Field Calculations for Bunch Compressors (non-1D and 1D)

1 Calculation Methods

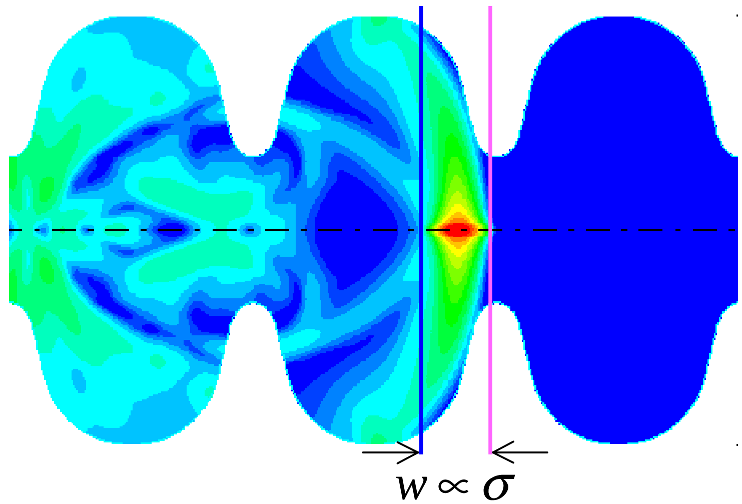
1.1 Direct Time Domain Calculation

e.g. FDTD - method with particle tracking
PIC codes (Particle In Cell)

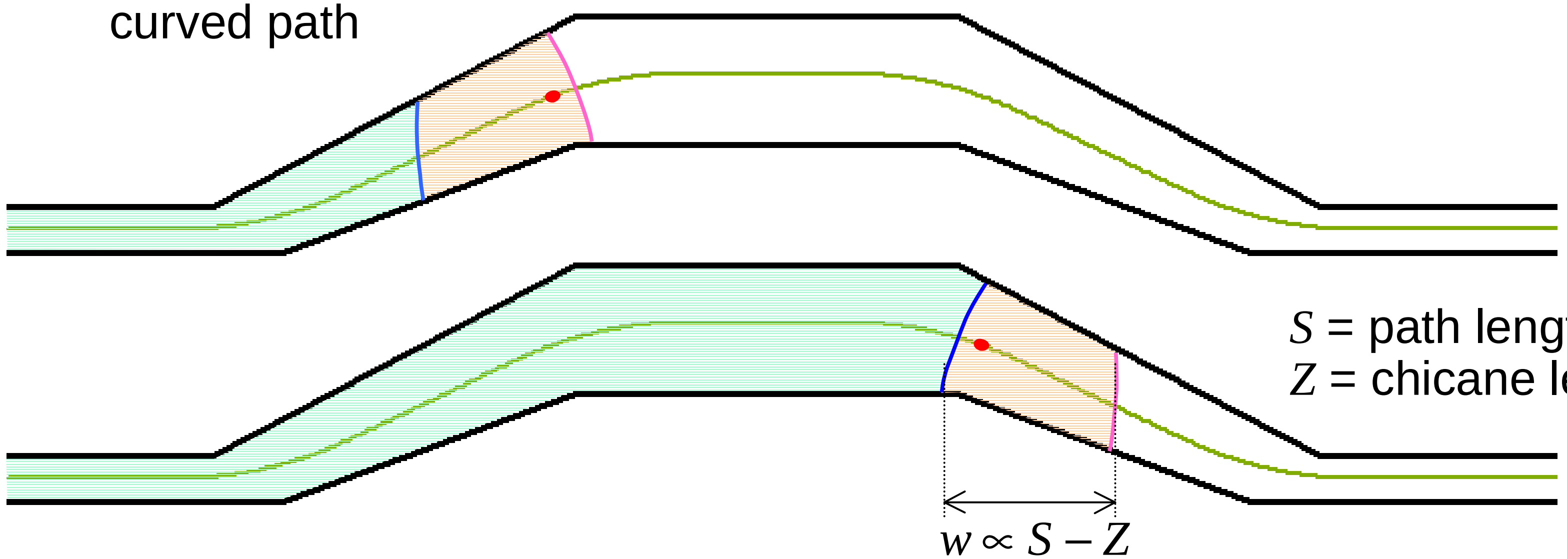
a) calculation window
b) dispersion

a) calculation window

linear motion
($v = c_0$)

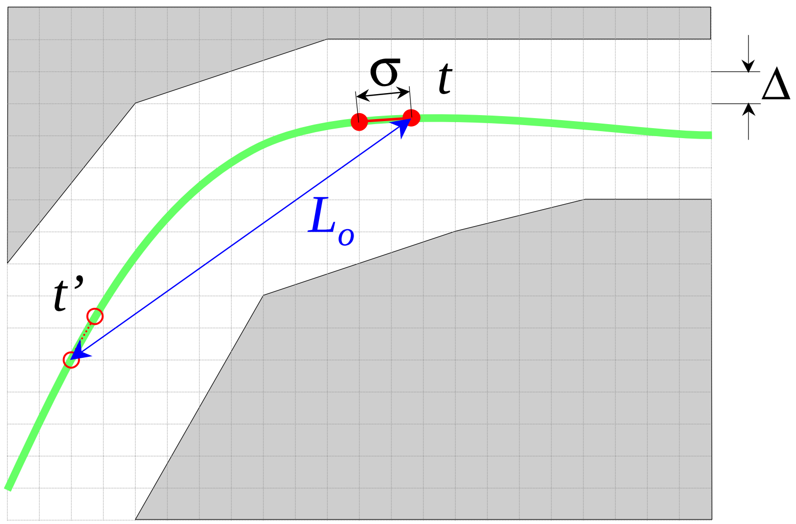


curved path



S = path length
Z = chicane length

b) dispersion



numerical integration:

$$\left\|\vec{k}\right\| = \frac{\omega}{c_0} + O\left(\Delta^N\right)$$

Δ = step width

N = order of integration (e.g. FDTD N=2)

error $\propto \frac{L_o \Delta^N}{\sigma^{N+1}}$ e.g. FDTD: $\frac{L_o \Delta^2}{\sigma^3} \lesssim 1$

typical interaction length: $L_o = \sqrt[3]{24 R_0^2 \sigma}$

e.g. $\sigma = 100\,\mu\text{m},\ R_0 = 1\,\text{m},\ \Rightarrow \Delta \propto 3\,\mu\text{m}$

algorithms with low numerical dispersion needed:

α) compensation for one direction of propagation

β) higher order algorithms error $\propto \frac{L_o \Delta^N}{\sigma^{N+1}}$

1.2 Retarded Sources

a) point charge (Liénerd-Wiechert fields)

exact solution known;

problems: very fast time structure of 1-particle field $\tau \propto R_0 / \gamma^3 c_0$
singularity at source point

} very many particles needed;

b) distributed sources

eg. $\vec{J}(\vec{r},t)$ and $\rho(\vec{r},t)$ known

$$\Phi(\vec{r},t) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\vec{r}\,',t')}{\left\|\vec{r}-\vec{r}\,'\right\|} dV'$$

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}\,',t')}{\left\|\vec{r}-\vec{r}\,'\right\|} dV'$$

$$\vec{E}(\vec{r},t) = -\nabla\Phi - \partial_t \vec{A}$$

$$\vec{B}(\vec{r},t) = \nabla \times \vec{A}$$

problems: $\vec{J}$ and ρ have to be known for all points (x,y,z,t);
3d integration for every observation point

c) simplified distributed sources

source distribution → ensemble of simplified source distributions "sub-bunches"

source distribution → tracking of test particles initial conditions

ensemble of simplified source distributions "sub-bunches" ↓ **fields** ↓ tracking of test particles

paths of sub-bunches ↑ (self consistent simulation)

computer codes: N.N. (R.Li, Jefferson Lab)
TraFiC4 (M.Dohlus, T.Limberg DESY, A.Kabel SLAC)

1.3 Tracking

a) partial cancellation of transverse forces

Transverse Effects of Microbunch Radiative Interaction
Y.S.Derbenev, V.D.Shieltsev

$$x'' + (K^2 - n)x = \frac{K\Delta\mathcal{E} + F_x}{\mathcal{E}}$$

$$K\Delta\mathcal{E} = K\Delta\mathcal{E}_0 + Kq_0 \int \mathrm{CSR}\, dt + Kq_0(\Phi_0 - \Phi)$$

$$F_x = \ldots + q_0 K\, \vec{v} \cdot \vec{A}$$

singular for 1D beams (~ln(r))
singularities cancel !

$K(s) = 1/R_0(s)$ = curvature

$\Delta\mathcal{E}$ = energy offset

$F_x(s,x,t)$ = transverse force

Φ = scalar potential

$\vec{A}$ = vector potential

approach 1 (TRAFIC4): calculate forces, solve EOM

$$x'' + (K^2 - n)x = \frac{K\Delta\mathcal{E} + F_x}{\mathcal{E}}$$

avoid singularities:
real transverse beam dimensions have to be taken into account

"cancellation":
F_x calculated by field solver
ΔE calculated by EOM solver

approach 2 (R.Li): modified LHS, EOM

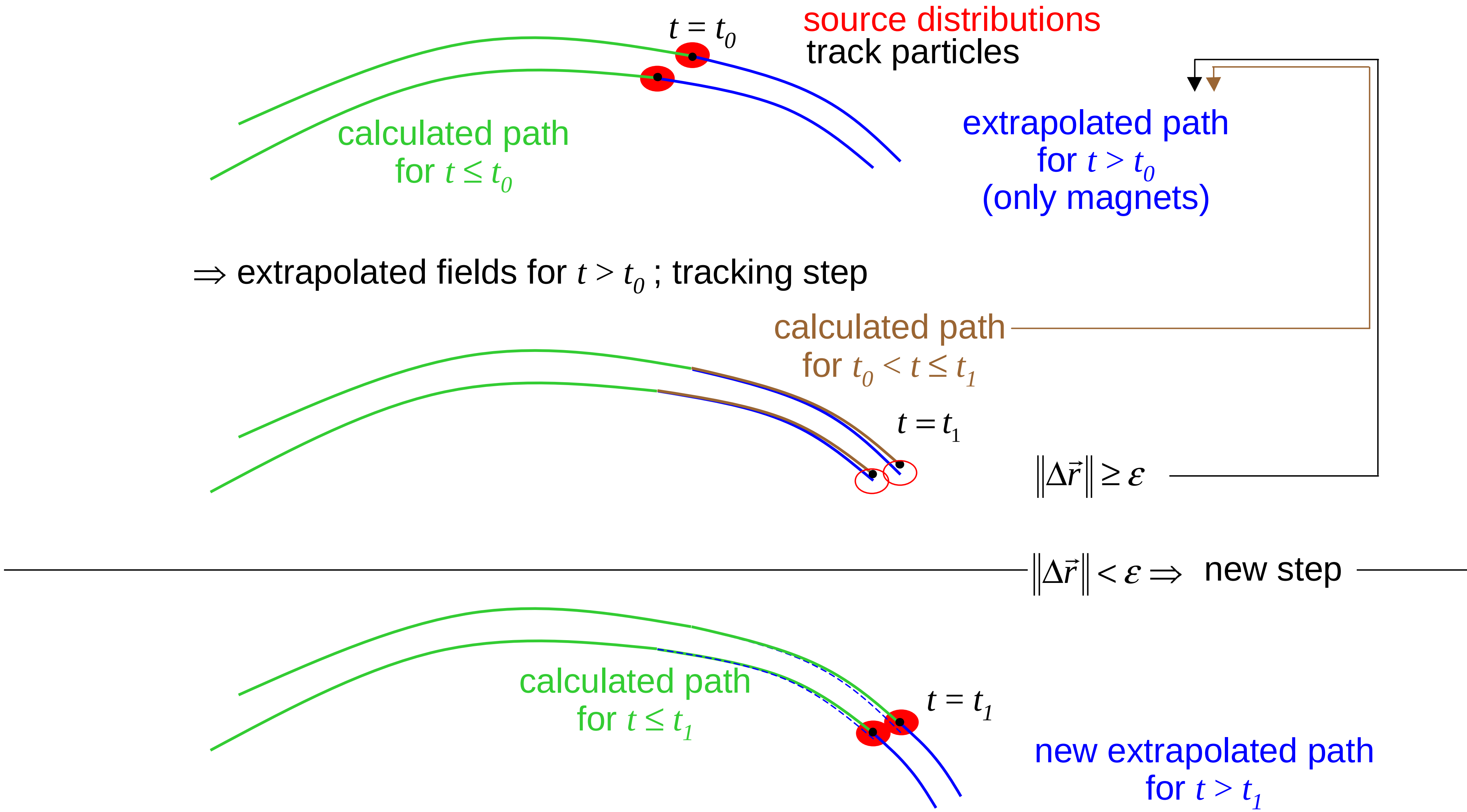
$$x'' + (K^2 - n)x = \frac{K\Delta\hat{\mathcal{E}} + \hat{F}_x}{\mathcal{E}}$$

$$K\Delta\hat{\mathcal{E}} = K\Delta\mathcal{E}_0 + Kq_0\Phi_0 + Kq_0\int \mathrm{CSR}\,dt$$

$$\hat{F}_x = \ldots + q_0 K\,\vec{v}\cdot\vec{A} - Kq_0\Phi$$

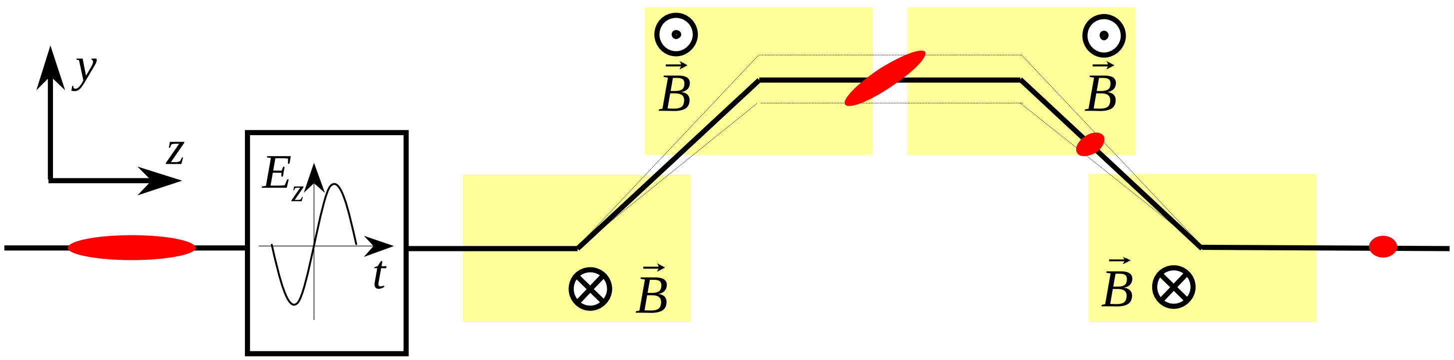
"cancellation" calculated by field solver

b) self consistent tracking



example: bunch compressor

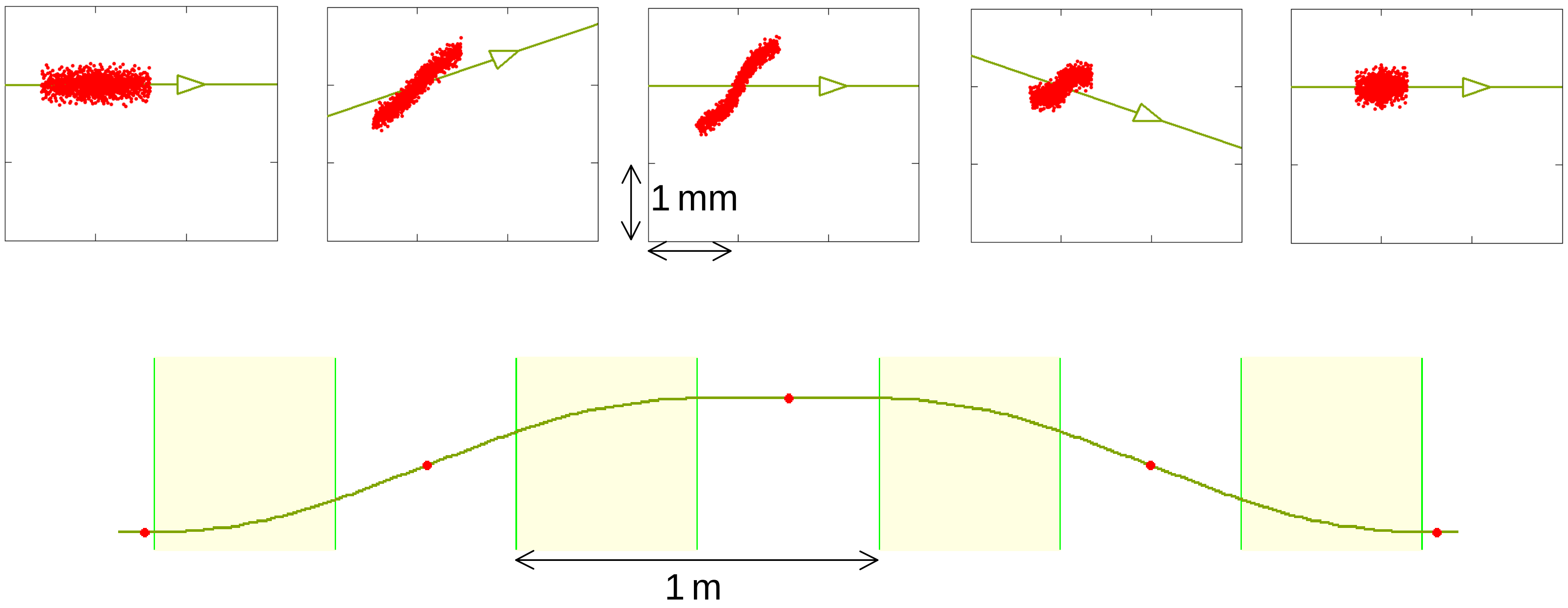
principle:



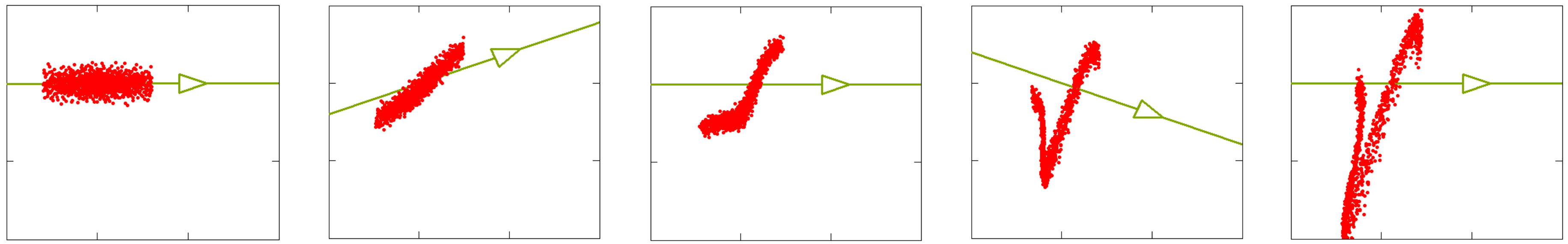
problem: EM fields generated by the bunch

e.g. energy loss of the bunch due to CSR

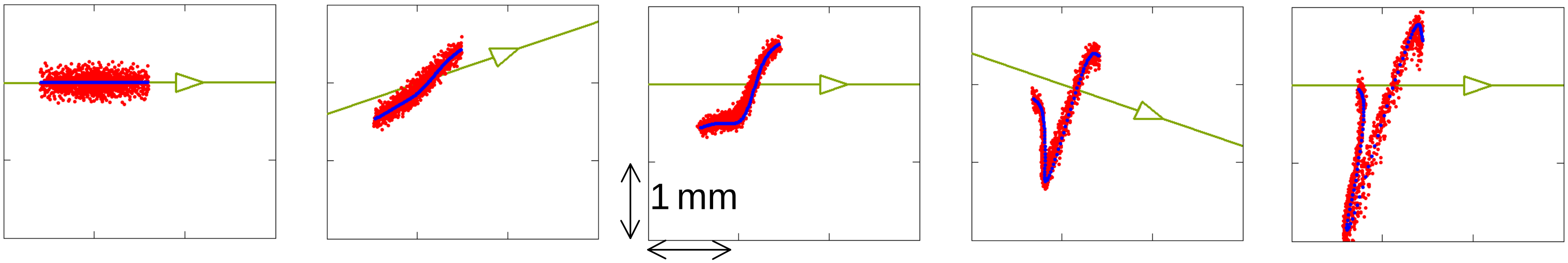
ΔE=f(particle position), bump is not closed → emittance growth without self-interaction

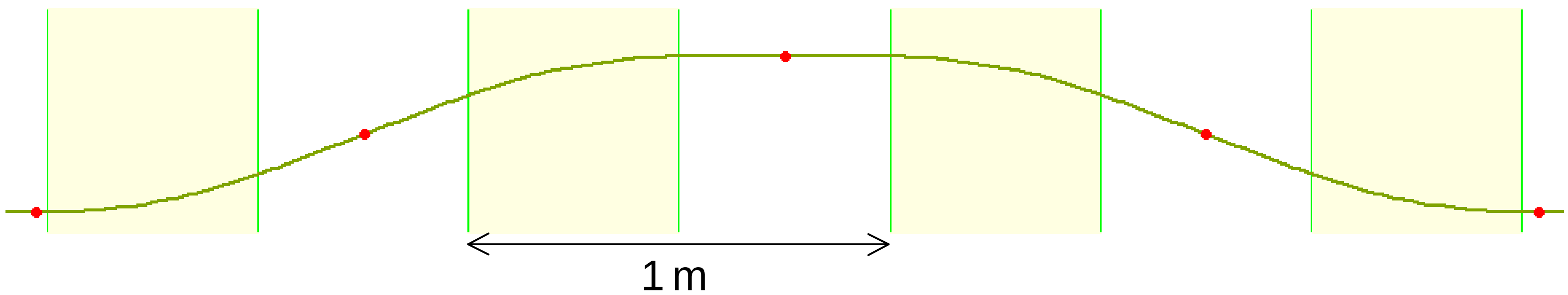


with self-interaction

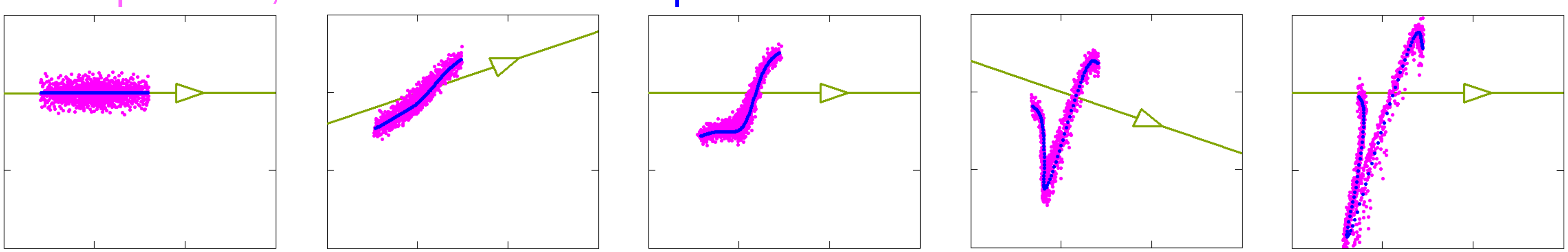


80 particles, self consistent
1000 particles, self consistent



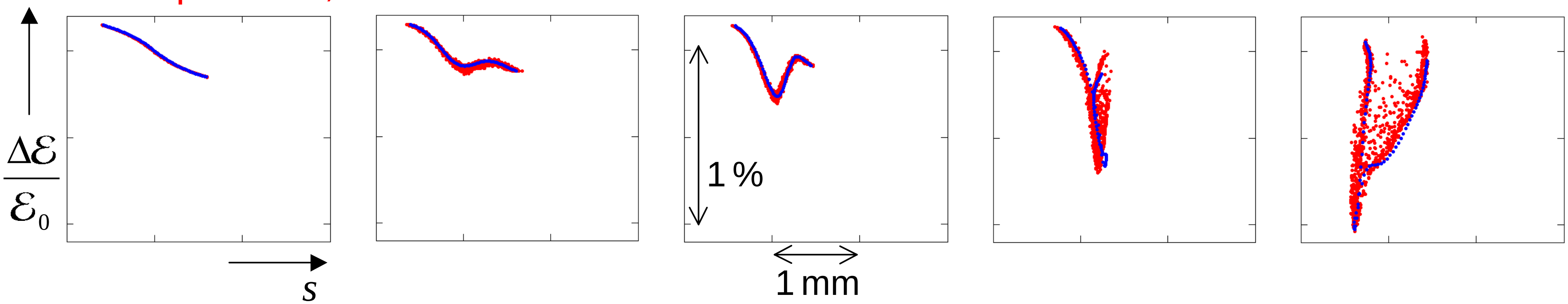


80 particles, self consistent
1000 particles, tracked in field of 80 particles

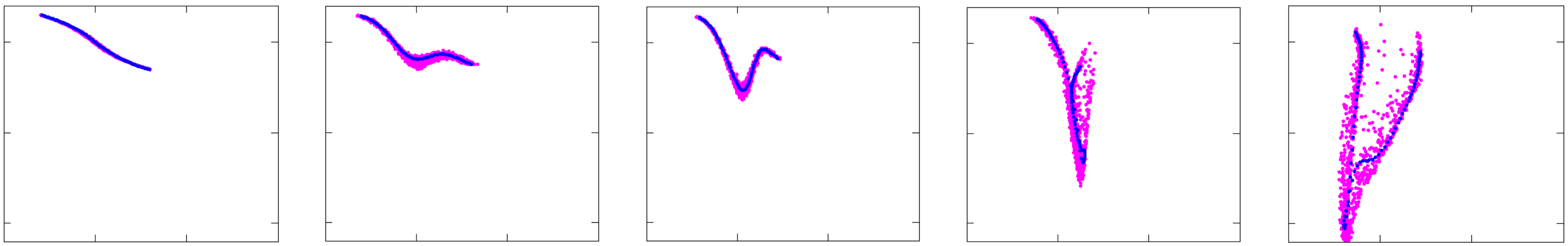


longitudinal phase space

80 particles, self consistent
1000 particles, self consistent

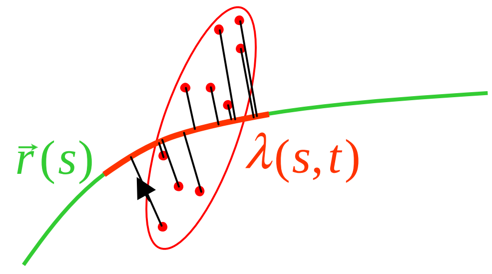


80 particles, self consistent
1000 particles, tracked in field of 80 particles



1.4 1D-Approach

e.g. Elegant
M. Borland



neglect transverse forces

neglect transverse beam dimesions

neglect Φ: $\Delta\hat{\mathcal{E}} = \Delta\mathcal{E}_0 + q_0 \int \mathrm{CSR}\, dt$

neglect deformation of retarded distribution: $\lambda(s,t') \approx \lambda(s\text{-}c_0 t)$

(local rigid bunch approximation)

$$\mathrm{CSR}(s,t) = \int \lambda'(\hat{s} - c_0 t') K(s,\hat{s}) d\hat{s}$$

$$\begin{bmatrix} x \\ x' \\ z \\ \Delta\hat{\mathcal{E}}/\mathcal{E}_0 \end{bmatrix}_{s+\delta s} = R(s, s+\delta s) \begin{bmatrix} x \\ x' \\ z \\ \Delta\hat{\mathcal{E}}/\mathcal{E}_0 \end{bmatrix}_{s} + \frac{q_0}{\mathcal{E}_0} \frac{\delta s}{c_0} \mathrm{CSR}(s+z, s/c_0) \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

or: sub-bunch approach

$$\lambda(s,t) \approx \sum q_\nu \lambda_{(\mathrm{sub})}(s - z_\nu(t) - c_0 t)$$

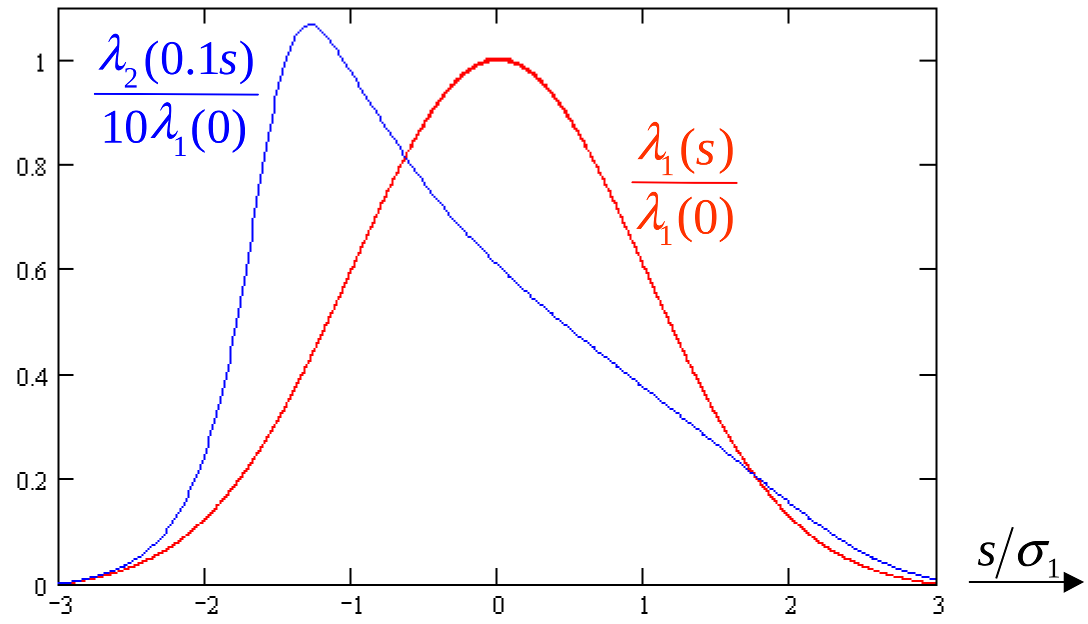
$$\int \lambda_{(\mathrm{sub})}(s)\,ds = 1$$

$$\mathrm{CSR}_{(\mathrm{sub})}(s,t) = \int \lambda'_{(\mathrm{sub})}(\hat{s} - c_0 t')\,K(s,\hat{s})\,d\hat{s}$$

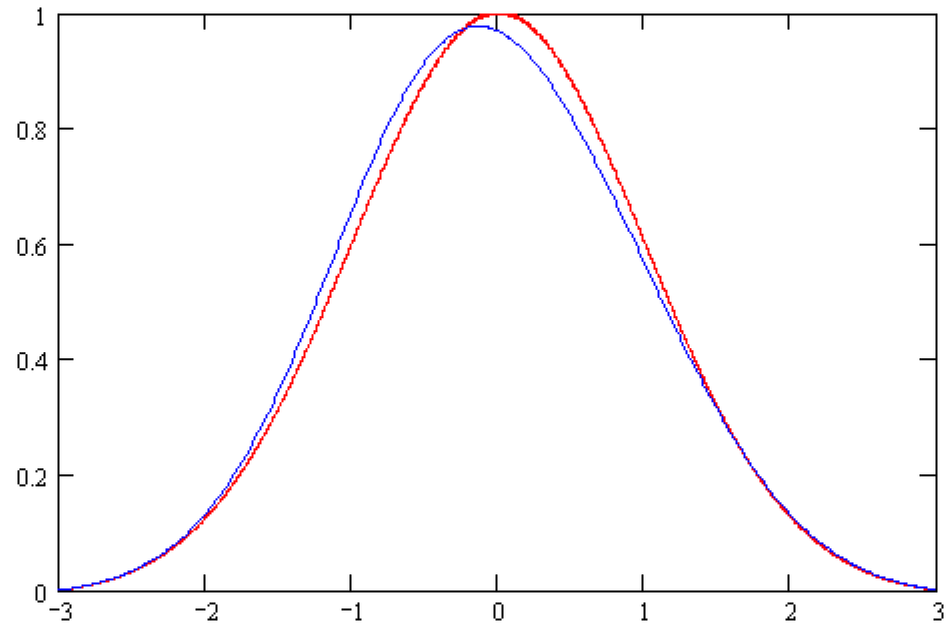
pseudo Green's function

$$\mathrm{CSR}(s,t) \approx \sum q_\nu \mathrm{CSR}_{(\mathrm{sub})}(s, t + z_\nu / c_0)$$

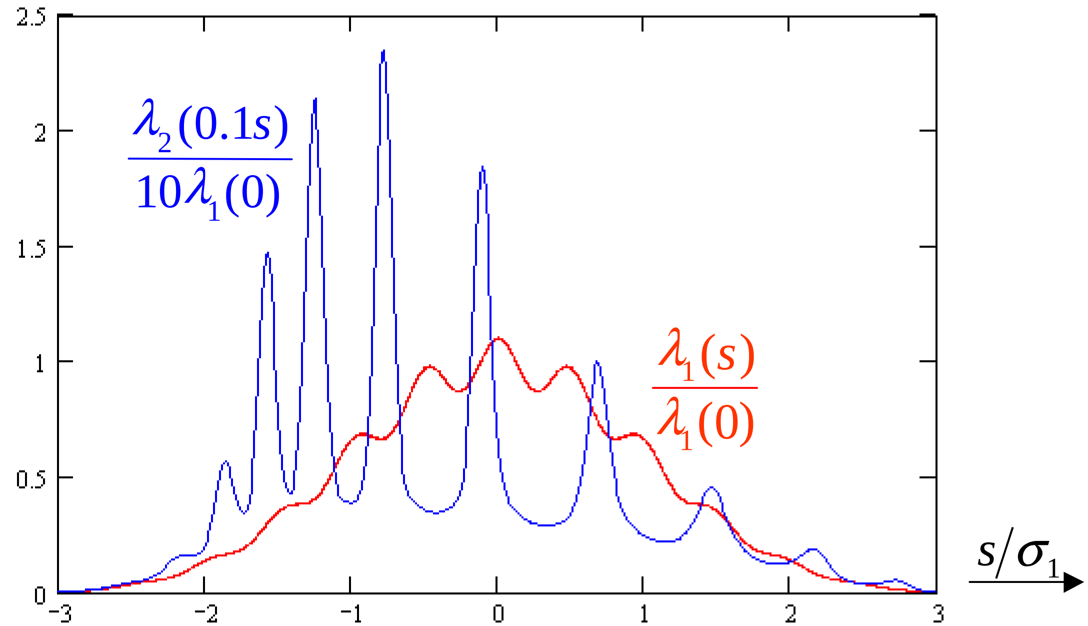
e.g. benchmark example
gaussian bunch
$q = 1\,\text{nC},\ \mathcal{E} = 500\,\text{MeV},\ \sigma_1 = 200\,\mu\text{m}$



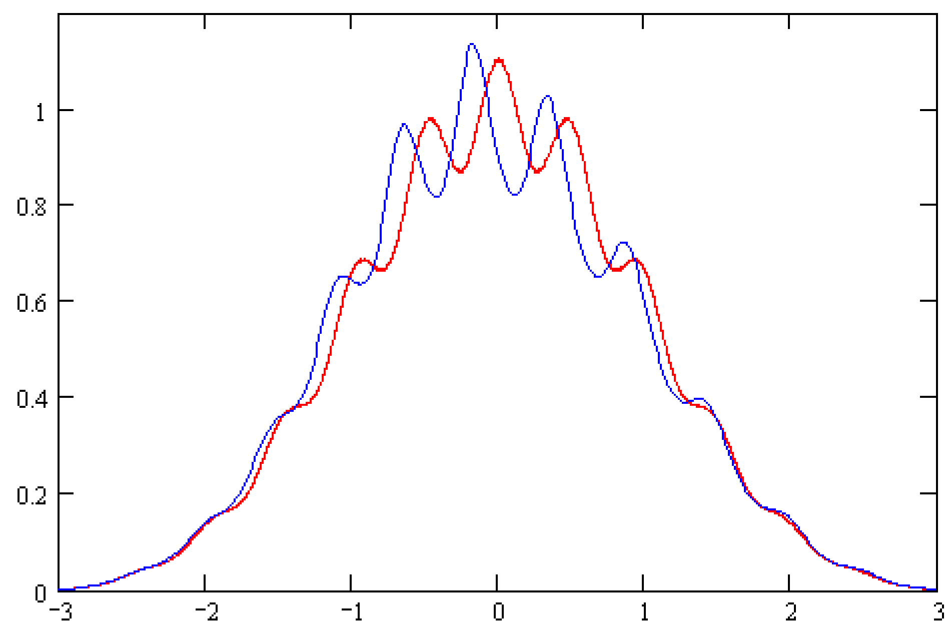
benchmark example
gaussian bunch
$q = 1\,\text{nC},\ \mathcal{E} = 5\,\text{GeV},\ \sigma_1 = 200\,\mu\text{m}$



modulated gaussian bunch
$q = 1\,\text{nC},\ \mathcal{E} = 500\,\text{MeV},\ \sigma_1 = 200\,\mu\text{m}$
initial modulation = 10 %
initial wavelength = 200 μm



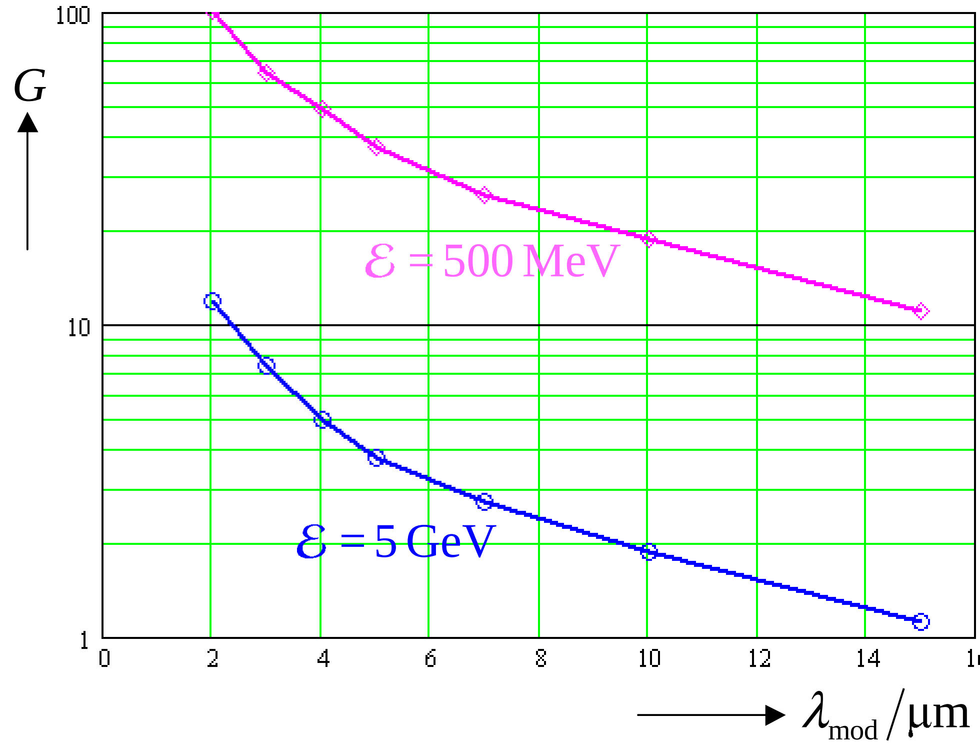
$\mathcal{E} = 5\,\text{GeV}$
same initial distribution



small signal gain

benchmark example

$q = 1\,\text{nC},\ \sigma_1 = 200\,\mu\text{m},\ \sigma_{\mathcal{E}}=0$



wavelength after compression

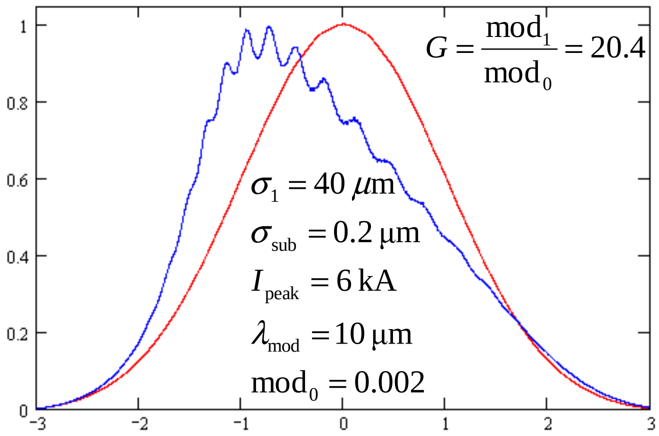
$\sigma_{\text{sub}} = 0.2\,\mu\text{m}$

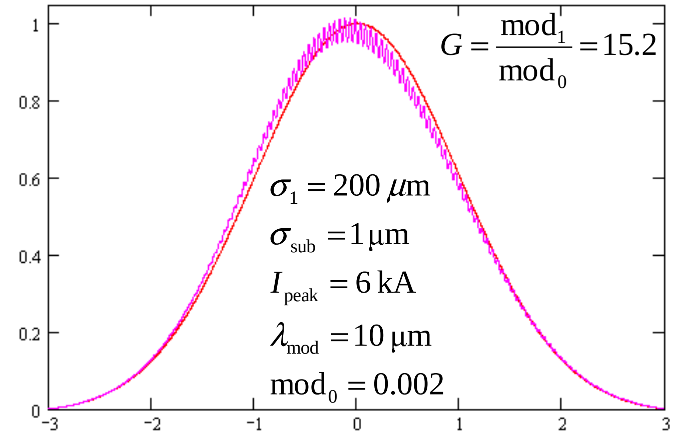
maximal gain limited by: uncorrelated energy spread $\lambda_{\max} \approx 2\pi R_{56} \dfrac{\sigma_{\mathcal{E}}}{\mathcal{E}}$

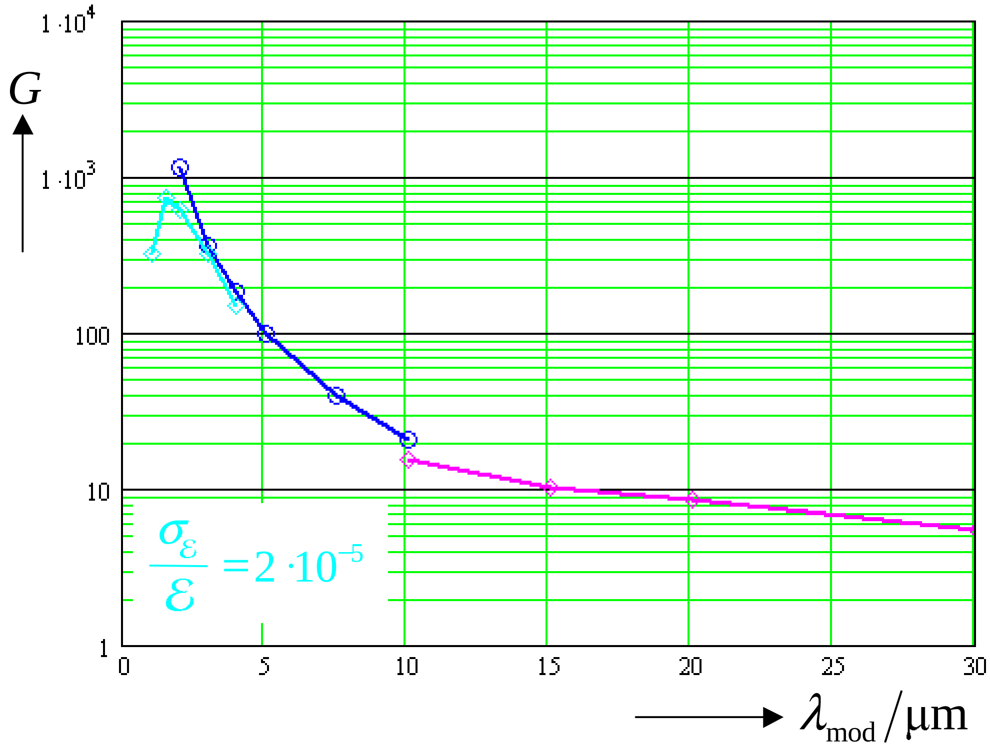
e.g.: $R_{56} = 2.5\,\text{cm}\,,\ \dfrac{\sigma_{\mathcal{E}}}{\mathcal{E}} = 2\cdot 10^{-5} \ \rightarrow \lambda_{\max} \approx 3\,\mu\text{m}$

sub bunch length of numerical simulation

small signal gain,
no compression,
$\mathcal{E} = 5\,\mathrm{GeV}$







2 Effects

2.1 Compression Work

2.2 Surface Impedance

2.3 Longitudinal Field

2.4 Transverse Effects

2.5 Transients

2.6 Shielding

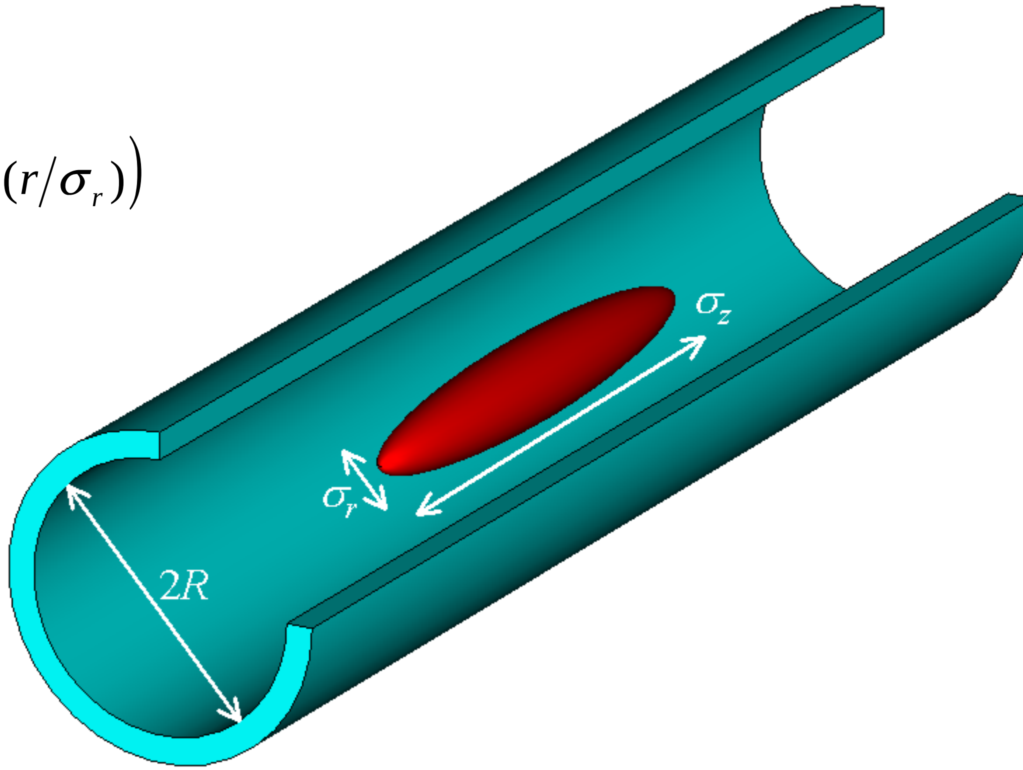
2.1 Compression Work

$$\rho(r,z) = \frac{q}{\sqrt{2\pi}\sigma_z\sigma_r^2} g(z/\sigma_z) g(r/\sigma_r)$$

$\gamma \to \infty$

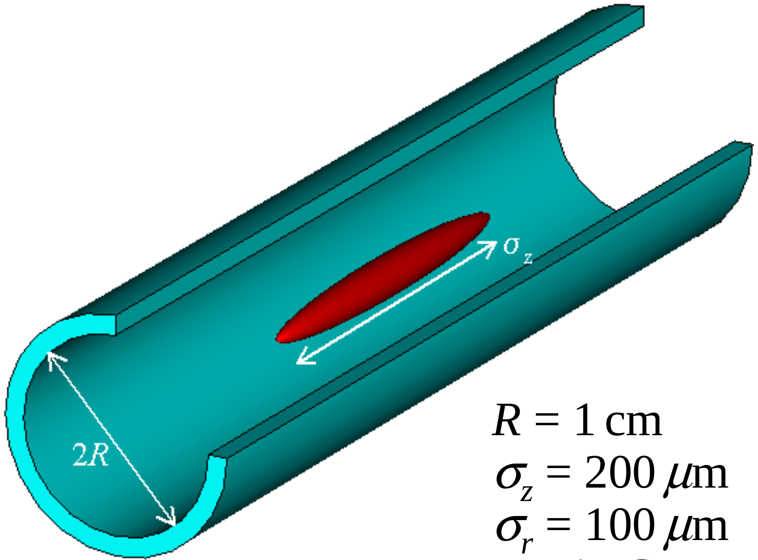
$$E_r(r,z) = \frac{q}{2\pi\varepsilon_0 r\sigma_z} g(z/\sigma_z)\left(1-\sqrt{2\pi}\,g(r/\sigma_r)\right)$$

$$B_\varphi(r,z) = c_0 E_r(r,z)$$



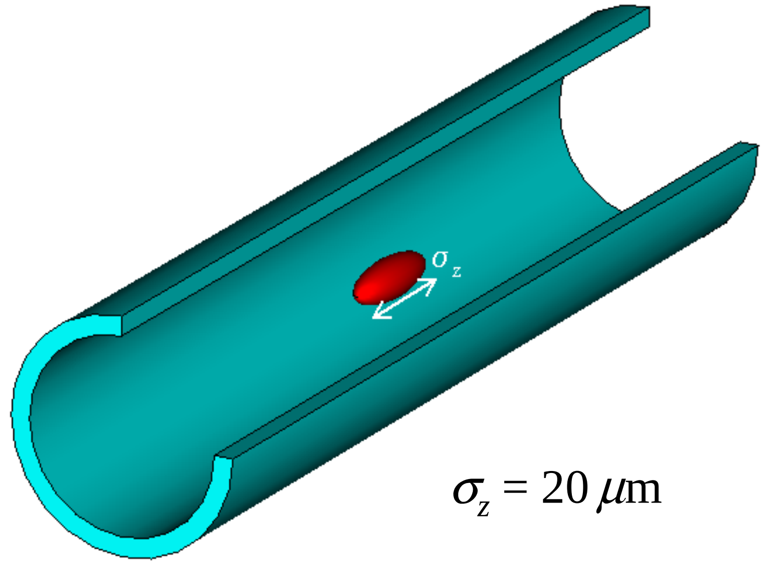
$\gamma >> R/\sigma_z$

$$W_{tot} = W_e + W_m = \frac{q^2}{4\pi^{3/2}\varepsilon_0\sigma_z}\ln\left(\frac{R}{1.5\sigma_r}\right)$$



$R = 1\,\text{cm}$
$\sigma_z = 200\,\mu\text{m}$
$\sigma_r = 100\,\mu\text{m}$
$q = 1\,\text{nC}$

$\Rightarrow$



$\sigma_z = 20\,\mu\text{m}$

$W_{\text{tot}} = 0.107\,\text{mJ}$ $W_{\text{tot}} = 1.065\,\text{mJ}$

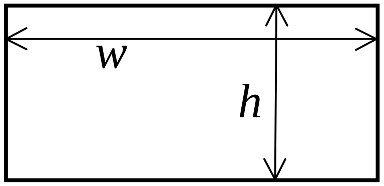
compression work $\Delta W_{\text{tot}} = 0.958\,\text{mJ}$ change of potential energy

CSR: $R_0 = 10\,\text{m},\ \sigma_z = 20\,\mu\text{m},\ L = 0.5\,\text{m}$

$P = 375\,\text{kW}$ $P\,L/c_0 = 0.625\,\text{mJ}$

2.2 Surface Impedance

flat chamber



$$Z \approx \frac{1}{2w} \frac{Z_s}{1 + jk_0 \frac{h}{2} \frac{Z_s}{Z_0}}$$

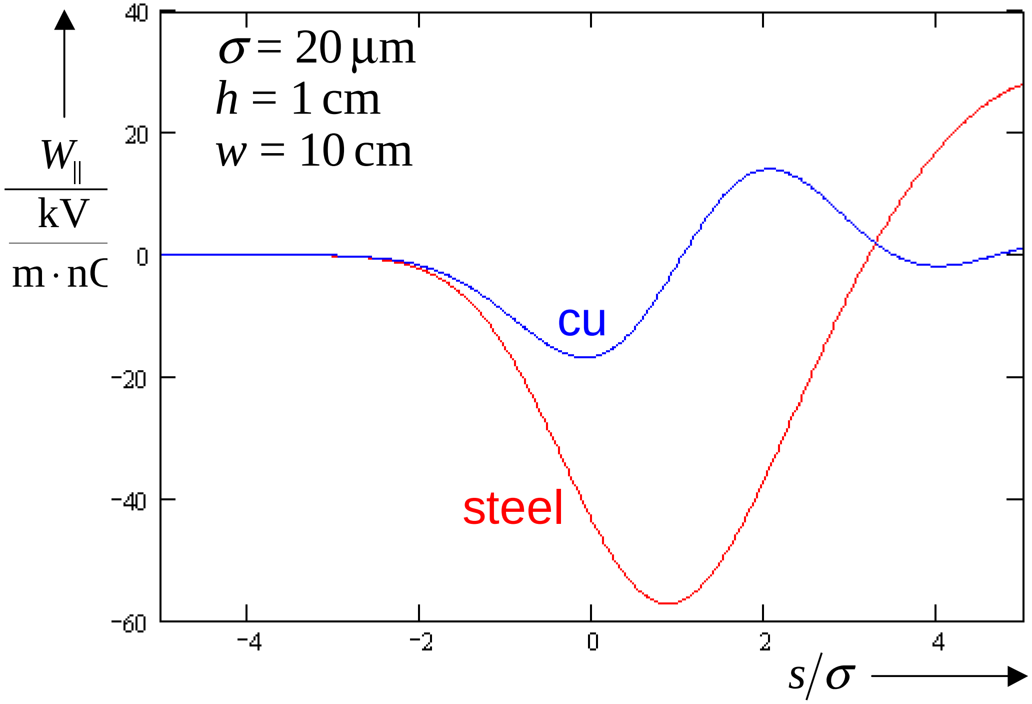
CSR field:

$$E_{\mathrm{csr}} \propto \frac{0.044}{R_0^{2/3} \sigma^{4/3}} \frac{q}{\varepsilon_0}$$

$\sigma = 20\,\mu\mathrm{m}$
$R_0 = 10\,\mathrm{m}$

$$\frac{E_{\mathrm{csr}}}{q} \propto 175 \frac{\mathrm{kV}}{\mathrm{m}} \frac{1}{\mathrm{nC}}$$

resistive wall wake:



2.3 Longitudinal Field

$$\vec{E}\cdot\vec{e}_{\parallel} = -\nabla\Phi\cdot\vec{e}_{\parallel} - \frac{\partial}{\partial t}\vec{A}\cdot\vec{e}_{\parallel}$$

$$= -\left(\gamma^{-2} + \beta^{2}\right)\nabla\Phi\cdot\vec{e}_{\parallel} - \frac{\partial}{\partial t}\vec{A}\cdot\vec{e}_{\parallel}$$

$$\nabla\Phi\cdot\vec{e}_{\parallel} = \frac{1}{v}\left(\frac{d\Phi}{dt} - \frac{\partial\Phi}{\partial t}\right)$$

$$\vec{E}\cdot\vec{e}_{\parallel} = \frac{1}{\gamma^{2}}\frac{1}{v}\frac{\partial\Phi}{\partial t} - \frac{1}{v}\frac{d\Phi}{dt} + \frac{1}{v}\frac{\partial}{\partial t}\left(\beta^{2}\Phi - \vec{A}\cdot\vec{v}\right)$$

"space charge" | potential | "CSR"

Longitudinal CSR Field of a Thin Beam

rigid bunch model: $\lambda(s-vt)$

$\vec{r}(s)$

observer: $\vec{r}_o$

$\vec{v}_o = v\vec{e}_o$

$$E_{\text{CSR}} = \frac{1}{\beta c_0}\frac{\partial}{\partial t}\left(\beta^2\Phi - \vec{A}\cdot\vec{v}\right)$$

$$E_{\text{CSR}} = \frac{-\beta^2}{4\pi\varepsilon_0}\int \lambda'(s+\beta R - vt)\frac{1-\vec{e}_o\cdot\vec{e}}{R}\,ds$$

$$\vec{e}(s) = \frac{d}{ds}\vec{r}(s)$$

$$R(s) = \left\|\vec{r}_o - \vec{r}(s)\right\|$$

Longitudinal Field of a Thin Beam on a Circular Path

$$\vec{E}\cdot\vec{e}_\varphi = \frac{1}{\gamma^2}\frac{1}{v}\frac{\partial\Phi}{\partial t} - \frac{1}{v}\frac{d\Phi}{dt} + E_c G(c_0 t/\sigma)$$

with $$E_c = \frac{1}{\sqrt[3]{3}(2\pi)^{3/2}}\frac{1}{R_0^{2/3}\sigma^{4/3}}\frac{q}{\varepsilon_0}$$

$$G(w) = \sqrt{2\pi}\int_0^\infty \frac{g'(x+w)}{\sqrt[3]{x}}dx$$

coherent radiated power:

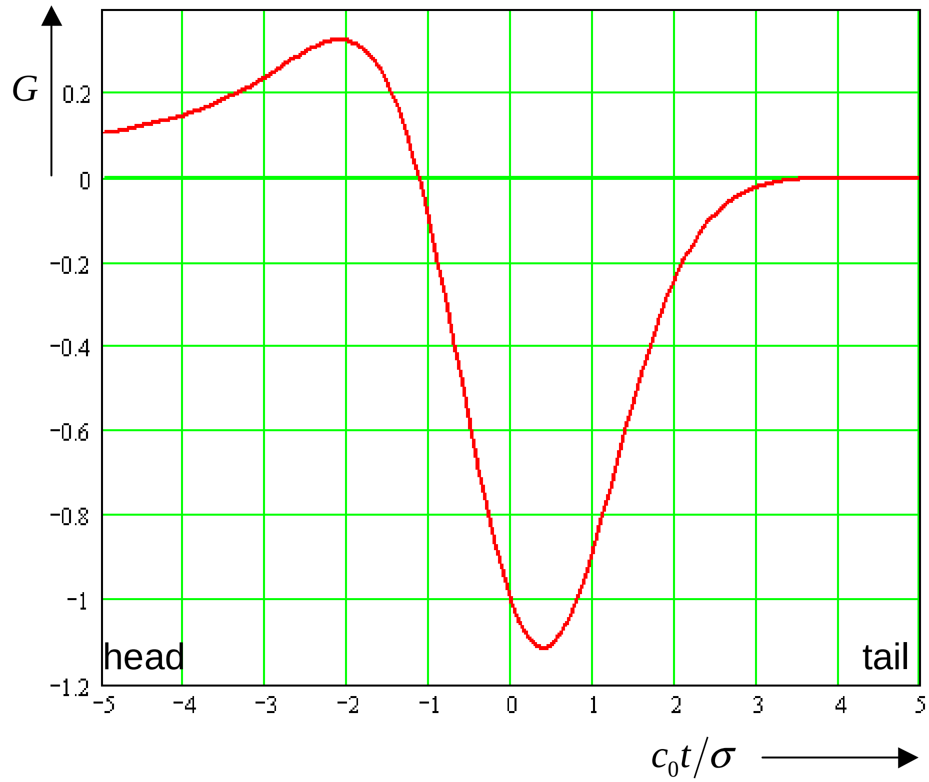
$$P \approx \frac{\Gamma(5/6)}{4\pi^{3/2}\cdot\sqrt[3]{6}}\frac{q^2 c_0}{\varepsilon_0}\frac{1}{R_0^{2/3}\sigma^{4/3}}$$

e.g.: $q = 1\,\mathrm{nC}$

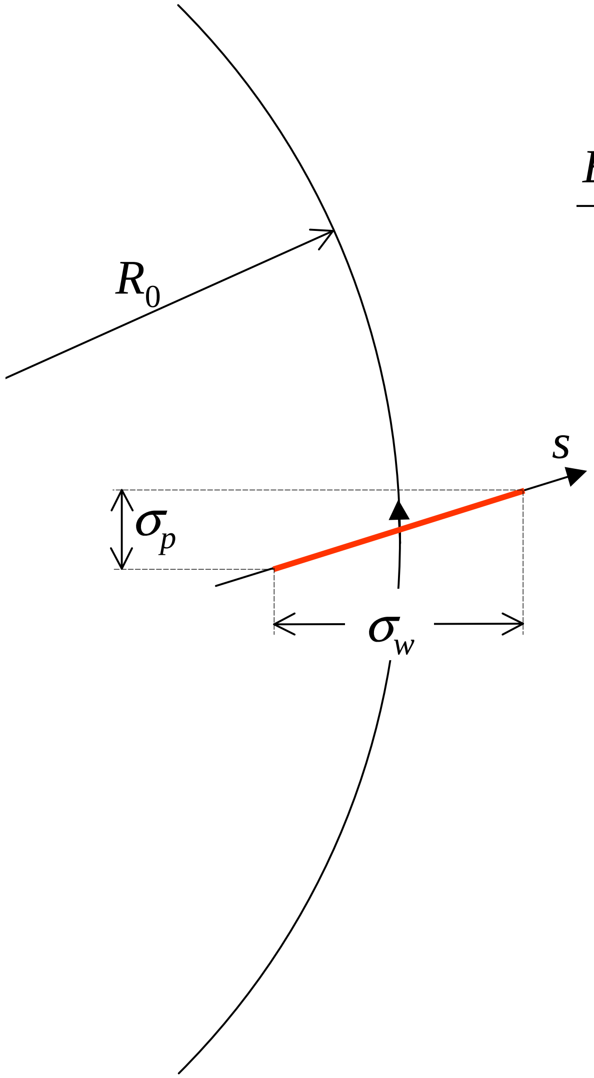
$R_0 = 10\,\mathrm{m}$

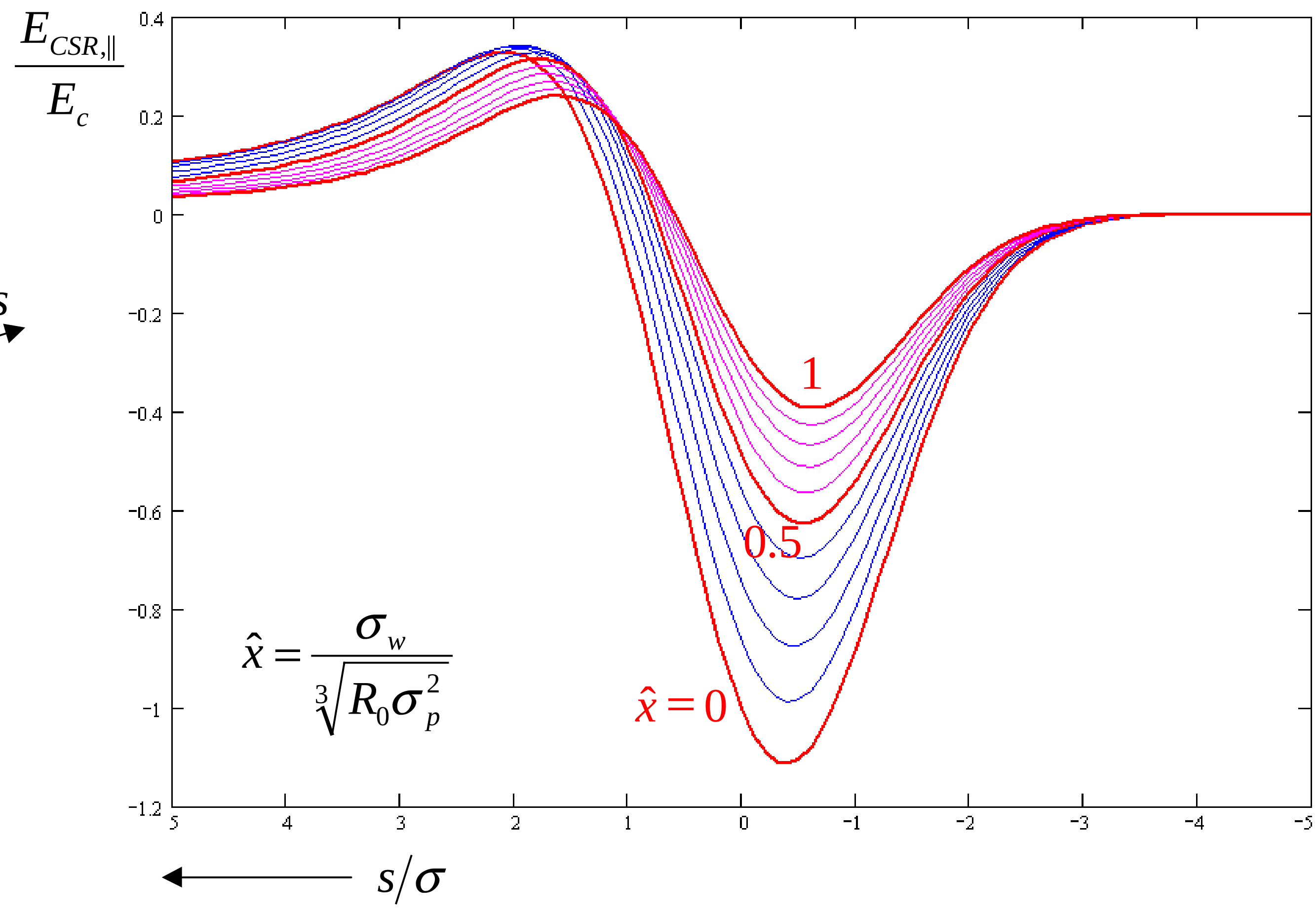
$\sigma = 20\,\mu\mathrm{m}$

$P \approx 375\,\mathrm{kW}$



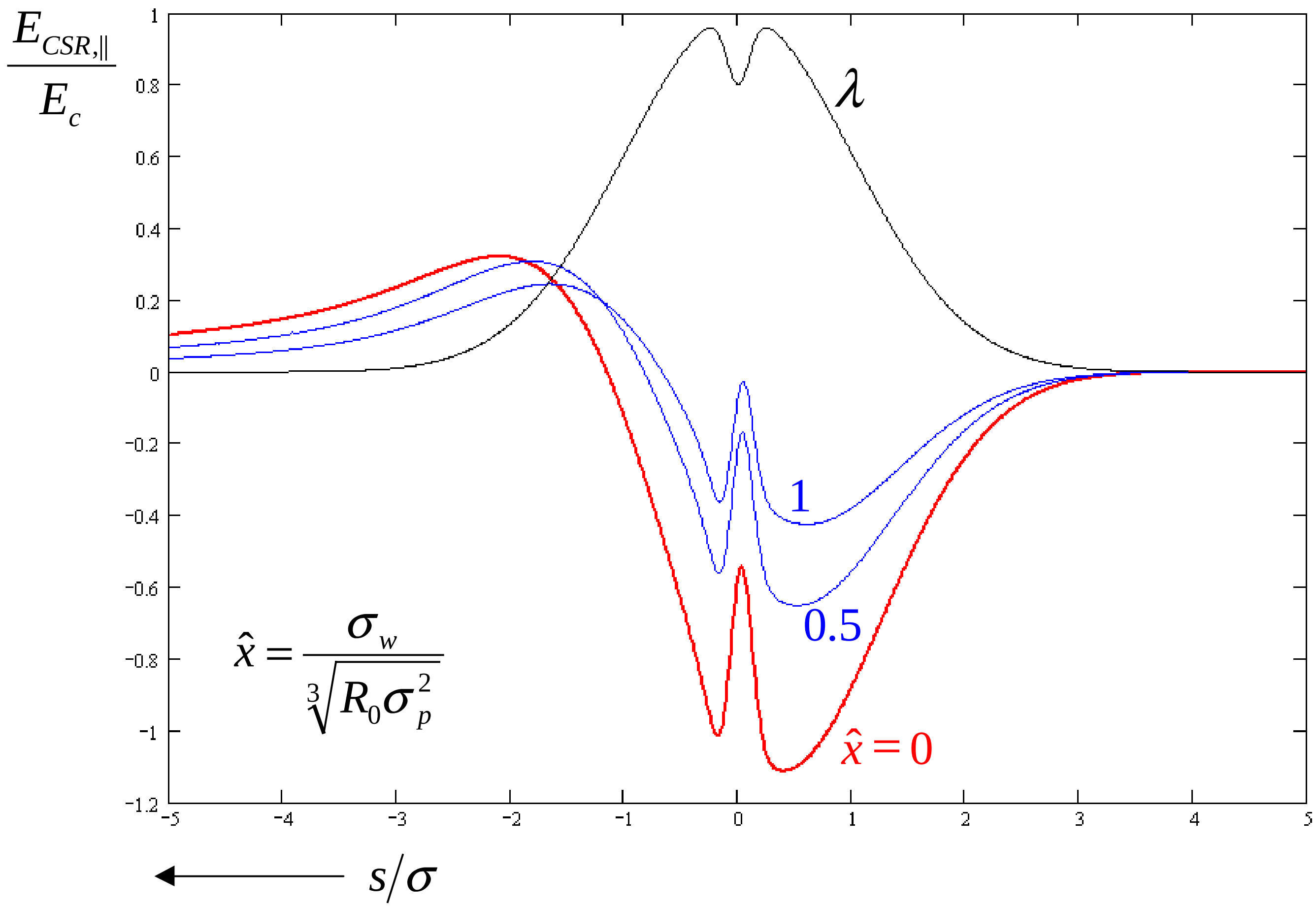
CSR Field of a Tilted Thin Beam





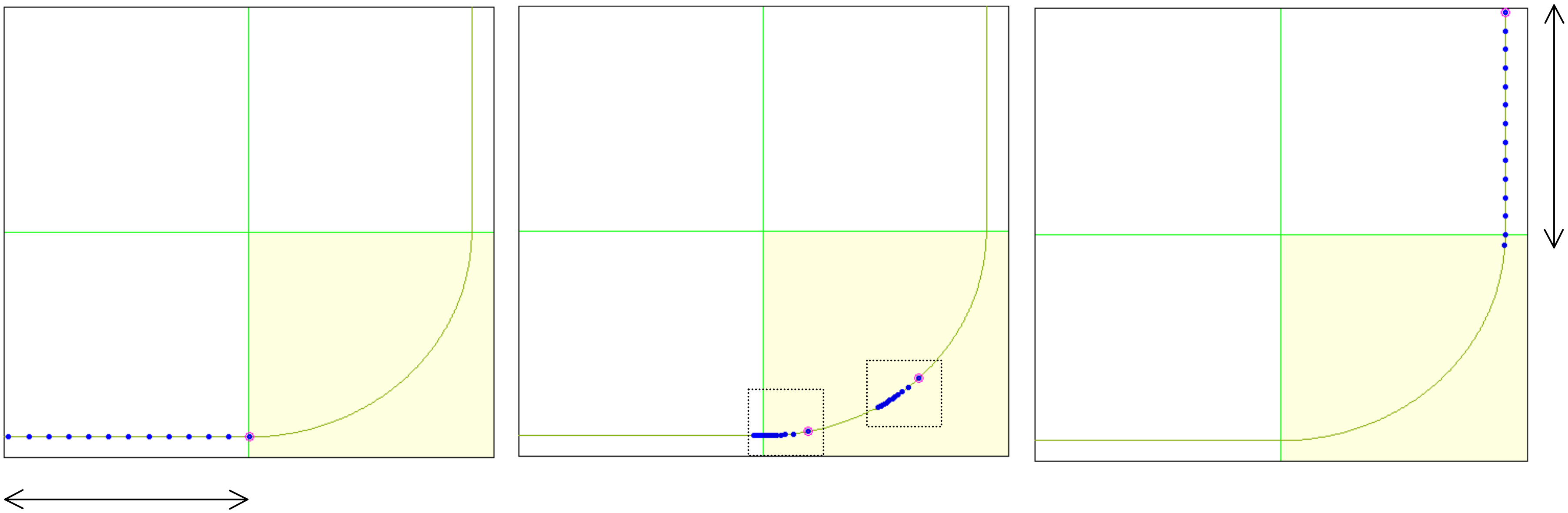
e.g. $R_0 = 10\text{ m}, \sigma_p = 100\,\mu\text{m}, \sigma_w = 2\text{ mm} \rightarrow \hat{x} = 0.43$

… CSR Field of a Tilted Thin Beam

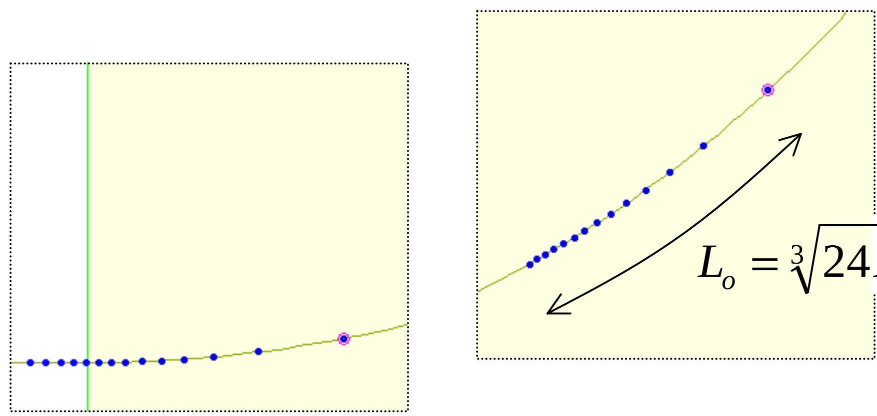


2.5 Transients

retarded particles, seen from the head particle:



$$L = \frac{l}{1-\beta} \approx 2\gamma^2 l$$



$$L_o = \sqrt[3]{24 R_0^2 l} \quad \text{for} \quad l >> R_0/\gamma^3$$

Example:

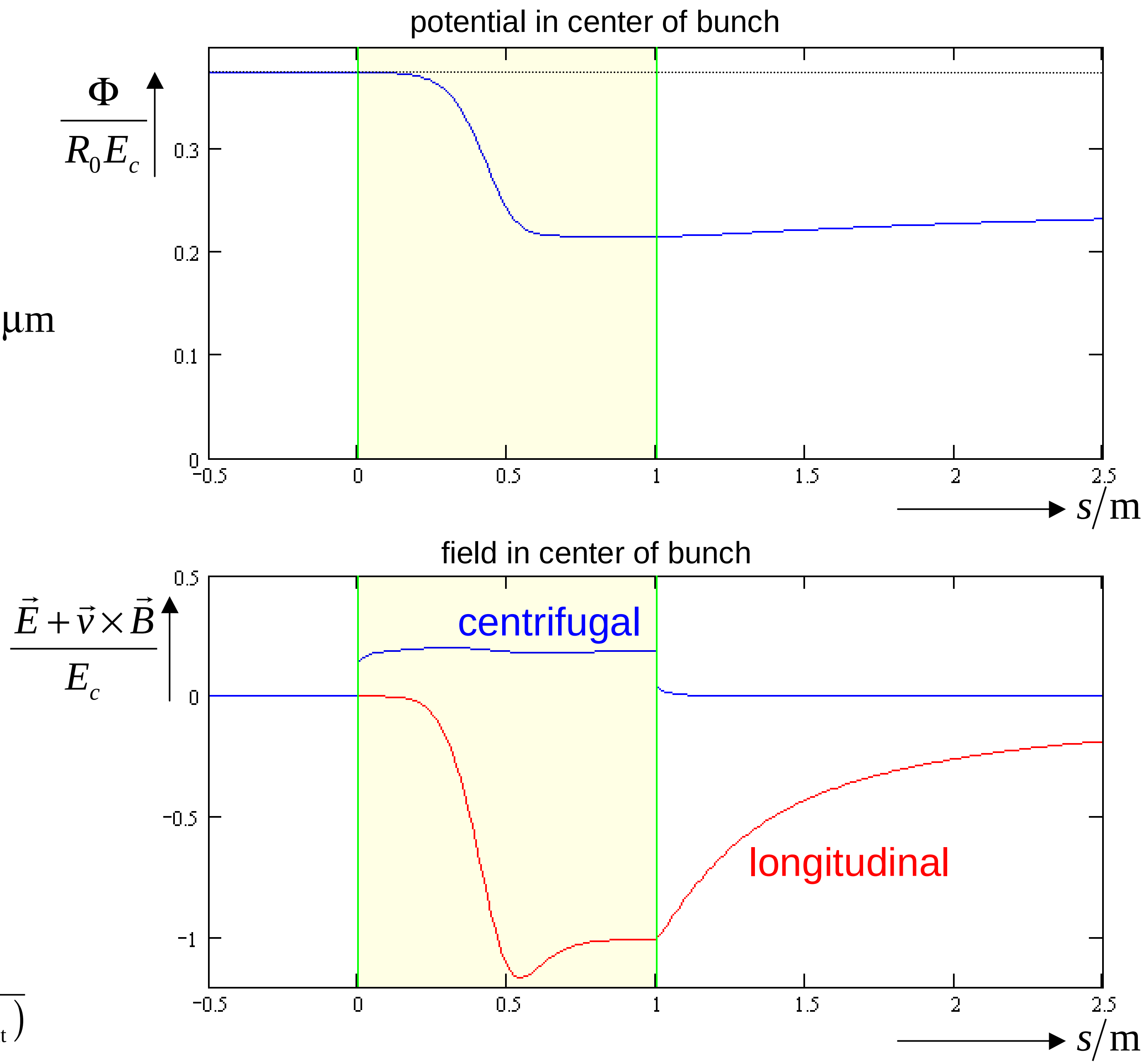
arc radius $R_0 = 10\,\text{m}$

arc length $L = 1\,\text{m}$

$\gamma = 10^4$

bunch length $\sigma = 100\,\mu\text{m}$

bunch radius $10\,\mu\text{m}$

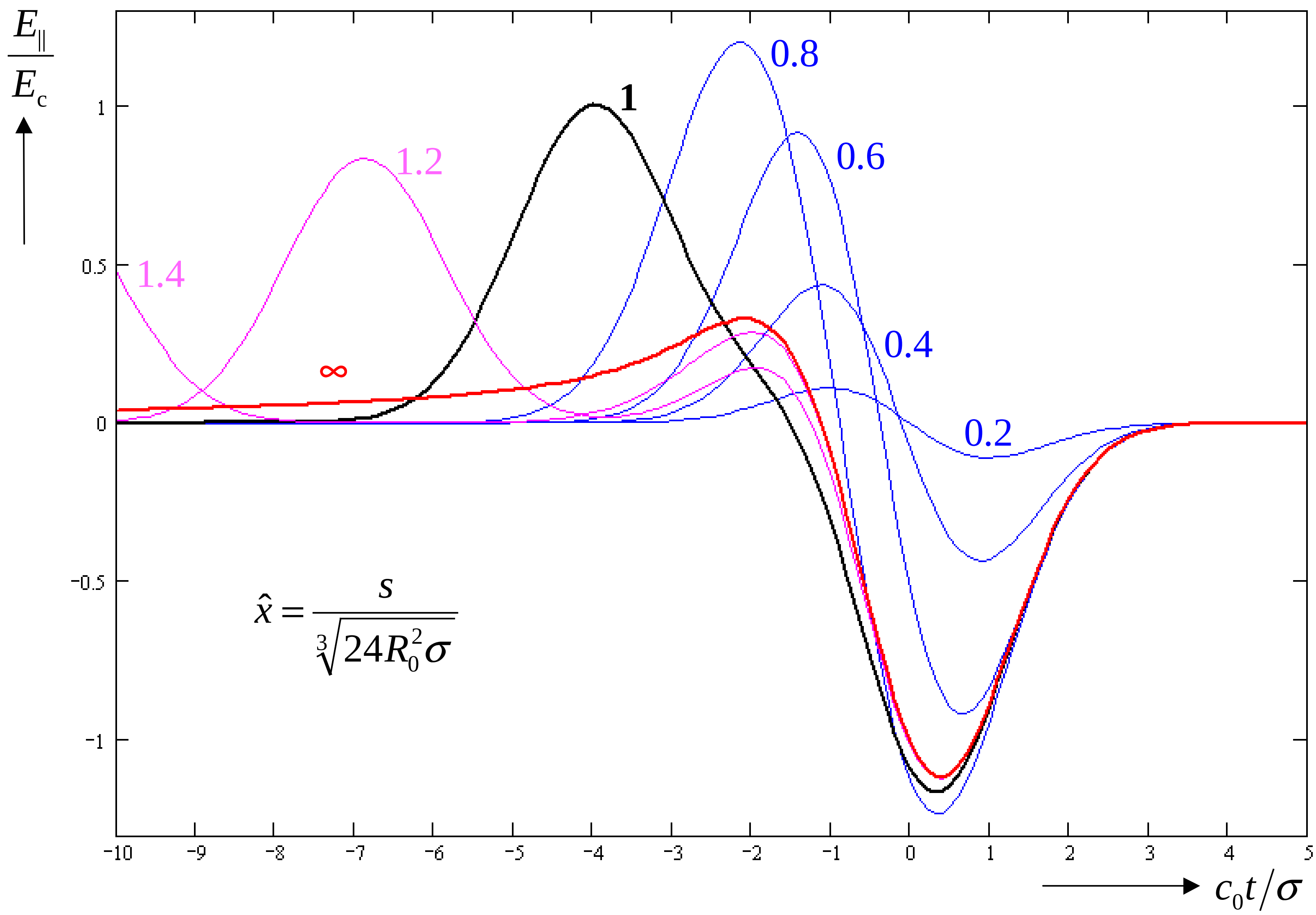


$$L_o = \sqrt[3]{24 R_0^2 \sigma} \approx 0.621\,\text{m}$$

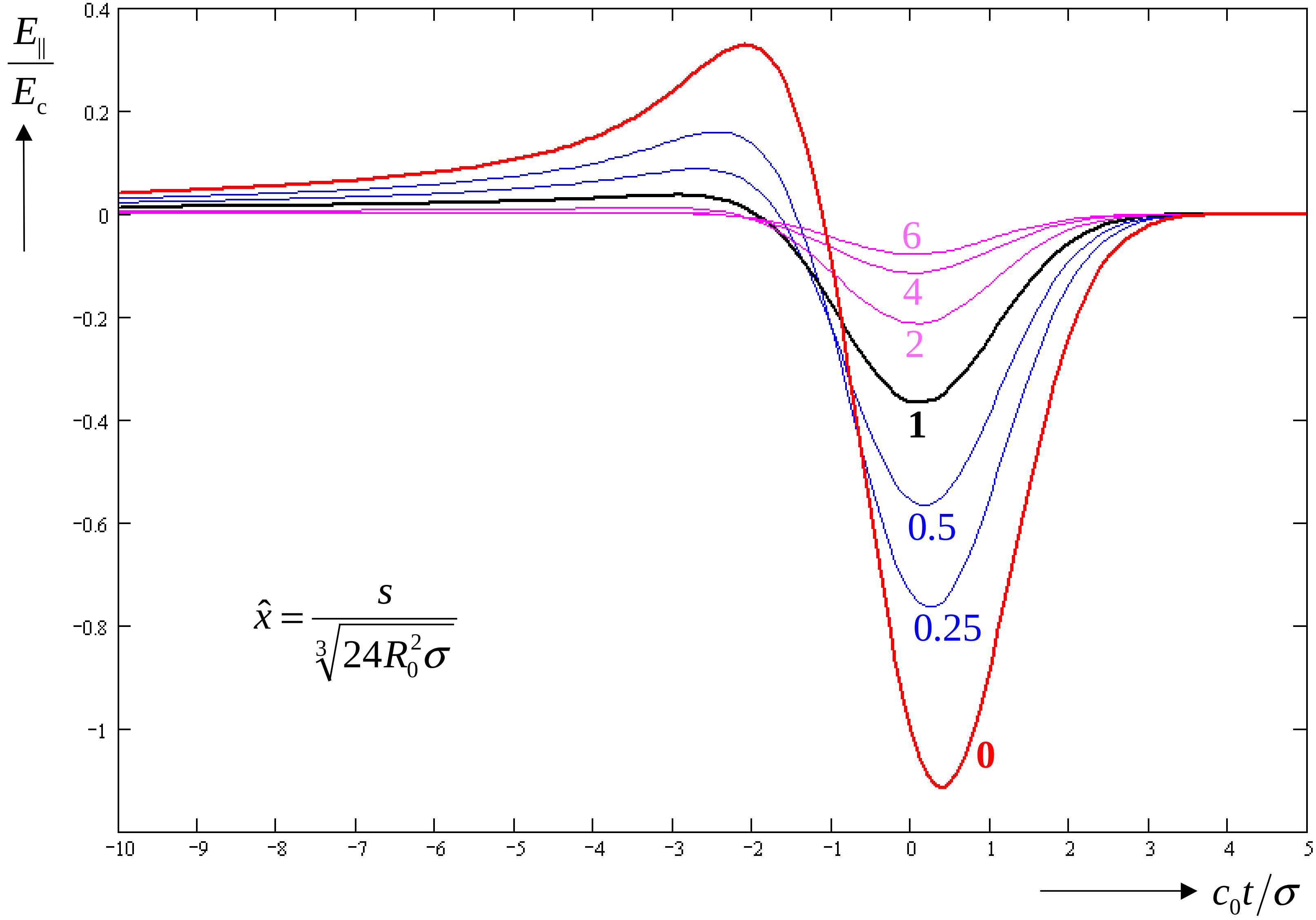
for $L_o << s - s_{exit} << L_\gamma$

$$E_\parallel \approx \frac{-\lambda}{4\pi\varepsilon_0 (s - s_{exit})}$$

Transient Longitudinal Field: Injection



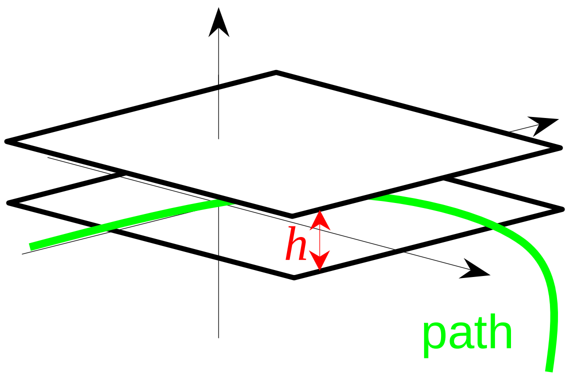
Transient Longitudinal Field: Ejection

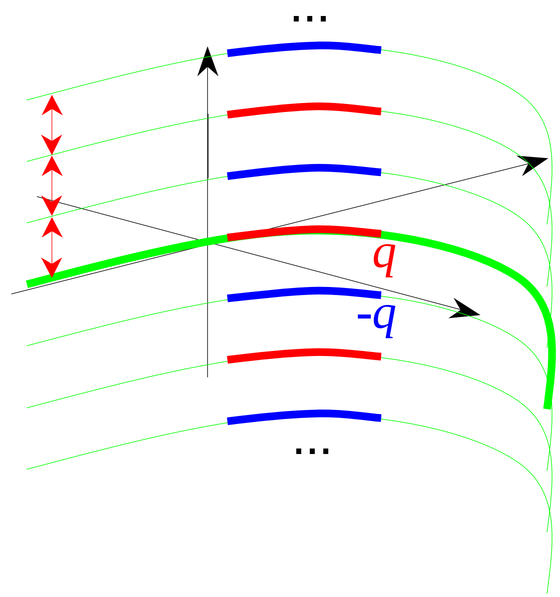


Martin Dohlus Deutsches Elektronen Synchrotron ICFA Workshop Jan. 2002

2.6 Shielding

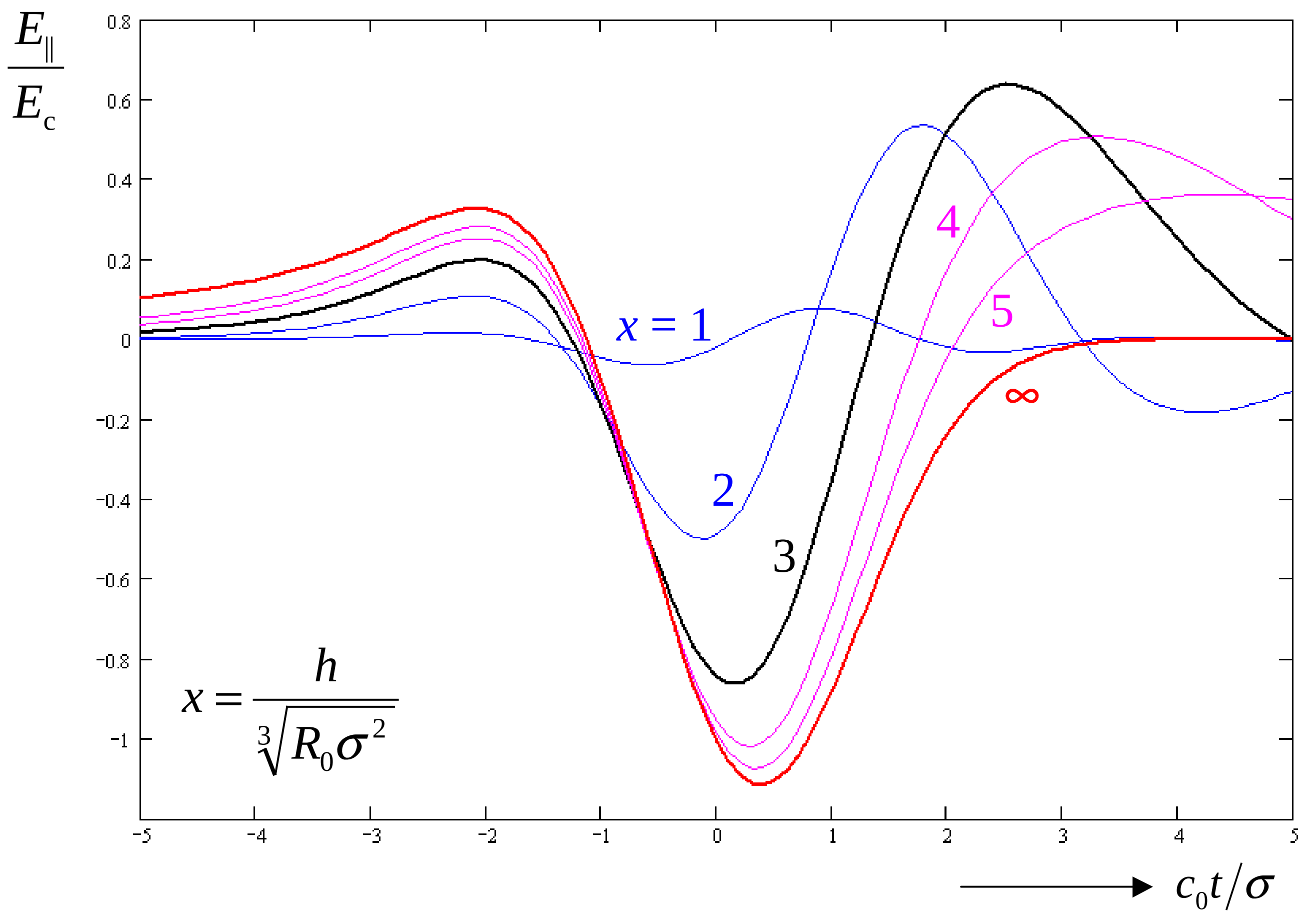
a) shielding by horizontal conducting planes:





general path
transient effects

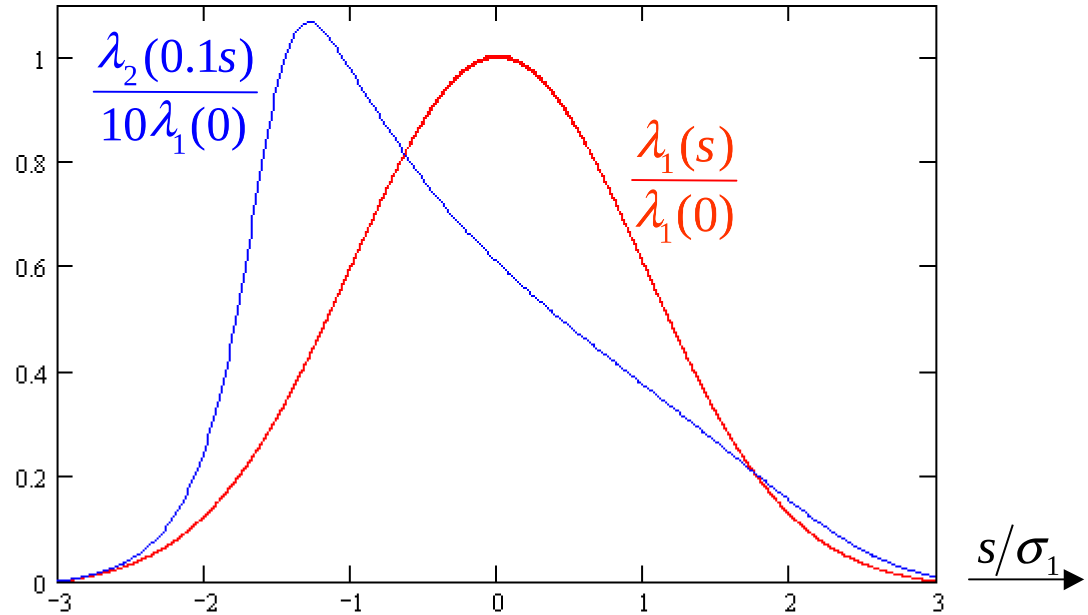
Shielding: Longitudinal Field



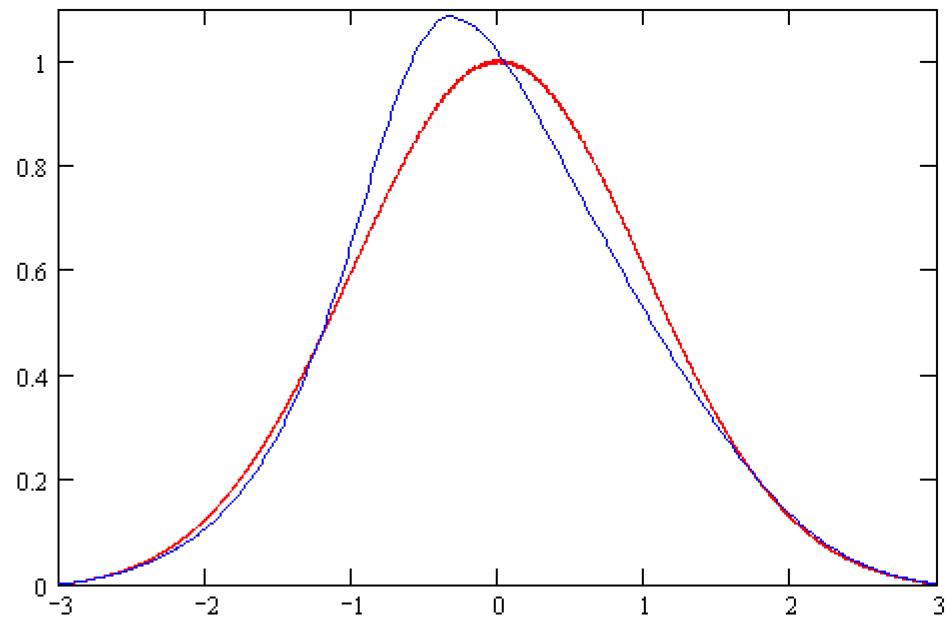
e.g.: $R_0 = 10\,\text{m}, \sigma = 100\,\mu\text{m}, h = 1\text{cm} \rightarrow \hat{h} = 2.1$

c) benchmark example (1D approach):

gaussian bunch
$q = 1\,\mathrm{nC},\ \mathcal{E} = 500\,\mathrm{MeV},\ \sigma_1 = 200\,\mu\mathrm{m}$
$\rightarrow \sigma_2 = 20\,\mu\mathrm{m}$



shielding by horizontal conducting planes, $h = 1\mathrm{cm}$



gaussian bunch

$q = 1\,\text{nC},\ \mathcal{E} = 500\,\text{MeV},\ \sigma_1 = 200\,\mu\text{m}$

$\rightarrow \sigma_2 = 20\,\mu\text{m}$