From $H = c\sqrt{(\vec{p} - e\vec{A})^2 + m^2c^2} + e\phi$

Eq. of Motion

$$\frac{d\left[\gamma m c^2 \vec{\beta} - e\vec{A}\right]}{c\,dt} = -e\nabla(\phi - \vec{\beta}\cdot\vec{A})$$

momentum
due to current

On Circular Orbit

$$\frac{d(\gamma m v_r)}{dt} = \underset{\uparrow \text{ centrifugal force}}{\frac{\gamma m v_\theta^2}{r}} + F_r$$

~~[illegible]~~

Bunched Beam

$$\frac{d(\gamma m v_r)}{dt} = \left(\frac{\beta c P_\theta}{r} - \underset{\uparrow \text{ centrifugal space charge force}}{\frac{e \beta c A_\theta}{r}} \right) \cancel{+} - e \frac{\partial(\phi - \vec{\beta}\cdot\vec{A})}{\partial r}$$

If treating $\boxed{\gamma mc^2 + e\phi = \gamma_{eff}\, mc^2}$

$$\boxed{\gamma_{eff} = \gamma + \frac{e\phi}{mc^2} \\ \Downarrow \\ \tilde{\phi}}$$

In circular orbit,

$$\frac{d(\gamma+\phi)\beta_r}{c\,dt} - \beta_\theta\left[\boxed{\frac{(\gamma+\tilde{\phi})\beta_\theta}{r}} - \frac{r_0\beta_\theta}{R}\right]$$

$$= \frac{e}{mc^2}\left[-\frac{\partial(\phi-\vec{\beta}\cdot\vec{A})}{\partial r} + \beta_\theta \boxed{\frac{A_\theta - \beta_\theta\phi}{r}} - \frac{d(A_r - \beta_r\phi)}{c\,dt}\right]$$

$$\frac{d(\gamma+\phi)\beta_\theta}{c\,dt} + \beta_r\left[\frac{(\gamma+\tilde{\phi})\beta_\theta}{r} - \frac{r_0\beta_\theta}{R}\right]$$

$$= \frac{e}{mc^2}\left[\boxed{-\frac{\partial(\phi-\vec{\beta}\cdot\vec{A})}{r\,\partial\theta}} - \beta_\theta\frac{A_r - \beta_r\phi}{r} - \frac{d(A_\theta - \beta_\theta\phi)}{c\,dt}\right]$$

$$\boxed{\frac{d(\gamma+\tilde{\phi})}{c\,dt} = \frac{e}{mc^2}\frac{\partial(\phi-\vec{\beta}\cdot\vec{A})}{c\,\partial t}.}$$