# Klystron Instability of a Relativistic Electron Beam in a Bunch Compressor

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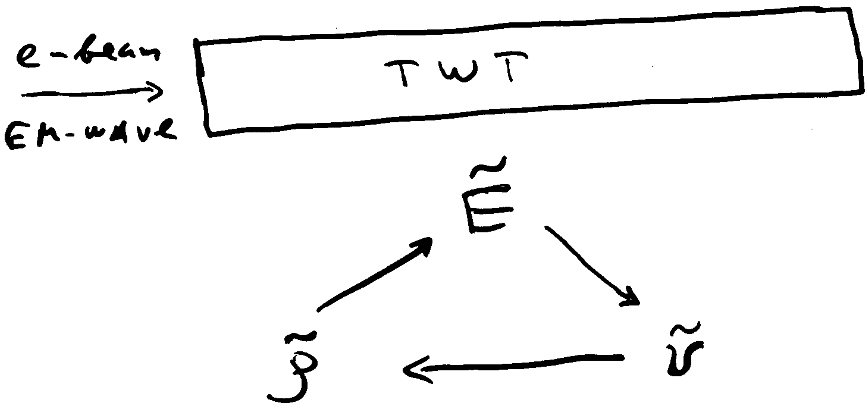
*/ CSR Workshop, Zeuthen, January 15, 2002 /*

# Klystron vs. TWT

## TWT



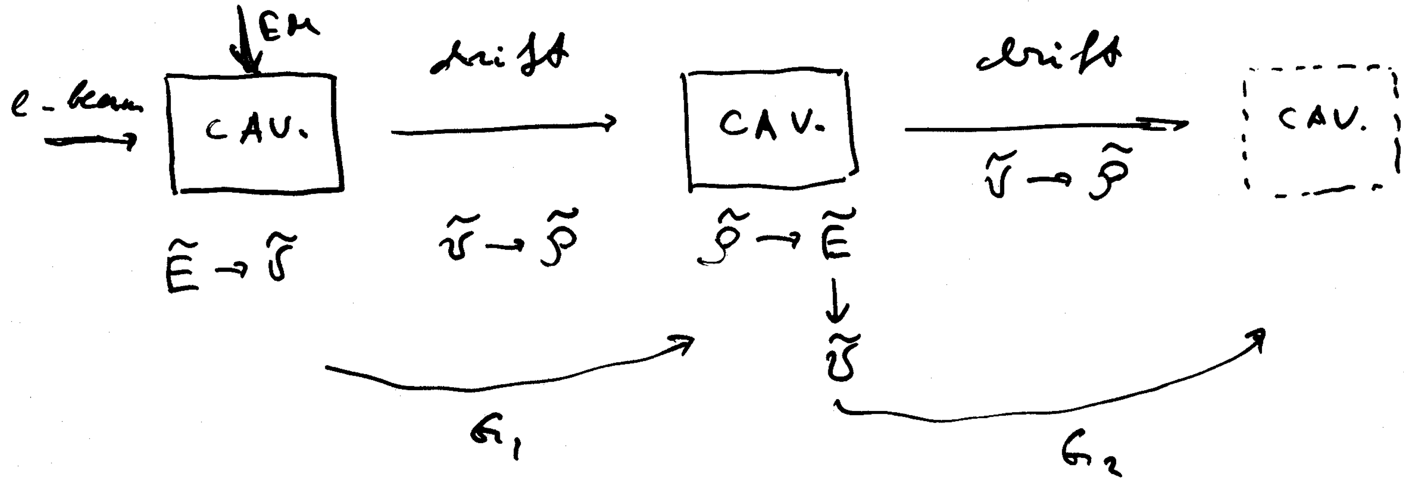

$$G = A e^{\Lambda(I_0) s}$$

Positive feedback → exponential growth
phase relations are important

Similar mechanism for instabilities of relativistic electron beams ($\tilde{v} \to \tilde{\gamma}$).

( Sign of mass, phase of $Z(\omega)$ are important. )

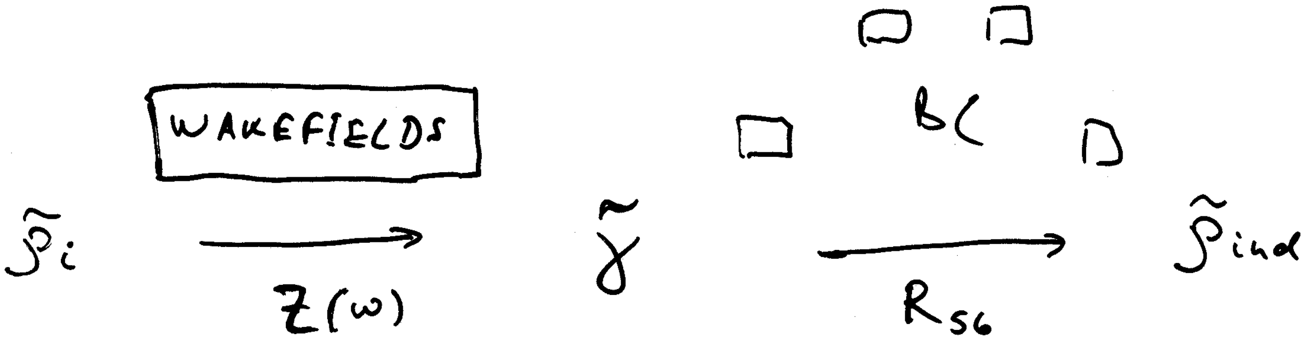
# Klystron




Phases are not important.

$$G = G_1 \cdot G_2 \cdots G_n \sim I_0^n$$

# BC

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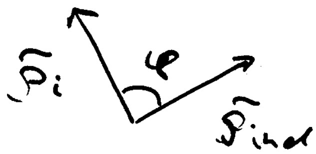
$|\tilde{\rho}_{ind}| \simeq |\tilde{\rho}_i|$




Phases are important but this case is not interesting

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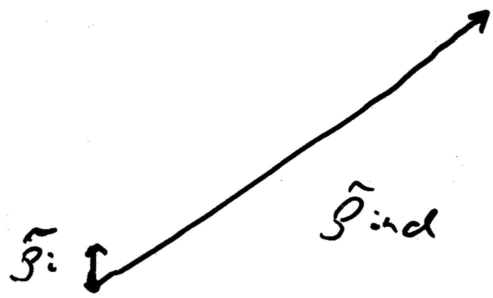
$|\tilde{\rho}_{ind}| \gg |\tilde{\rho}_i|$

$$G \triangleq \frac{|\tilde{\rho}_{ind}|}{|\tilde{\rho}_i|} \gg 1$$




Don't care about phases!

Only $|Z(\omega)|$, $|R_{56}|$ are important

## Previous works:

T. Limberg, Ph. Piot, and E.A. Schneidmiller, "An Analysis of Longitudinal Phase Space Fragmentation at the TESLA Test Facility",
FEL 2000 Conf., TESLA-FEL-2000-05

S. Heifets and G. Stupakov, "Beam Instability and Microbunching due to Coherent Synchrotron Radiation",
PAC 2001, SLAC-PUB-8761 (2001)

M. Borland, "Start-to-End Simulation of SASE FELs from the Gun through the Undulator",
FEL 2001 Conf.

S. Heifets and G. Stupakov, "Microbunching due to Coherent Synchrotron Radiation in a Bunch Compressor",
SLAC-PUB-8988 (2001)

E.L. Saldin, E.A. Schneidmiller and M.V. Yurkov, "Longitudinal Phase Space Distortions in Magnetic Bunch Compressors",
FEL 2001 Conf., DESY-01-129 (2001)

S. Heifets, "Vlasov Equation with Coherent Synchrotron Radiation",
SLAC-PUB-9054 (2001)

## Compression of the beam with linear energy chirp and superimposed sinusoidal modulation

**Distribution before compression:**

$$f(z, \delta\gamma) = \frac{I_0}{\sqrt{2\pi}\sigma_\gamma} \exp\left\{-\frac{[\delta\gamma - h\gamma_0 z + \Delta\gamma \sin(kz)]^2}{2\sigma_\gamma^2}\right\} \ ,$$

$$\delta\gamma = (\mathcal{E} - \mathcal{E}_0)/(mc^2) \ , \quad \sigma_\gamma = \sigma_{\mathcal{E}}/(mc^2) \ , \quad h = d\delta\gamma/(\gamma_0 dz)$$

$$k = 2\pi/\lambda$$

**Compression:**

$$z_{\mathrm{f}} = z_{\mathrm{i}} + R_{56}\frac{\delta\gamma}{\gamma_0}$$

$$I(z) = \int_{-\infty}^{\infty} f(z, \delta\gamma) d\delta\gamma$$

$$I(z) = CI_0 \left[1 + 2\sum_{n=1}^{\infty} J_n\left(nCkR_{56}\frac{\Delta\gamma}{\gamma_0}\right) \exp\left(-\frac{1}{2}n^2C^2k^2R_{56}^2\frac{\sigma_\gamma^2}{\gamma_0^2}\right)\cos(nCkz)\right]$$

$$C = \frac{1}{1 + hR_{56}}$$

$R_{56} < 0$ **for a chicane**

**Linear approximation:**

$$Ck|R_{56}|\Delta\gamma/\gamma_0 \ll 1$$

$$I(z) \simeq CI_0 \left[1 + \rho_{\rm ind}\ {\rm sgn}(R_{56}) \cos(Ckz)\right]$$

$$\rho_{\rm ind} = Ck|R_{56}| \frac{\Delta\gamma}{\gamma_0} \exp\left(-\frac{1}{2}C^2k^2R_{56}^2\frac{\sigma_\gamma^2}{\gamma_0^2}\right)$$

## Wakefields upstream of a bunch compressor

### Initial modulation

$$I(z) = I_0 \left[1 + \rho_{\rm i} \cos(kz)\right]$$

### Wakefields:

$$\Delta\gamma = \frac{|Z(k)|}{Z_0} \frac{I_0}{I_{\rm A}} \rho_{\rm i}$$

$$Z_0 = 377\ \Omega\ , \qquad I_{\rm A} = 17 kA$$

### High gain approximation:

$$\rho_{\rm ind} \gg \rho_{\rm i}\ , \qquad G \simeq \frac{\rho_{\rm ind}}{\rho_{\rm i}} \gg 1$$

$$G = Ck|R_{56}| \frac{I_0}{\gamma_0 I_{\rm A}} \frac{|Z(k)|}{Z_0} \exp\left(-\frac{1}{2} C^2 k^2 R_{56}^2 \frac{\sigma_\gamma^2}{\gamma_0^2}\right)$$

### Broadband wakefields:

$$k_{\rm opt}^{-1} \simeq \frac{\sigma_\gamma}{\gamma_0} |R_{56}| C$$

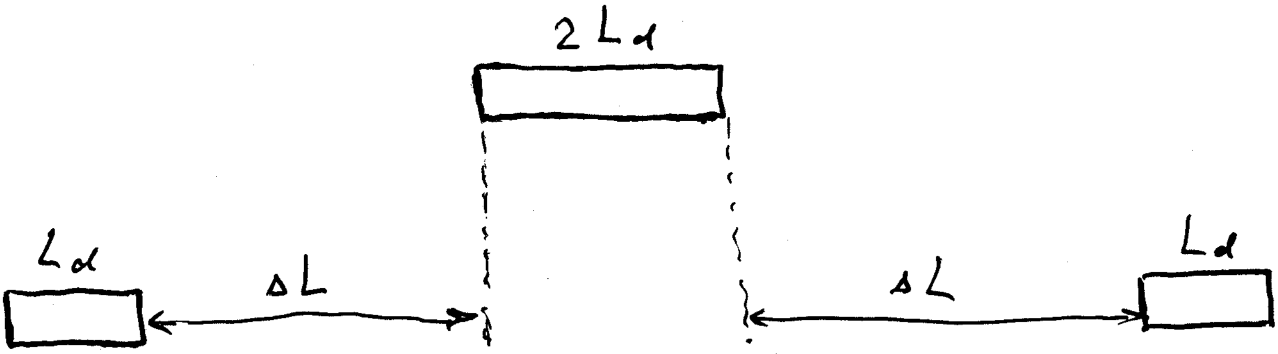
$$G_{\rm max} \simeq \frac{I_0}{\sigma_\gamma I_{\rm A}} \frac{|Z(k_{\rm opt})|}{Z_0}$$

### Emittance does not influence longitudinal dynamics if

$$k \epsilon S_{\rm c} / \beta \ll 1$$

# CSR in the bunch compressor chicane

**The model:**




$$\Delta L \gg L_{\mathrm{d}} \ , \qquad \frac{L_{\mathrm{d}}}{R} \ll 1 \ , \qquad R_{56} \simeq -2\Delta L \frac{L_{\mathrm{d}}^2}{R^2}$$

**Steady-state CSR:**

$$L_{\mathrm{d}} \gg (24R^2/k)^{1/3}$$

**Thin beam, no shielding:**

$$(R/b^3)^{1/2} \ll k \ll (R/\sigma_\perp^3)^{1/2}$$

**CSR impedance:**

$$\frac{|Z(k)|}{Z_0} = \frac{|Z_{\mathrm{l}}(k)| L_{\mathrm{d}}}{Z_0} = \frac{2\Gamma(2/3)}{3^{1/3}} \frac{L_{\mathrm{d}} k^{1/3}}{R^{2/3}}$$

**No compression ($h = 0$, $C = 1$). Transverse forses not included. Influence of emittance on longitudinal dynamics neglected.**

## Simple estimate

$$k_{\rm opt}^{-1} \simeq \frac{\sigma_\gamma}{\gamma_0}|R_{56}|$$

$$G_{\rm max} \simeq \frac{I_0}{\sigma_\gamma I_{\rm A}} \frac{|Z(k_{\rm opt})|}{Z_0}$$

$$\frac{|Z(k)|}{Z_0} = \frac{|Z_{\rm l}(k)| L_{\rm d}}{Z_0} = \frac{2\Gamma(2/3)}{3^{1/3}} \frac{L_{\rm d} k^{1/3}}{R^{2/3}}$$

**Effect of first dipole:**

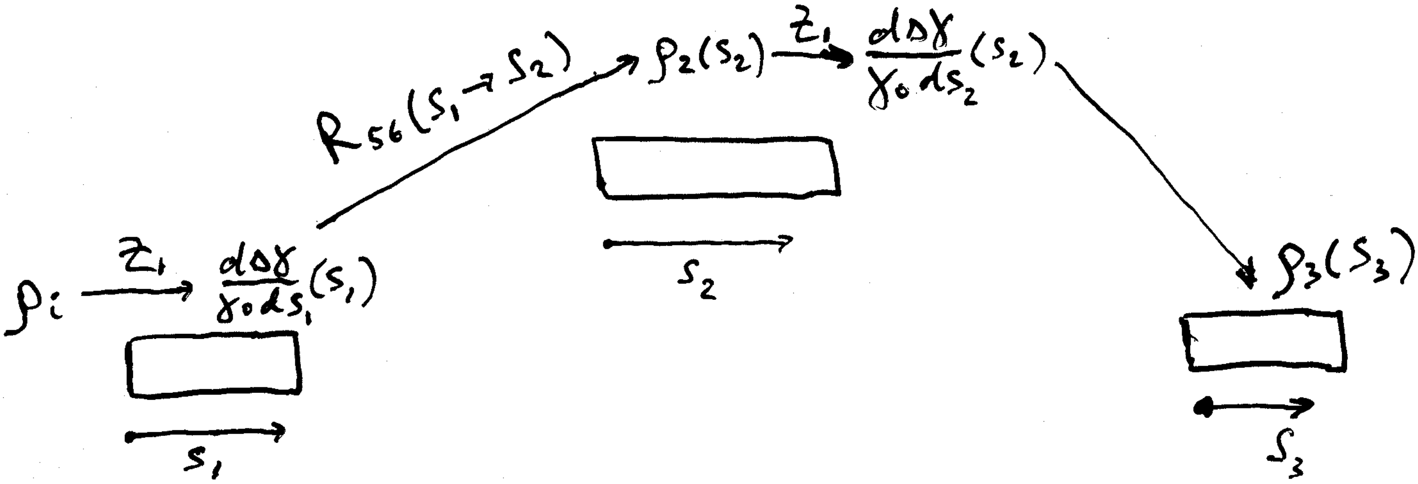
$$G_{\rm max} \simeq g_0$$

$$g_0 = \frac{I_0}{\sigma_\gamma I_{\rm A}} \left(\frac{\gamma_0}{\sigma_\gamma}\right)^{1/3} \frac{L_{\rm d}}{(R^2|R_{56}|)^{1/3}}$$

**Actually, we have two-stage amplification:**

$$G_{\rm max} \simeq g_0^2$$

## "Cold" beam $\quad k \ll \gamma_0/(\sigma_\gamma |R_{56}|)$




$$\frac{d\Delta\gamma}{\gamma_0 ds_1} = \frac{|Z_1(k)|}{Z_0}\frac{I_0}{\gamma_0 I_\mathrm{A}}\rho_\mathrm{i}$$

$$\rho_2(s_2) = -k\int_0^{L_\mathrm{d}} ds_1 \frac{d\Delta\gamma}{\gamma_0 ds_1} R_{56}(s_1 \to s_2)$$

$$R_{56}(s_1 \to s_2) \simeq -\frac{\Delta L}{R^2}(L_\mathrm{d} - s_1)s_2$$

$$\rho_2(s_2) = \frac{1}{2}\frac{|Z_1(k)|}{Z_0}\frac{I_0}{\gamma_0 I_\mathrm{A}}\frac{k\Delta L L_\mathrm{d}^2}{R^2}\rho_\mathrm{i} s_2$$

$$\frac{d\Delta\gamma}{\gamma_0 ds_2} = \frac{|Z_1(k)|}{Z_0}\frac{I_0}{\gamma_0 I_\mathrm{A}}\rho_2(s_2)$$

$$\rho_3(s_3) = -k\int_0^{2L_\mathrm{d}} ds_2 \frac{d\Delta\gamma}{\gamma_0 ds_2} R_{56}(s_2 \to s_3)$$

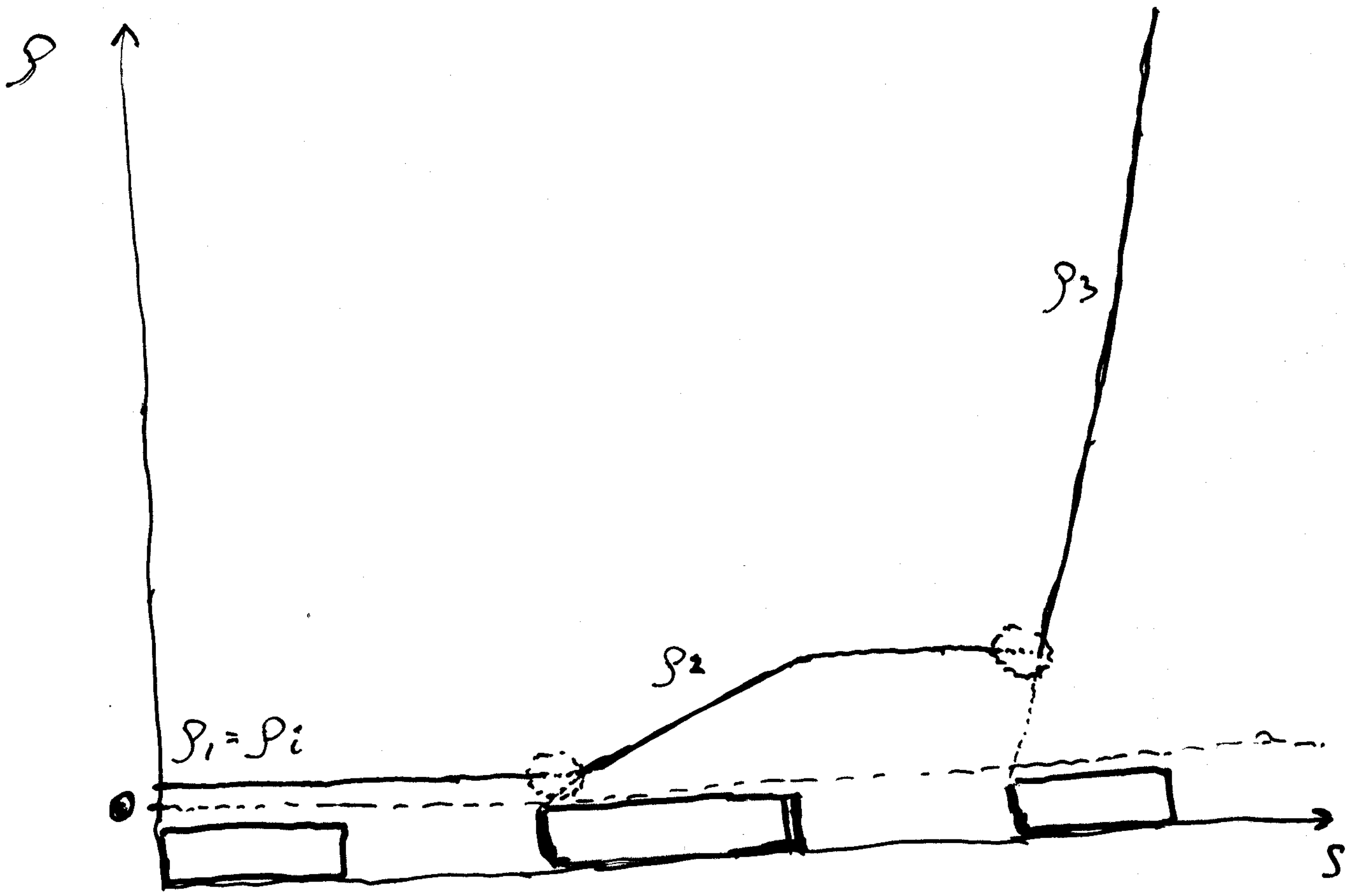
$$R_{56}(s_2 \to s_3) \simeq -\frac{\Delta L}{R^2}(2L_\mathrm{d} - s_2)s_3$$

**Finally:**

$$G = \frac{\rho_3(s_3 = L_\mathrm{d})}{\rho_\mathrm{i}} = \frac{2}{3}\frac{|Z_1(k)|^2}{Z_0^2}\left(\frac{I_0}{\gamma_0 I_\mathrm{A}}\right)^2 \frac{k^2 \Delta L^2 L_\mathrm{d}^6}{R^4}$$

**or**

$$G = \frac{2\Gamma^2(2/3)}{3^{5/3}}\left(\frac{I_0}{\gamma_0 I_\mathrm{A}}\right)^2 \frac{k^{8/3}|R_{56}|^2 L_\mathrm{d}^2}{R^{4/3}}$$



## Gaussian energy spread

$$G = \frac{2\Gamma^2(2/3)}{3^{5/3}} g_0^2 f(\hat{k})$$

$$g_0 = \frac{I_0}{\sigma_\gamma I_\mathrm{A}} \left(\frac{\gamma_0}{\sigma_\gamma}\right)^{1/3} \frac{L_\mathrm{d}}{(R^2|R_{56}|)^{1/3}}$$

$$\hat{k} = \frac{\sigma_\gamma}{\gamma_0}|R_{56}|k$$

$$f(\hat{k}) = 3\hat{k}^{2/3}\exp\left(-\hat{k}^2/2\right)\left[1 + \frac{\sqrt{\pi}}{2}\frac{\hat{k}^2 - 2}{\hat{k}}\exp\left(\hat{k}^2/4\right)\mathrm{erf}(\hat{k}/2)\right]$$

$$\hat{k} \ll 1 \quad : \qquad f(\hat{k}) \simeq \hat{k}^{8/3}$$

$$\hat{k}_\mathrm{opt} = 2.15 \qquad\qquad f_\mathrm{max} = 1.98$$

$$G_\mathrm{max} = 1.16 g_0^2$$

# Proposed Benchmark Example (comments, suggestions and questions may be addressed to either members of the Program Committee)

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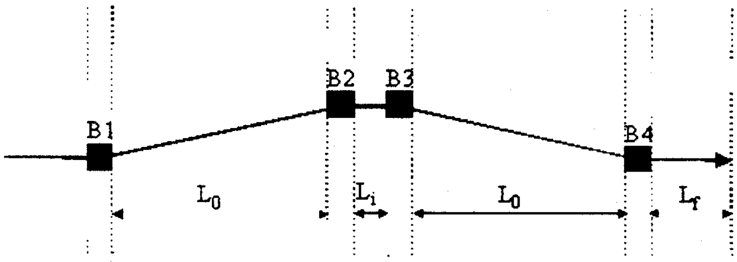
### The Case: A simple unshielded Four-Bend Chicane

## Chicane description:

The example consists of a simple four-bend chicane with parameters similar to the one required for the compression stages of the LCLS (at 5 GeV) or TESLA XFEL (at 500 MeV). It is meant to be a compromise between academic benchmarking and more practical issues. The compressor is depicted in the figure below and its parameters are gathered in the first table. Click on the graphics to download a MAD input deck.




| *Parameters* | *Symbol* | *Value* | *Unit* |
|---|---|---|---|
| Bend magnet length (projected) | $L_b$ | 0.5 | m |
| Drift length B1->B2 and B3->B4 (projected) | $L_0$ | 5.0 | m |
| Drift length B2->B3 | $L_i$ | 1.0 | m |
| Post chicane drift | $L_f$ | 2.0 | m |
| Bend radius of each dipole magnet | R | 10.35 | m |
| Bending Angle | $\phi$ | 2.77 | deg |
| Momentum compaction | $R_{56}$ | -25 | mm |
| 2nd order momentum compaction | $T_{566}$ | +37.5 | mm |
| Total projected length of chicane | $L_{tot}$ | 13.0 | m |
| Vertical half gap of bends | g | 2.5,5 | mm |

Note 1: Beam entrance and exit angles of the bends in degree: E1(B1)=0., E2(B1)= 2.77, E1(B2)=2.77, E2(B2)=0.

E1(B3)=0., E2(B3)=-2.77, E1(B4)=-2.77,E2(B4)=0.

Note 2: In the sign conventions used for $R_{56}$ and $T_{566}$ the bunch head is at z<0.

Note 3: We assume hard edge magnets. The vertical gap is specified for participants interested in comparing calculations taking into account shielding effects.

## The electron beam description:

The input electron beam will test two different examples: (1) a uniform, and (2) a Gaussian distribution for the temporal profile, where the initial rms length is the same in both case ($FWHM_{uniform}$=2Sqrt[3]*rms). The transverse phasespace is assumed to be gaussian in either case. The beam should have a perfectly linear time-energy "chirp" (the bunch head has lower energy than the tail). Therefore the time and energy distribution will be identical. In addition a very small uncorrelated ("slice") energy spread should be added with, for example, a Gaussian distribution.

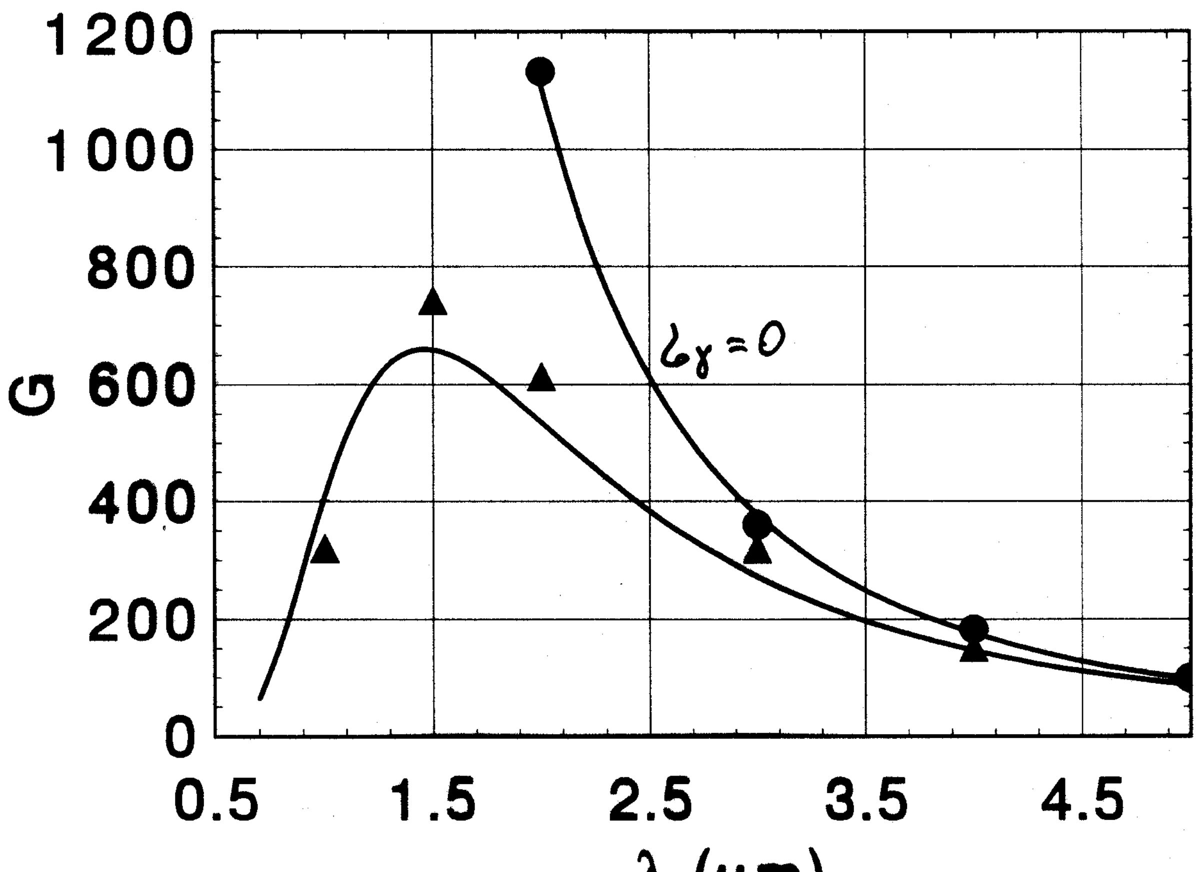
| *Parameter* | *Symbol* | *Value* | *Unit* |
|---|---|---|---|
| Nominal energy | $E_0$ | 0.5/5.0 | GeV |
| bunch charge | Q | 0.5, 1.0 | nC |
| incoherent rms energy spread | $(\Delta E)_{u\text{-}rms}$ | 10 | keV |
| linear energy-z correlation | a | +36.0 | $m^{-1}$ |
| total initial rms relative energy spread | $(\Delta E/E_0)_{rms}$ | 0.720 | % |
| initial rms bunch length | $\sigma_i$ | 200 | μm |
| final rms bunch length | $\sigma_f$ | 20 | μm |
| initial normalized rms emittance | $\varepsilon_{n,x}$ / $\varepsilon_{n,y}$ | 1.0 / 1.0 | mm-mrad |
| initial betatron functions at 1st bend entrance | $\beta_x$ / $\beta_y$ | 40 / 13 | m |
| initial alpha-function at 1st bend entrance | $\alpha_x$ / $\alpha_y$ | +2.6 / + 1.0 | |

Note: the linear energy-z correlation is defined as: $(\Delta E/E_0)(z) = (\Delta E/E_0)_{u\text{-}rms} + a\,z$ (i.e., the bunch head (z<0) is at lower energy than the tail)

In order to make even more basic comparisons, we propose to calculate the above case without compression. In this case, use a Gaussian 1-nC electron bunch with 20 μm rms bunchlength, no correlated energy spread, and an uncorrelated rms spread of 2*E-5 at an energy of 5 GeV and emittances of $\varepsilon_{n,x,y}$=1 mm-mrad. The input Twiss functions should be $\beta_{x,y}$=10 m, $\alpha_{x,y}$=0. $I_0$= 6kA

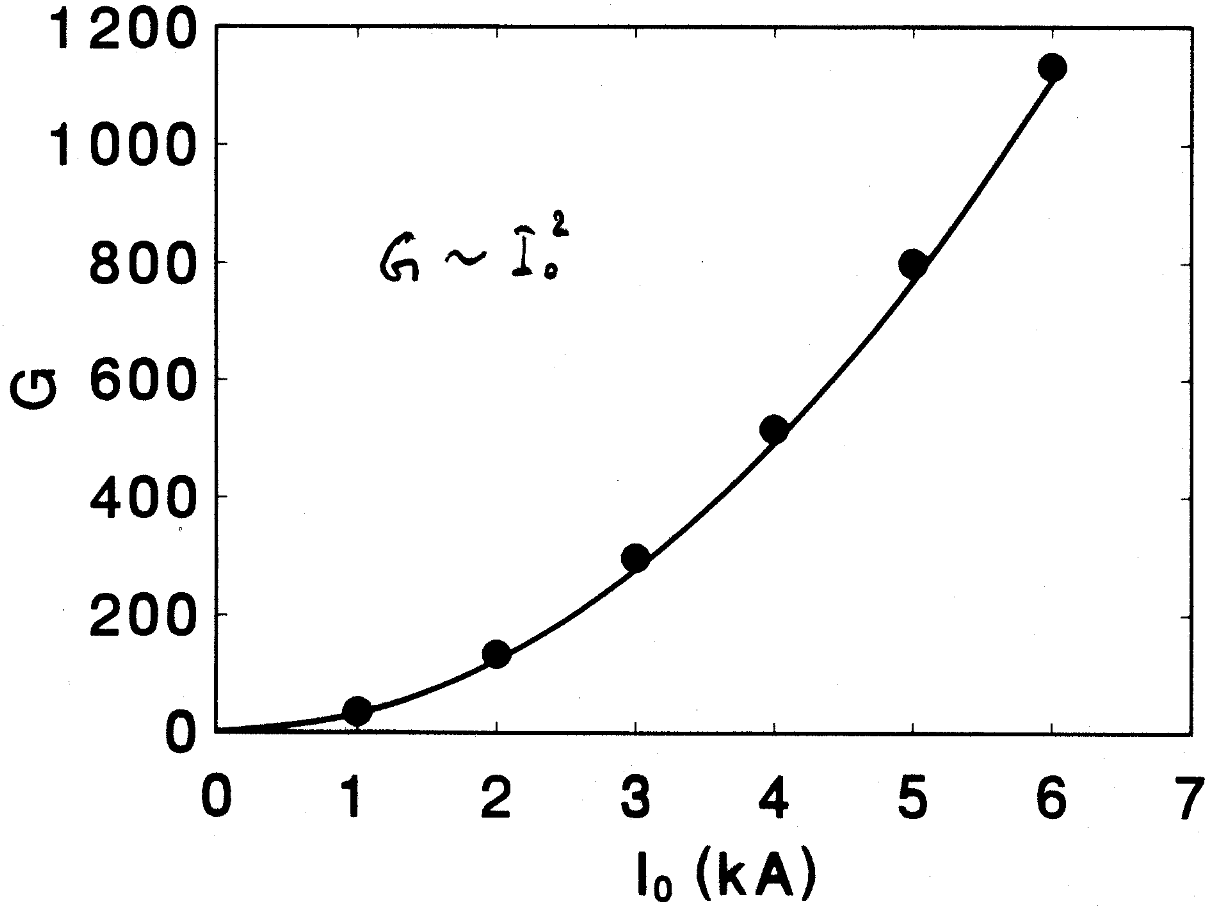
## Suggested output for comparison among the various codes

It will be helpful to look at various parameters at the end of the chicane (+Lf), and also as they evolve along the beamline. Such parameters are the CSR-induced energy loss and rms spread, bunch length, the emittance, and the Twiss parameters. The initial and final temporal and energy distributions will also be important. In addition, bunch length-sliced values of emittance, slice-centroid positions, and slice energy and energy spread will be useful, if accessible.




▲,● M. Dohlus (1-D simulation code, transients included).

$\lambda = 2\ \mu m$



## Estimation of emittance effect

$$\frac{dz}{ds} = -\frac{x}{R}$$

**For a typical offset**

$$x \simeq \sqrt{\epsilon\beta}$$

**the maximal change of coordinate $z$ is**

$$\Delta z \simeq \frac{\sqrt{\epsilon\beta}L_\mathrm{d}}{R}$$

**We require $k\Delta z \ll 1$, or**

$$\frac{k\sqrt{\epsilon\beta}L_\mathrm{d}}{R} \ll 1$$

**Emittance does nothing when**

$$\sqrt{\epsilon\beta} \ll \frac{\sigma_\gamma}{\gamma_0}\Delta L\frac{L_\mathrm{d}}{R}$$

**In the opposite case the cut-off is defined by emittance:**

$$k \simeq \frac{R}{\sqrt{\epsilon\beta}L_\mathrm{d}}$$

# Shot noise

**Broadband amplifier** $\Delta\omega \simeq \omega_0$

**Schottky formula:**

$$< i^2 >= \frac{eI_0 \Delta\omega}{\pi}$$

**Relative initial fluctuations**

$$< \rho_{\mathrm{i}}^2 >_{\mathrm{sh}} = \frac{< i^2 >}{I_0^2} \simeq \frac{e\omega_0}{\pi I_0} \simeq \frac{1}{N_\lambda}$$

**or**

$$(\rho_{\mathrm{i}})_{\mathrm{sh}} \simeq \frac{1}{\sqrt{N_\lambda}}$$

**Typically**

$$(\rho_{\mathrm{i}})_{\mathrm{sh}} \simeq 10^{-4}$$

**At the amplifier exit:**

$$\rho_{\mathrm{f}} \simeq \frac{G_{\mathrm{max}}}{\sqrt{N_\lambda}}$$

**Simulations should be carefully done. If $N_b$ is an actual number of particles per bunch and $N_m$ is a number of macroparticles, distributed randomly, then the effect will be overestimated by a factor**

$$\sqrt{\frac{N_b}{N_m}}$$

Energy modulation in the last dipole:

$$\frac{\Delta\gamma}{\gamma_0} \simeq \frac{L_d K^{1/3}}{R^{2/3}} \cdot \frac{I_0}{\gamma_0 I_A} \cdot \rho_3 \approx 6 \cdot 10^{-4} \cdot \rho_3$$

If it saturates ( $\rho_3 \approx 1$ ) $\rightarrow 6 \cdot 10^{-4}$

then it is of the order of FEL parameter ($\rho$)

at 1 Å - could be dangerous

(A positive thing : peak current increases)