# Transverse CSR Force – Critical Analysis

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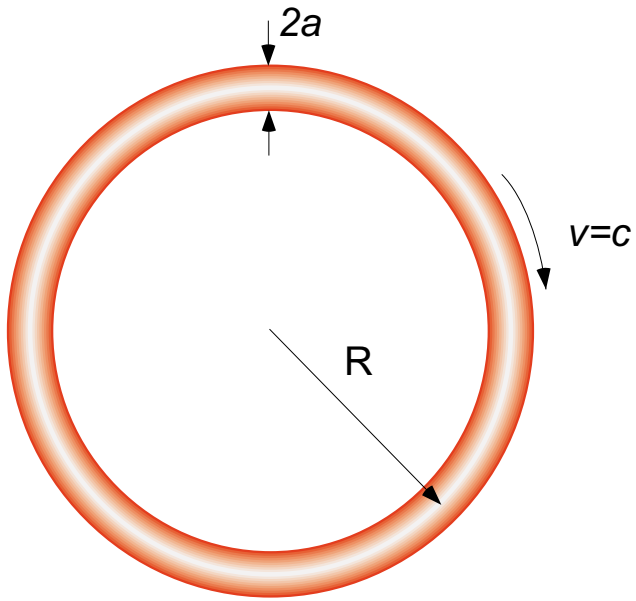
CSR Workshop 2002
Berlin, Jan. 14-18, 2002

Let us consider a coasting beam with charge per unit length $\lambda$




Uniformly charged circular beam
in vacuum

## Talman force

- Talman calculates a centrifugal space-charge force *per unit charge*, CSCF, (PRL, 86):

$$F_\perp \sim \frac{\lambda}{R} \ln \frac{R}{a}$$

- He notices that the force varies over the bunch radius and $\partial F_\perp / \partial r$ should be large. Hence, there should be a strong focusing effect for the beam particles.
- Exact expression for $F_\perp$ (Stupakov, 98)

$$F_\perp = \frac{\lambda}{R}\left(-1 - \frac{r^2}{a^2} + 2\ln\frac{8R}{a}\right)$$

$$\frac{\partial F_\perp}{\partial r} = -\frac{\lambda}{R}\frac{2r}{a^2} \sim \frac{\lambda}{Ra}$$

## Lee Analysis

- E. P. Lee, *Cancellation of the Centrifugal Space-Charge Force* (PA, 90). He points out the importance of the energy variation of a particle during betatron oscillations. It turns our that this effect almost completely cancels out the focusing effect of $F_\perp$.

- Equation for $x$

$$x'' + Kx = \frac{eF_\perp(x)}{E} + \frac{1}{R(s)}\frac{\Delta E}{E},$$

The transverse force shifts the orbit, but so does the energy variation. $\Delta E$ is the particle energy variation, $\Delta E = -e\phi(x)$,

$$x'' + Kx = \frac{e}{E}\left(F_\perp - \frac{\phi}{R}\right).$$

- He shows by direct calculation for two cases that

$$\frac{\partial}{\partial r}\left(F_\perp - \frac{\phi}{R}\right) \sim \frac{\lambda}{R^2}$$

But he does not give an expression for $F_\perp$ other than saying $F_\perp \sim O(\lambda/R)$.

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**Y. Derbenev and V. Shiltsev**, *Transverse effects of Microbunch Radiative Interaction*, Fermilab-TM-1974, 1996.

They find a *centripetal force* for a general case of a bunched beam

$$F_{\perp \text{eff}} = -\frac{2\lambda}{R}$$

G. Stupakov, *Effect of Centrifugal Transverse Wakefield for Microbuch in Bend*, Arcidosso conference, 1998.

- Calculated the force and the potential inside a coasting beam, but missed a term in $\phi$ (it is a *constant*)

$$\phi = \frac{\lambda}{R}\left(-\frac{r^2}{a^2} + 1 + 2\ln\frac{8R}{a}\right)$$

- Found agreement with Lee, but disagreed with Derbenev/Shiltsev. Concluded that $F_\perp \sim (\lambda/R)\ln\frac{R}{a}$, and overestimated emittance growth due to $F_\perp$.

- With corrected potential

$$F_\perp - \frac{\phi}{R} = -2\frac{\lambda}{R}$$

  The same holds for a bunched beam.

R. Li, *Analysis and Simulation of CSR Effects*, 2000.

A thorough analysis of the transverse force, claims agreement with Derbenev/Shiltsev formula and computer simulations.

The bottom line:

$$F_\perp \sim \frac{\lambda}{R} \ln \frac{R}{a}$$

but

$$\Delta E/e \sim \frac{\lambda}{R} \ln \frac{R}{a}$$

The effect of the transverse force on the orbit would be to expand the circle. However, it is almost cancelled by the orbit shift due to the energy change. The net result is a decrease of the orbit radius as if the force is $-2\lambda/R$.

This cancellation still seems magic to me.

Questions: Is it is universal? How about transient wakes, do we have cancellation there? Wake in a pipe?

In any case, the effect of the transverse force in our bunch compressors seems to be small.