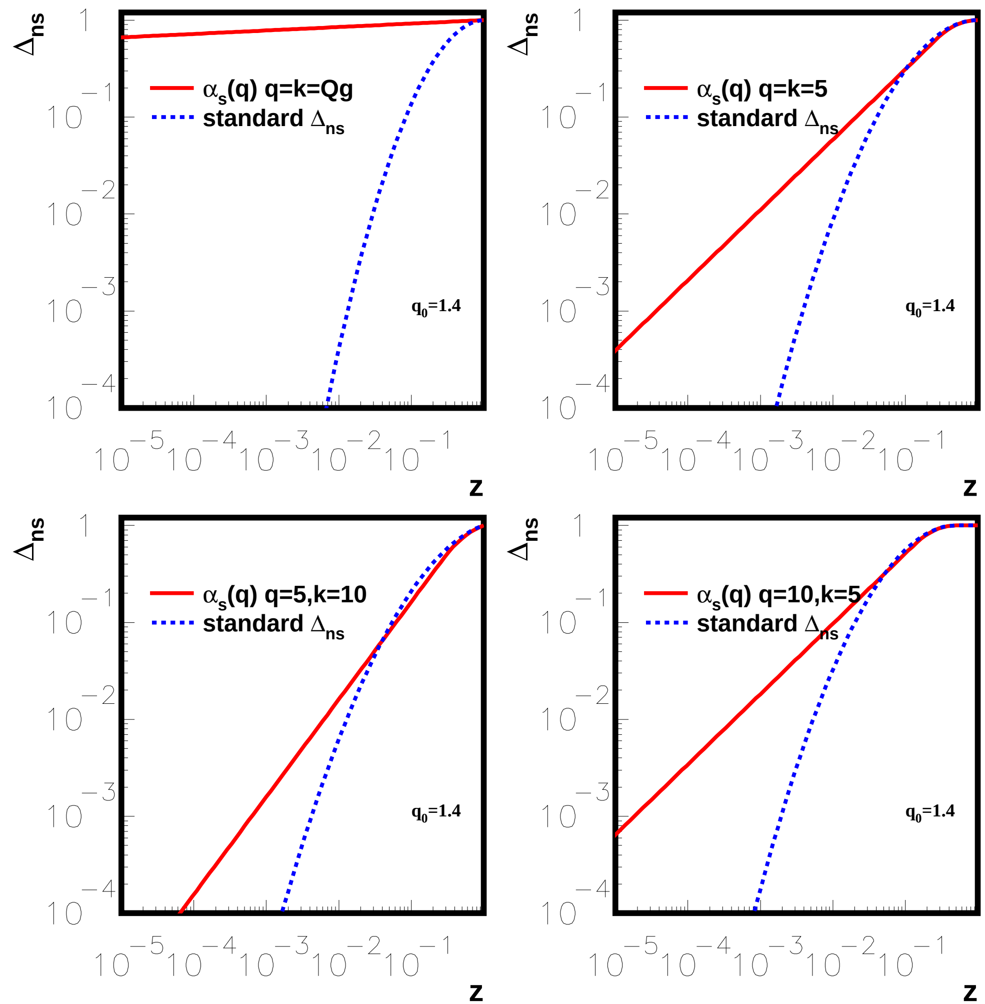
$$\tilde{P} = \frac{\bar{\alpha}_s(q(1-z))}{1-z} + \frac{\bar{\alpha}_s(q)}{z}\Delta_{ns}(z,q,k_t)$$

$$\log\Delta_{ns} = -\int_0^1 \frac{dz'}{z'}$$
$$\int \frac{dq^2}{q^2}\alpha_s(q)\Theta(k_t - q)\Theta(q - z'q_t)$$

$$\log\Delta_{ns} = \cdots \int_{(z'q_t)^2}^{k_t^2} \frac{dq^2}{q^2}\frac{1}{\log(q/\Lambda_{QCD})}$$

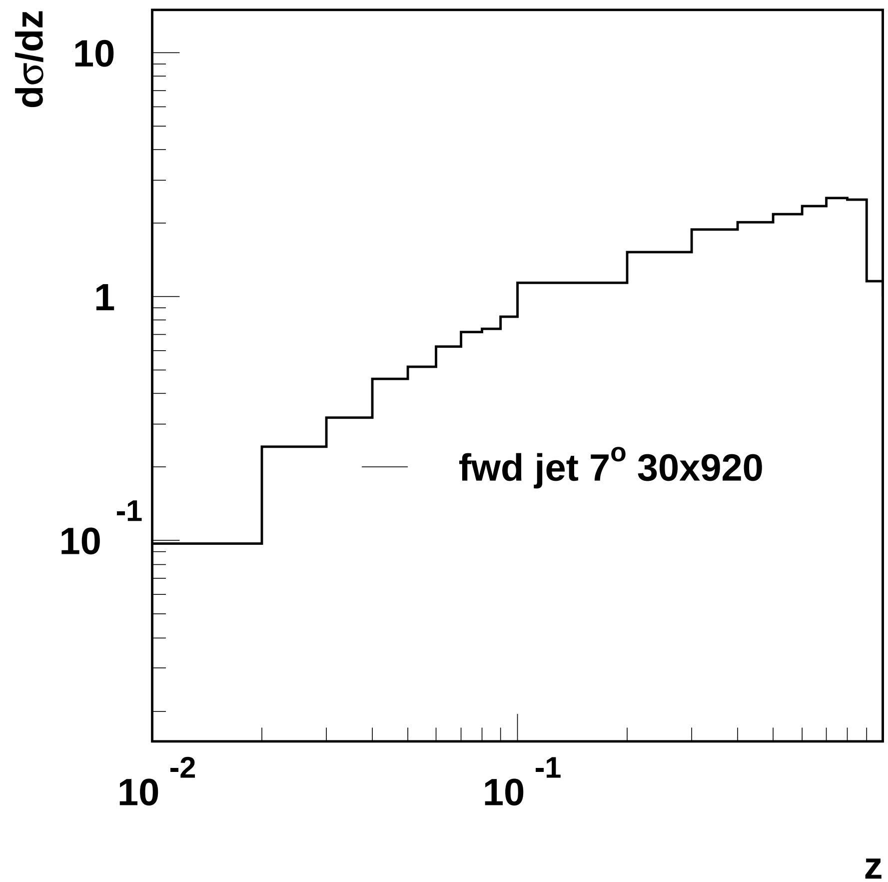
worry: lower limit $z'q_t \ll \Lambda_{QCD}$:

introduce cutoff q_0.



Problems in small x evolution

- only singular terms in splitting function $P_g{}^g \sim \bar{\alpha}_\mathrm{s} \left(\frac{1}{z}\Delta_{ns} + \frac{1}{1-z}\right)$
- similar in BFKL
- conceptual problems with non-singular terms
$P_{gg} \sim \bar{\alpha}_\mathrm{s} \left(\frac{1}{z}\Delta_{ns} - 2 + z(1-z) + \frac{1}{1-z}\right)$
- study z values in forward jet events with CASCADE
- at HERA z values ~ 0.5
- need to consider non - singular terms in splitting function



CCFM including full splitting fucntion

together with G.P. Salam

- **improve splitting function**

$$P_{gg} \sim \bar{\alpha}_s \left(\frac{1}{z} \Delta_{ns} + \frac{1}{1-z} \right)$$

- **to include non-singular terms**

$$P_{gg} \sim \bar{\alpha}_s \left(\frac{1}{z} \Delta_{ns} - 2 + z(1-z) + \frac{1}{1-z} \right)$$

- **new attempt (G. Salam):**

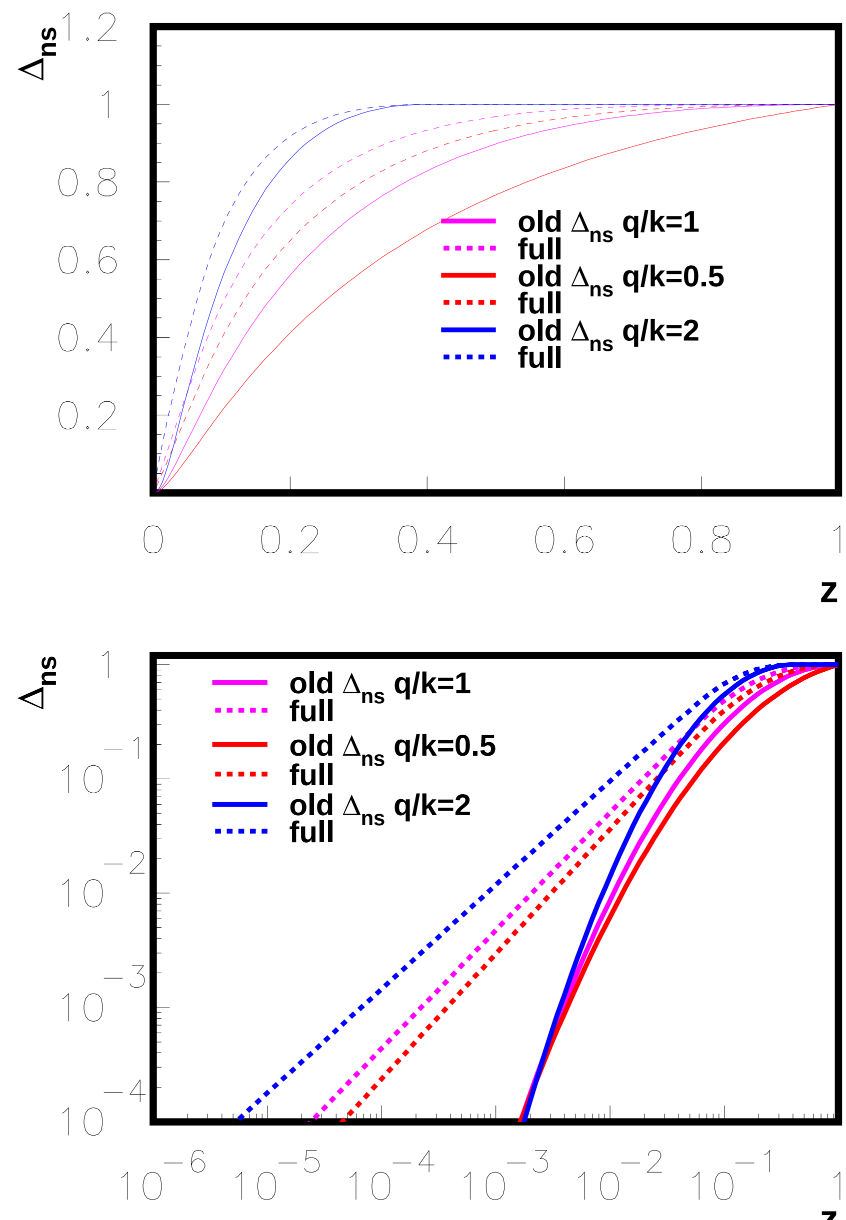
$$P = \bar{\alpha}_s \left(\frac{(1-z)}{z} + (1-B) z (1-z) \right) \Delta_{ns} + \bar{\alpha}_s \left(\frac{z}{1-z} + B z (1-z) \right)$$

- **need also new Sudakov:**

$$\log \Delta_s = - \int_0^1 \frac{dq'^2}{q'^2} dz' \bar{\alpha}_s \left(\frac{z'}{1-z'} + \frac{z(1-z)}{2} \right)$$

and new non-Sudakov

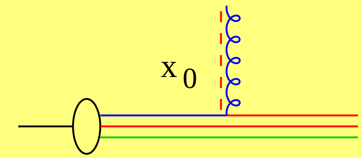
$$\log \Delta_{ns} = -\bar{\alpha}_s(k) \int \int dz' \frac{dq'^2}{q'^2} \left(\frac{1-z}{z'} + (1-B) z (1-z) \right)$$



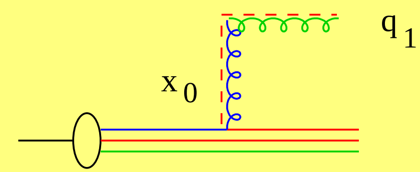
MC solution of CCFM eq: Forward Evolution

following SMALLX of G. Marchesini and B. Webber NPB 349 (1991) 617, NPB 386 (1992) 215

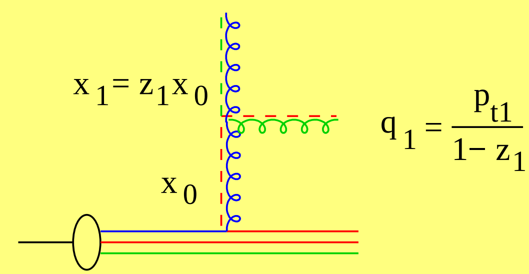
1. **select starting** x_0, k_{t0} **from** $\mathcal{A}_0(x, k_{t0})$



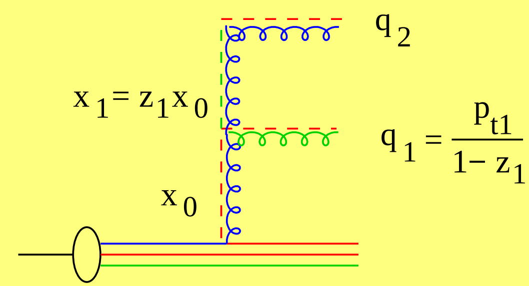
2. **select** q_1 **from Sudakov** **(incl. angular ordering)**
Sudakov gives probability for non-radiation between Q_0 **and** q_1



3. **select** z_1 **from** $\tilde{P}$
calculate x_1**, and finally** $p_{t1} = q_1 \cdot (1 - z_1)$
and k_{t1}



4. **select** q_2 **from Sudakov** $\Delta_s(q_2, z_1 q_1)$
repeat step 2 and 3
until next step would result in
$q_n > \bar{q}$ **(max. angle)**



Forward or backward evolution where is the problem ?

SMALLX **Monte Carlo:**

- forward evolution used to solve CCFM equation
- obtain unintegrated gluon density
- produces also partons
- weighted events produced
- large fluctuations
- difficult to extend to $p\bar{p}$

Better

backward evolution

- starting from hard scattering
- use unintegrated gluon density
- parton and hadron level
- very efficient
- unweighted events
- easy to extend to $p\bar{p}$

... Only need to formulate backward evolution for CCFM ...

CCFM backward evolution

together with G.P. Salam, EPJC 19, 351 (2001)

Backward evolution needs differential form of CCFM eq.

(G. Marchesini NPB 445 (1995) 49)

- $$\bar{q}^2 \frac{d}{d\bar{q}^2} \frac{x\mathcal{A}(x,k_t,\bar{q})}{\Delta_s(\bar{q},Q_0)} = \int dz \frac{d\phi}{2\pi} \frac{\tilde{P}(z,\bar{q}/z,k_t)}{\Delta_s(\bar{q},Q_0)} \, x'\mathcal{A}(x',k_t',\bar{q}/z)$$
- **similar form to DGLAP**

BUT **angular ordering complicates things**

- $\bar{q}/z$ **instead of** $\bar{q}$ **in DGLAP**

And **non-Sudakov also plays a role**

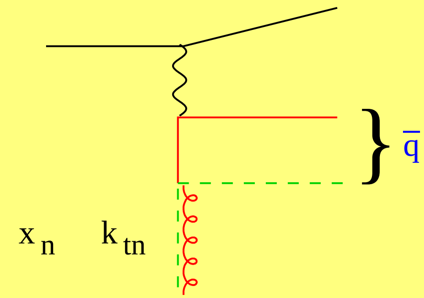
unintegrated gluon density $\mathcal{A}(x,k_t,\bar{q})$

- **from forward evolution**

The CCFM Backward Evolution

1.

starting with quark - box: $\sigma(\gamma^* g^* \to q\bar{q})$
select x_n, k_{tn} **from** $\mathcal{A}(x, k_t, \bar{q})$
with $\bar{q}$ **given by quark box:** $\bar{q} \sim \sqrt{\hat{s} + Q_t^2}$

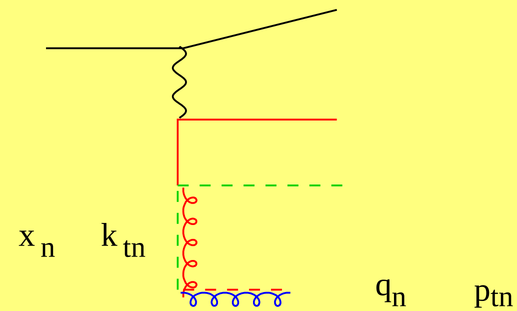


2.

select q_n **from** *bwd evolution* **Sudakov:**

$$\log\left(\mathcal{P}_{no\ rad}(\bar{q}, q')\right) = -\int_{q'}^{\bar{q}} \frac{dq^2}{q^2} \int dz \frac{d\phi}{2\pi} \tilde{P}(z, q/z, k_t) \frac{x'\mathcal{A}(x', k_t', q/z)}{x\mathcal{A}(x, k_t, q)}$$

with $\bar{q}$ **from previous step,** $q' = z_n q_n$

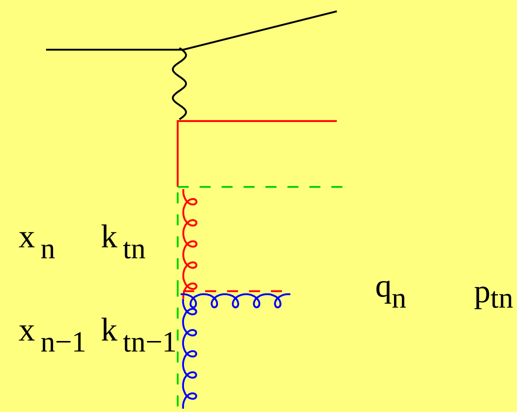


3.

select z_n **from**

$$\tilde{P} = \frac{\bar{\alpha}_s(q_n(1-z_n))}{1-z_n} + \frac{\bar{\alpha}_s(k_{tn})}{z_n} \Delta_{ns}(z_n, k_{tn}, q_n)$$

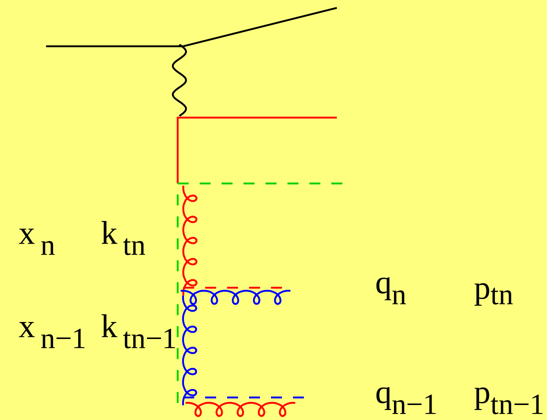
with k_{tn}, q_n **already known**
calculate x_{n-1}**, and**
finally $p_{tn} = q_n \cdot (1 - z_n)$ **and** k_{tn-1}



The CCFM Backward Evolution (cont'd)

4.

repeat step 2 and 3 until ...
stopping condition is reached:

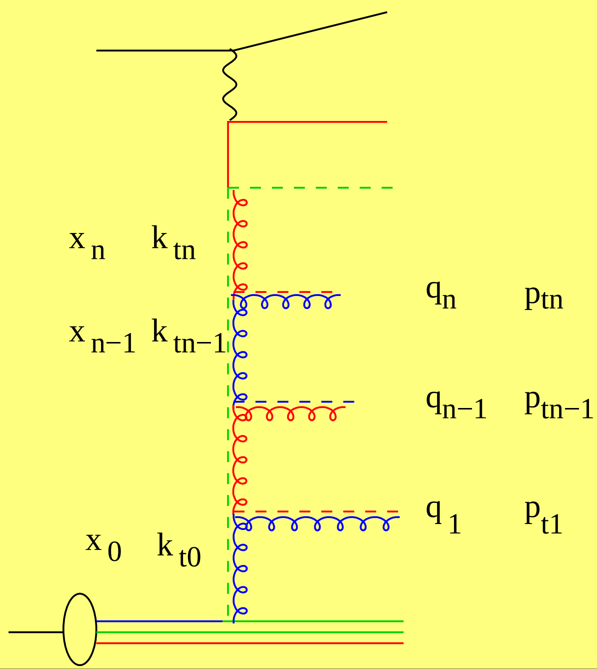


5.

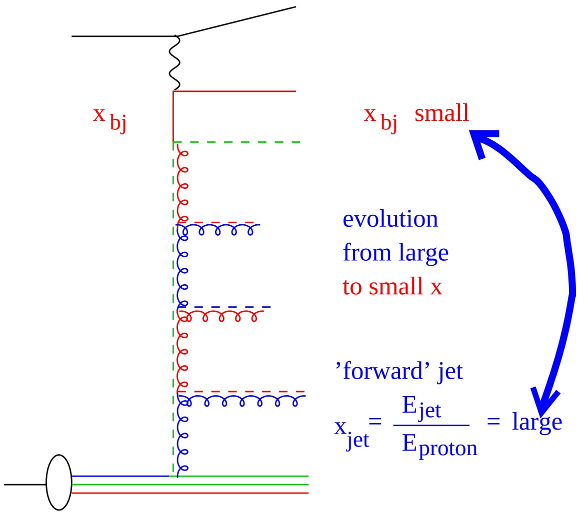
Stop evolution with probability:

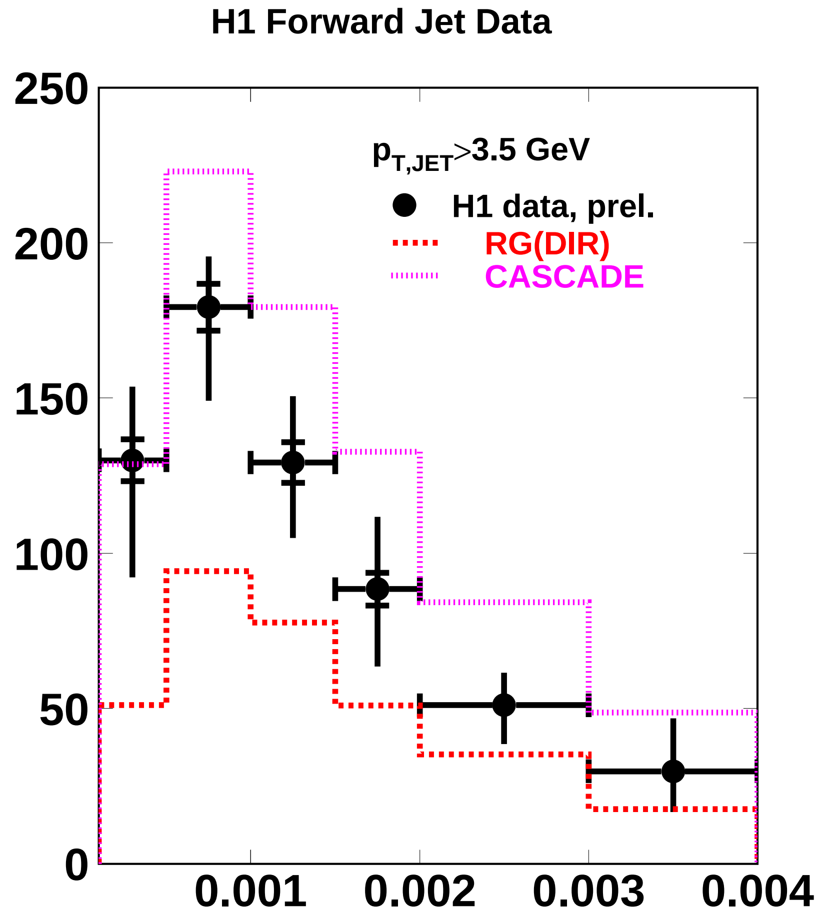
$$P = \frac{\mathcal{A}_0(x_n, k_{tn}, \bar{q})}{\mathcal{A}(x_n, k_{tn}, \bar{q})} = \frac{\mathcal{A}_0(x_n, k_{tn})\Delta_s(\bar{q}, Q_0)}{\mathcal{A}(x_n, k_{tn}, \bar{q})}$$

- **construct x_0 and k_{t0}**
- **construct P-remnant system**
- **add on primordial k_t to q di-q system**



Parton dynamics at small x: Forward Jets I





DIS : $5\ \text{GeV}^2 < Q^2 < 75\ \text{GeV}^2$

forward jet (incl. k_t algorithm)

$7.03^o < \theta_{jet} < 20.0^o$

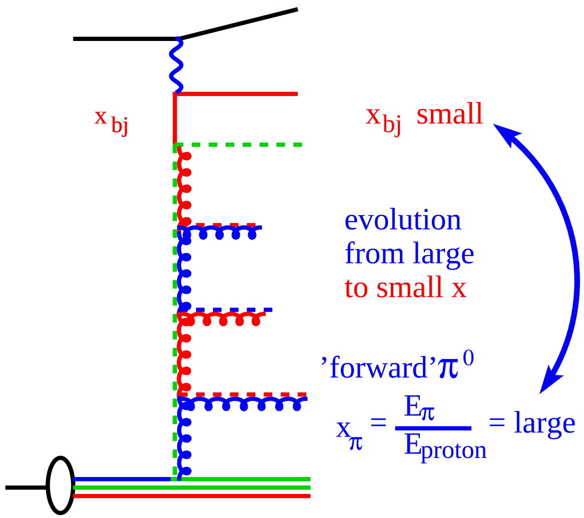
$x_{jet} > 0.035$

$0.5 < \frac{p_{t\ jet}^2}{Q^2} < 2$

DGLAP too small, need k_t factorisation with CCFM

why CASCADE too large ? quarks ?

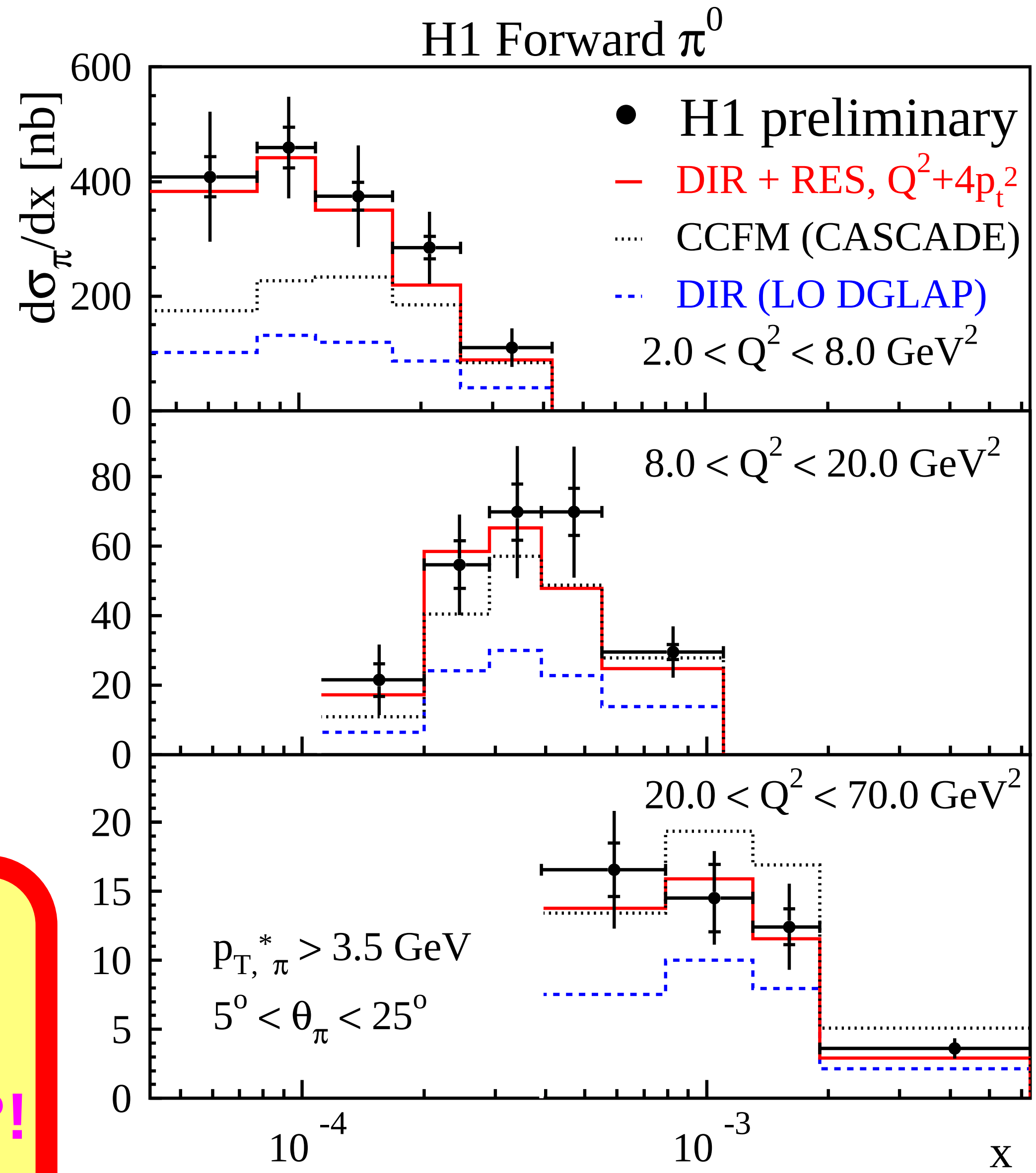
Parton dynamics at small x: Forward π^0 I



DIS : forward π^0 (instead of jet)

$5^o < \theta_\pi < 25.0^o$

$x_\pi > 0.01$



DGLAP too small, need:

- **resolved virtual photons ?!?**
- **CCFM too small at small x !?!**
- **WHY ?!?**

$b\bar{b}$ production in DIS at HERA: H1 and ZEUS

H1(prel.)

$2 < Q^2 < 100$ **GeV^2**, $0.1 < y < 0.8$,

$p_t^\mu > 2$ **GeV**, $35^o < \theta^\mu < 130^o$

visible x-section $ep \to e' b\bar{b}X \to \mu X$**:**

$\sigma = 39 \pm 8(stat.) \pm 10(syst.)$ pb

NLO: $\sigma = 11 \pm 2$ **pb**

CASCADE $\sigma(ep \to e' b\bar{b}X) = 15$ pb

R_{MC}(**H1**) $= \frac{\sigma_{measured}}{\sigma_{MC}} = 2.6$

ZEUS(prel.) ICHEP 2002

$Q^2 > 2$ **GeV^2**, $0.05 < y < 0.7$,

$E_{T,jet}^{Breit} > 6$ **GeV**, $-2 < \eta_{jet}^{lab} < 2.5$

$p^\mu > 2$ **GeV**, $30^o < \theta^\mu < 160^o$

x-section $ep \to e' b\bar{b}X \to e' jet \mu X$**:**

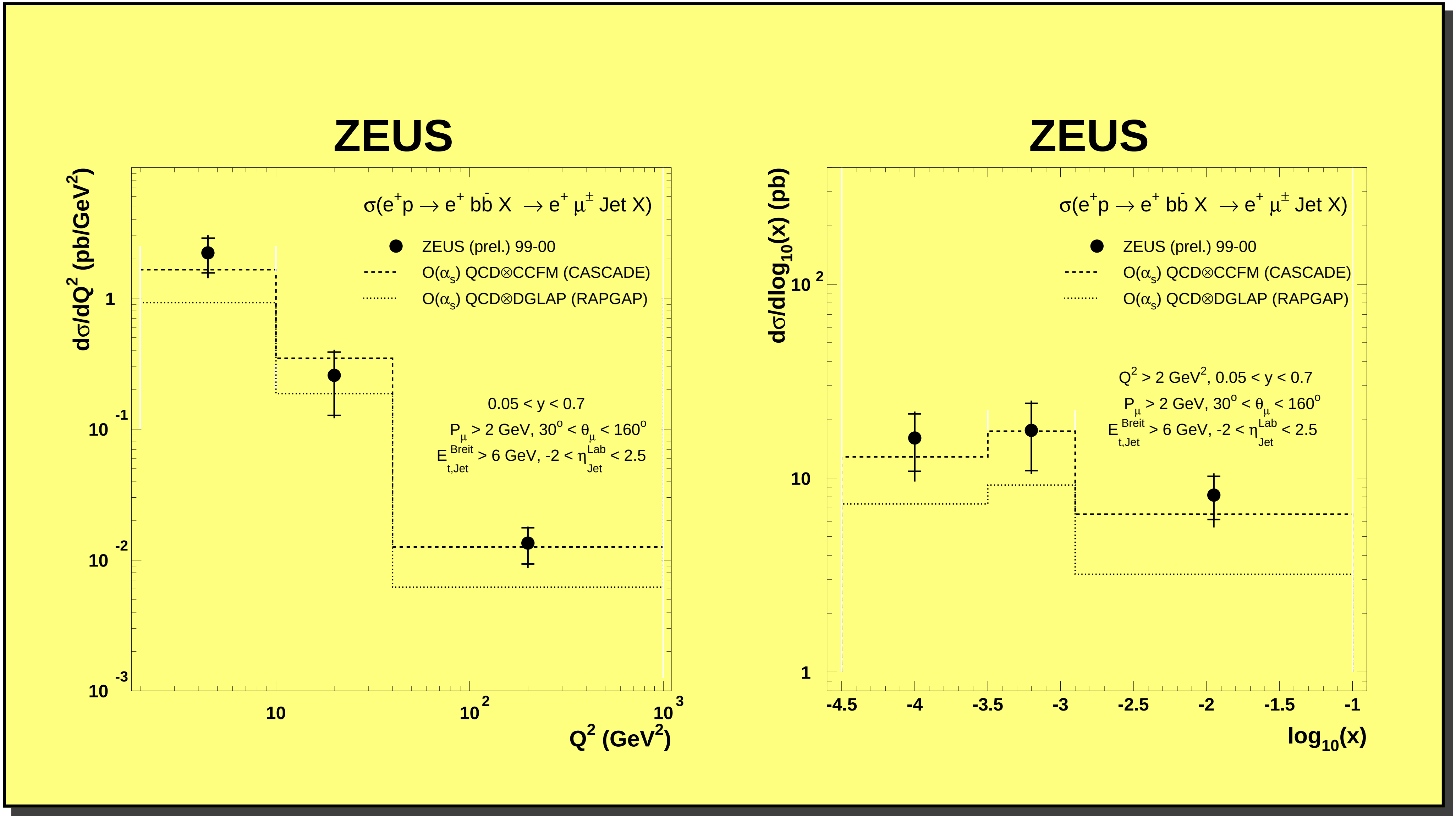
$\sigma = 38.7 \pm 7.7(stat.)_{5.0}^{6.1}(syst.)$ pb

NLO: $\sigma = 28.1 \pm 2$ **pb**

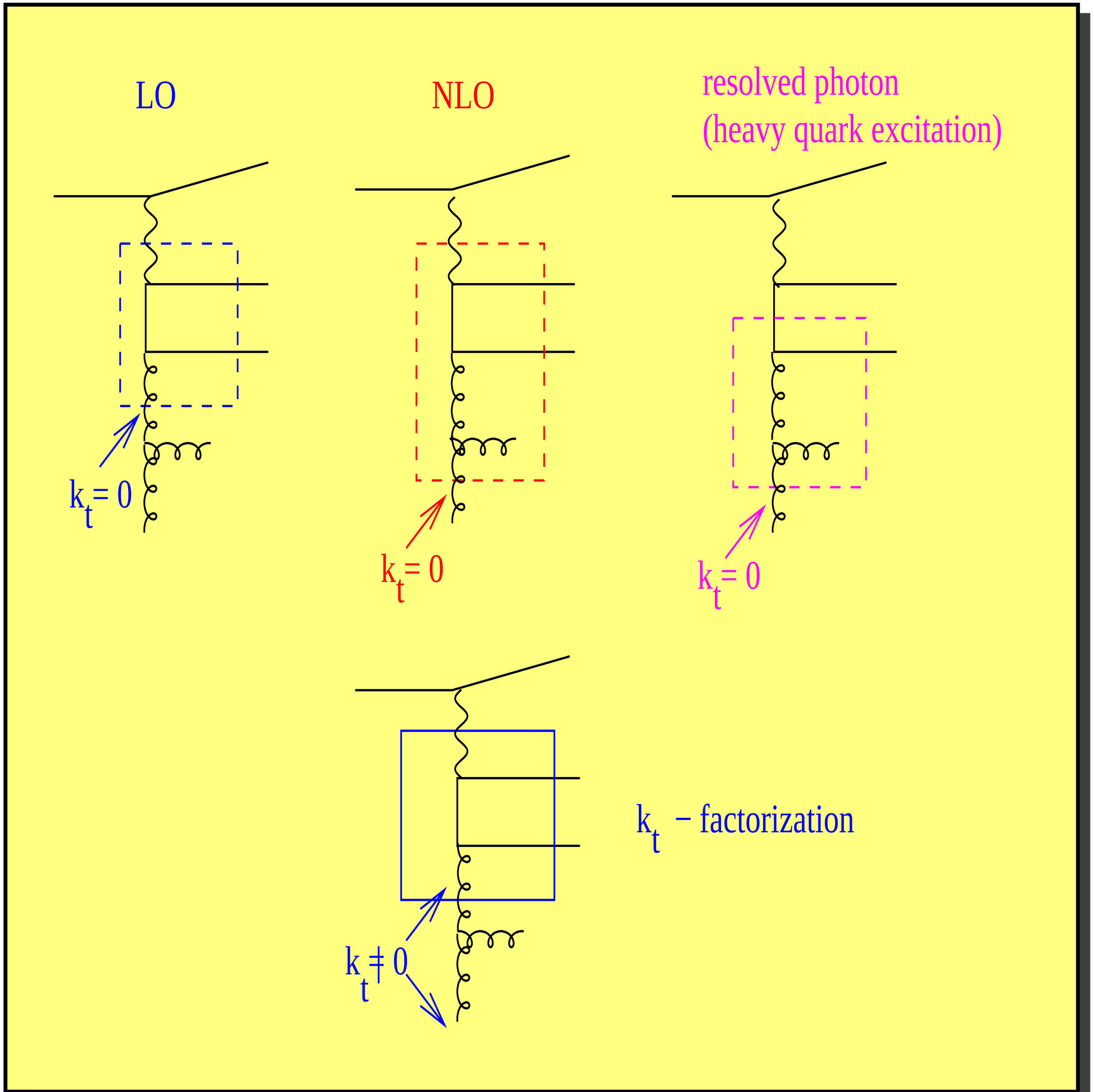
CASCADE $\sigma(ep \to e' b\bar{b}X) = 35$ pb

R_{MC}(**ZEUS**) $= \frac{\sigma_{measured}}{\sigma_{MC}} = 1.1$

$b\bar{b}$ production in DIS at HERA: ZEUS



Resolved - γ, NLO and k_t - factorization



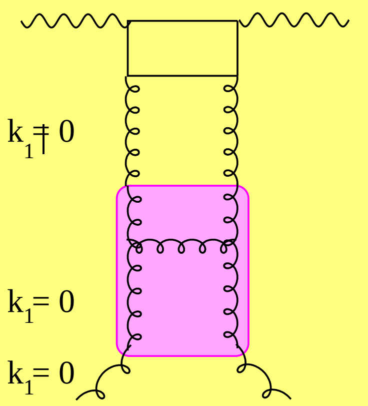
k_t **factorization**

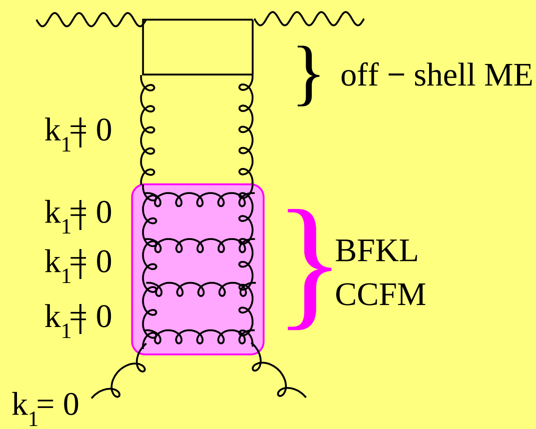
- **NLO corrections**
- **anomalous γ**
- **even in NLO**
- **includes NNLO**
- **includes NNNLO**
- **includes NNNNLO**

k_t **factorization has no problem with:**

- **negative weights....**
- **matching to PS**
- **matching to hadronsiation**

k_t - and collinear factorization





off - shell matrix element

➤ **1-loop correction to Born approximation**

➤ **high energy limit of NLO !!!**

$$\sigma = \int \frac{dz}{z} d^2 k_t \hat{\sigma}\left(\frac{x}{z}, k_t\right) \mathcal{F}(z, k_t)$$

$$\mathcal{F}(z, k_t) = \int \frac{dz}{z} \tilde{\mathcal{F}}(x/z, k_t; Q_0) \bar{f}_0(z, Q_0)$$

factorize k_t dependence in $\mathcal{F}$ and insert in σ

➤ **improved coefficient and splitting functions to all order**