QCD Analysis of Polarized Deep Inelastic Scattering Data and New Polarized Parton Distributions

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Outline:

- **Motivation**
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Motivation

Why and for Which Purpose do we study Polarized Deep Inelastic Scattering ?

- Short distance structure of nucleon spin
- Test of perturbative QCD: Λ_{QCD}
- Test of fundamental and less fundamental sum rules
- Does QCD describe polarized nucleons non-perturbatively?
  Parton distributions from Experiment
  vs. Lattice Moments

Is there a Spin Crisis?

Motivation

WHAT IS THE NUCLEON'S SPIN MADE OFF ?

EMC (1987):

$$\sum_{i=1}^{3} [\Delta q_i + \Delta \overline{q}_i] << \frac{1}{2} \qquad \text{Today} : 0.14 \quad \mathrm{Q}^2 = 4\ \mathrm{GeV}^2$$

Violation of the Ellis-Jaffe sum rule :

$$\int_0^1 dx g_1^{ep(n)}(x) = \frac{g_A}{12}\left[\pm 1 + \frac{5}{3}\frac{3(F/D)-1}{(F/D)+1}\right]$$

The E-J sumrule is non-fundamental, since :

$$\int_0^1 dx g_1(x) = \frac{1}{6} I_3^N (F+D) + \frac{1}{36}(3F-D) + \frac{2}{9}\Gamma_0^{5,N}$$

(e.g. Van Neerven, Ravindran, 2001)

Non-conservation of the axial current, due to the ABJ-anomaly, leads to a scale dependence of $\Gamma_0^{5,N}$, which cannot be associated to a hadron quantum number.

Motivation

What is the Nucleon's Spin made off ?

$$\sum_{i=1}^{3} [\Delta q_i + \Delta \overline{q}_i] + L_q + [\Delta G + L_g] = \frac{1}{2}$$

$\Delta q_i, \Delta \overline{q}_i, \Delta G$: from polarized DIS

L_q, L_G : (with ENORMOUS effort and luck) from:
DI non-forward scattering.

Motivation

Sum Rules & Integral Relations

Twist 2

$$g_2(x,Q^2) = -g_1(x,Q^2) + \int_x^1 \frac{dz}{z} g_1(z,Q^2)$$

Wandzura, Wilczek, 1977

$$g_3(x,Q^2) = 2x \int_x^1 \frac{dz}{z^2} g_4(z,Q^2) \quad \textit{Blümlein, Kochelev, 1996}$$

$$g_4(x,Q^2) = 2x g_5(x,Q^2) \quad \textit{Dicus, 1972}$$

Twist 3

$$g_1(x,Q^2) = \frac{4M^2x^2}{Q^2}\left[g_2(x,Q^2) - 2\int_x^1 \frac{dz}{z} g_2(z,Q^2)\right]$$

$$\frac{4M^2x^2}{Q^2} g_3(x,Q^2) = \left(1 + \frac{4M^2x^2}{Q^2}\right) g_4(x,Q^2) + 3\int_x^1 \frac{dz}{z} g_4(z,Q^2)$$

$$2x g_5(x,Q^2) = -\int_x^1 \frac{dz}{z} g_4(z,Q^2)$$

Blümlein, Tkabladze, 1998

$\Rightarrow$ Transverse Spin or Electro-Weak Interactions

Motivation

- A number of QCD analyses for polarized data performed so far :
  - T.Gehrmann and W.J.Stirling (GS), Phys.Rev.**D53**(1996)6100.
  - G.Altarelli et al. (ABFR), Nucl.Phys.**B496**(1997)337.
  - Y.Goto et al. (AAC), Phys.Rev.**D62**(2000)034017.
  - M.Glück et al. (GRSV), Phys.Rev.**D63**(2001)094005.
  - E.Leader et al. (LSS), Eur.Phys.J.**C23**(2002)479.

  - E154 Collaboration, Phys.Lett.**B405**(1997)180.
  - SMC Collaboration, Phys.Rev.**D58**(1998)112002.

  However, no reliable parametrization of the error bands for the polarized parton densities are given.
- We aim at parametrizations of polarized densities and their fully correlated 1σ error bands which are directly applicable to determine 'experimental' errors for other polarized observables.
- Such an analysis has a value of its own within the framework of spin physics in order to understand the spin puzzle.
- Comparison of the QCD analysis results with results from recent lattice simulations concerning both QCD parameters and low order moments.

Evolution in MELLIN space

- The polarized structure function $g_1(x, Q^2)$ represented in terms of a MELLIN convolution of polarized parton densities Δf_j and Wilson coefficients ΔC_j:

$$
\begin{aligned}
g_1(x, Q^2) \quad = \quad & \frac{1}{2} \sum_{j=1}^{N_f} e_j^2 \int_x^1 \frac{dz}{z} \left[\frac{1}{N_f} \Delta\Sigma \left(\frac{x}{z}, \mu_f^2 \right) \Delta C_q^S \left(z, \frac{Q^2}{\mu_f^2} \right) \right. \\
& + \Delta G \left(\frac{x}{z}, \mu_f^2 \right) \Delta C_G \left(z, \frac{Q^2}{\mu_f^2} \right) \\
& \left. + \Delta q_j^{NS} \left(\frac{x}{z}, \mu_f^2 \right) \Delta C_q^{NS} \left(z, \frac{Q^2}{\mu_f^2} \right) \right] ,
\end{aligned}
$$

with the singlet density $\Delta\Sigma$

$$
\Delta\Sigma \left(z, \mu_f^2 \right) \quad = \quad \sum_{j=1}^{N_f} \left[\Delta q_j \left(z, \mu_f^2 \right) + \Delta \bar{q}_j \left(z, \mu_f^2 \right) \right] ,
$$

the gluon density ΔG,
the non–singlet density Δq_j^{NS}

$$
\begin{aligned}
\Delta q_j^{NS} \left(z, \mu_f^2 \right) \quad = \quad & \Delta q_j \left(z, \mu_f^2 \right) + \Delta \bar{q}_j \left(z, \mu_f^2 \right) \\
& - \frac{1}{N_f} \Delta\Sigma \left(z, \mu_f^2 \right) ,
\end{aligned}
$$

and the factorization scale μ_f.

- The above quantities also depend on the renormalization scale μ_r of the strong coupling constant $a_s(\mu_r^2) = g_s^2(\mu_r^2)/(16\pi^2)$. The observable $g_1(x, Q^2)$ is independent of the choice of both scales.

Evolution in Mellin space (cont'd)

- $a_s(\mu_r)$ is obtained as the solution of

$$\mu_r^2 \frac{da_s(\mu_r^2)}{d\mu_r^2} = -\beta_0 a_s^2(\mu_r^2) - \beta_1 a_s^3(\mu_r^2) + \mathcal{O}(a_s^4),$$

where the coefficients of the β–function are given by (in the $\overline{\mathrm{MS}}$ scheme)

$$\begin{aligned}\beta_0 &= \frac{11}{3}C_A - \frac{4}{3}T_F N_f,\\ \beta_1 &= \frac{34}{3}C_A^2 - \frac{20}{3}C_A T_F N_f - 4C_F T_F N_f,\end{aligned}$$

and

$$C_A = 3, \quad T_F = 1/2, \quad C_F = 4/3.$$

- $\Lambda_{QCD}^{\overline{\mathrm{MS}}}$ is given by:

$$\begin{aligned}\Lambda_{\mathrm{QCD}}^{\overline{\mathrm{MS}}} &= \mu_r \exp\left\{-\frac{1}{2}\left[\frac{1}{\beta_0 a_s(\mu_r^2)}\right.\right.\\ &\left.\left.\qquad -\frac{\beta_1}{\beta_0^2}\log\left(\frac{1}{\beta_0 a_s(\mu_r^2)} + \frac{\beta_1}{\beta_0}\right)\right]\right\}.\end{aligned}$$

$\Longrightarrow$ We extract $\Lambda_{QCD}^{(4)}$ from the data and choose $N_f = 4$ whereas the polarized structure function $g_1(x, Q^2)$ is presented using only the three light flavors.

Evolution in MELLIN space (cont'd)

- The evolution equations are given by

$$\frac{\partial \Delta q_i^{\rm NS}(x,Q^2)}{\partial \log Q^2} = \Delta P^-_{NS}(x,a_s) \otimes \Delta q_i^{\rm NS}(x,Q^2)$$

$$\frac{\partial}{\partial \log Q^2}\begin{pmatrix} \Delta\Sigma(x,Q^2) \\ \Delta G(x,Q^2) \end{pmatrix} = \Delta \boldsymbol{P}(x,a_s) \otimes \begin{pmatrix} \Delta\Sigma(x,Q^2) \\ \Delta G(x,Q^2) \end{pmatrix}$$

with

$$\begin{aligned} \Delta P^-_{\rm NS}(x,a_s) &= a_s \Delta P^{(0)}_{\rm NS}(x) + a_s^2 \Delta P^{-(1)}_{\rm NS}(x) + \mathcal{O}(a_s^3) \\ \Delta \boldsymbol{P}(x,a_s) &\equiv \begin{pmatrix} \Delta P_{qq}(x,Q^2) & \Delta P_{qg}(x,Q^2) \\ \Delta P_{gq}(x,Q^2) & \Delta P_{gg}(x,Q^2) \end{pmatrix} \\ &= a_s \Delta \boldsymbol{P}^{(0)}(x) + a_s^2 \Delta \boldsymbol{P}^{(1)}(x) + \mathcal{O}(a_s^3) \end{aligned}$$

and $\otimes$ the MELLIN convolution

$$[A \otimes B](x) = \int_0^1 dx_1 dx_2 \delta(x - x_1 x_2) A(x_1) B(x_2)$$

- The polarized Wilson coefficient functions $\Delta C_i(x, \alpha_s(Q^2))$ and the polarized splitting functions $\Delta P_{ij}(x, \alpha_s(Q^2))$ are known in the $\overline{MS}$ scheme up to NLO. [W.L. van Neerven and E.B. Zijlstra, Nucl. Phys. B417 (1994) 61, R. Mertig and W.L. van Neerven, Z. Phys. C70 (1996) 637, W. Vogelsang, Phys. Rev. D54 (1996) 2023]

$\Longrightarrow$ A complete NLO QCD Analysis possible.

Evolution in MELLIN space (cont'd)

- The evolution equations are solved analytically in MELLIN–N space:

  $\longrightarrow$ A MELLIN–transformation is performed

$$\mathbf{M}[f](N) = \int_0^1 dx x^{N-1} f(x), \quad N \,\epsilon\, \mathbf{N} \,,$$

  which turns the MELLIN convolution $\otimes$ into an ordinary product.

- The non–singlet solution:

$$\Delta q^{\rm NS}(N, a_s) = \left(\frac{a_s}{a_0}\right)^{-P_{\rm NS}^{(0)}/\beta_0} \left[1 - \frac{1}{\beta_0}(a_s - a_0) \times \left(P_{\rm NS}^{-(1)} - \frac{\beta_1}{\beta_0} P_{\rm NS}^{(0)}\right)\right] \Delta q^{\rm NS}(N, a_0)$$

  and the singlet solution:

$$\begin{pmatrix} \Delta\Sigma(N, a_s) \\ \Delta G(N, a_s) \end{pmatrix} = \left[\mathbf{1} + a_s \boldsymbol{U}_1(N)\right] \boldsymbol{L}(N, a_s, a_0) \left[\mathbf{1} - a_0 \boldsymbol{U}_1(N)\right] \times \begin{pmatrix} \Delta\Sigma(N, a_0) \\ \Delta G(N, a_0) \end{pmatrix} ,$$

  where $a_s = a_s(Q^2)$ and $a_0 = a_s(Q_0^2)$.

$\Longrightarrow$ The input and the evolution parts factorize.

[W.Furmanski and R.Petronzio, Z.Phys.**C11**(1982)293, M.Glück, E.Reya, and A.Vogt, Z.Phys.**C48**(1990)471, J.Blümlein and A.Vogt, Phys.Rev.**D58**(1998)014020.]

Evolution in Mellin space (cont'd)

- The Leading Order singlet evolution matrix is given by

$$\boldsymbol{L}(a_s, a_0, N) = \boldsymbol{e}_-(N)\left(\frac{a_s}{a_0}\right)^{-r_-(N)} + \boldsymbol{e}_+(N)\left(\frac{a_s}{a_0}\right)^{-r_+(N)}$$

with the eigenvalues

$$r_\pm = \frac{1}{\beta_0}\left[tr(\boldsymbol{P}^{(0)}) \pm \sqrt{tr(\boldsymbol{P}^{(0)})^2 - det_2(\boldsymbol{P}^{(0)})}\right]$$

and the eigenvectors

$$\boldsymbol{e}_\pm = \frac{\boldsymbol{P}^{(0)}/\beta_0 - r_\mp \boldsymbol{1}}{r_\pm - r_\mp}\,.$$

- The Next–to–Leading Order singlet solution is obtained from the LO singlet solution through the matrix $\boldsymbol{U}_1(N)$

$$\begin{aligned}\boldsymbol{U}_1(N) \quad &= \quad -\boldsymbol{e}_-\boldsymbol{R}_1\boldsymbol{e}_- - \boldsymbol{e}_+\boldsymbol{R}_1\boldsymbol{e}_+ + \frac{\boldsymbol{e}_+\boldsymbol{R}_1\boldsymbol{e}_-}{r_- - r_+ - 1}\\ &\quad + \frac{\boldsymbol{e}_-\boldsymbol{R}_1\boldsymbol{e}_+}{r_+ - r_- - 1}\end{aligned}$$

with

$$\boldsymbol{R}_1 = [\boldsymbol{P}^{(1)} - (\beta_1/\beta_0)\boldsymbol{P}^{(0)}]/\beta_0.$$

Evolution in Mellin space (cont'd)

- The input densities

  $$\Delta\Sigma(N, a_0),\ \Delta G(N, a_0),\ \text{and}\ \Delta q_i^{NS}(N, a_0)$$

  are evolved to the scale Q^2, respectively to the coupling $\alpha_s(Q^2)$. An inverse Mellin–transformation to x–space is then performed by a contour integral in the complex plane around all singularities:

  $$\Delta f(x) = \frac{1}{\pi} \int_0^\infty dz \, \mathrm{Im} \left[\exp(i\phi) x^{-c(z)} \Delta f[c(z)] \right] .$$

  (Path: $c(z) = c_1 + \rho[\cos(\phi) + i \sin(\phi)]$, with

  $c_1 = 1.1, \ \rho \geq 0$, and $\phi = \frac{3}{4}\pi$.)

- The function $\Delta f(x)$ finally depends on the parameters of the parton distributions chosen at the input scale Q_0^2 and on Λ_{QCD}. These parameters are determined by the fit to the data.

Parametrization

- General choice for the parametrization of the polarized parton distributions at Q_0^2:

$$x\Delta q_i(x, Q_0^2) = \eta_i A_i x^{a_i}(1-x)^{b_i}(1+\gamma_i x+\rho_i x^{\frac{1}{2}})$$

- Normalization:

$$A_i^{-1} = \left(1+\gamma_i \frac{a_i}{a_i+b_i+1}\right)\frac{\Gamma(a_i)\Gamma(b_i+1)}{\Gamma(a_i+b_i+1)} + \rho_i \frac{\Gamma(a_i+0.5)\Gamma(b_i+1)}{\Gamma(a_i+b_i+1.5)}$$

such that $$\int_0^1 dx \Delta q_i(x, Q_0^2) = \eta_i$$

are the first moment of $\Delta q_i(x, Q_0^2)$.

- The polarized parton distributions to be fitted are:

$$\Delta u_v \ , \ \Delta d_v \ , \ \Delta \bar{q} \ , \ \Delta G \ ,$$

where the index v denotes the $valence$ quark.

Note : $\Delta q + \Delta \bar{q} = \Delta q_v + 2\Delta \bar{q}$.

Choice of Parameters

- $Q_0^2 = 4.0\, GeV^2$
- SU(3) flavour symmetry assumed

  η_{u_v} and η_{d_v} determined from the SU(3) parameters F and D involved in the matrix elements describing the neutron and hyperon β-decays:

  $$\eta_{u_v} = 2F = 0.926 \; ; \; \eta_{d_v} = F - D = -0.341$$
  $$g_A/g_V = \eta_{u_v} - \eta_{d_v} = F + D = 1.267$$

- Flavour symmetric sea assumed

  $$\Delta\bar{u}(x, Q_0^2) = \Delta\bar{d}(x, Q_0^2) = \Delta\bar{s}(x, Q_0^2) = \Delta\bar{q}(x, Q_0^2)$$

- For Δu_v and Δd_v: $\rho = 0$

  $$x\Delta q_i(x, Q_0^2) = \eta_i A_i x^{a_i}(1-x)^{b_i}(1+\gamma_i x)$$

- For $\Delta\bar{q}$ and ΔG: $\gamma = \rho = 0$ (Gluon A)

  $$x\Delta q_i(x, Q_0^2) = \eta_i A_i x^{a_i}(1-x)^{b_i}$$

- The remaining 12 parameters to be determined are:

  Δu_v: a_u , b_u , γ_u Δd_v: a_d , b_d , γ_d

  $\Delta\bar{q}$: $\eta_{\bar{q}}$, $a_{\bar{q}}$, $b_{\bar{q}}$ ΔG: η_G , a_G , b_G

Note:

$$x\Delta q_i(x, Q_0^2) = \eta_i A_i x^{a_i}(1-x)^{b_i}(1+\gamma_i x + \rho_i x^{\frac{1}{2}})$$

Choice of Parameters

- Parameters which have been fixed since the data do not constrain those parameters well enough:
  - For Δu_v and Δd_v: γ

$$x\Delta q_i(x, Q_0^2) = \eta_i A_i x^{a_i}(1-x)^{b_i}(1+\gamma_i x)$$

  - For $\Delta \bar{q}$ and ΔG: b

$$x\Delta q_i(x, Q_0^2) = \eta_i A_i x^{a_i}(1-x)^{b_i}$$

- Relations adopted between the parameters a_i and b_i for $\Delta \bar{q}$ and ΔG:

$$a_G = a_{\bar{q}} + C, \quad with \quad 0.5 < C < 1.0.$$

$$\left(\frac{b_{\bar{q}}}{b_G}\right)^{pol} = \left(\frac{b_{\bar{q}}}{b_G}\right)^{unpol}$$

$\Longrightarrow$ Essential to respect Positivity for $\Delta \bar{q}$ and ΔG.

- No Positivity constraint assumed for Δu_v and Δd_v.

$\Longrightarrow$ Finally 7 parameters are left free to be determined in the fit. In addition Λ_{QCD} is fitted. $\longrightarrow (7+1)$

Note:

$$x\Delta q_i(x, Q_0^2) = \eta_i A_i x^{a_i}(1-x)^{b_i}(1+\gamma_i x + \rho_i x^{\frac{1}{2}})$$

The World Data

Published Experimental Data above $Q^2 = 1.0\ GeV^2$

Experiment	x–range	Q^2–range $[GeV^2]$	number of data points: g_1/F_1 or A_1	number of data points: g_1
E143(p)	0.027 – 0.749	1.17 – 9.52	82	28
HERMES(p)	0.028 – 0.660	1.13 – 7.46	39	39
E155(p)	0.015 – 0.750	1.22 – 34.72	24	24
SMC(p)	0.005 – 0.480	1.30 – 58.0	59	12
EMC(p)	0.015 – 0.466	3.50 – 29.5	10	10
proton			214	113
E143(d)	0.027 – 0.749	1.17 – 9.52	82	28
E155(d)	0.015 – 0.750	1.22 – 34.79	24	24
SMC(d)	0.005 – 0.479	1.30 – 54.8	65	12
deuteron			171	64
E142(n)	0.035 – 0.466	1.10 – 5.50	30	8
HERMES(n)	0.033 – 0.464	1.22 – 5.25	9	9
E154(n)	0.017 – 0.564	1.20 – 15.0	11	17
neutron			50	34
total			435	211

$$g_1/F_1 \approx \frac{1}{(1+\gamma^2)} A_1 \ , \quad where \quad \gamma^2 = Q^2/\nu^2$$

$$F_1 = \frac{(1+\gamma^2)}{2x(1+R)} F_2$$

F_2–Parametrization: NMC, M. Arneodo et al., Phys. Lett. **B364** (1995) 107.

R–Parametrization: SLAC, L. Withlow et al., Phys. Lett. **B250** (1990) 193.

From the measured $A_{||}(x,Q^2)$ to $g_1(x,Q^2)$

- Cross Section Asymmetry $A_{||}$:

$$A_{||} = \frac{\sigma^{\uparrow\uparrow} - \sigma^{\uparrow\downarrow}}{\sigma^{\uparrow\uparrow} + \sigma^{\uparrow\downarrow}} .$$

- From $A_{||}$ to A_1 or g_1/F_1:

$$\begin{aligned} A_1 &= \frac{A_{||}}{D} - \eta A_2 , \\ \frac{g_1}{F_1} &= \frac{1}{(1+\gamma^2)} \left[\frac{A_{||}}{D} + (\gamma - \eta) A_2 \right] , \\ \frac{g_1}{F_1} &= \frac{1}{(1+\gamma^2)} [A_1 + \gamma A_2] \approx \frac{1}{(1+\gamma^2)} A_1 . \end{aligned}$$

  A_2 is measured to be small. Its contribution to A_1 or g_1/F_1 can be neglected. D is the virtual photon depolarization factor. γ and η are kinematic factors.

- From g_1/F_1 to g_1:

$$\begin{aligned} g_1(x,Q^2) &= g_1/F_1 \times F_1(x,Q^2), \\ F_1(x,Q^2) &= \frac{(1+\gamma^2)}{2x(1+R(x,Q^2))} F_2(x,Q^2), \\ R(x,Q^2) &= \sigma_L/\sigma_T, \quad \gamma^2 = Q^2/\nu^2. \end{aligned}$$

  F_2–Parametrization: NMC, M. Arneodo et al., Phys. Lett. **B364** (1995) 107.

  R–Parametrization: SLAC, L. Withlow et al., Phys. Lett. **B250** (1990) 193.

What about the Errors?

⇒ Problem: Systematic errors are known to be partly correlated which would lead to an overestimation of the errors when added in quadrature with the statistical ones.

- Statistical Errors:
  To treat all data sets on the same footing statistical errors are taken only. Accept only fits with a Positive Definite Covariance Matrix.

  $\Longrightarrow$ Calculate the Fully Correlated 1σ Error Bands by Gaussian error propagation.

- Systematic Uncertainties:
  Allow for a Relative Normalization Shift between the different data sets within the normalization uncertainties quoted by the experiments (fitted and then fixed).

$$\chi^2 = \sum_{i=1}^{n^{exp}} \left[\frac{(N_i - 1)^2}{(\Delta N_i)^2} + \sum_{j=1}^{n^{data}} \frac{(N_i g_{1,j}^{data} - g_{1,j}^{theor})^2}{(\Delta g_{1,j}^{data})^2} \right]$$

$\Longrightarrow$ Thereby accounting for the main systematic uncertainties (luminosity and beam and target polarization).

Gaussian Error Propagation

In the treatment used in our analysis the evolved polarized parton densities are linear functions of the input densities for all parameters, except Λ_{QCD}.

Let $f(x, Q^2; a_i|_{i=1}^k)$ be the evolved density at Q^2 depending on the fitted parameters $a_i|_{i=1}^k$ at the input scale Q_0^2. Then its fully correlated error Δf as given by Gaussian error propagation is

$$\Delta f(x, Q^2) = \left[\sum_{i=1}^{k} \left(\frac{\partial f}{\partial a_i} \right)^2 C(a_i, a_i) + \sum_{i \neq j=1}^{k} \left(\frac{\partial f}{\partial a_i} \frac{\partial f}{\partial a_j} \right) C(a_i, a_j) \right]^{\frac{1}{2}} .$$

$C(a_i, a_j)$ are the elements of the covariance matrix determined in the QCD analysis at the input scale Q_0^2.

$\Longrightarrow$ All what is needed are the gradients $\partial f / \partial a_i$ w.r.t. the parameters a_i. They can be calculated analytically at the input scale Q_0^2. Their value at Q^2 is then given by evolution.

Error Propagation in $\mathrm{MELLIN\text{-}N}$ space

The general form of the derivative of the MELLIN moment $\mathbf{M}[f(a)](N)$ w.r.t. parameter a for complex values of N is

$$\frac{\partial \mathbf{M}[f(a)](N)}{\partial a} = F(a) \times \mathbf{M}[f(a)](N),$$

- For Δu_v and Δd_v:

$$\begin{aligned} F(a_i) &= \psi(N-1+a_i) - \psi(N+a_i+b_i) + \frac{\gamma_i(b_i+1)}{(N+a_i+b_i)(N+a_i+b_i+\gamma_i(N-1+a_i))} \\ &\quad - \psi(a_i) + \psi(a_i+b_i+1) \\ &\quad - \frac{\gamma_i(b_i+1)}{(a_i+b_i+1)(a_i+b_i+1+\gamma_i a_i)}\,, \\ F(b_i) &= \psi(b_i+1) - \psi(N+a_i+b_i) - \frac{\gamma_i(N-1+a_i)}{(N+a_i+b_i)(N+a_i+b_i+\gamma_i(N-1+a_i))} \\ &\quad - \psi(b_i+1) + \psi(a_i+b_i+1) \\ &\quad + \frac{\gamma_i a_i}{(a_i+b_i+1)(a_i+b_i+1+\gamma_i a_i)} \end{aligned}$$

Note:

$$x\Delta q_i(x, Q_0^2) = \eta_i A_i x^{a_i}(1-x)^{b_i}(1+\gamma_i x)$$

Error Propagation in $\mathrm{MELLIN\text{-}N}$ space (cont'd)

- For $\Delta\bar{q}$ and ΔG:

$$
\begin{aligned}
F(\eta_i) &= \frac{1}{\eta_i}\,, \\
F(a_i) &= \psi(N-1+a_i) \;-\; \psi(N+a_i+b_i) \\
&\quad -\;\psi(a_i)\;+\;\psi(a_i+b_i+1).
\end{aligned}
$$

Note: $x\Delta q_i(x,Q_0^2) = \eta_i A_i x^{a_i}(1-x)^{b_i}$

with $\psi(z) = d/dz(log\Gamma(z))$ the EULER ψ–function.

$\Longrightarrow$ The gradients evolved in $\mathrm{MELLIN\text{-}N}$ space are then transformed back to x-space and can be used according to the error propagation equation.

- When fitting Λ_{QCD} its gradient has to be determined numerically due to non–linear and iterative aspects in the calculation of $\alpha_s(Q^2,\Lambda_{QCD})$:

$$\frac{\partial f(x,Q^2,\Lambda)}{\partial\Lambda} = \frac{f(x,Q^2,\Lambda+\delta) - f(x,Q^2,\Lambda-\delta)}{2\delta}$$

with $\delta \sim 10\ MeV$.

Error Propagation in x-space

The gradients at the input scale Q_0^2 w.r.t. the parameters of the input densities

$$\Delta f_i = x\Delta q_i(x, Q_0^2) = \eta_i A_i x^{a_i}(1-x)^{b_i}(1+\gamma_i x+\rho_i x^{\frac{1}{2}})$$

in x space are (here given w.r.t. all parameters):

$$\begin{aligned}
\frac{\partial \Delta f_i}{\partial \eta_i} &= \frac{1}{\eta_i}\Delta f_i \ , \\
\frac{\partial \Delta f_i}{\partial a_i} &= \left(log(x) - \frac{1}{T}\frac{\partial T}{\partial a_i}\right)\Delta f_i \ , \\
\frac{\partial \Delta f_i}{\partial b_i} &= \left(log(1-x) - \frac{1}{T}\frac{\partial T}{\partial b_i}\right)\Delta f_i \ , \\
\frac{\partial \Delta f_i}{\partial \gamma_i} &= \left(\frac{x}{1+\gamma_i x+\rho_i x^{\frac{1}{2}}} - \frac{1}{T}\frac{\partial T}{\partial \gamma_i}\right)\Delta f_i \ , \\
\frac{\partial \Delta f_i}{\partial \rho_i} &= \left(\frac{x^{\frac{1}{2}}}{1+\gamma_i x+\rho_i x^{\frac{1}{2}}} - \frac{1}{T}\frac{\partial T}{\partial \rho_i}\right)\Delta f_i \ .
\end{aligned}$$

with

$$\begin{aligned}
T &= B(a_i, b_i+1)\left(1+\frac{\gamma_i a_i}{1+a_i+b_i}\right) \\
&\quad + \gamma_i B(a_i+\frac{1}{2}, b_i+1) \ ,
\end{aligned}$$

and

Error Propagation in x-space (cont'd)

$$
\begin{aligned}
\frac{\partial T}{\partial a_i} &= \left[\psi(a_i) - \psi(a_i + b_i + 1)\right] B(a_i, b_i + 1) \times \\
&\left(1 + \frac{\gamma_i a_i}{1 + a_i + b_i}\right) + B(a_i, b_i + 1) \times \\
&\left(\frac{\gamma_i a_i}{(1 + a_i + b_i)^2}\right) + \left[\psi(a_i + \frac{1}{2}) - \psi(a_i + b_i + \frac{3}{2})\right] \\
&\times \rho_i B(a_i + \frac{1}{2}, b_i + 1) \ , \\
\frac{\partial T}{\partial b_i} &= \left[\psi(b_i + 1) - \psi(a_i + b_i + 1)\right] B(a_i, b_i + 1) \times \\
&\left(1 + \frac{\gamma_i a_i}{1 + a_i + b_i}\right) - B(a_i, b_i + 1) \times \\
&\left(\frac{\gamma_i a_i}{(1 + a_i + b_i)^2}\right) + \left[\psi(b_i + 1) - \psi(a_i + b_i + \frac{3}{2})\right] \\
&\times \rho_i B(a_i + \frac{1}{2}, b_i + 1) \ , \\
\frac{\partial T}{\partial \gamma_i} &= B(a_i, b_i + 1) \left(\frac{a_i}{1 + a_i + b_i}\right) \ , \\
\frac{\partial T}{\partial \rho_i} &= B(a_i + \frac{1}{2}, b_i + 1) \ .
\end{aligned}
$$

with $B(z)$ the β–function for complex arguments.

$\Longrightarrow$ Both approaches give the same error contours at the input scale Q_0^2.

Parameter Values at $Q_0^2 = 4.0\,GeV^2$

7+1 Parameter Fit based on the Asymmetry Data:

	Scenario 1			
	LO		NLO	
	value	error	value	error
$\Lambda_{QCD}^{(4)}, MeV$	203	120	235	53
η_{u_v}	0.926	fixed	0.926	fixed
a_{u_v}	0.197	0.013	0.294	0.035
b_{u_v}	2.403	0.107	3.167	0.212
$\gamma_{u_v}(*)$	21.34	fixed	27.22	fixed
η_{d_v}	-0.341	fixed	-0.341	fixed
a_{d_v}	0.190	0.049	0.254	0.111
b_{d_v}	3.240	0.884	3.420	1.332
$\gamma_{d_v}(*)$	30.80	fixed	19.06	fixed
η_{sea}	-0.353	0.054	-0.447	0.082
a_{sea}	0.367	0.048	0.424	0.062
$b_{sea}(*)$	8.51	fixed	8.93	fixed
η_G	1.281	0.816	1.026	0.554
a_G	$a_{sea} + 0.9$		$a_{sea} + 1.0$	
$b_G(*)$	5.91	fixed	5.51	fixed
χ^2 / NDF	1.02		0.90	

$\Rightarrow$ The parameters marked by (*) have been fitted first and then fixed since the present data do not constrain their values well enough.

$\Rightarrow$ Scenario 2 : $a_G = a_{sea} + 0.6\ (LO)$

$$a_G = a_{sea} + 0.5\ (NLO)$$

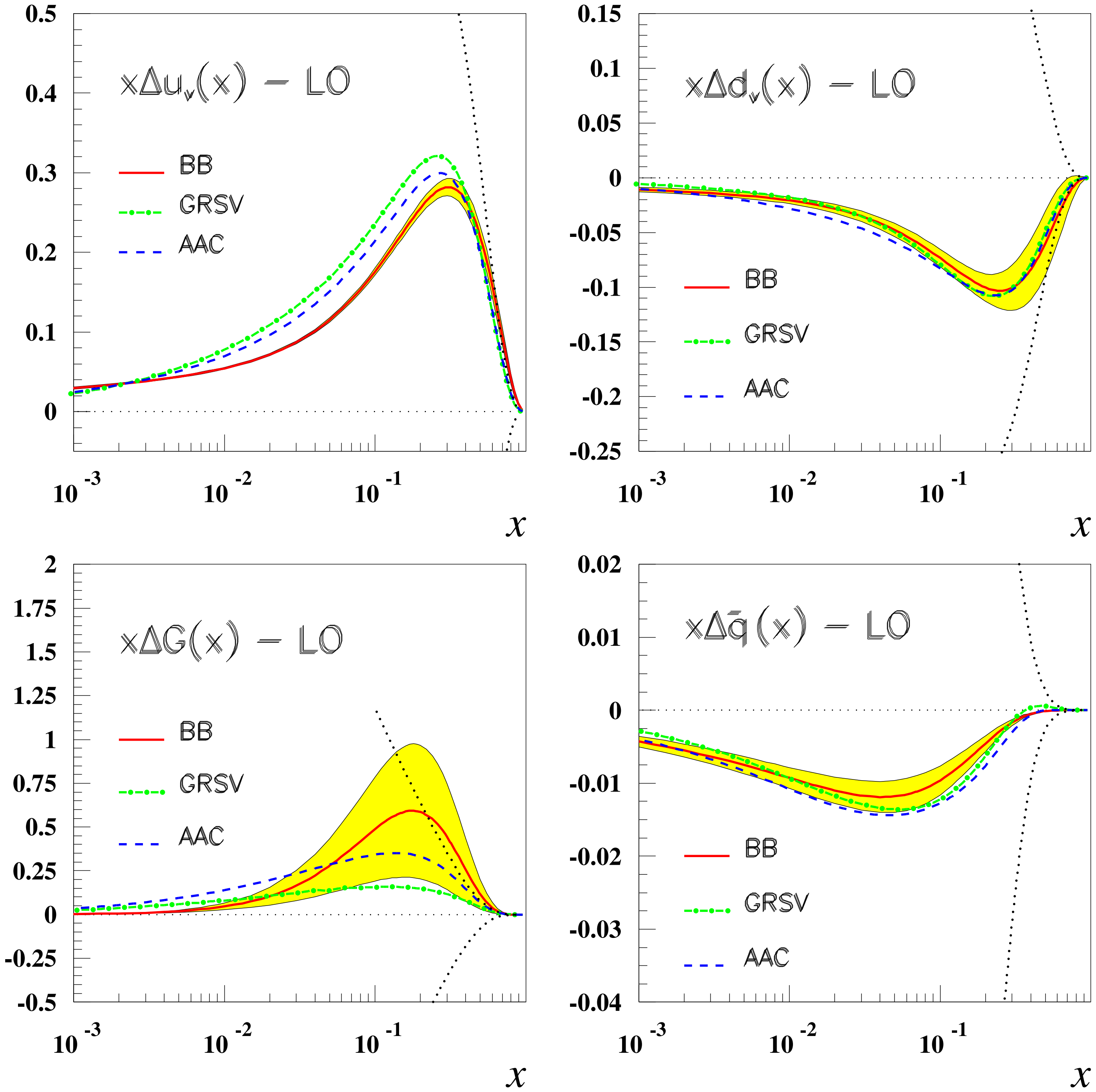
Covariance Matrices at $Q_0^2 = 4.0\ GeV^2$ – $7+1$ Parameter Fit - Scenario 1

LO								
	$\Lambda_{QCD}^{(4)}$	a_{u_v}	b_{u_v}	a_{d_v}	b_{d_v}	η_{sea}	a_{sea}	η_G
$\Lambda_{QCD}^{(4)}$	1.43E-2							
a_{u_v}	-2.05E-5	1.80E-4						
b_{u_v}	-9.07E-5	3.91E-4	1.15E-2					
a_{d_v}	1.10E-4	1.03E-5	-2.40E-3	2.43E-3				
b_{d_v}	-4.65E-5	-7.92E-3	-6.86E-3	5.48E-3	7.82E-01			
η_{sea}	1.02E-4	-4.46E-4	-2.84E-3	9.85E-4	2.82E-2	2.94E-3		
a_{sea}	-4.31E-5	1.58E-4	1.33E-3	-5.96E-4	-9.32E-3	-2.58E-4	2.29E-3	
η_G	-1.03E-3	2.02E-3	1.58E-2	-2.78E-3	-1.61E-1	-1.59E-2	9.56E-3	6.65E-1

NLO								
	$\Lambda_{QCD}^{(4)}$	a_{u_v}	b_{u_v}	a_{d_v}	b_{d_v}	η_{sea}	a_{sea}	η_G
$\Lambda_{QCD}^{(4)}$	2.81E-3							
a_{u_v}	2.71E-5	1.22E-3						
b_{u_v}	-1.30E-4	5.10E-3	4.50E-2					
a_{d_v}	-3.35E-4	-5.17E-4	-3.23E-3	1.23E-2				
b_{d_v}	-6.22E-4	-1.27E-2	4.65E-2	8.29E-2	1.78E-0			
η_{sea}	-5.30E-5	-2.13E-3	-1.12E-2	5.19E-3	4.74E-2	6.77E-3		
a_{sea}	-4.85E-6	9.07E-4	4.49E-3	-3.78E-3	-2.98E-2	-2.39E-3	3.82E-3	
η_G	4.03E-4	1.41E-2	6.71E-2	-3.07E-2	-2.22E-1	-3.78E-2	1.90E-2	3.07E-1

Pol. Parton Densities at $Q_0^2 = 4.0\,GeV^2$

- 7+1 Parameter Fit based on the Asymmetry Data:

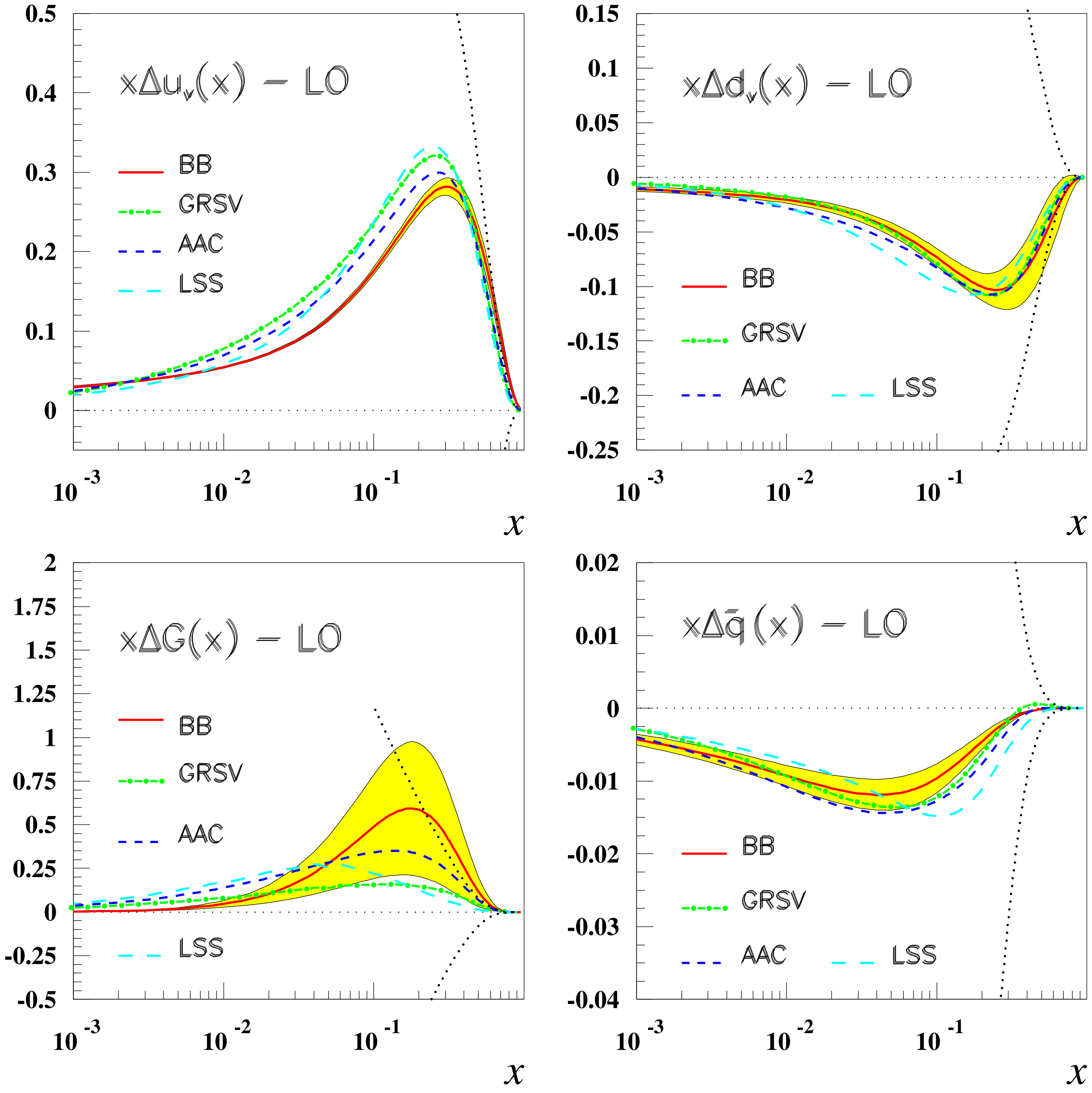


⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation at the input scale Q_0^2.

⇒ Dark dotted line: Unpolarized Parton Distribution ('Positivity Limit') taken from GRV.

Pol. Parton Densities at $Q_0^2 = 4.0\,GeV^2$

- 7+1 Parameter Fit based on the Asymmetry Data:

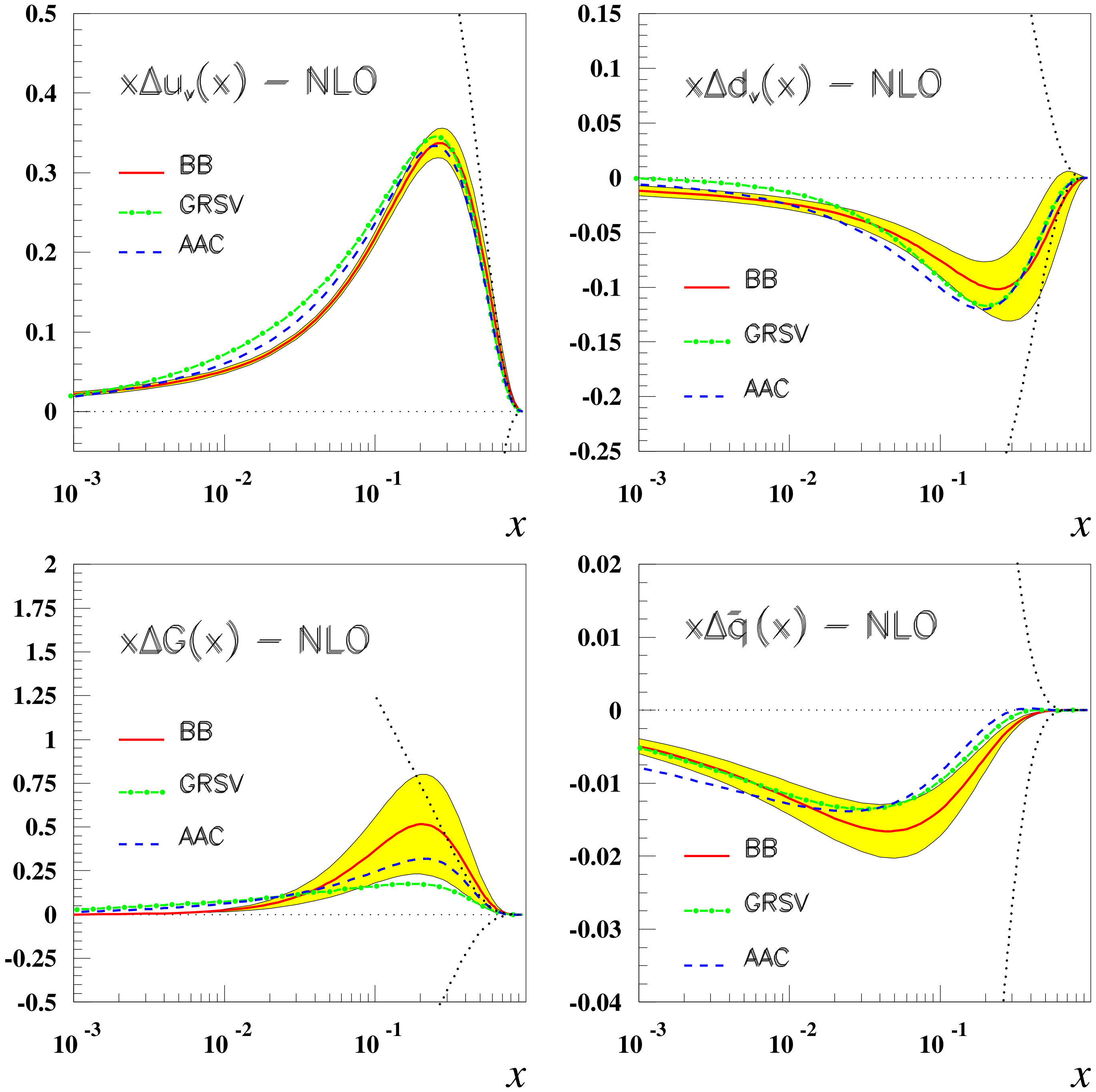


⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation at the input scale Q_0^2.

⇒ Dark dotted line: Unpolarized Parton Distribution ('Positivity Limit') taken from GRV.

Pol. Parton Densities at $Q_0^2 = 4.0\, GeV^2$

- 7+1 Parameter Fit based on the Asymmetry Data:

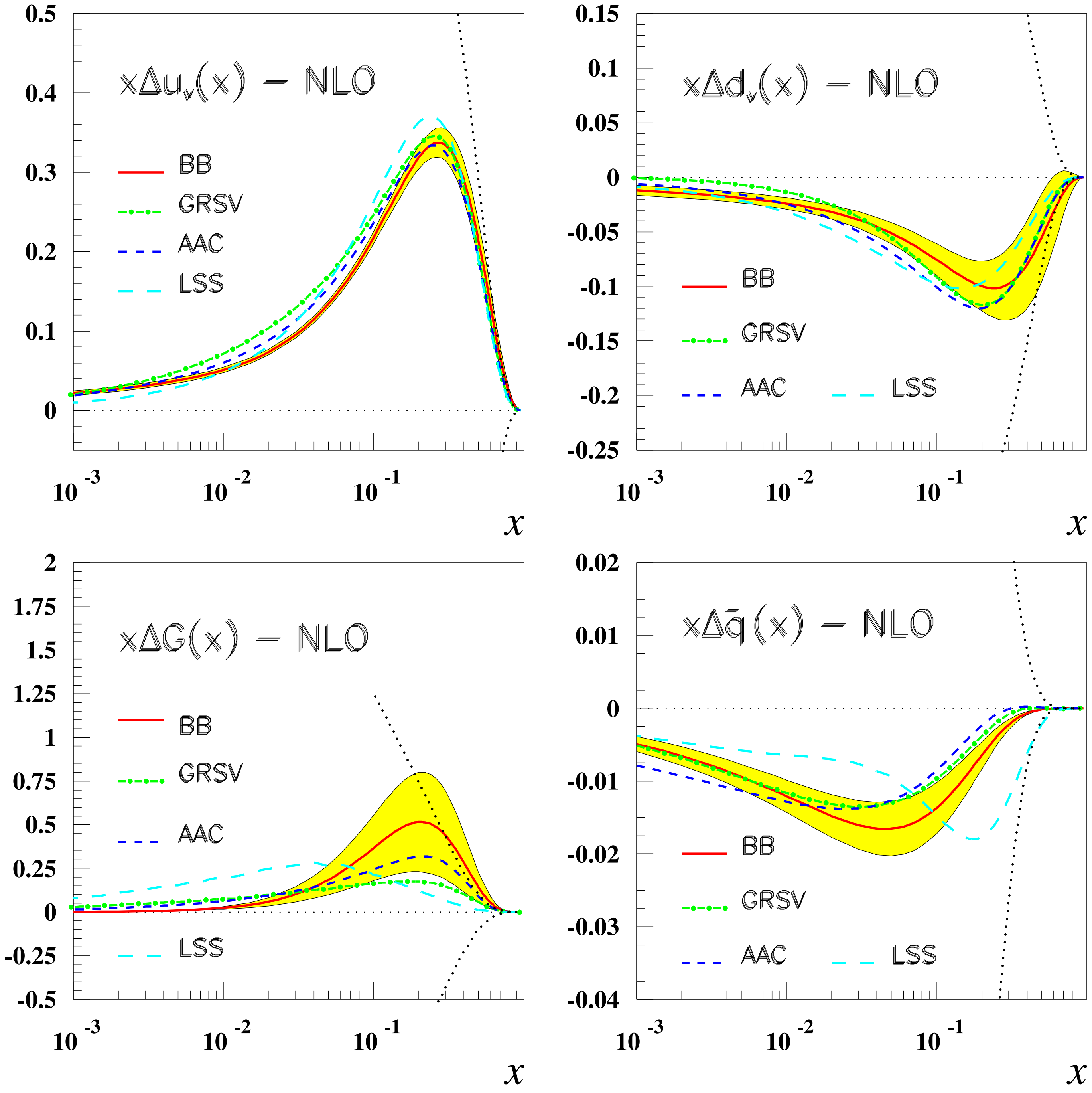


⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation at the input scale Q_0^2.

⇒ Dark dotted line: Unpolarized Parton Distribution ('Positivity Limit') taken from GRV.

Pol. Parton Densities at $Q_0^2 = 4.0\,GeV^2$

- 7+1 Parameter Fit based on the Asymmetry Data:

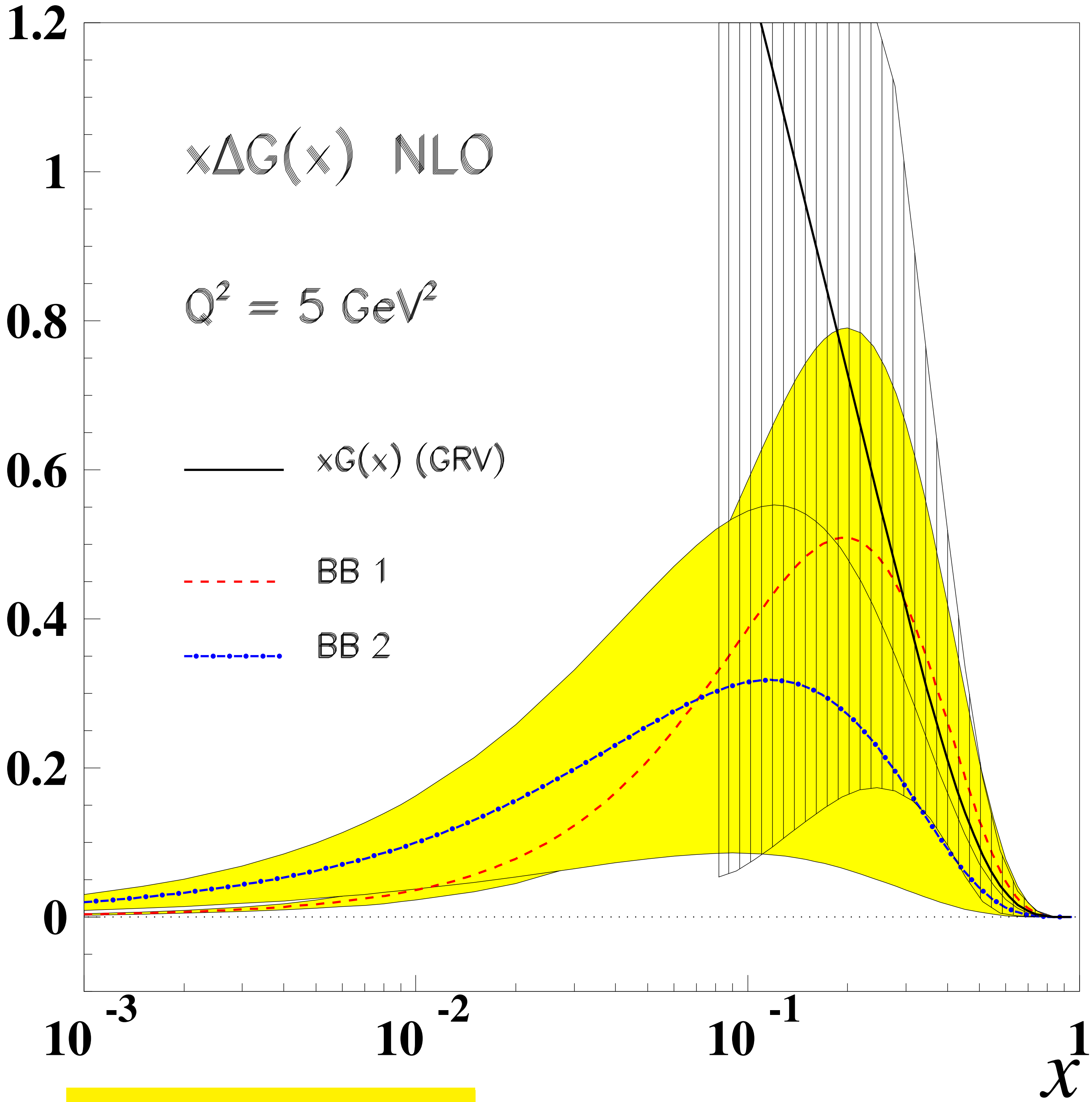


$\Rightarrow$ Yellow error band: Fully correlated 1σ Gaussian error propagation at the input scale Q_0^2.

$\Rightarrow$ Dark dotted line: Unpolarized Parton Distribution ('Positivity Limit') taken from GRV.

The Polarized Gluon at $Q_0^2 = 5.0\,GeV^2$

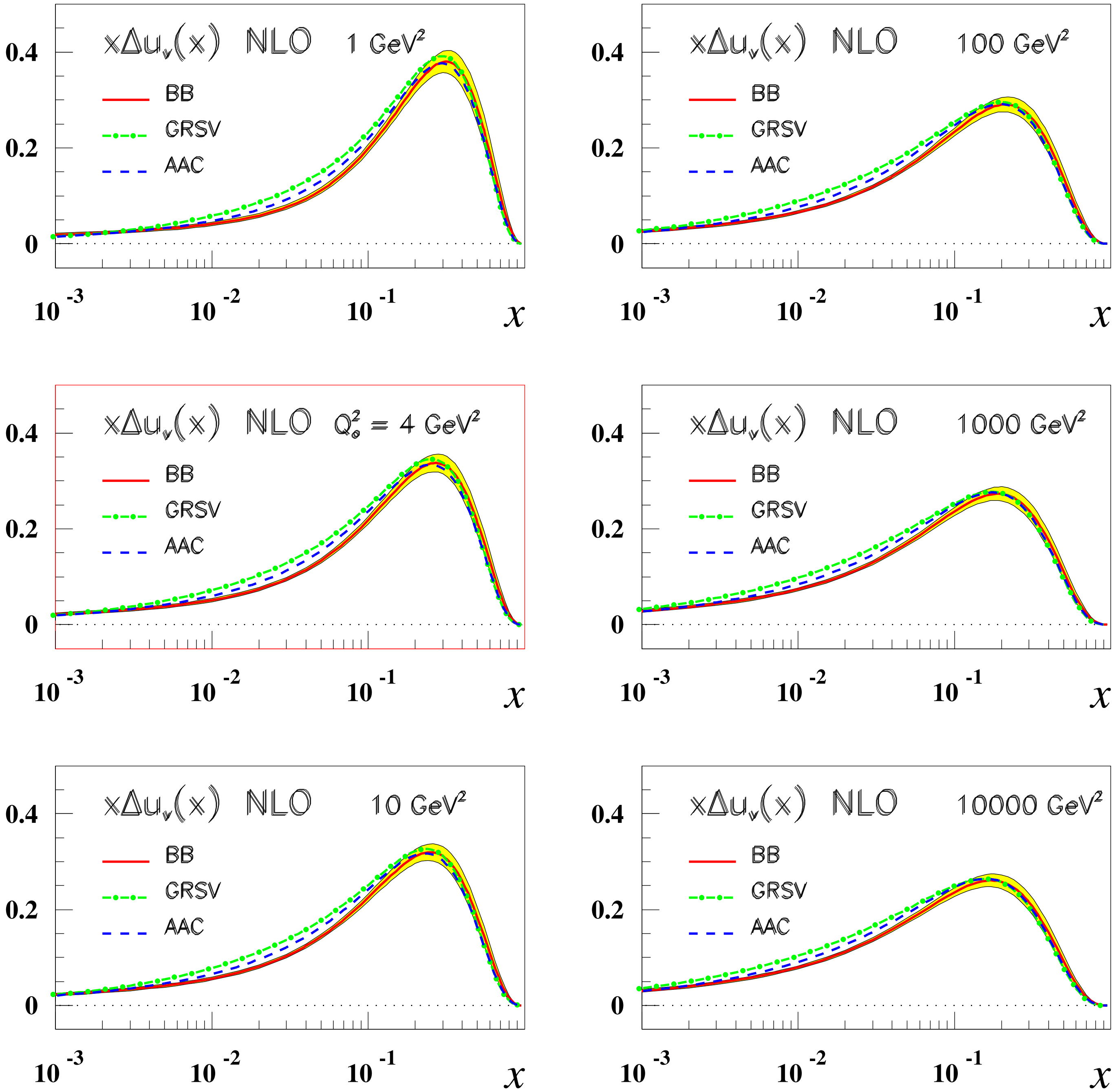
- 7+1 Parameter Fit based on the Asymmetry Data:



⇒ Yellow error bands: Fully correlated 1σ Gaussian error propagation at $Q^2 = 5.0\,GeV^2$.
⇒ Hatched Area: Error Band taken from H1 and laid over the GRV curve.

Evolution of Polarized Parton Densities

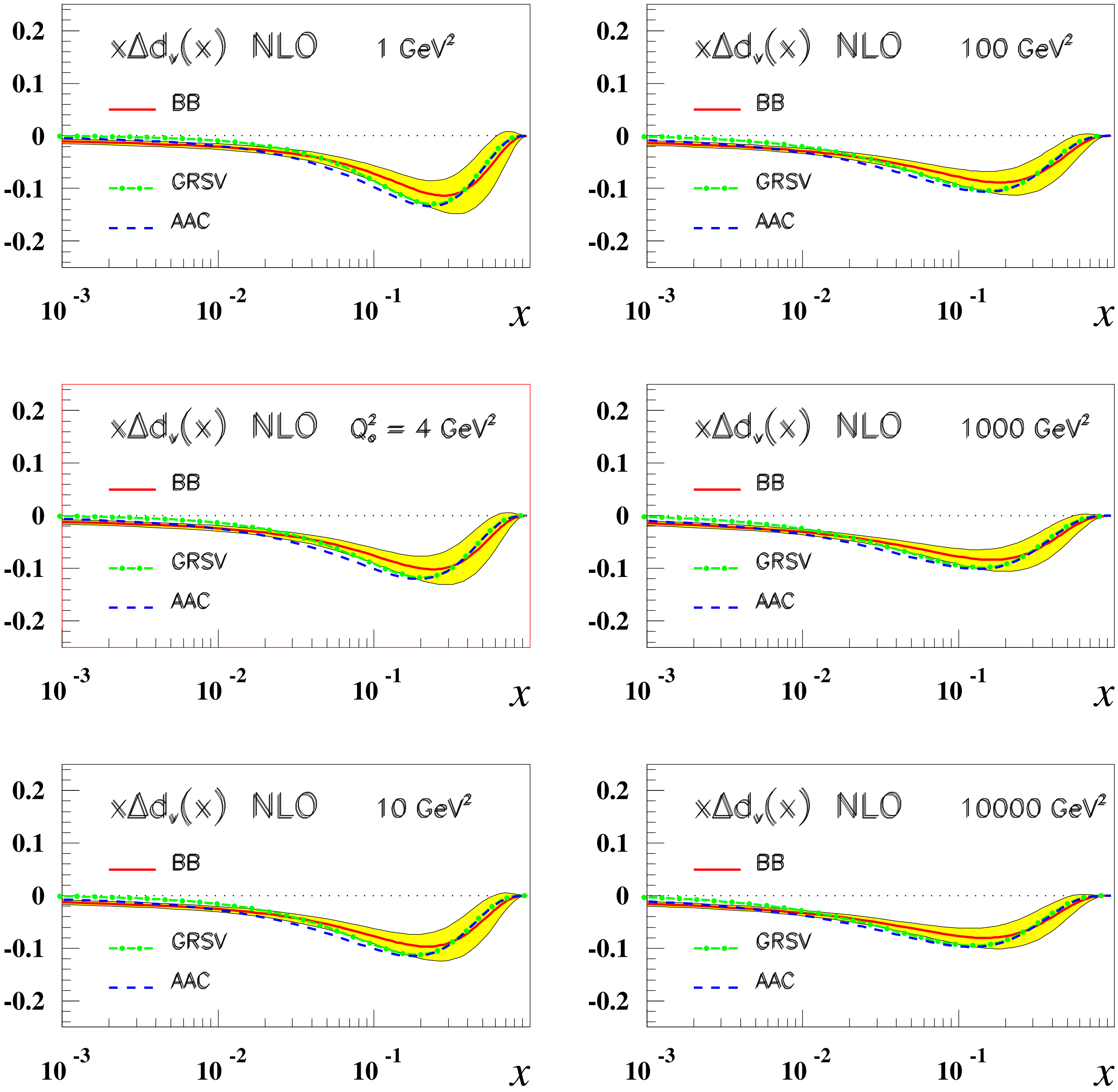
- 7+1 Parameter Fit based on the Asymmetry Data:



⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

Evolution of Polarized Parton Densities

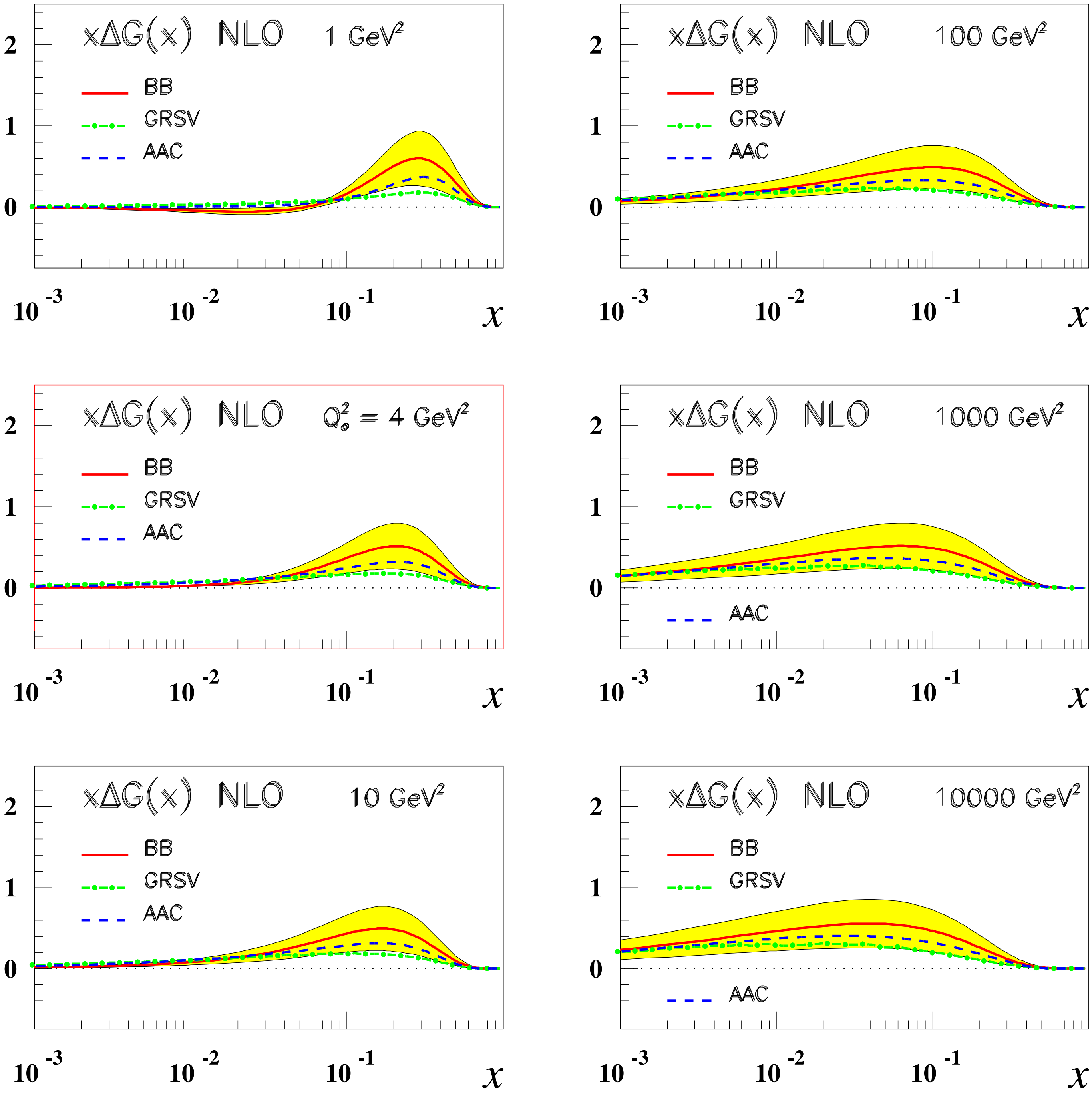
- 7+1 Parameter Fit based on the Asymmetry Data:



⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

Evolution of Polarized Parton Densities

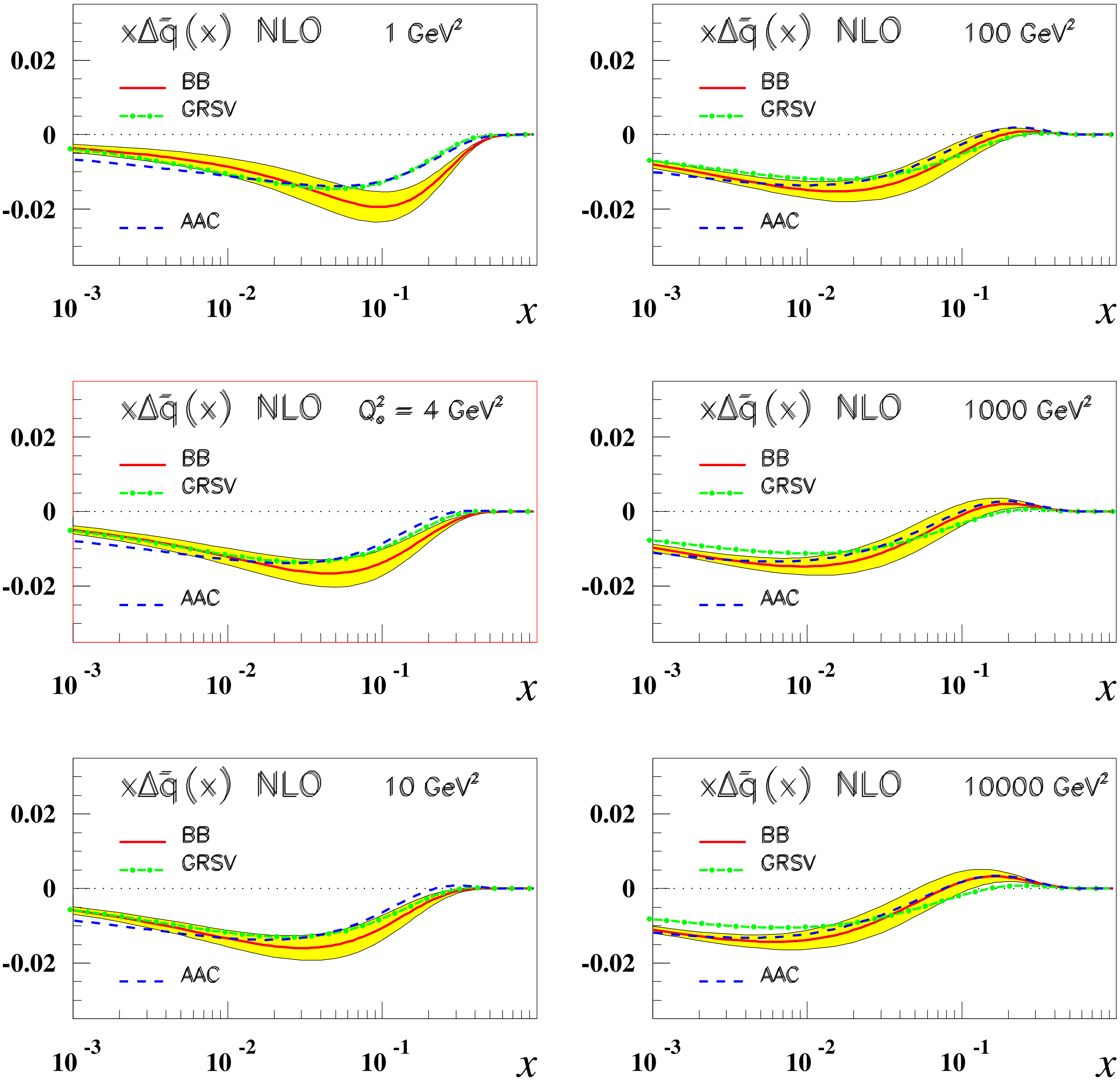
- 7+1 Parameter Fit based on the Asymmetry Data:



$\Rightarrow$ Yellow error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

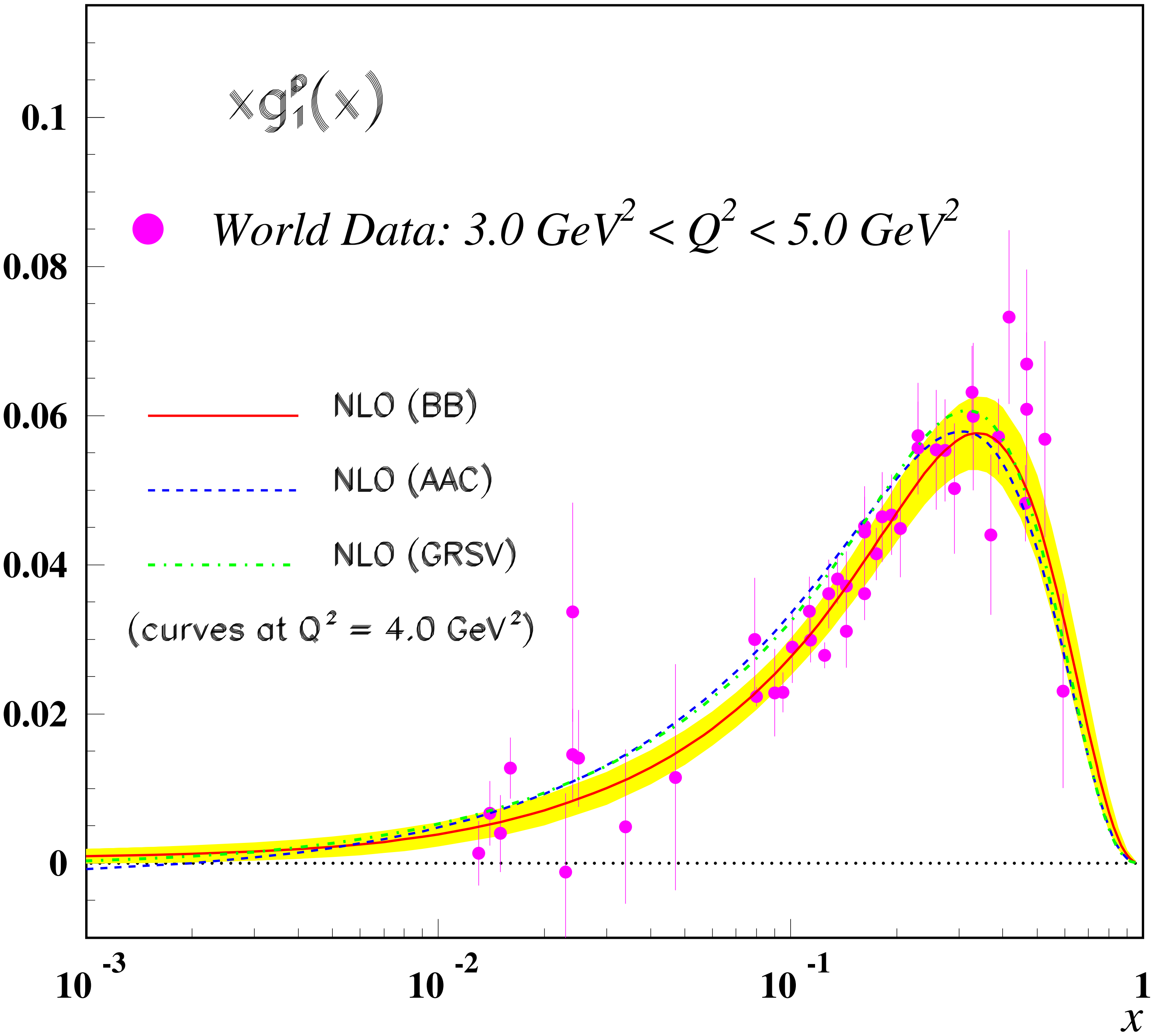
Evolution of Polarized Parton Densities

- 7+1 Parameter Fit based on the Asymmetry Data:



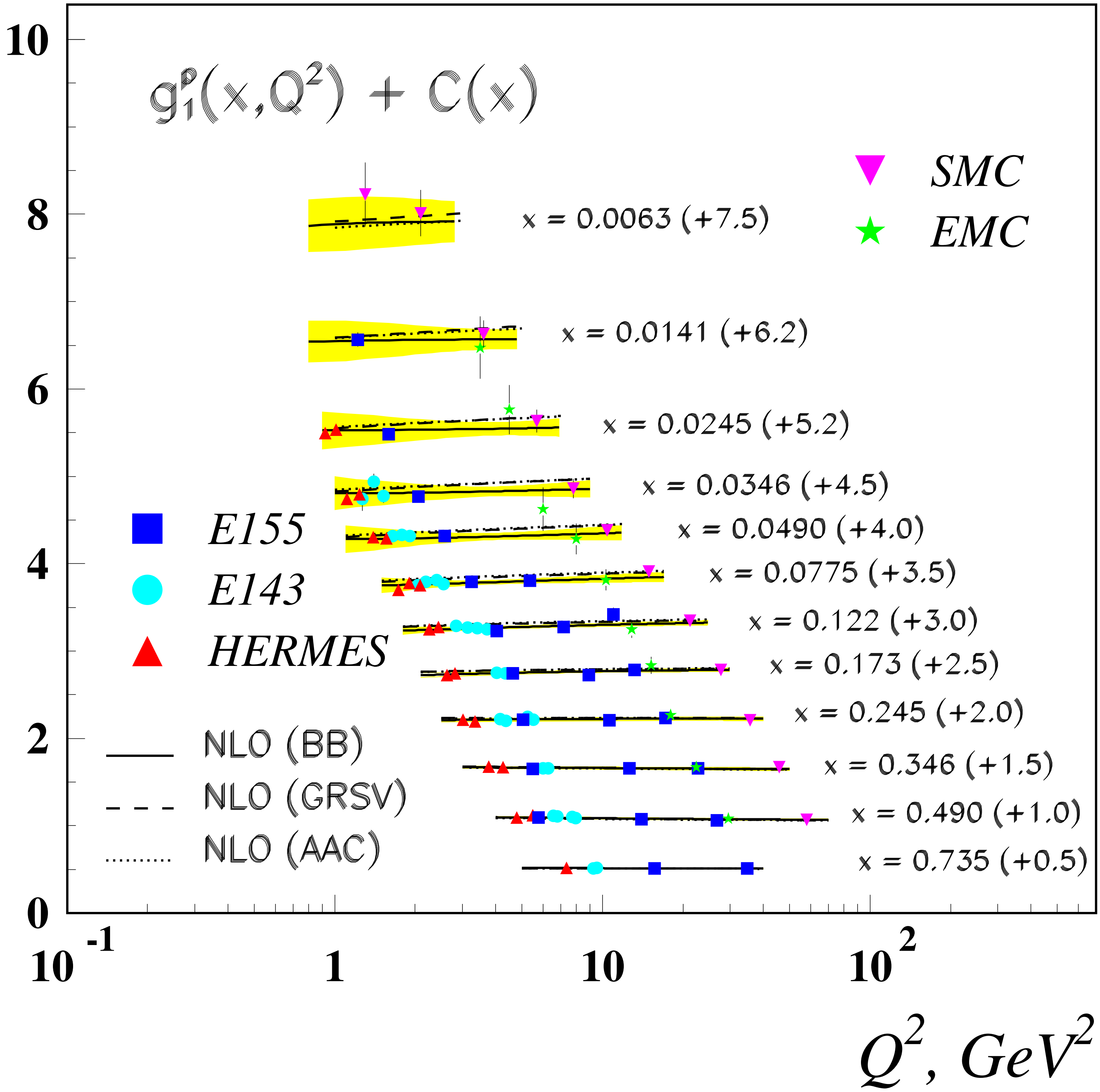
⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

$xg_1^p(x)$ from Measured Asymmetry Data



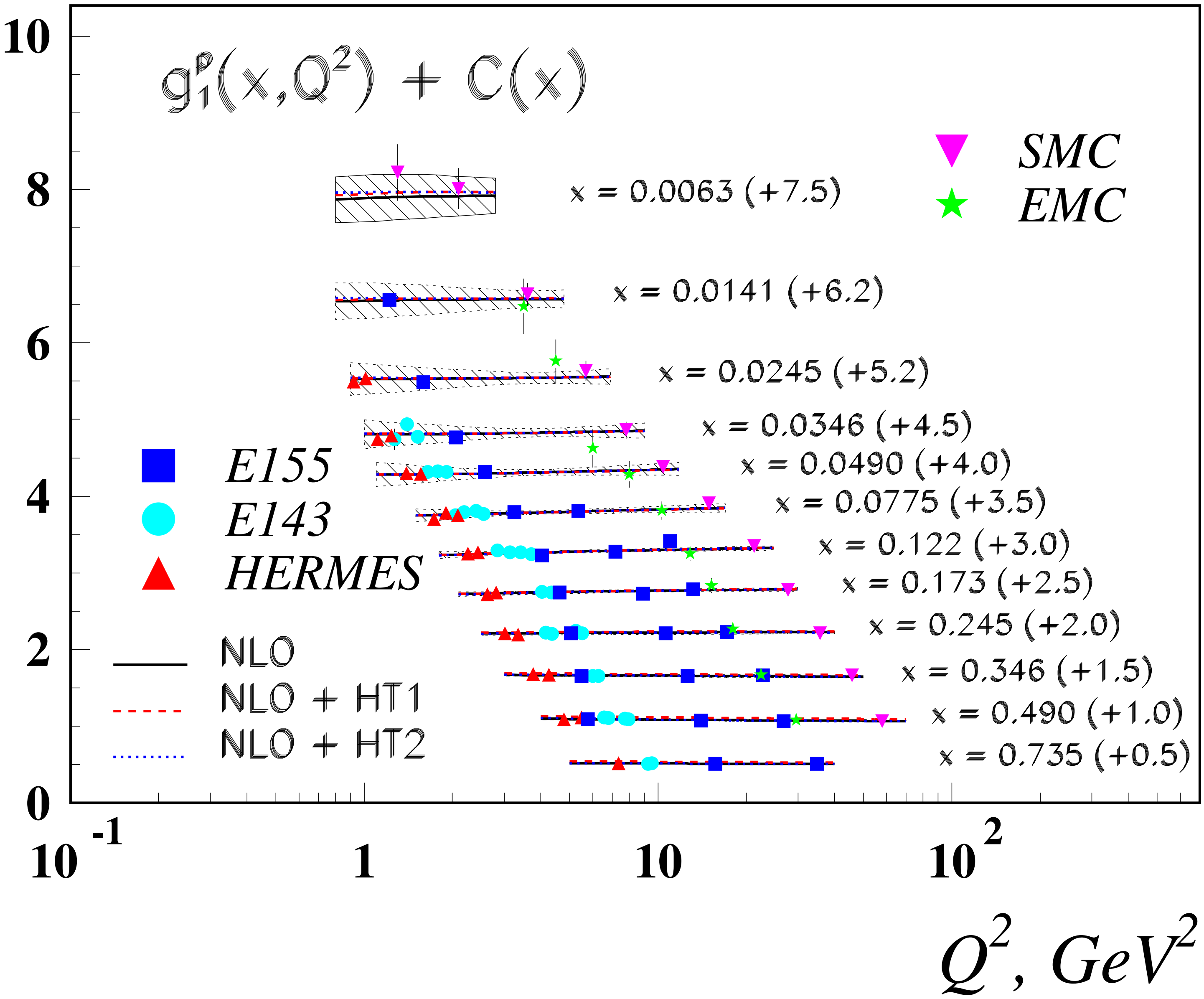
⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

$g_1^p(x)$ **versus** Q^2



$\Rightarrow$ Yellow error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

$g_1^p(x)$ + Higher Twist - Scenario 1



⇒ Hatched error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

- Higher Twist Contribution: $g_1(x,Q^2)[1 + HT(x,Q^2)]$
  - HT1: $1/Q^2(x^a(1-x)^b)$
  - HT2: $1/Q^2(a + bx + cx^2)$

7+1 parameter NLO fit: $\Lambda^{(4)}_{QCD} \Rightarrow \alpha_s(M_Z^2)$

$\Lambda^{(4)}_{QCD}$ [Gev]	Scenario 1		Scenario 2	
	value	error	value	error
FS/RS=1.0/1.0	0.235	± 0.053	0.240	± 0.060
FS/RS=0.5/1.0	0.188	− 0.047	0.195	− 0.045
FS/RS=2.0/1.0	0.296	+ 0.061	0.298	+ 0.058
FS/RS=1.0/0.5	0.349	+ 0.114	0.363	+ 0.123
FS/RS=1.0/2.0	0.174	− 0.061	0.174	− 0.066

- Sc. 1:
  $$\alpha_s(M_Z^2) = 0.113 \begin{matrix} +0.004 \\ -0.004 \end{matrix} \begin{matrix} +0.004 \\ -0.004 \end{matrix} \begin{matrix} +0.008 \\ -0.005 \end{matrix}$$
  (fit) (fac) (ren)
- Sc. 2:
  $$\alpha_s(M_Z^2) = 0.114 \begin{matrix} +0.004 \\ -0.005 \end{matrix} \begin{matrix} +0.004 \\ -0.004 \end{matrix} \begin{matrix} +0.008 \\ -0.006 \end{matrix}$$

- SMC: $0.121 \pm 0.002(stat) \pm 0.006(syst + theor)$

  E154: $0.108 - 0.116 \quad (bad\ for \geq 0.120)$

  ABFR: $0.120 \begin{matrix} +0.004 \\ -0.005 \end{matrix} (exp) \begin{matrix} +0.009 \\ -0.006 \end{matrix} (theor)$

⇒ world average (PDG): 0.118 ± 0.002

⇒ H1 + BCDMS data: [Eur.Phys.J. **C21**(2001)33]

$0.1150 \pm 0.0017(exp) \begin{matrix} +0.0009 \\ -0.0005 \end{matrix} (model) \pm 0.0050(thy)$

Fac. Scheme Invariant Combinations

- Instead of PROCESS-INDEPENDENT SCHEME-DEPENDENT Evolution Equations for PARTONS one may think of PROCESS-DEPENDENT SCHEME-INDEPENDENT Evolution Equations for OBSERVABLES, F_A, F_B.

$\Rightarrow$ The input densities are measured! Control over the input directly.
$\Rightarrow$ No ΔG–Ansatz necessary.
$\Rightarrow$ A one parameter fit only – Λ_{QCD}.

Evolution Equations : [J. Blümlein, V. Ravindran, and W. L. van Neerven, Nucl. Phys **B586** (2000) 349.]

$$\frac{\partial}{\partial t}\left(\begin{array}{c} F_A^N \\ F_B^N \end{array}\right) = -\frac{1}{4}\left(\begin{array}{cc} K_{AA}^N & K_{AB}^N \\ K_{BA}^N & K_{BB}^N \end{array}\right)\left(\begin{array}{c} F_A^N \\ F_B^N \end{array}\right)$$

evolution variable :

$$t = -\frac{2}{\beta_0}\log\left(\frac{a_s(Q^2)}{a_s(Q_0^2)}\right)$$

$\Longrightarrow$ The evolution kernels K_{IJ}^N are also Physical Quantities! The Factorization Scheme Independence holds order by order.

The Renormalization Scale Dependence disappears only with more higher orders.

$\Longrightarrow$ A possible choice: $F_A = g_1$ and $F_B = \partial g_1/\partial t$.

System : $g_1(x,Q^2), \partial g_1/\partial t(x,Q^2)$

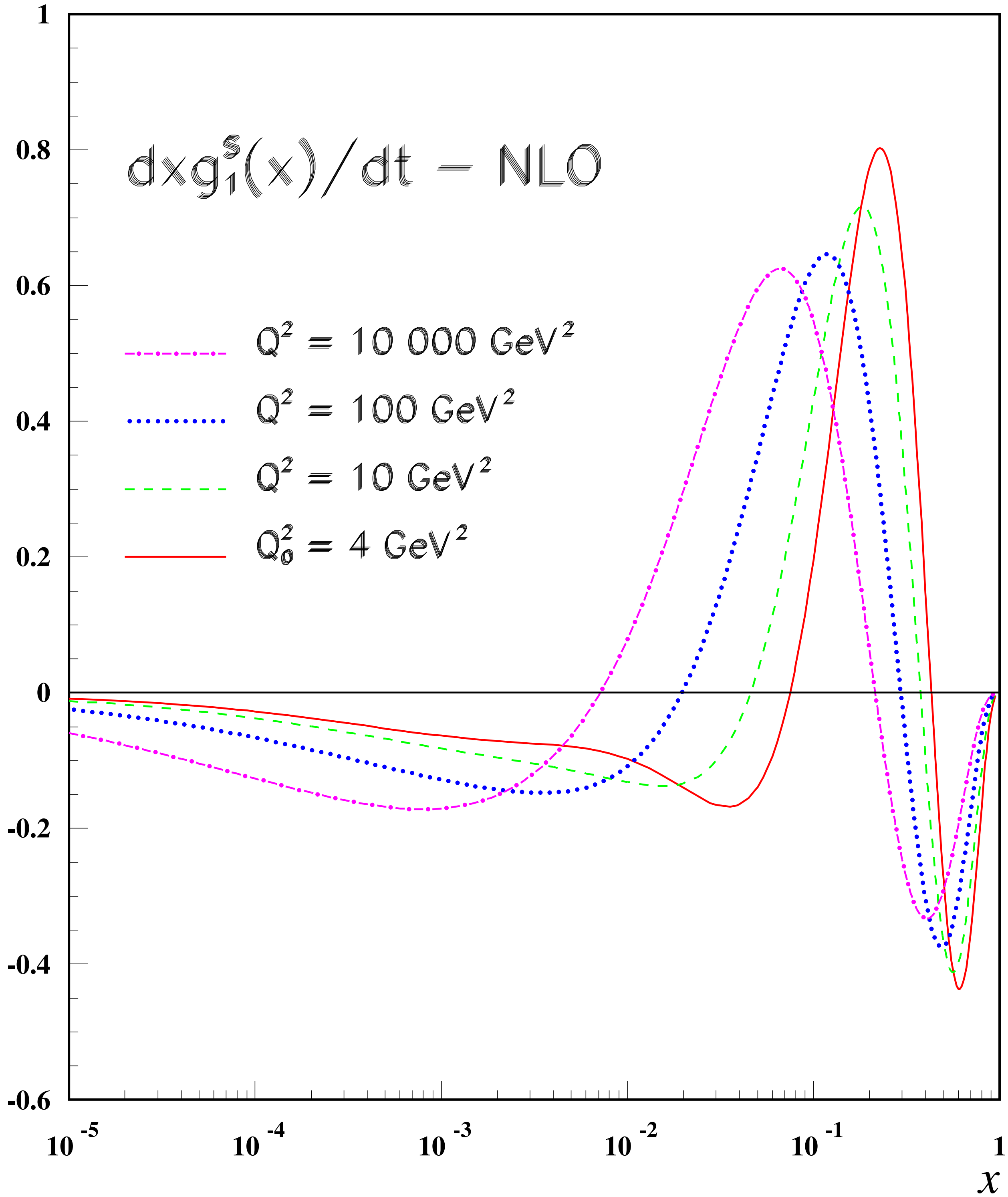
Leading Order :

$$
\begin{aligned}
K_{22}^{N(0)} &= 0 \\
K_{2d}^{N(0)} &= -4 \\
K_{d2}^{N(0)} &= \frac{1}{4}\left(\gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} - \gamma_{qg}^{N(0)}\gamma_{gq}^{N(0)}\right) \\
K_{dd}^{N(0)} &= \gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)}
\end{aligned}
$$

Next-to-Leading Order : [W. Furmanski and R. Petronzio, Z. Phys. **C 11** (1982) 293.]

$$
\begin{aligned}
K_{22}^{N(1)} &= K_{2d}^{N(1)} = 0 \\
K_{d2}^{N(1)} &= \frac{1}{4}\left[\gamma_{gg}^{N(0)}\gamma_{qq}^{N(1)} + \gamma_{gg}^{N(1)}\gamma_{qq}^{N(0)} - \gamma_{qg}^{N(1)}\gamma_{gq}^{N(0)} - \gamma_{qg}^{N(0)}\gamma_{gq}^{N(1)}\right] \\
&\quad -\frac{\beta_1}{2\beta_0}\left(\gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} - \gamma_{gq}^{N(0)}\gamma_{qg}^{N(0)}\right) \\
&\quad +\frac{\beta_0}{2}C_{2,q}^{N(1)}\left(\gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)} - 2\beta_0\right) \\
&\quad -\frac{\beta_0}{2}\frac{C_{2,g}^{N(1)}}{\gamma_{qg}^{N(0)}}\left[\gamma_{qq}^{N(0)2} - \gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} + 2\gamma_{qg}^{N(0)}\gamma_{gq}^{N(0)} - 2\beta_0\gamma_{qq}^{N(0)}\right] \\
&\quad -\frac{\beta_0}{2}\left(\gamma_{qq}^{N(1)} - \frac{\gamma_{qq}^{N(0)}\gamma_{qg}^{N(1)}}{\gamma_{qg}^{N(0)}}\right) \\
K_{dd}^{N(1)} &= \gamma_{qq}^{N(1)} + \gamma_{gg}^{N(1)} - \frac{\beta_1}{\beta_0}\left(\gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)}\right) + 4\beta_0 C_{2,q}^{N(1)} - 2\beta_1 \\
&\quad -\frac{2\beta_0}{\gamma_{qg}^{N(0)}}\left[C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} - 2\beta_0\right) - \gamma_{qg}^{N(1)}\right]
\end{aligned}
$$

$\partial x g_1^S/\partial t(x, Q^2)$ and shift of $\Lambda_{QCD}^{(4)}$



$$Sc1: \ \Lambda_{QCD}^{(4)} : 0.235 \rightarrow 0.223 \ , \ \alpha_s(M_Z^2) : 0.113 \rightarrow 0.112$$

$$Sc2: \ \Lambda_{QCD}^{(4)} : 0.240 \rightarrow 0.228 \ , \ \alpha_s(M_Z^2) : 0.114 \rightarrow 0.113$$

Comparison of Moments

Δf	n	QCD Scenario 1	lattice results	
		moment at $Q^2 = 4\,GeV^2$	QCDSF	LHPC/ SESAM
Δu_v	–1	0.926 ± 0.071	0.889(29)	0.860(69)
	0	0.163 ± 0.014	0.198(8)	0.242(22)
	1	0.055 ± 0.006	0.041(9)	0.116(42)
Δd_v	–1	-0.341 ± 0.123	-0.236(27)	-0.171(43)
	0	-0.047 ± 0.021	-0.048(3)	-0.029(13)
	1	-0.015 ± 0.009	-0.028(2)	0.001(25)
Δu–Δd	–1	1.267 ± 0.142	1.14(3)	1.031(81)
	0	0.210 ± 0.025	0.246(9)	0.271(25)
	1	0.070 ± 0.011	0.069(9)	0.115(49)

$\Longrightarrow \Gamma_{\Delta f}(Q^2) = \int_0^1 x^{n+1} \Delta f(x, Q^2) dx$

Lattice simulation: Scale $\mu^2 = 1/a^2 \sim 4\,GeV^2$. For the $n = 0, 1$ values of the QCDSF Coll. no continuum extrapolation was performed.

[**Refs:** M.Göckeler et al., QCDSF Coll., Phys.Rev. **D53** (1996) 2317; Phys.Lett. **B414** (1997) 340; hep-ph/9711245; Phys.Rev. **D63** (2001) 074506; S.Capitani et al., Nucl.Phys.(Proc. Suppl.) **B79** (1999) 548; S.Güsken et al., SESAM Coll., hep-lat/9901009; D.Dolgov et al., LHPC and SESAM Coll., hep-lat/0201021.]

Comparison of Moments (2)

	BB Scenario 1	AAC	GRSV	ABFR
Δu_v	0.926 ± 0.071	0.921	0.928	$\eta_u =$
Δd_v	-0.341 ± 0.123	–0.341	–0.342	0.692
Δu	0.851 ± 0.075	0.859	0.840	$\eta_d =$
Δd	-0.415 ± 0.124	–0.404	–0.430	-0.418
$\Delta \overline{q}$	-0.074 ± 0.017	–0.063	–0.088	
ΔG	1.026 ± 0.549	0.683	0.808	1.262

Comparison of the first moments of the polarized parton densities in NLO in the $\overline{\mathrm{MS}}$ scheme at $Q^2 = 4$ GeV^2 for different sets of recent parton parameterizations. For the ABFR-analysis the values $\eta_{u,d}$ are the first moments of $\Delta u + \Delta\overline{u}$ and $\Delta d + \Delta\overline{d}$, respectively, and $\Delta s + \Delta\overline{s} = -0.081$.

'Prediction' of Moments

	n	QCD Scenario 1	
		value at $Q^2 = 4\,GeV^2$	value out of measured range
Δu	–1	0.851 ± 0.075	0.152|4E$-$4
	0	0.160 ± 0.014	8E$-$4\|3E$-$4
	1	0.055 ± 0.006	1E$-$5\|3E$-$4
	2	0.024 ± 0.003	0|3E$-$4
Δd	–1	-0.415 ± 0.124	-0.144|-7E$-$5
	0	-0.050 ± 0.022	-7E$-$4\|-6E$-$5
	1	-0.015 ± 0.009	-1E$-$5\|-5E$-$5
	2	-0.006 ± 0.005	0|-5E$-$5
$\Delta \overline{q}$	–1	-0.074 ± 0.017	-0.04|0
	0	-0.003 ± 0.001	-2E$-$4|0
	1	-4E-4 ± 1E-4	0|0
	2	-8E-5 ± 2E-5	0|0
ΔG	–1	1.026 ± 0.549	0.04|1E$-$5
	0	0.184 ± 0.103	5E$-$4\|1E$-$5
	1	0.050 ± 0.028	1E$-$5\|1E$-$5
	2	0.017 ± 0.010	0|1E$-$5

Conclusions

- A LO and NLO QCD Analysis of the current World-Data of Polarized Structure Functions was performed.

- New parametrizations of the parton densities including their fully correlated 1σ Error Bands were derived. They are available via a fast `FORTRAN` program for the range:

$$1\ GeV^2 < Q^2 < 10^6\ GeV^2 \text{ and } 10^{-9} < x < 1.$$

- The following values for $\alpha_s(M_Z^2)$ were obtained:

  – Scenario 1:

  $$\alpha_s(M_Z^2) = 0.113 \begin{array}{l} +0.004 \\ -0.004 \end{array} (\text{fit}) \begin{array}{l} +0.004 \\ -0.004 \end{array} (\text{fac}) \begin{array}{l} +0.008 \\ -0.005 \end{array} (\text{ren})\ ,$$

  – Scenario 2:

  $$\alpha_s(M_Z^2) = 0.114 \begin{array}{l} +0.004 \\ -0.005 \end{array} (\text{fit}) \begin{array}{l} +0.004 \\ -0.004 \end{array} (\text{fac}) \begin{array}{l} +0.008 \\ -0.006 \end{array} (\text{ren})\ ,$$

  compatible with results from other QCD analyses and with the world average.

Conclusions (cont'd)

- First steps in a factor. scheme invariant QCD evolution based on the structure function $g_1(x, Q^2)$ and $\partial g_1(x, Q^2)/\partial \log Q^2$ were performed yielding similar results for $\alpha_s(M_Z^2)$.

  Such an analysis is a very promising way to proceed in the future, since it allows to extract Λ_{QCD} fixing all the input distributions by direct measurement.

- Comparing the lowest moments with values from Lattice simulations the errors improved during recent years and the values became closer. The chiral extrapolation $m_\pi^2 \rightarrow 0$ seems to be flat. However, more work has yet to be done in the future on systematic effects and even more precise experimental data are welcome to improve precision.

- The evanescent spin puzzle lead to both a much deeper experimental and theoretical understanding of the nucleon at short distances, and, hopefully will in the future.