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# The inclusion of NLO matrix elements into Monte Carlos

DESY Seminar, DESY, 2/3/2004

# QCD at the Born level gives qualitative information

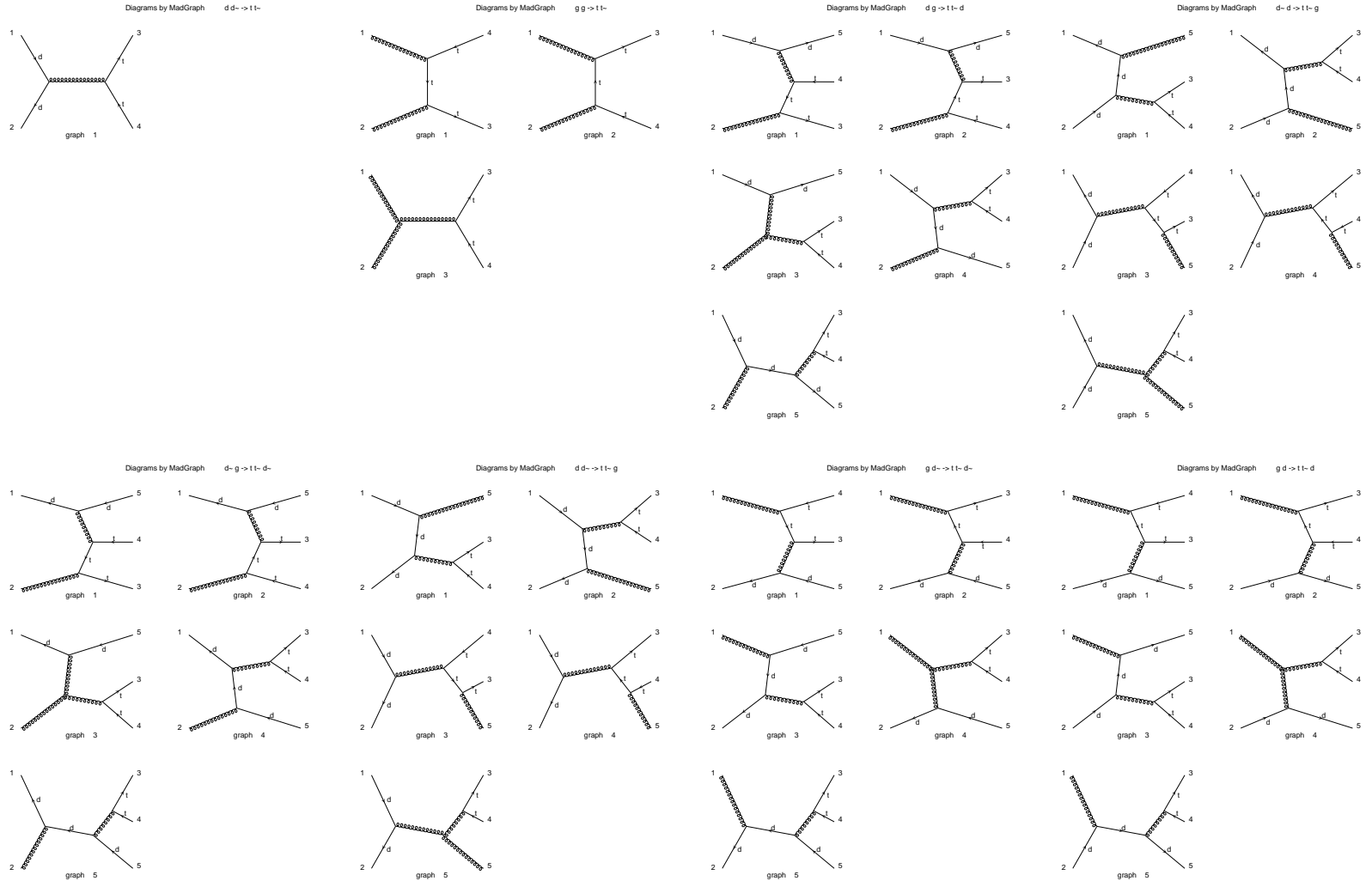
- ◆ Each parton is identified with a physical particle
- ◆ Total rates are largely underpredicted
- ◆ Only basic kinematic features are described
- ◆ Jets have no structure
- ◆ Scale dependence can be very large

Basically all realistic predictions are obtained either with perturbative  $N^k\text{LO}$  computations or with MC simulations, which have complementary strengths

This complementarity is due to the vastly different techniques used to evaluate the relevant Feynman diagrams, upon which  $N^k\text{LO}$  computations *and* MC simulations are based

# $N^k$ LO predictions imply...

a lot of diagrams to compute (here  $t\bar{t}$  production at NLO)



...and the loop diagrams are not even depicted here

# N<sup>k</sup>LO results may appear disappointing

- Somewhere the cross section is negative  $\implies$  needs to be resummed
- The computations are at the parton level (the description of parton-to-hadron transition is fairly primitive)
- The computations are difficult: “fully-exclusive” NNLO not achieved yet, more than five-leg, one-loop results unavailable in QCD
- The concept of event doesn't make sense (there's no parameter-free unweighting)

But, they are absolutely crucial in order to:

- ◆ Get a decent estimate of the total rates
- ◆ Reduce the impact of unphysical mass scales, and determine the unknowns of the theory, such as  $\alpha_s$  and PDFs
- ◆ Correctly account for hard emissions

# A different approach: Parton Shower Monte Carlo

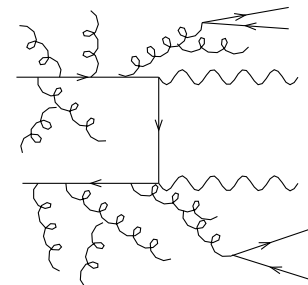
Key observation: soft and collinear emissions factorize

$$\mathcal{M}_{n+1}(p_1, \dots, p_{n+1}) \xrightarrow{p_1^0 \rightarrow 0} \frac{\alpha_S}{2\pi} \sum_{\substack{i < j \\ i, j=2}}^{n+1} \frac{p_i \cdot p_j}{p_i \cdot p_1 p_j \cdot p_1} \mathcal{M}_n^{(ij)}(p_2, \dots, p_{n+1})$$

$$\mathcal{M}_{n+1}(p_1, p_2, \dots, p_{n+1}) \xrightarrow{\vec{p}_1 || \vec{p}_2} \frac{4\pi\alpha_S}{p_1 \cdot p_2} P_{a_1(a_1+a_2)}(z) \mathcal{M}_n(p_1 + p_2, \dots, p_{n+1})$$

Thus, the emission of several soft and/or collinear gluons and quarks can be treated in process-independent manner. This is done through the definition of the Sudakov form factors

$$\Delta(t) = \exp \left[ - \int_{t_0}^t \frac{dt}{t} \int_{\varepsilon(t)}^{1-\varepsilon(t)} dz \frac{\alpha_S}{2\pi} P(z) \right] \longrightarrow$$



The problem in MC evolution (showering): given  $(t_1, z_1)$ , get  $(t_2, z_2)$ . This is done by solving  $\Delta(t_2)/\Delta(t_1) = R$  (and a similar equation that only involves the AP splitting functions). Initial-state (backward) evolution is treated similarly

# MC results may appear realistic

- The cross section is positive: MC's effectively perform resummations
- Hadron and parton levels are both available  $\implies$  still the most popular way to correct NLO results at hadron level
- The computations are trivial: any extra emission costs a few nsec of CPU time
- Events are thought to be a faithful representation of what's going on in detectors

Unfortunately, they have serious drawbacks:

- ◆ Cannot simulate the emissions of hard partons
- ◆ Cannot go beyond the LO in the computation of total rates

These problems arise from the fact that matrix elements are computed **exactly only at the LO**. Emissions contributing to beyond LO are *estimated* using a **soft/collinear approximation**

# NLO versus MC

	<i>Good</i>	<i>Bad</i>	<i>Users</i>
NLO	Hard emissions Total rates	Soft&coll emissions Hadronization No events	Theorists
MC	Soft&coll emissions Hadronization Outputs events	Hard emissions Total rates	Experimentalists

In other words:  $\text{NLO} \cap \text{MC} = \emptyset$

A formalism incorporating NLO *and* MC should combine their *Good* features, avoiding the *Bad* ones. However, the radical differences between the two approaches made QCDists wonder whether such a combination was possible

# Motivations for matching NLO and MC

A formalism with all the **Good** features is certainly desirable, and its definition is a challenging theoretical problem. But, are there compelling physical motivations?

- It is not unlikely that new physics signals will emerge from counting experiments, which require firm control on SM signal and background simulations
- The high-energy regime of the Tevatron and the LHC implies the relevance of **multi-jet, multi-scale processes, with large  $K$ -factors**
- Standard MC's don't perform well in predicting multi-jet observables, and the practice of multiplying the results by inclusive  $K$ -factors is just wrong. This may lead to **major errors in the strategies for searches**
- Multi-scale processes are badly predicted by fixed-order computations. Results matching these computations with resummed ones are mandatory (**a procedure largely successful at LEP**)
- The hadronization procedure in NLO computations is extremely naive, and strictly speaking can be applied only at very large  $p_T$ 's



# Objectives

Our aim is to develop a practical method for combining **existing** parton shower MC programs with NLO perturbative calculations; the resulting object is called **NLOwPS**. Let's start with some *definitions*

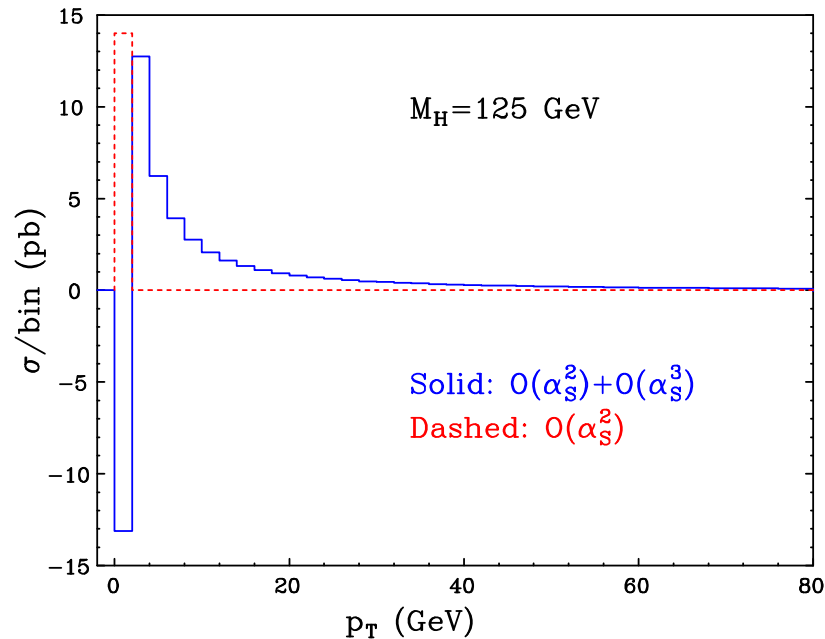
- ◆ Total rates are accurate to NLO
- ◆ Hard emissions are treated as in NLO computations
- ◆ Soft/collinear emissions are treated as in MC
- ◆ NLO results are recovered upon expansion of NLOwPS results in  $\alpha_s$ .  
In other words: there is no **double counting** in NLOwPS
- ◆ The matching between hard- and soft/collinear-emission regions is smooth
- ◆ The output is a set of events, which are fully exclusive
- ◆ MC hadronization models are adopted

NLOwPS is not positive definite, and some events may have **negative weights**. From the user's point of view, **it works just like an ordinary MC**

Warning: a related, but different, procedure aims at incorporating multi-leg, real emission diagrams into MC's – virtual diagrams are thus not included

# What does NLO mean?

Consider Higgs production:



$$\frac{d\sigma}{dp_T} = (A\alpha_s^2 + B\alpha_s^3) \delta(p_T) + C(p_T)\alpha_s^3$$
$$\int_{p_T^{min}}^{\infty} dp_T \frac{d\sigma}{dp_T} = \mathcal{C}_3 \alpha_s^3, \quad p_T^{min} > 0$$

$$= \mathcal{D}_2 \alpha_s^2 + \mathcal{D}_3 \alpha_s^3, \quad p_T^{min} = 0$$

$$p_T^{min} > 0 \Rightarrow \text{LO}, \quad p_T^{min} = 0 \Rightarrow \text{NLO}$$

The answer depends on the observable, and even on the kinematic range considered.  
So this definition cannot be adopted in the context of event generators

- $N^k$ LO accuracy in event generators is defined by the number  $k$  of extra gluons (either virtual or real) wrt the LO contribution (hopefully we all agree on LO definition)

# The actual NLOwPS's

- MC@NLO (Webber & SF; Nason, Webber & SF)

Based on NLO subtraction method

Formulated in general, interfaced to Herwig

Processes implemented:  $H_1 H_2 \longrightarrow W^+ W^-, W^\pm Z, ZZ, b\bar{b}, t\bar{t}, H^0, W^\pm, Z/\gamma$

- $\Phi$ -veto (Dobbs & Lefebvre)

Based on NLO slicing method

Avoids negative weights, at the price of double counting

Processes implemented:  $H_1 H_2 \longrightarrow Z, W^\pm$

- GRACE\_LLsub (Kurihara *et al*)

Based on NLO hybrid slicing method, computes ME's numerically

Double counts, unless the parton shower is not tuned

Process implemented:  $H_1 H_2 \longrightarrow Z$

A proposal by Collins aims at including NLL effects in showers, but lacks gluon emission so far.  $\Phi$ -veto is based on an old proposal by Baer&Reno; jets in DIS have been considered by Pötter&Schörner using a similar method. Soper&Krämer implemented  $e^+ e^- \rightarrow 3$  jets (but without a realistic MC)

# A simple way to understand NLOwPS

A system  $S$  moves along a line between 0 and 1. It can radiate “photons”, whose energy we denote with  $x$ .  $S$  can undergo several further emissions; on the other hand, one photon cannot branch. Internal degrees of freedom of  $S$  are understood

$$\begin{aligned} \left(\frac{d\sigma}{dx}\right)_B &= B\delta(x) & \longleftrightarrow & \text{Diagram 1: A horizontal line from } x=0 \text{ to } x=1 \text{ with a pink dot at } x=0. \\ \left(\frac{d\sigma}{dx}\right)_V &= \alpha_S \left(\frac{B}{2\epsilon} + V\right) \delta(x) & \longleftrightarrow & \text{Diagram 2: A horizontal line from } x=0 \text{ to } x=1 \text{ with a pink dot at } x=0 \text{ and a starburst above it.} \\ \left(\frac{d\sigma}{dx}\right)_R &= \alpha_S \frac{R(x)}{x} & \longleftrightarrow & \text{Diagram 3: A horizontal line from } x=0 \text{ to } x=1 \text{ with a pink dot at } x > 0 \text{ and a wavy line from } x=0 \text{ to the dot.} \end{aligned}$$

where  $\lim_{x \rightarrow 0} R(x) = B$  as in QCD. An NLO prediction:

$$\frac{d\sigma}{dO} = \lim_{\epsilon \rightarrow 0} \int_0^1 dx x^{-2\epsilon} \delta(O - O(S, x)) \left[ \left(\frac{d\sigma}{dx}\right)_B + \left(\frac{d\sigma}{dx}\right)_V + \left(\frac{d\sigma}{dx}\right)_R \right]$$

with  $\lim_{x \rightarrow 0} O(S, x) = O(S, 0)$  (infrared safeness). Note the kinematics:

$$\text{B\&V} \implies O(S, 0), \quad R \implies O(S, x)$$

# The computation of the NLO cross section I

## ■ SLICING

$$\left(\frac{d\sigma}{dO}\right)_{NLOslice} = \int_{\delta}^1 dx \left\{ \delta(O - O(S, x)) \frac{\alpha_s R(x)}{x} + \delta(O - O(S, 0)) \left[ B + \alpha_s (B \log \delta + V) \right] \right\}$$

## ■ SUBTRACTION

$$\left(\frac{d\sigma}{dO}\right)_{NLOsubt} = \int_0^1 dx \left\{ \delta(O - O(S, x)) \frac{\alpha_s R(x)}{x} + \delta(O - O(S, 0)) \left( B + \alpha_s V - \frac{\alpha_s B}{x} \right) \right\}$$

$$B \& V \implies O(S, 0), \quad R \implies O(S, x)$$

# The computation of the NLO cross section II

$$\left(\frac{d\sigma}{dO}\right)_{NLOsubt} = \int_0^1 dx \left\{ \delta(O - O(S, x)) \frac{\alpha_s R(x)}{x} + \delta(O - O(S, 0)) \left( B + \alpha_s V - \frac{\alpha_s B}{x} \right) \right\}$$

Upon integration in  $x$ , the bin of  $O(S, x)$  gets a weight

$$w_{\mathbb{H}}(x) = \frac{\alpha_s R(x)}{x}$$

and the bin of  $O(S, 0)$  gets a weight

$$w_{\mathbb{S}}(x) = B + \alpha_s V - \frac{\alpha_s B}{x}$$

The divergence of  $w_{\mathbb{H}}(x)$  and  $w_{\mathbb{S}}(x)$  for  $x \rightarrow 0$  is the reason for:

- 1) numerical instabilities
- 2) the impossibility of getting unweighted events in NLO computations

# The toy MC

The system can undergo an arbitrary number of emissions, with probability controlled by the Sudakov form factor

$$\Delta(x_1, x_2) = \exp \left[ -\alpha_s \int_{x_1}^{x_2} dz \frac{Q(z)}{z} \right]$$

i.e., the probability that no photon be emitted with energy  $x_1 < x < x_2$ . The function  $Q(z)$  parametrizes beyond-LL effects, with

$$0 \leq Q(z) \leq 1, \quad \lim_{z \rightarrow 0} Q(z) = 1$$

The Born cross section

$$\left( \frac{d\sigma}{dx} \right)_B = B \delta(x)$$

gives the overall normalization ( $B$ ) and initial condition ( $(S, 0)$ ) for the shower. Apart from the trivial normalization, this can be formally embedded in the generating functional (i.e., the history of all possible showers)

$$\mathcal{F}_{\text{MC}}(S, 0)$$

# NLO $\oplus$ MC $\longrightarrow$ NLOwPS?

Naive first try: use the NLO kinematic configurations as initial conditions for showers, rather than for filling the histograms

◆  $\delta(O - O(S, 0)) \longrightarrow$  start the MC with 0 emissions:  $\mathcal{F}_{\text{MC}}(S, 0)$

◆  $\delta(O - O(S, x)) \longrightarrow$  start the MC with 1 emission at  $x$ :  $\mathcal{F}_{\text{MC}}(S, x)$

$$\mathcal{F}_{\text{naive}} = \int_0^1 dx \left[ \mathcal{F}_{\text{MC}}(S, x) \frac{\alpha_S R(x)}{x} + \mathcal{F}_{\text{MC}}(S, 0) \left( B + \alpha_S V - \frac{\alpha_S B}{x} \right) \right]$$

It doesn't work:

- Cancellations between  $(S, x)$  and  $(S, 0)$  contributions occur **after the shower**: hopeless from the practical point of view; and, unweighting is still impossible
- $(d\sigma/dO)_{\text{naive}} - (d\sigma/dO)_{\text{NLO}} = \mathcal{O}(\alpha_S)$ . In words: **double counting**

The problem is a fundamental one: **KLN cancellation** is achieved in standard MC's through **unitarity**, and embedded in Sudakovs. This is no longer possible: IR singularities **do appear in hard ME's**



# MC@NLO: modified subtraction I

Get rid of the MC  $\mathcal{O}(\alpha_s)$  contributions by an extra subtraction of  $\mathcal{O}(\alpha_s)$

$$\mathcal{F}_{\text{MC@NLO}} = \int_0^1 dx \left[ \mathcal{F}_{\text{MC}}(S, x) \frac{\alpha_s [R(x) - BQ(x)]}{x} + \mathcal{F}_{\text{MC}}(S, 0) \left( B + \alpha_s V + \frac{\alpha_s B [Q(x) - 1]}{x} \right) \right]$$

where the two (one for branching, one for no-branching probability) new terms are sensibly chosen:

$$\left( \frac{d\sigma}{dx} \right)_{\text{MC}} = \alpha_s B \frac{Q(x)}{x} + \mathcal{O}(\alpha_s^2)$$

$Q(x)$  is MC-dependent (i.e., Pythia's and Herwig's differ), but  $Q(x) \rightarrow 1$  for  $x \rightarrow 0$  always holds

By explicit computation,  $(d\sigma/dO)_{\text{MC@NLO}} - (d\sigma/dO)_{\text{NLO}} = \mathcal{O}(\alpha_s^2)$ , and therefore there is no double counting

Furthermore  $\longrightarrow$

# MC@NLO: modified subtraction II

Let's look at the weights of  $\mathcal{F}_{\text{MC}}(S, x)$  and  $\mathcal{F}_{\text{MC}}(S, 0)$

$$w_{\mathbb{H}}(x) = \frac{\alpha_s [R(x) - BQ(x)]}{x}$$

$$w_{\mathbb{S}}(x) = B + \alpha_s V + \frac{\alpha_s B [Q(x) - 1]}{x}$$

They don't diverge any longer for  $x \rightarrow 0$

The MC provides local, observable-independent, counterterms  $\implies$  greater numerical stability, unweighting possible

MC@NLO can thus be minimally seen as a way to stabilize NLO computations, through the construction of a simplified MC whose only aim is to furnish the local counterterms. In this sense, the generalization to NNLO should not be too difficult.

# Modified subtraction in QCD

**Strategy:** Take the toy model seriously, and literally translate it in QCD language

$$\mathcal{F}_{\text{MC@NLO}} = \sum_{ab} \int dx_1 dx_2 d\phi_3 f_a(x_1) f_b(x_2) \left[ \mathcal{F}_{\text{MC}}^{(2 \rightarrow 3)} \left( \mathcal{M}_{ab}^{(h)}(x_1, x_2, \phi_3) - \mathcal{M}_{ab}^{(\text{MC})}(x_1, x_2, \phi_3) \right) + \mathcal{F}_{\text{MC}}^{(2 \rightarrow 2)} \left( \mathcal{M}_{ab}^{(b,v,c)}(x_1, x_2, \phi_2) - \mathcal{M}_{ab}^{(c.t.)}(x_1, x_2, \phi_3) + \mathcal{M}_{ab}^{(\text{MC})}(x_1, x_2, \phi_3) \right) \right]$$

Since it is literal...

$$\frac{\alpha_S R(x)}{x} \leftrightarrow \mathcal{M}_{ab}^{(h)}, \quad \frac{\alpha_S BQ(x)}{x} \leftrightarrow \mathcal{M}_{ab}^{(\text{MC})}$$

it works only if  $\mathcal{M}_{ab}^{(\text{MC})}$  is a **local counterterm** of  $\mathcal{M}_{ab}^{(h)}$ . This is **not** the case: large-angle soft gluon emission in MC's is not described by eikonals

Fortunately, the problem is not a serious one: we can still use existing MC's. Formally, some observables will get extra power-suppressed contributions; practically, the effects are invisible

# What's the problem with the soft limit?

From perturbative computations, we expect the following formula to hold

$$d\sigma_{2\rightarrow 3} \xrightarrow{E\rightarrow 0} \frac{\alpha_S}{E^2} \frac{1}{1 - \cos^2 \theta} d\sigma_{2\rightarrow 2}$$

Using the MC (HERWIG) showering variables, we find instead

$$d\sigma_{2\rightarrow 3} \xrightarrow{E\rightarrow 0} \frac{\alpha_S}{E^2} \left[ \frac{2\Theta(\cos \theta > -1/3)}{(1 - \cos \theta)(3 + \cos \theta)} + \frac{2\Theta(\cos \theta < 1/3)}{(1 + \cos \theta)(3 - \cos \theta)} \right] d\sigma_{2\rightarrow 2}$$

MC's are not designed to produce fixed-order results. As such, the initial conditions for the showers are chosen in order to maximize the efficiency, and the coverage of the phase space. However, it is legitimate to ask why MC's can describe physics, and still disagree with QCD

$$\int_{-1+\epsilon}^{1-\epsilon} d\cos \theta \left[ \frac{2\Theta(\cos \theta > -1/3)}{(1 - \cos \theta)(3 + \cos \theta)} + \frac{2\Theta(\cos \theta < 1/3)}{(1 + \cos \theta)(3 - \cos \theta)} - \frac{1}{1 - \cos^2 \theta} \right] \xrightarrow{\epsilon\rightarrow 0} 0$$

This equation is the answer: the total amount of “soft” energy given by the MC is in agreement with QCD. Physical observables must be independent of the angular distributions of soft gluons (beware of non-global logs)

# MC@NLO: summary

1. Choose your favourite MC (Herwig, Pythia), and compute analytically the “NLO cross section”, i.e., the first emission. This is an **observable-independent**, **process-independent** procedure, which is done once and for all
2. Combine the LO+NLO matrix elements of the process to be implemented according to the universal, **observable-independent**, **subtraction-based** formalism of SF, Kunszt, Signer for cancelling IR divergences. All counterterm, virtual, and LO contributions must have a unique kinematics (**achieved through a projection**)
3. Add and subtract the MC counterterms, computed in step 1, to the quantity computed in step 2. The resulting expression allows to generate the hard kinematic configurations, which are eventually fed into the MC showers as **initial conditions**

Some of these features are shared with multi-leg generators, implemented according to CKKW prescription: however, NLOwPS's don't have any dependence upon unphysical parameters

# From the user's point of view

Almost nothing changes. MC@NLO works identically to Herwig (the same analysis routines can be used), except for the fact that hard partonic processes are generated by a companion piece of code, at the beginning of the run rather than on an event-by-event basis (the same happens for multileg ME generators interfaced to MCs)

- Unweighted event generation achieved
- Weighted event generation possible (currently not implemented)
- MC@NLO shape identical to MC shape in soft/collinear regions
- $\text{MC@NLO}/\text{NLO}=1$  in hard regions
- There are negative-weight events

Negative weights don't mean negative cross sections. They arise from a different mechanism wrt those at the NLO, and their number is fairly limited

# NLOwPS: $\Phi$ -veto

Exploit a proposal by Baer&Reno to get rid of the soft/collinear configurations:

$$B + \alpha_s (B \log \delta_0 + V) = 0 \quad \Longrightarrow \quad \delta_0 = \exp \left[ - (B + \alpha_s V) / \alpha_s B \right]$$

Another parameter  $\delta_{PS} > \delta_0$  separates the shower region from the hard region (Pötter, Schörner, Dobbs)

$$\mathcal{F}_{\Phi_{\text{veto}}} = \alpha_s \int_{\delta_{PS}}^1 dx \mathcal{F}_{\text{MC}}(S, x) \frac{R(x)}{x} + \alpha_s \mathcal{F}_{\text{MC}}(S, 0) \int_{\delta_0}^{\delta_{PS}} dx \frac{R(x)}{x}$$

- + Only positive weights
- + Doesn't need to know details of MC implementation
- Double counting for  $x < \delta_{PS}$ , and discontinuity at  $x = \delta_{PS}$  imply dependence upon  $\delta_{PS}$ , which is hidden by integration over Bjorken  $x$ 's
- Strictly speaking, the (perturbative) result is non-perturbative, since  $\delta_0 \sim \exp(-1/\alpha_s)$ 
  - Applied to: Z,  $W^\pm$  production

# NLO<sub>w</sub>PS: GRACE\_LLsub

Partition the phase space as in standard slicing, but subtract the MC contribution from the hard region:

$$\mathcal{F}_{\text{GRACE}} = \alpha_S \int_{\delta}^1 dx \mathcal{F}_{\text{MC}}(S, x) \frac{R(x) - B}{x} + \mathcal{F}_{\text{MC}}(S, 0) (B + \alpha_S V)$$

This *formally* coincides with MC@NLO, provided that

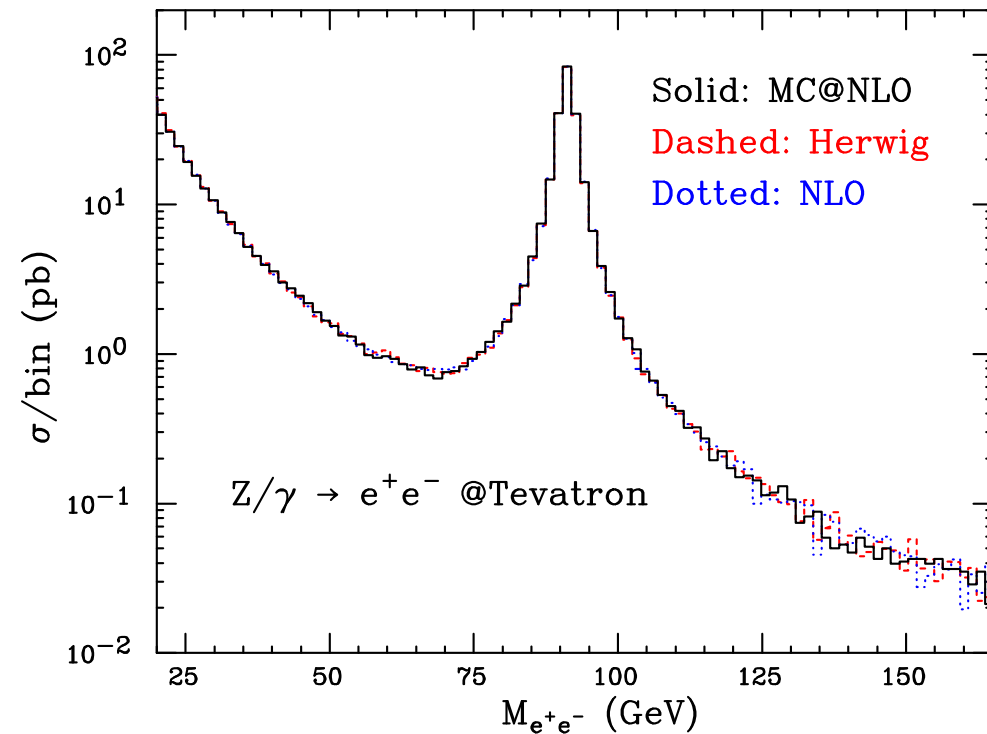
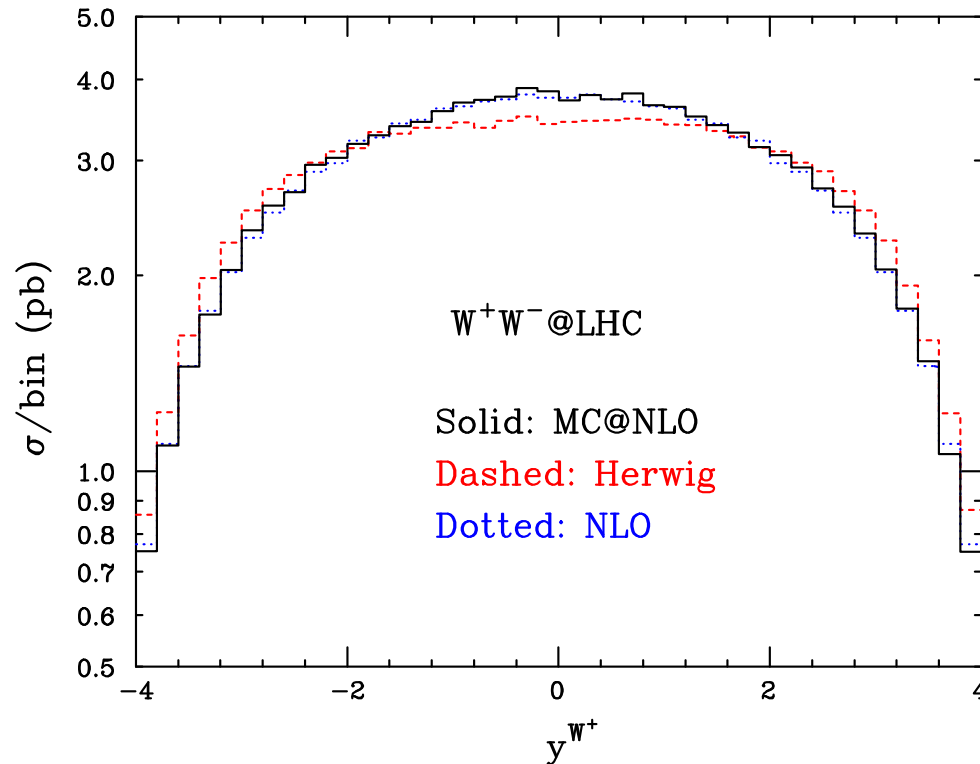
$$\delta \longrightarrow 0, \quad Q(x) \equiv 1$$

The second condition cannot, however, be imposed: it must naturally result from the MC implementation

- + *All matrix elements generated numerically*
- *Double counting* if  $Q(x)$  is not tuned
- *Tuning  $Q(x)$  implies the construction of an ad-hoc MC*
  - Applied to: Z production



# The first check: $\text{MC@NLO} \simeq \text{NLO}$



NLO is OK for these observables

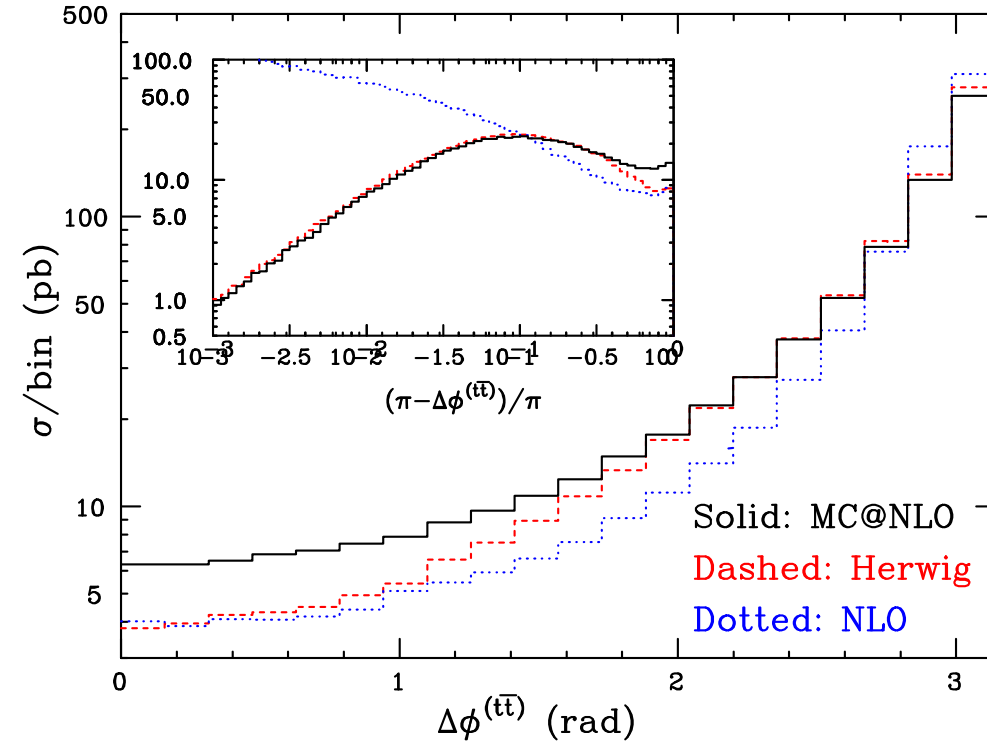
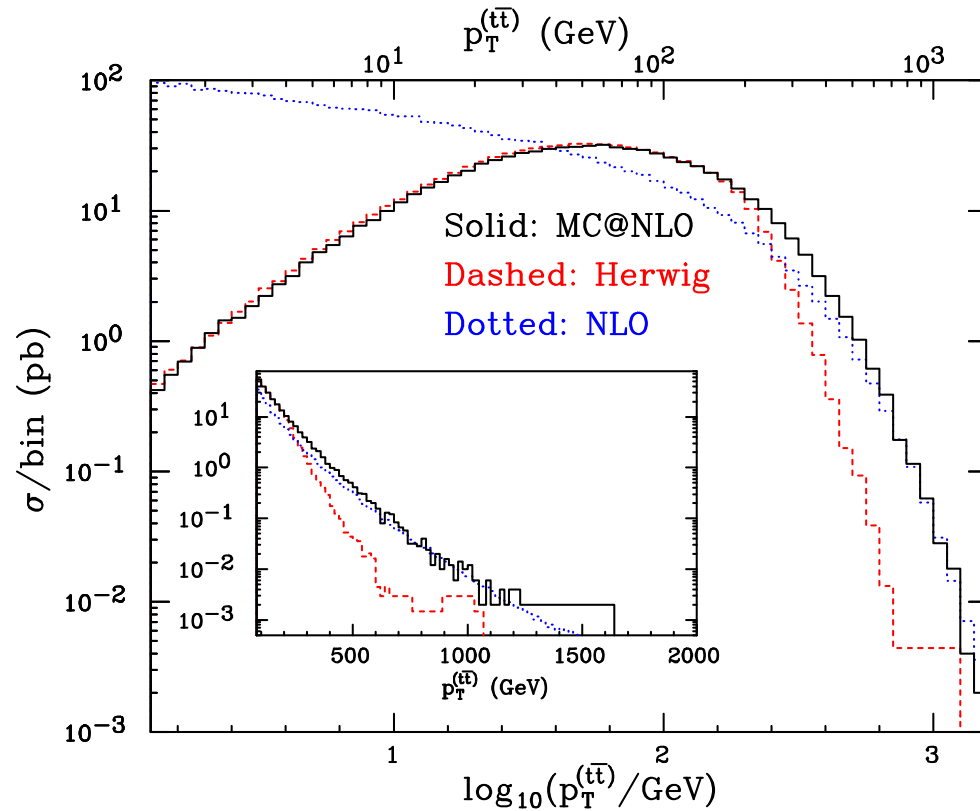
MC@NLO outputs a realistic final state, which matters when full detector simulation is included

Solid: MC@NLO

Dashed:  $\text{HERWIG} \times \frac{\sigma_{\text{NLO}}}{\sigma_{\text{LO}}}$

Dotted: NLO

# A highly non-trivial check: $t\bar{t}$ at the LHC



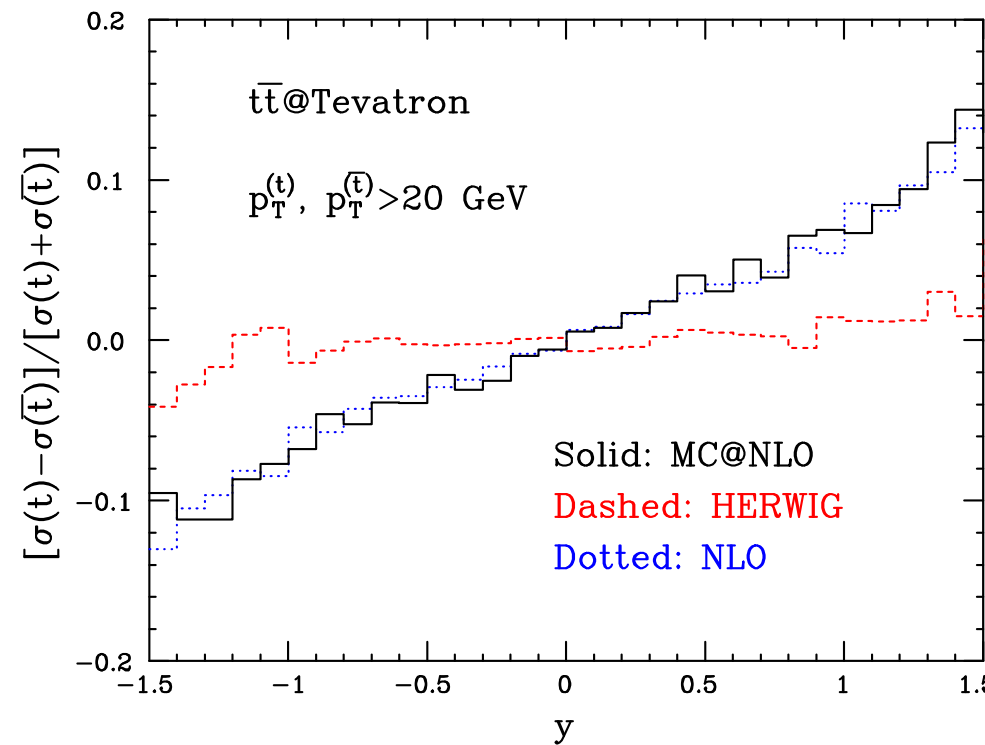
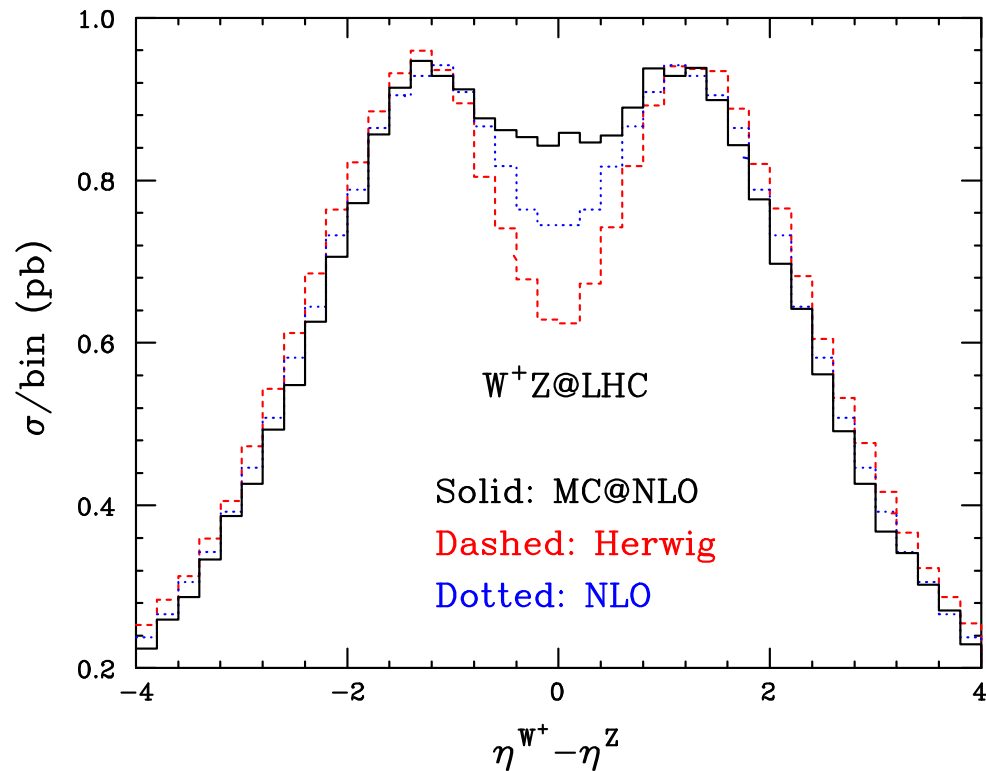
These correlations are problematic: soft and hard emissions are both relevant. MC@NLO does well, resumming large logarithms, and yet handling large-scale physics correctly

Solid: MC@NLO

Dashed:  $\text{HERWIG} \times \frac{\sigma_{NLO}}{\sigma_{LO}}$

Dotted: NLO

# New features in MC's



Radiation zero is further filled by MC@NLO

$t\bar{t}$  asymmetry is absent at the Born level, and thus also in standard MC's

Solid: MC@NLO

Dashed: HERWIG

Dotted: NLO

# An interlude: charm and bottom physics

Consider hadron collisions (just to simplify the notation)

$$\frac{d\sigma}{dp_T^2} = \sum_{i=2}^{\infty} a_i \alpha_S^i = \textcolor{red}{a_2} \alpha_S^2 + \textcolor{violet}{a_3} \alpha_S^3 + \textcolor{green}{a_4} \alpha_S^4 + \dots$$

LO      NLO      NNLO      N<sup>k</sup>LO

The computation of  $\textcolor{red}{a_2}$  is trivial, that of  $\textcolor{violet}{a_3}$  very difficult, that of  $\textcolor{green}{a_4}$  almost impossible  
 $\Rightarrow$  we have to live with NLO for a long while. Furthermore:

$$a_i = \sum_{k=0}^{i-2} a_i^{(i-2-k)} \log^{i-2-k} \frac{p_T^2}{m^2} \quad \Rightarrow \quad \textcolor{violet}{a_3} = a_3^{(0)} + a_3^{(1)} \log \frac{p_T^2}{m^2}$$

The coefficients  $a_i^{i-2-k}$  have a non-trivial  $p_T$  dependence, such that:

- ◆ When  $p_T \rightarrow 0$ , the coefficients  $a_i$  tend to a constant  $\rightarrow \textcolor{red}{a_k} \alpha_S^k \gg \textcolor{red}{a_{k+1}} \alpha_S^{k+1}$
- ◆ When  $p_T \gg m$ , the logs dominate in  $a_i \rightarrow \textcolor{red}{a_k} \alpha_S^k \simeq \textcolor{red}{a_{k+1}} \alpha_S^{k+1}$

When  $p_T \gg m$ , N<sup>k</sup>LO computations are useless

# The large- $p_T$ regime

Just keep the log terms: they are easy to compute to any order! (*resummation*)

$$\begin{aligned} \frac{d\sigma}{dp_T^2} &= \alpha_s^2 \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} r_i^{(j)} \alpha_s^j \left( \alpha_s \log \frac{p_T^2}{m^2} \right)^i = \alpha_s^2 \sum_{i=0}^{\infty} r_i^{(0)} \left( \alpha_s \log \frac{p_T^2}{m^2} \right)^i && \text{LL} \\ &+ \alpha_s^3 \sum_{i=0}^{\infty} r_i^{(1)} \left( \alpha_s \log \frac{p_T^2}{m^2} \right)^i && \text{NLL} \\ &+ \dots && \text{N}^k\text{LL} + \text{PST} \end{aligned}$$

The difficulties of the  $\text{N}^k\text{LO}$  computations are hidden in the  $\text{PST} \equiv (m/p_T)^a$  terms, which are irrelevant for  $p_T \gg m$ , but crucial for  $p_T \lesssim m$ . So the key question is:

What does  $p_T \gg m$  mean? (i.e., which are the  $p_T$  values involved?)

Roughly speaking, the neglected terms are of  $\mathcal{O}(m/p_T)$

■ In my opinion, resummed computations are only relevant at HERA to charm production for  $p_T^{(D)} \gtrsim 10$  GeV

My opinion is as good as anyone else's, since a quantitative statement is *impossible*

# The way out

Match the resummed computation with the fixed-order one, in such a way that either of them dominates in the relevant  $p_T$  region

◆ Example: FONLL (Cacciari, Greco & Nason)

$$\frac{d\sigma}{dp_T^2} = a_2 \alpha_s^2 + a_3 \alpha_s^3 + \alpha_s^2 \sum_{i=2}^{\infty} r_i^{(0)} \left( \alpha_s \log \frac{p_T^2}{m^2} \right)^i + \alpha_s^3 \sum_{i=1}^{\infty} r_i^{(1)} \left( \alpha_s \log \frac{p_T^2}{m^2} \right)^i$$

This is quite an achievement, but:

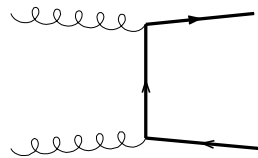
- Works only for single-inclusive  $p_T$  spectrum
- Hadronization is described by fragmentation
- Can't help in processing raw data

It is very likely that the major source of the discrepancies between data and theory for  $\gamma\gamma$  production in  $\gamma\gamma$  and  $ep$  collisions is due to extrapolations as predicted by MC's

Standard MC's are unreliable at very small  $p_T$ 's, where there are no data and the cross section peaks  $\Rightarrow$  Avoid extrapolations outside the visible region

# Why standard MC's fail at small $p_T$ 's

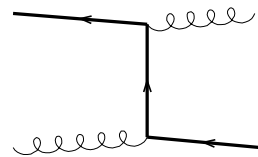
MC rule: if we aim to study any physical system, we start by producing it in the hard process  $\Rightarrow$



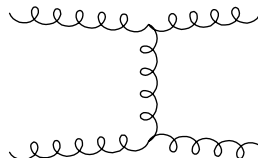
Flavour **C**reation

This is going to underestimate the rate by a factor of 4 (which is not so important), and to miss key kinematic features (which is crucial – see [R. Field](#))

So break the rule and add other hard processes



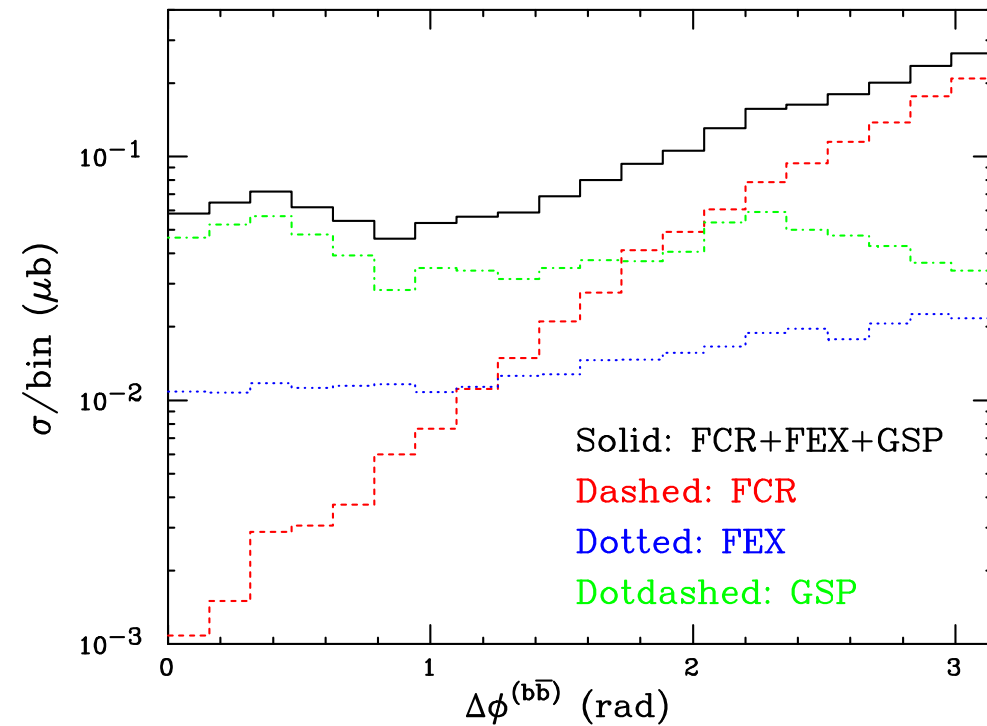
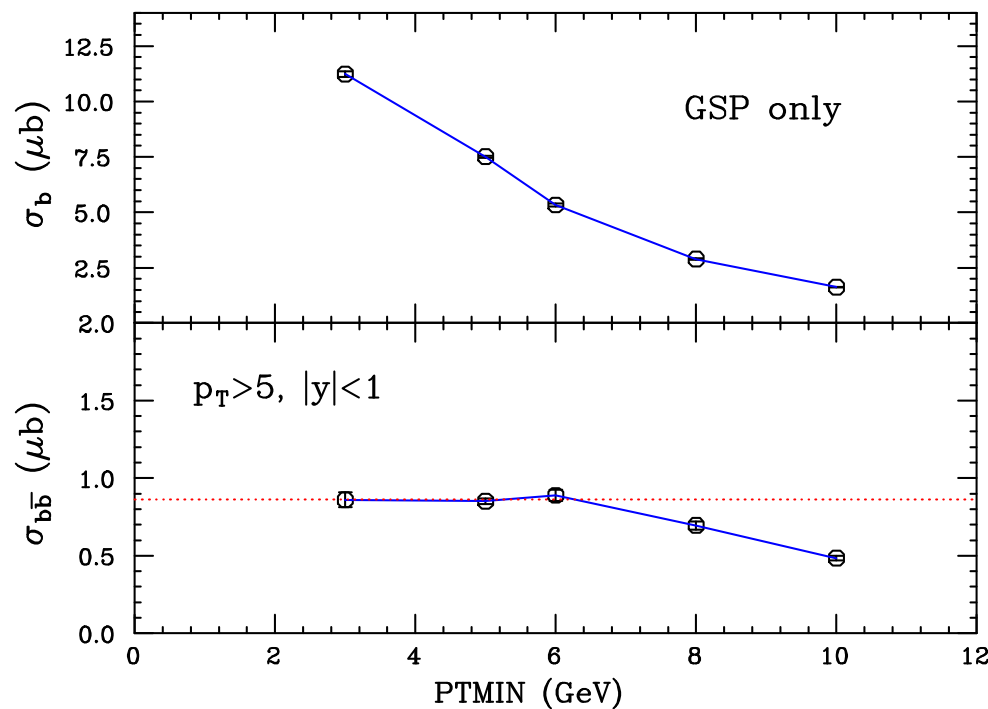
Flavour **E**Xcitation



Gluon **S**plitting

- In **FEX**, the missing  $Q$  or  $\bar{Q}$  results from initial-state radiation. A cutoff **PTMIN** avoids divergences in the matrix element
- In **GSP**, the  $Q$  and  $\bar{Q}$  result from final-state gluon splitting. **PTMIN** is again necessary to obtain finite results

# $b$ production with HERWIG



- The  $\text{PTMIN}$  dependence is worrisome in the case of single-inclusive observables
- FCR, FEX and GSP are complementary, and all must be generated
- GSP efficiency is extremely poor:  $10^{-4}$  within cuts for correlations

Reliability and efficiency rapidly degrade for smaller  $p_T$  cuts. In FEX, the dependence on bottom PDF is problematic. No standard MC can work for  $p_T \simeq 0$

All these problems are avoided with MC@NLO



# Why does MC@NLO work better?

MC@NLO is by definition a formalism that matches fixed-order and resummed results (in this sense, is analogue to FONLL), the latter obtained by means of the shower

## ◆ MC@NLO vs FONLL

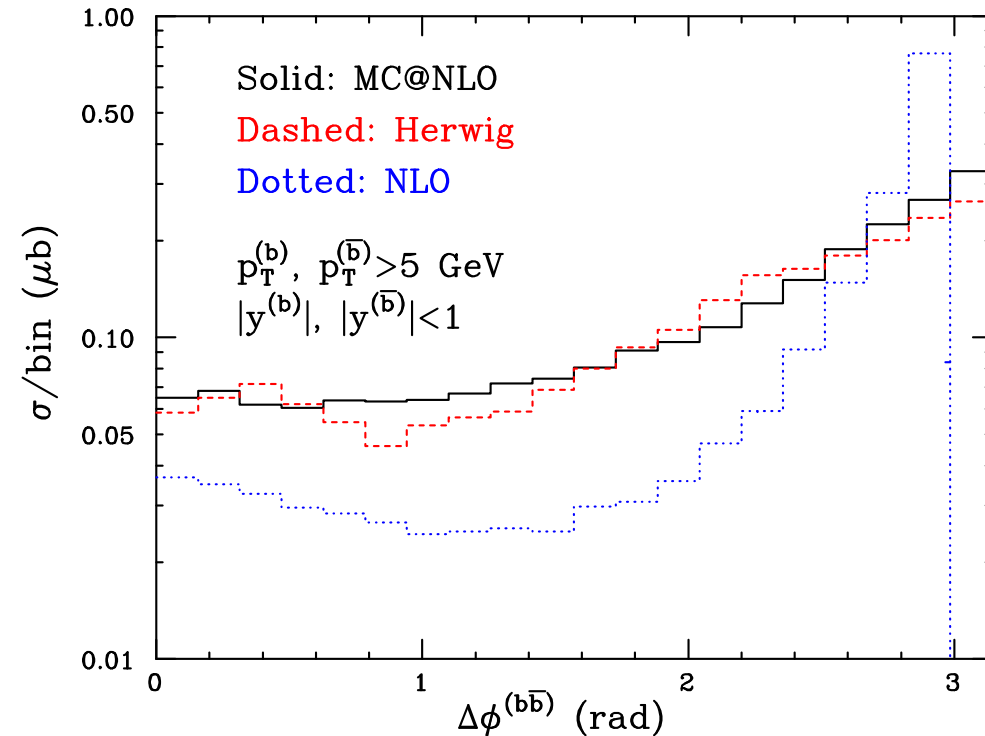
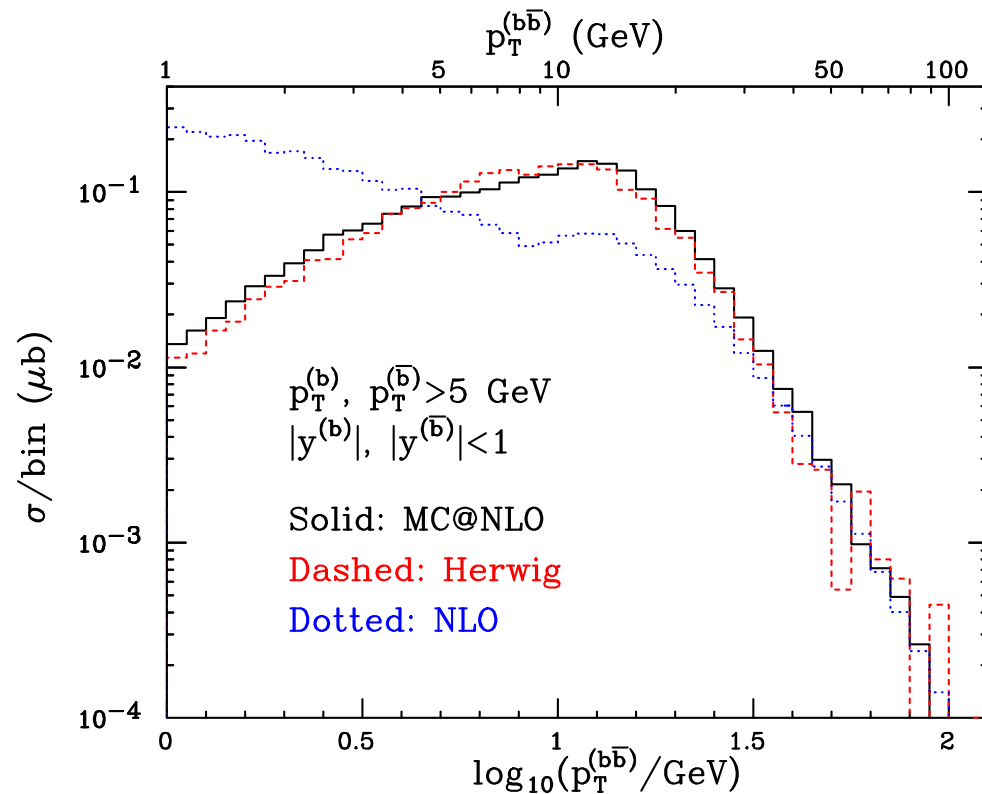
- + Fully realistic final state, hadronization, and decay
- + Works for any observable
- Formally less accurate in terms of logs

## ◆ MC@NLO vs standard MC's

- + No PTMIN dependence, no separate generation of FCR, FEX, and GSP
- + Reliable prediction of hard emission
- Misses some of the higher logs in GSP

MC@NLO can be used to obtain state-of-the-art theoretical predictions, and/or to treat raw data

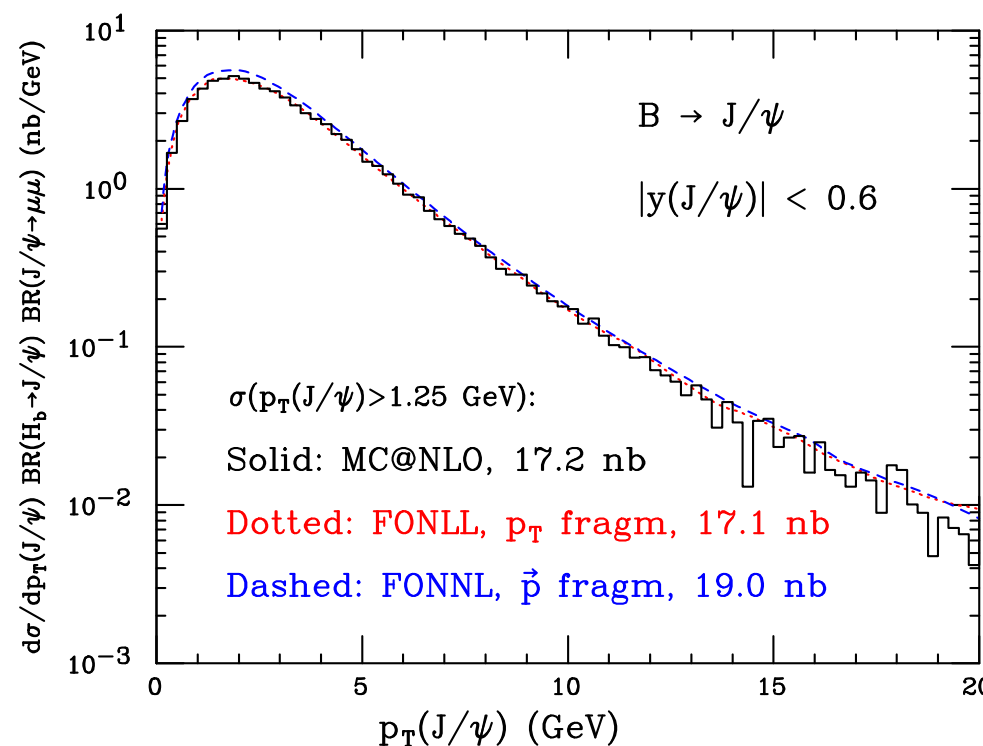
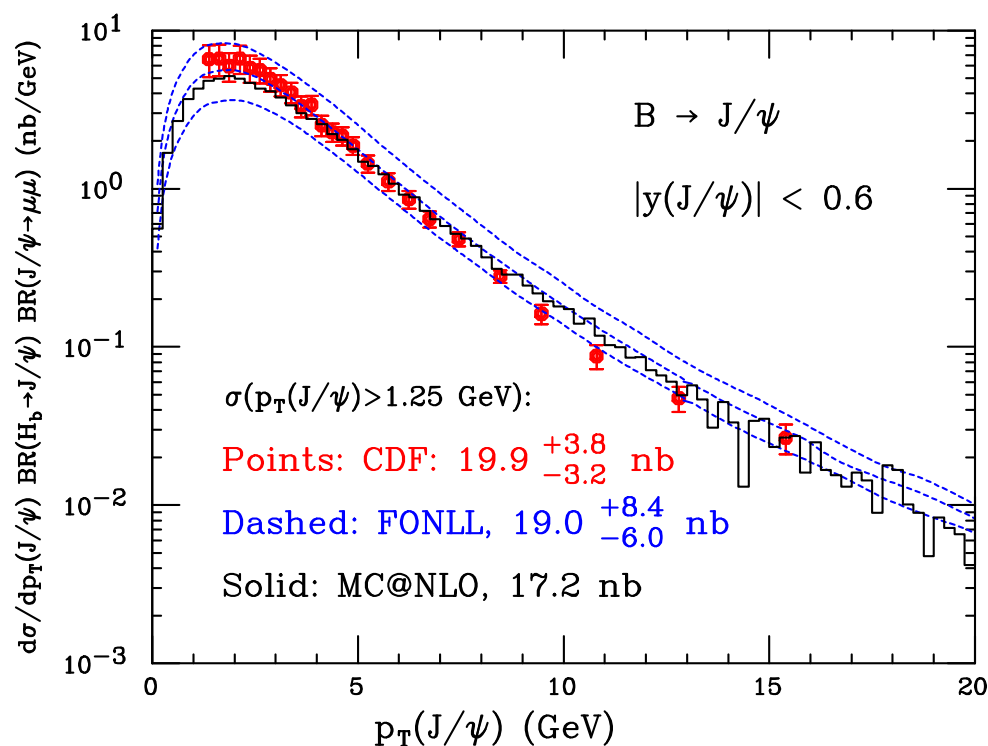
# $b\bar{b}$ correlations with MC@NLO



HERWIG does surprisingly well, but needs quite a lot of CPU (14 millions events – 1 million for MC@NLO). The hard emission effects are huge for  $b$  production, and cannot be neglected

Solid: MC@NLO  
Dashed: HERWIG  
Dotted: NLO

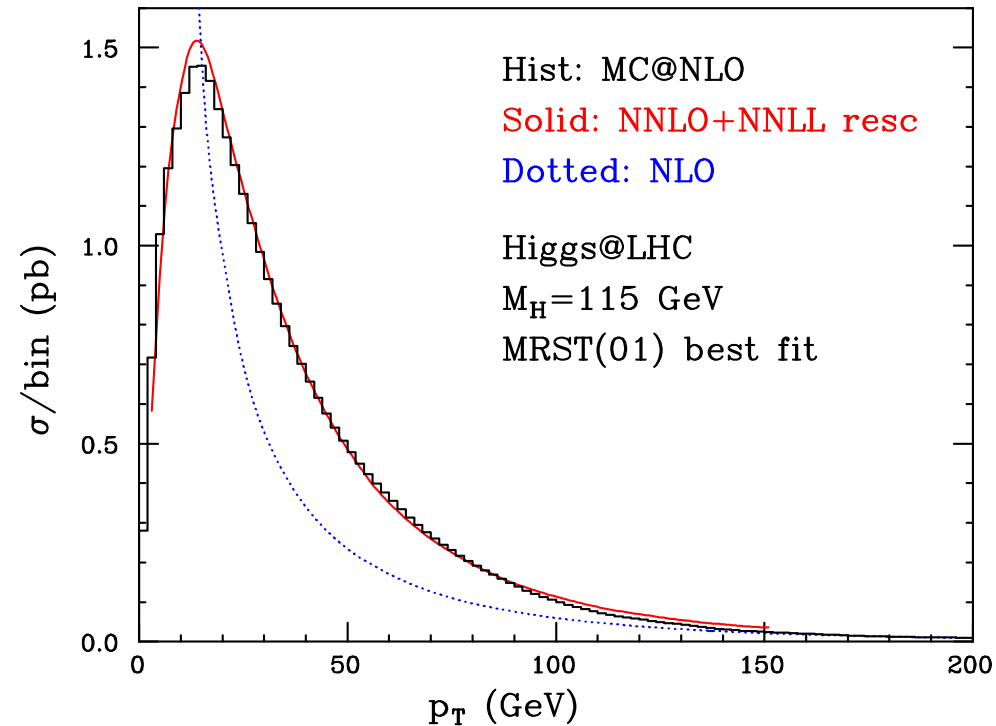
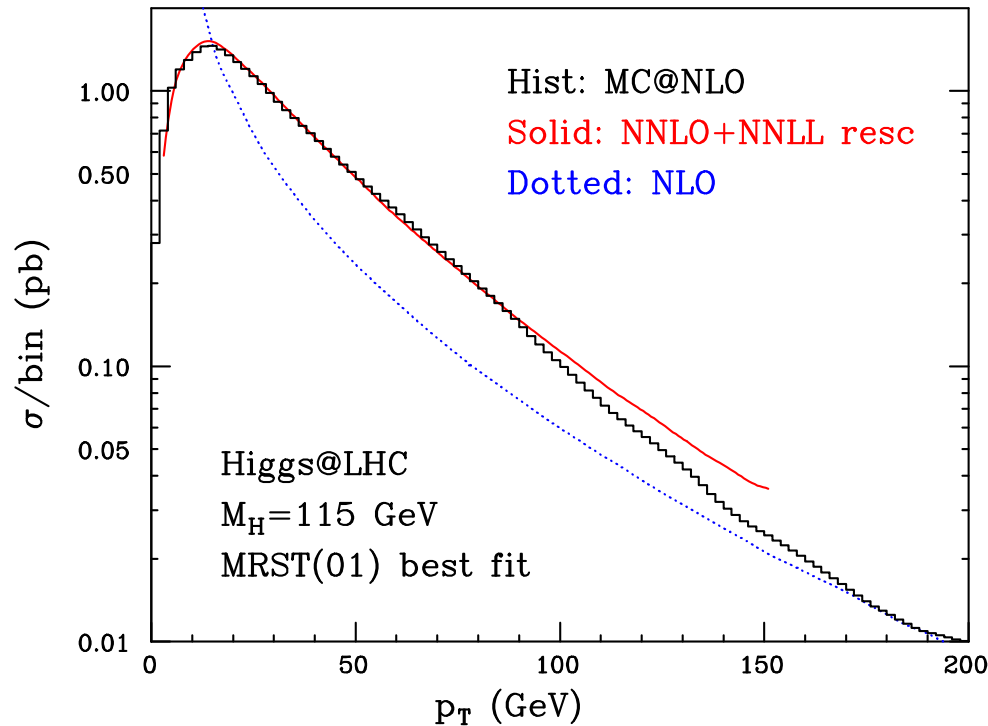
# Single-inclusive $b$ at the Tevatron



No significant discrepancy with data

- No PTMIN dependence in MC@NLO  $\Rightarrow$  solid predictions down to  $p_T = 0$ , no “perturbative-parameter tuning” (more work on  $b$  hadronization parameters needed)
- Full agreement with NLL+NLO computation (FONLL, Cacciari&Nason), if the large dependence (at small  $p_T$ ) on the hadronization scheme of the latter is taken into account

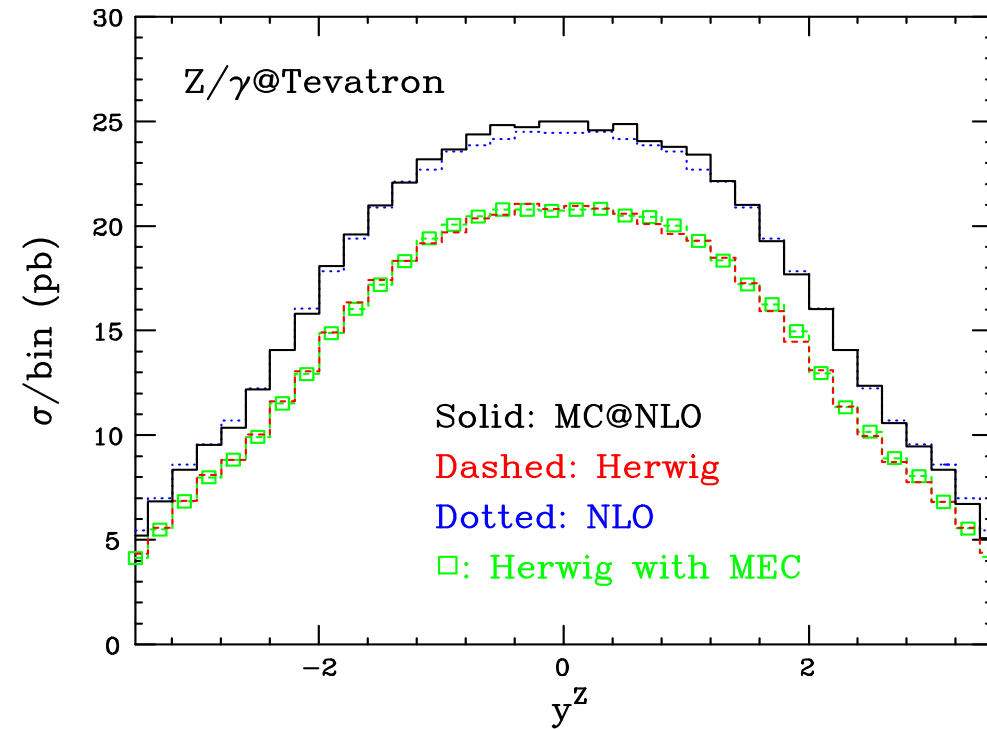
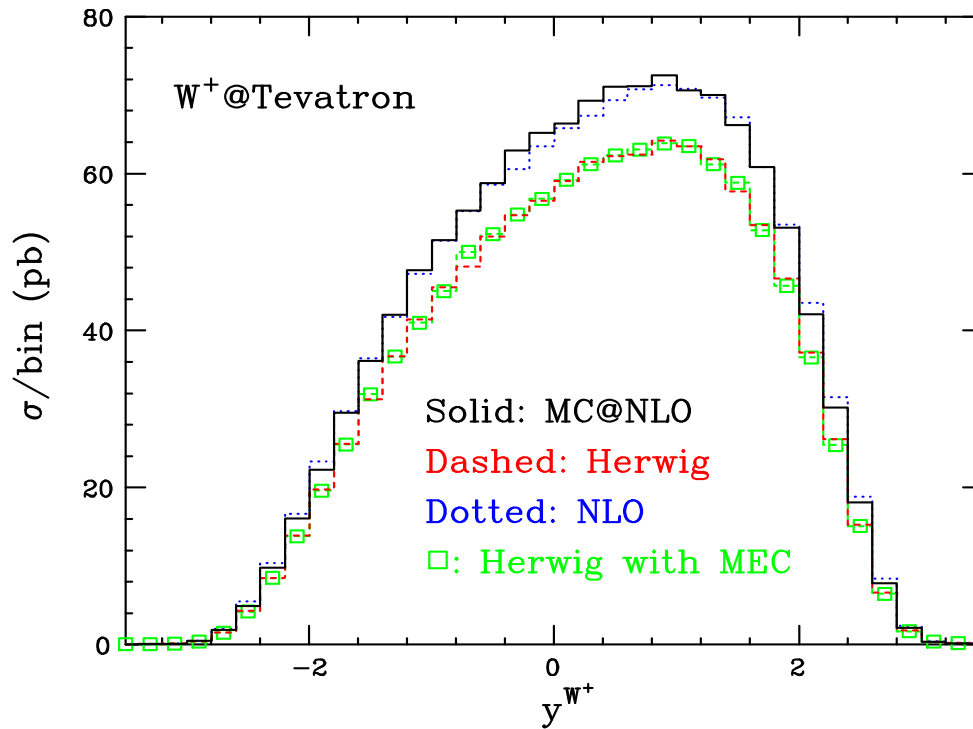
# Is the agreement with the resummed result accidental?



The same happens with Higgs. The result of Bozzi, Catani, de Florian, Grazzini has a matching condition similar to MC@NLO, in that it conserves the total rate

- ◆ The agreement with the analytically-resummed result improves when the logarithmic accuracy of the latter is increased  $\longrightarrow$  Herwig has more logs than you expect
- ◆ We can now apply any cuts we like (decay products, recoiling system) – a fully realistic jet-veto analysis is doable
- ◆ Beware: vastly different from Pythia!

# MC@NLO and luminosity monitors



There is a good agreement between MC@NLO and NLO. NNLO contributions could perhaps be included by following the procedure advocated by [Anastasiou, Dixon, Melnikov, Petriello](#), of multiplying by  $K^{(2)} = \sigma_{\text{NNLO}}/\sigma_{\text{NLO}}$

- However,  $|\text{MC@NLO} - \text{NLO}| = \mathcal{O}(1 - 2\%)$  – may change with larger statistics
- A careful analysis, including realistic experimental cuts, is therefore necessary to decide whether  $Z$  and  $W$  production can be used as parton luminosity monitors in an analysis aimed at the 1% precision

# Conclusions

NLOwPS's are theoretically well defined, and have reached the implementation stage. For them to become standard analysis tools *exp's feedback is essential*. NLOwPS's improve NLO computations and parton shower simulations in several respects

- MC@NLO is numerically more stable than NLO computations
- Realistic final states, including hadronization, are part of NLO predictions
- NLOwPS's are the *only way* in which  $K$ -factors can be embedded into MC's
- Hard radiation is incorporated in MC's, without the kinematical distortions of MEC

We are working to:

- ◆ Implement more processes (jets  $\longrightarrow$  firm estimate of hadronization corrections), and spin correlations where needed ( $V \rightarrow l_1 \bar{l}_2$  included in MC@NLO v2.3;  $VV$  or  $t\bar{t}$  next)
- ◆ Eliminate the negative weights (the formalism is almost done!)
- ◆ Increase the logarithmic accuracy of the showers

No work is currently being done on  $ep$  collisions (we lack manpower);  $Q\bar{Q}$  is the strongest case. Please encourage us to change this! (HERA-LHC workshop?)