Numerical Calculation of CSR Effects Using TraFiC^4

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2. CSR Effects
3. Considerations for Numerical Calculations
4. The Simulation Code TraFiC^4
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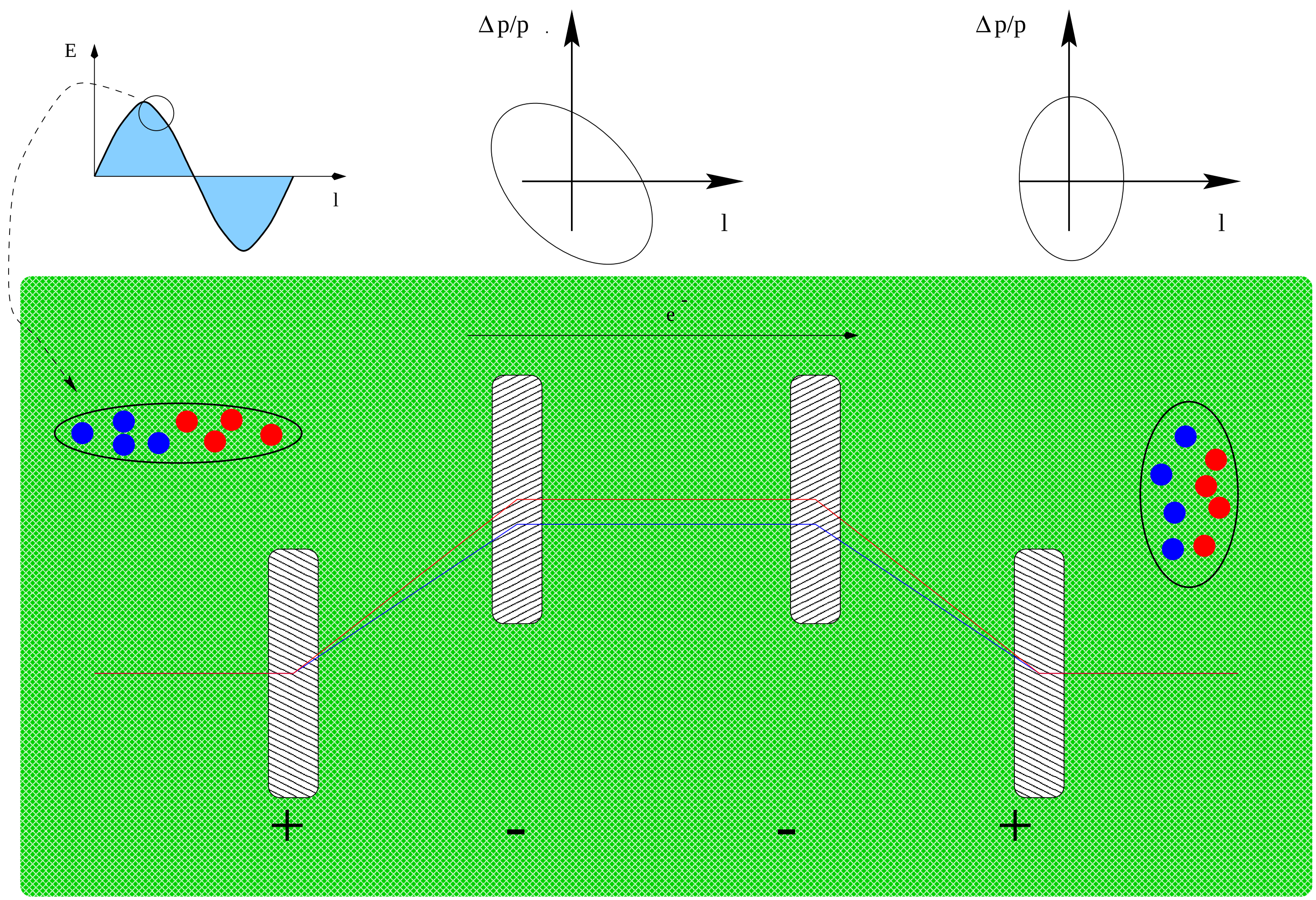
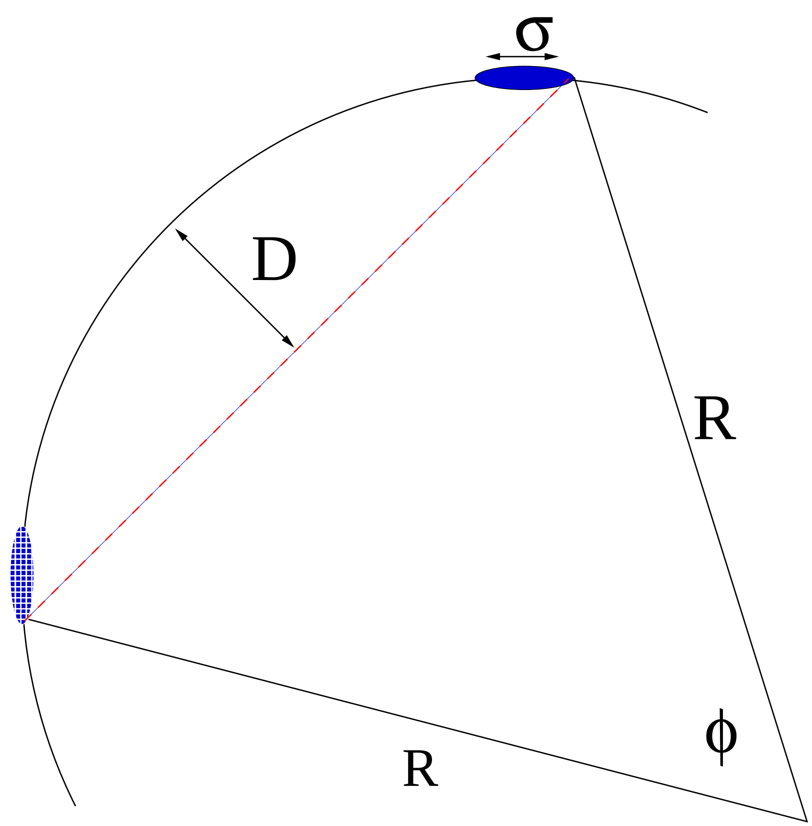
Magnetic Chicane



On curved trajectories: Fields from the tail of the bunch can interact with the head. purely geometrical effect, independent of energy: forces won't scale down (as space charge)



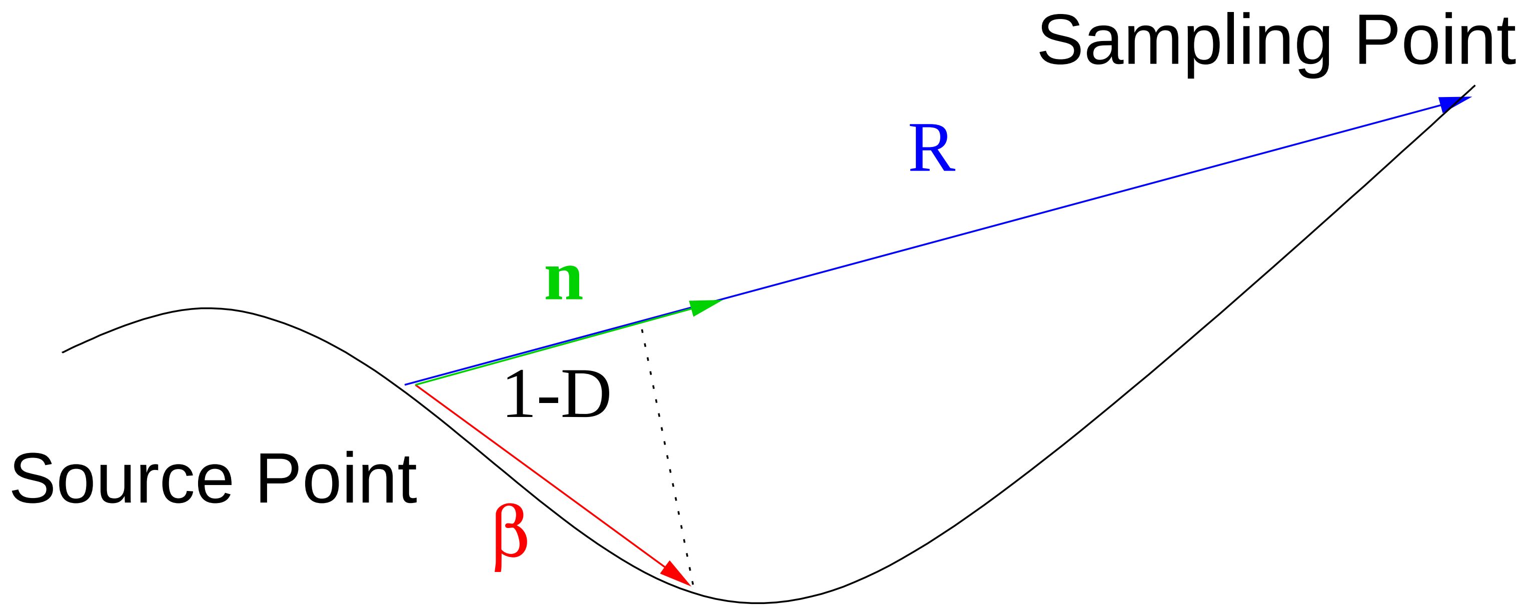
Overtaking condition:

$$R\varphi - \sigma = 2R\sin\frac{\varphi}{2} \Leftrightarrow L_{\mathsf{Ot}} \approx \sqrt[3]{24R^2\sigma}$$

$$D \approx \frac{L_{\mathsf{Ot}}^2}{8R}$$

$\longrightarrow$ Collective effects take place ("Coherent Synchrotron Radiation")

From Liénard-Wiechert Potentials:



$$1/q\boldsymbol{E}(\boldsymbol{x},t) = \left(\frac{\boldsymbol{n}-\boldsymbol{\beta}}{\gamma^2 D^3 R^2}\right)_{\text{retarded}} + \left(\frac{\boldsymbol{n}\times[(\boldsymbol{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}]}{D^3 R}\right)_{\text{retarded}}$$

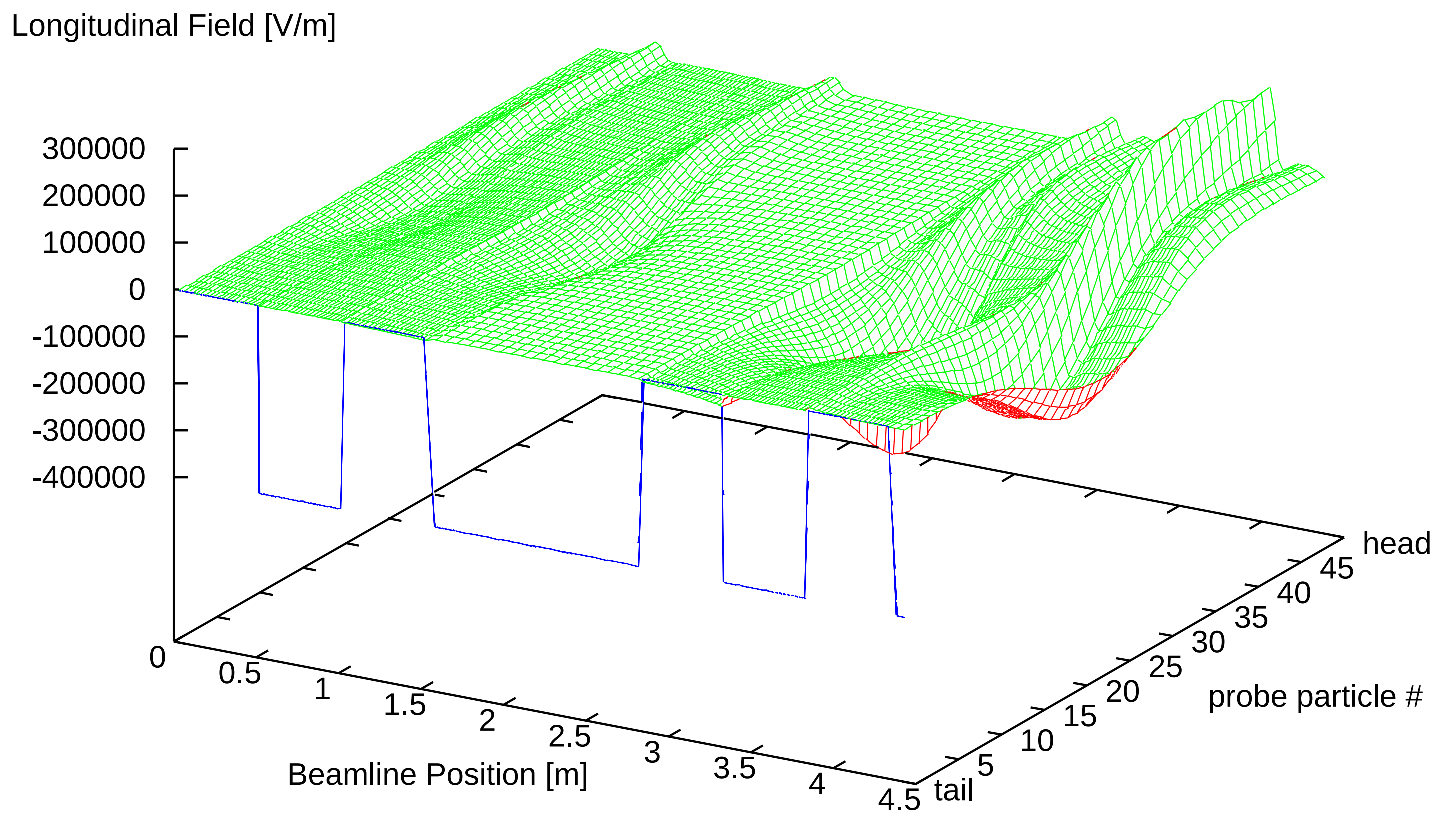
$$\boldsymbol{B}(\boldsymbol{x},t) = \boldsymbol{n}\times\boldsymbol{E}(\boldsymbol{x},t)$$

Forces on a Bend

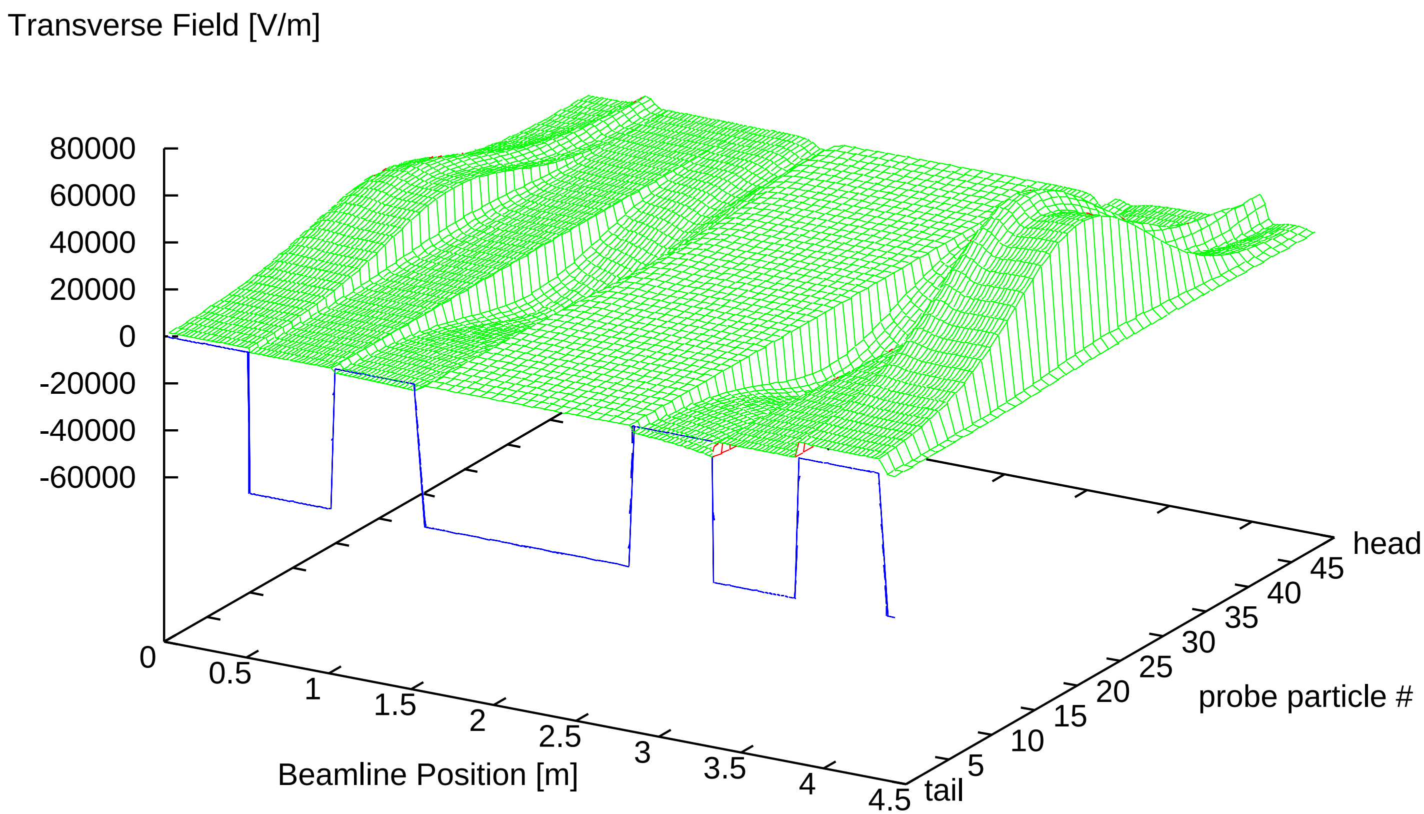
$\Rightarrow$ Coulomb-like forces and acceleration forces, acting both transversally and longitudinally.

Forces due to acceleration:

- instantaneous transverse force
- longitudinal force with slow build-up



Longitudinal Field [V/m]
300000
200000
100000
0
-100000
-200000
-300000
-400000
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
Beamline Position [m]
tail
5
10
15
20
25
30
35
40
45
head
probe particle #


Transverse Field [V/m]
80000
60000
40000
20000
0
-20000
-40000
-60000
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
Beamline Position [m]
tail
5
10
15
20
25
30
35
40
45
head
probe particle #

Transient Fields

496 *M. Dohlus, T. Limberg / Nucl. Instr. and Meth. in Phys. Res. A 393 (1997) 494–499*

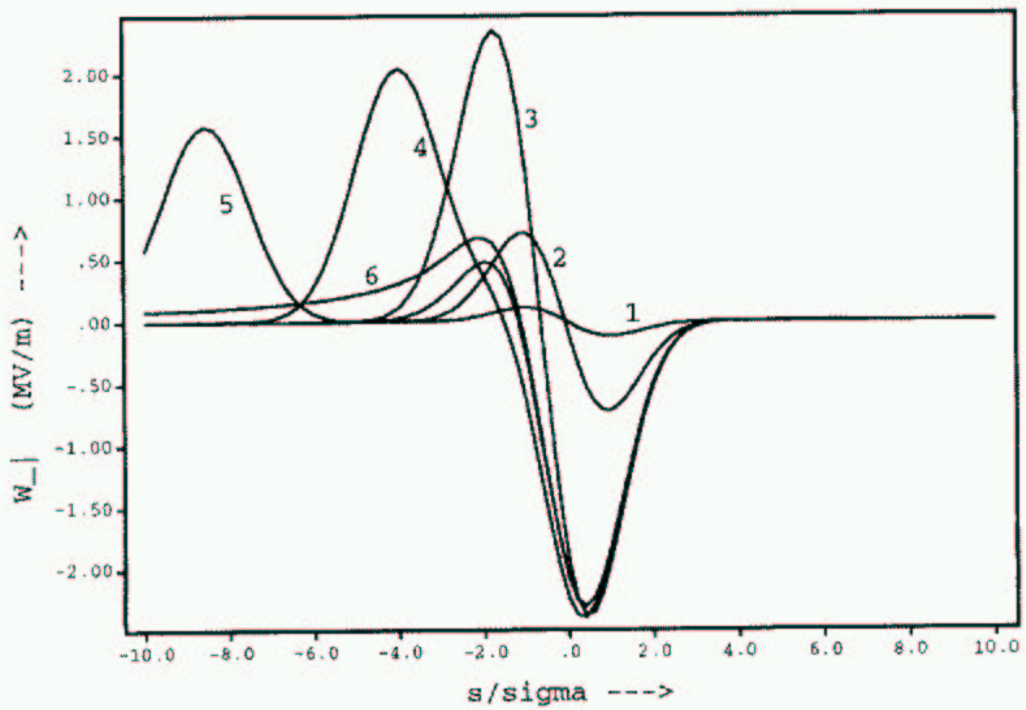


Fig. 2. Entering the magnet, longitudinal wake. Gaussian line bunch, $\sigma = 50\ \mu m$, $q = 1$ nC, $R = 1.5$ m; observation point: s = ordinate, $\partial r = 0$, $\partial z = 10\ \mu m$. Curve 1: 2 cm after begin of magnet; curve 2: 5 cm after begin of magnet; curve 3: 10 cm after begin of magnet; curve 4: 14 cm (interaction length); curve 5: 18 cm after begin of magnet; curve 6: steady state.

Similar behavior for exiting the bend: the fields need some time to decay

Transient Fields

M. Dohlus, T. Limberg / Nucl. Instr. and Meth. in Phys. Res. A 393 (1997) 494–499 497

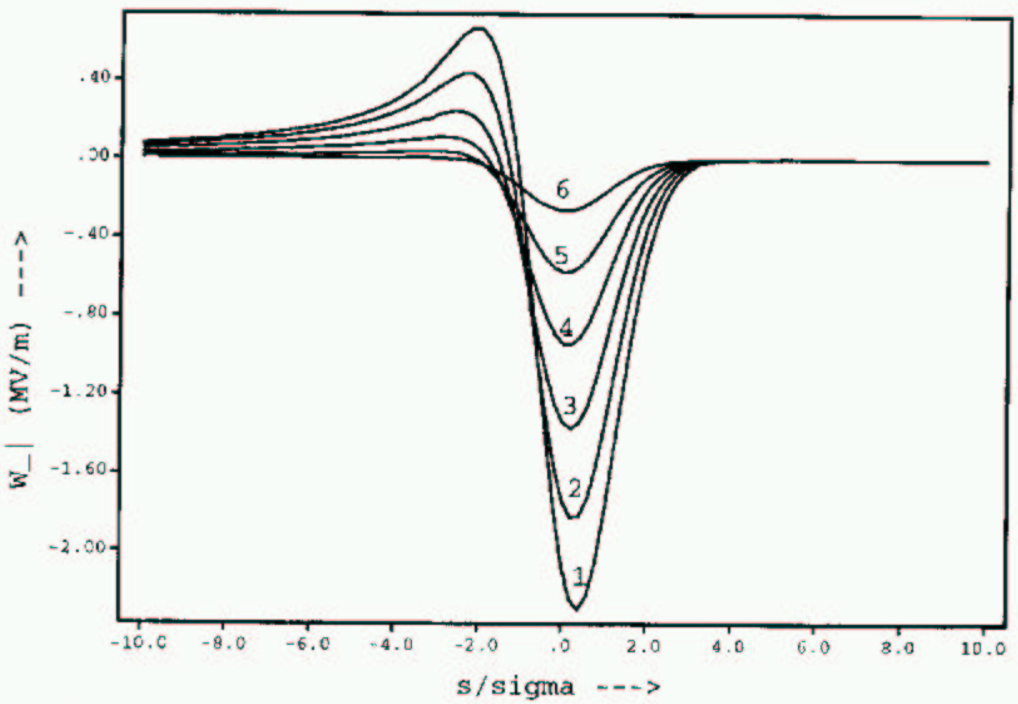
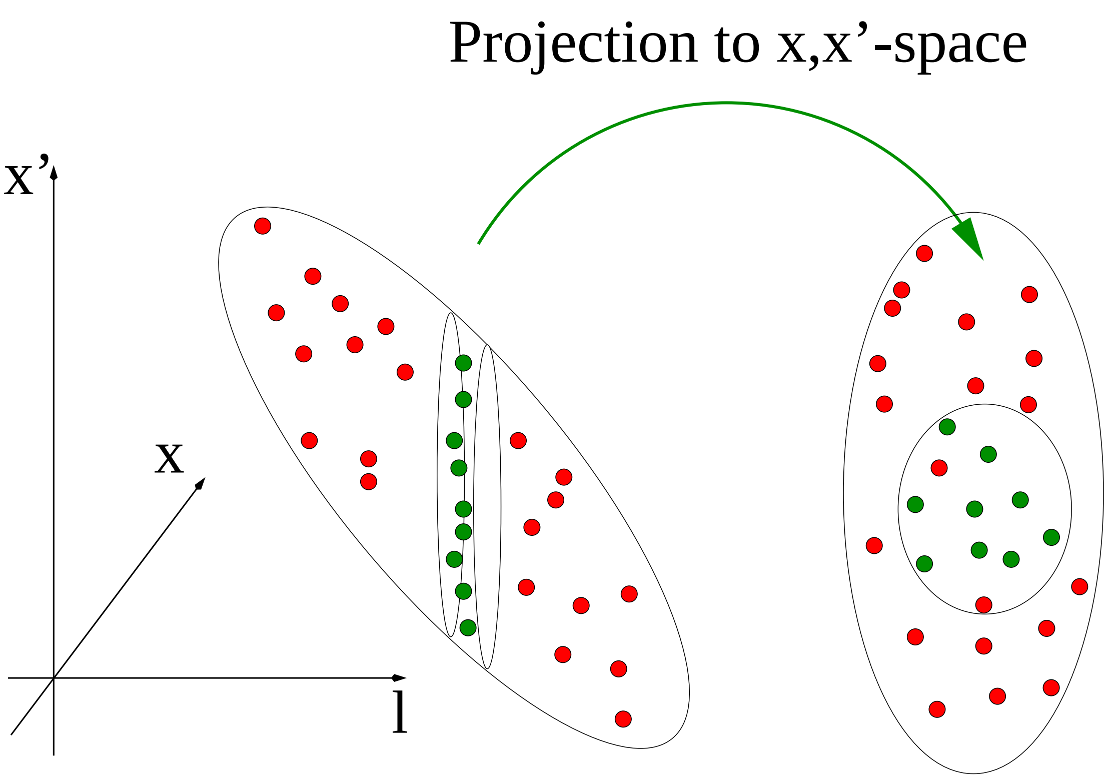


Fig. 3. End of magnet, longitudinal wake. Gaussian line bunch, $\sigma = 50\,\mu$m, $q = 1$ nC, $R = 1.5$ m; observation point: $s =$ ordinate, $\partial r = 0$, $\partial z = 10\,\mu$m. Curve 1: end of magnet; curve 2: 2 cm after end of magnet; curve 3: 5 cm after end of magnet; curve 4: 10 cm after end of magnet; curve 5: 20 cm after end of magnet; curve 6: 50 cm after end of magnet.

For FEL operation, beam quality is crucial.

The FEL process involves only particles within a certain longitudinal range ("slippage length").

This range may be much smaller than the bunch length:



For Phase 1 of TTF-FEL, the slippage length

$\Delta l \approx 10\mu\text{m} \ll l_{\text{final}} = 250\mu\text{m}$

$\rightarrow$ Projective emittance misleading for judging beam quality for FEL operation.

(But important for optics considerations)

Slice Emittance:

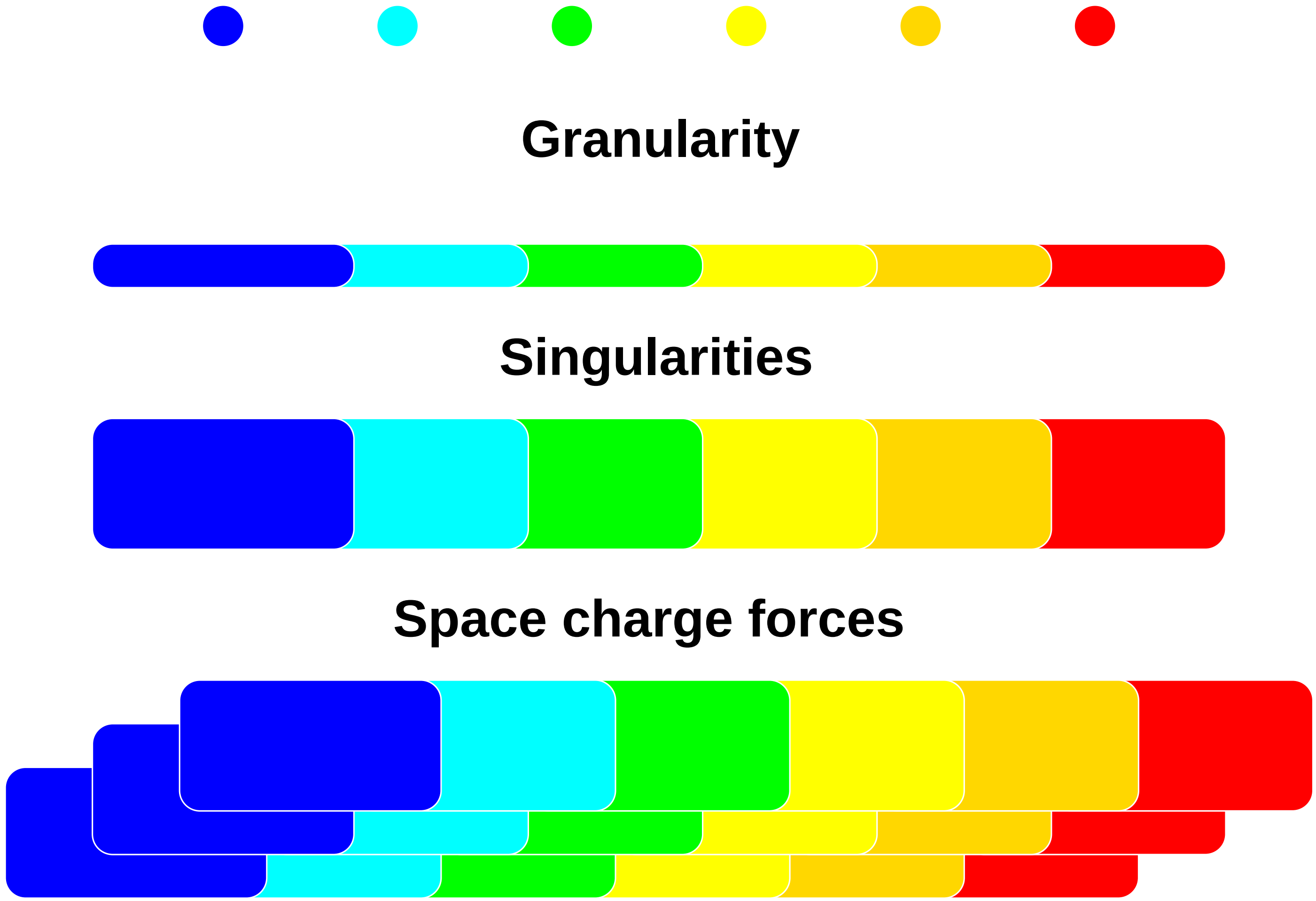
$$\epsilon(s_0) = \beta\gamma\sqrt{\langle x^2\rangle_c \langle x'^2\rangle_c - \langle xx'\rangle_c^2}$$

$$\text{where } \langle q\rangle = \frac{1}{N}\sum_{|s_i - s_0| < \frac{\Delta l}{2}} q_i$$

Emittance growth can take place by means of two mechanisms:

$$(\epsilon^2/2)' = \overbrace{\Phi\left(\partial^2_{x,x'}F(x,x'), \left\langle x^3\right\rangle, \ldots\right)}^{\text{Nonlinear forces \& uncorrelated energy spread}} + \frac{1}{R}\underbrace{\left(\left\langle x^2\right\rangle \langle \delta x'\rangle - \langle xx'\rangle \langle \delta x\rangle\right)}_{\text{Dispersion mismatch}}$$

Higher-dimensional particles are needed for simulations:



Shielding

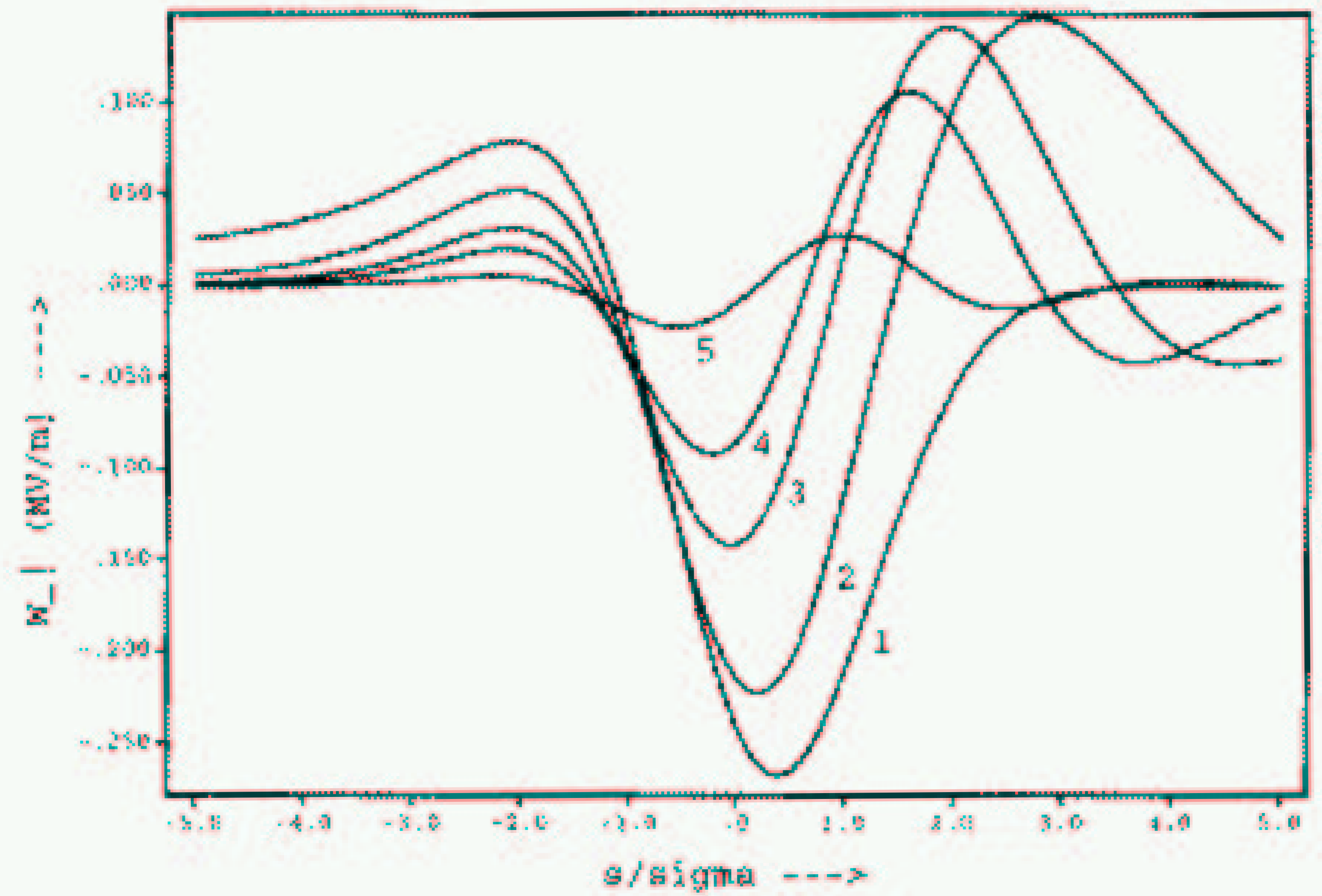


Fig. 4. Shielding by horizontal plates. Gaussian line bunch, $\sigma = 250\ \mu m$, $q = 1\ nC$, $R = 1.5\ m$; observation point: s = ordinate. $\delta r = 0$, $\delta z = 10\ \mu m$. Curve 1: without shielding; curve 2: 15 mm distance between plates; curve 3: 10 mm distance between plates; curve 4: 8 mm distance between plates; curve 5: 5 mm distance between plates.

What a Simulation Code Should Do:

- Handle retardation effects correctly $\longrightarrow$ use cartesian coordinates
- Calculate all fields from first principles
- Don't use linear approximations
- Consider the full six-dimensional phase space
- Don't use point particles $\longrightarrow$ use continuous charge distributions
- Use pointlinke probe particles
- Handle shielding

What TraFiC^4 Does:

- Handle retardation effects correctly $\longrightarrow$ use cartesian coordinates
- Calculate all fields from first principles
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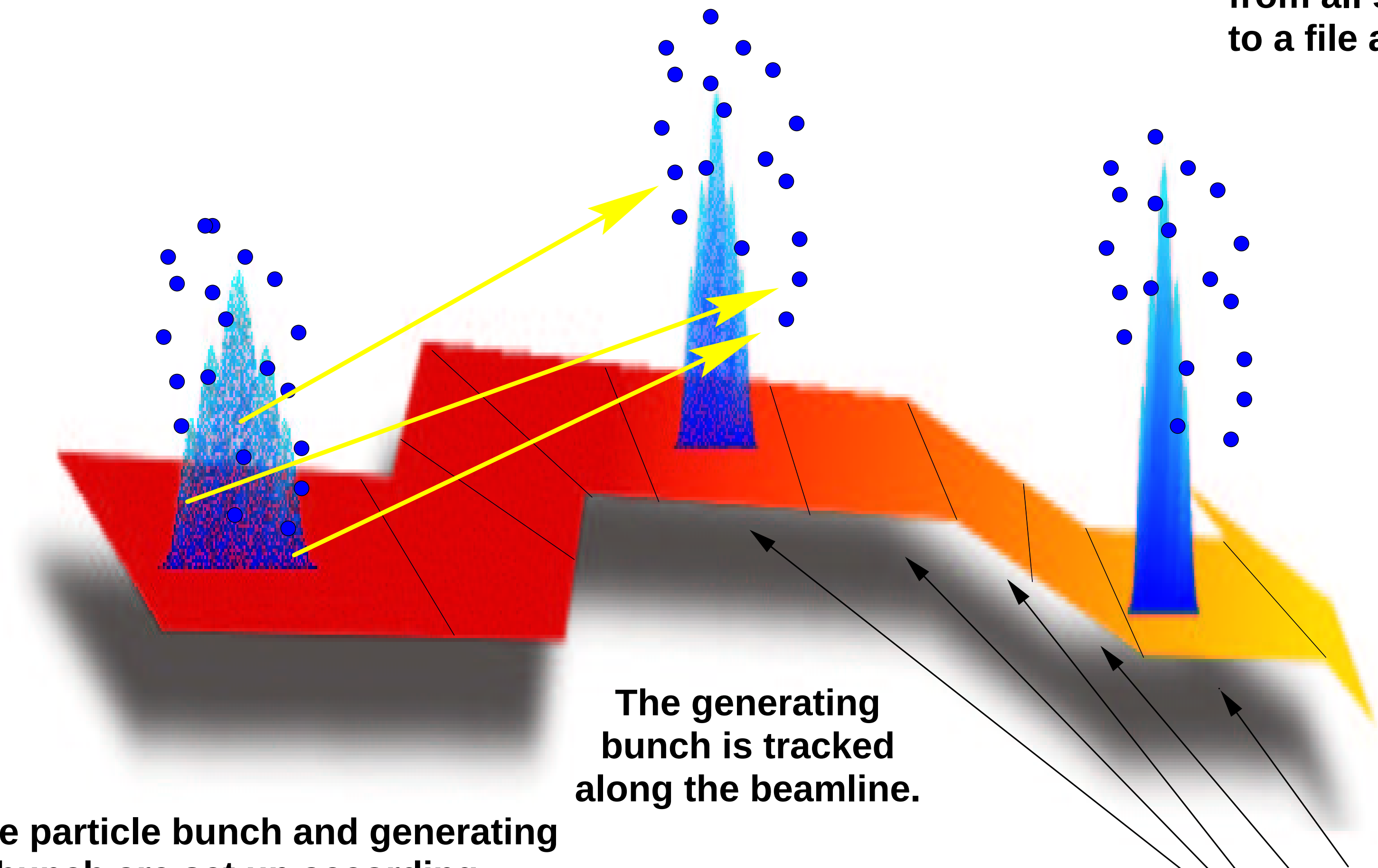
- Consider the full six-dimensional phase space
- Don't use point particles $\longrightarrow$ use continuous charge distributions
- Use pointlike probe particles
- Handle shielding

TraFiC^4= Track particles in the Fields of Continuous Charge distributions in Cartesian Coordinates. (Written by M. Dohlus, A. K., T. Limberg)

The Simulation Code TraFiC^4

In each slice and for each particle, the retarded position of each generating sub-bunch is found. The fields are calculated and are applied to the probe particles. The particles are tracked into the next slice.

The procedure is repeated up to the exit slice. The phase-space distribution from all slices is written to a file and postprocessed.

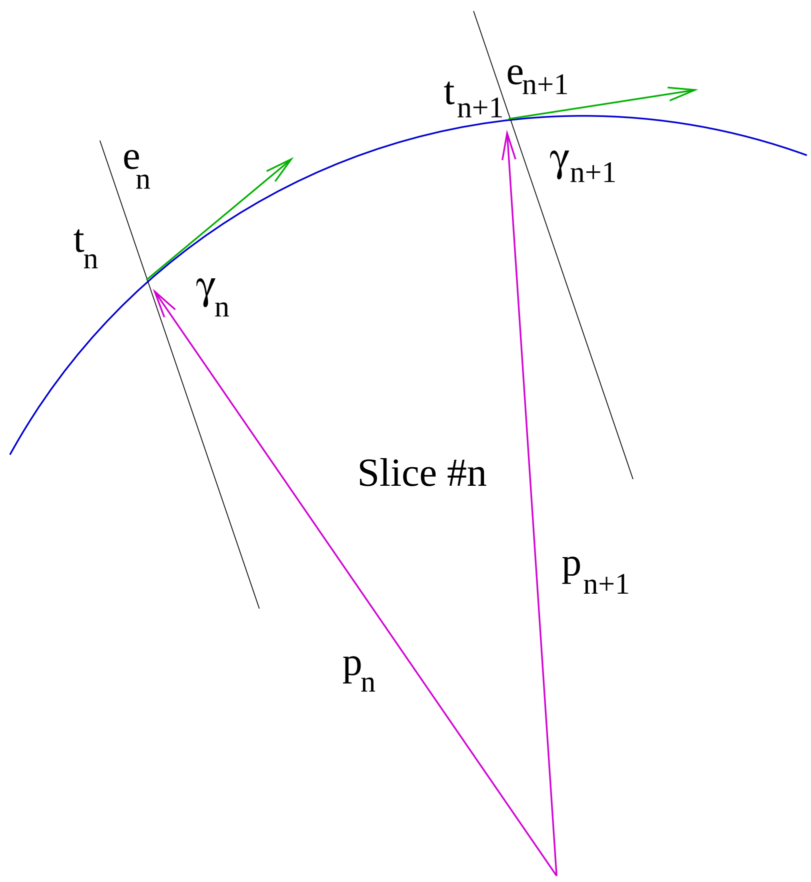


The generating bunch is tracked along the beamline.

Probe particle bunch and generating bunch are set up according to the optics of the beamline.

The beamline is divided into slices.

The Perturbative Kicks Procedure



The Perturbative Kicks Procedure

1. The beamline is separated into slices; the field-generating bunch is tracked and its parameters on the slices are stored
2. iteratively track probe particles:
   - To find the retarded position of the generating bunch, a fast binary lookup combined with "micro-tracking" through one slice is used
   - The resulting forces are applied to the probe particles:

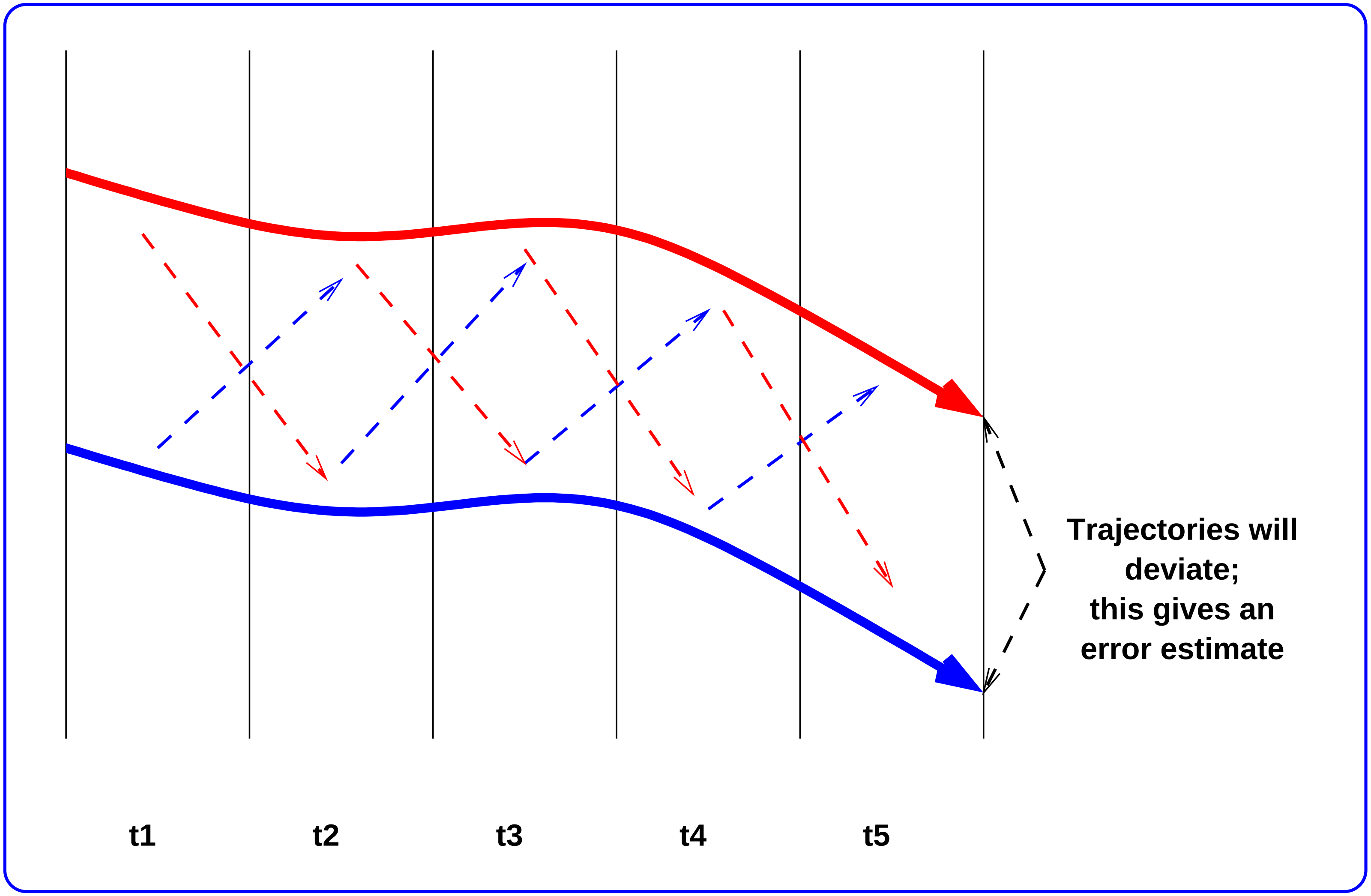
$$\begin{aligned} \gamma_{n+1} &= \gamma_n + \Delta s \frac{F_{\parallel}}{m} \\ \boldsymbol{e}_{n+1} &= \boldsymbol{e}_n + \Delta s \frac{\boldsymbol{F}_{\perp}}{m \beta_n^2 \gamma_n} \end{aligned}$$

   - The particle is tracked into the next slice

The Self-Consistent Tracking Algorithm

- We can use causality: only slices upstream will contribute
- Use a leapfrog alorithm: kicks are applied in turns
- Using two trajectories gives error estimate

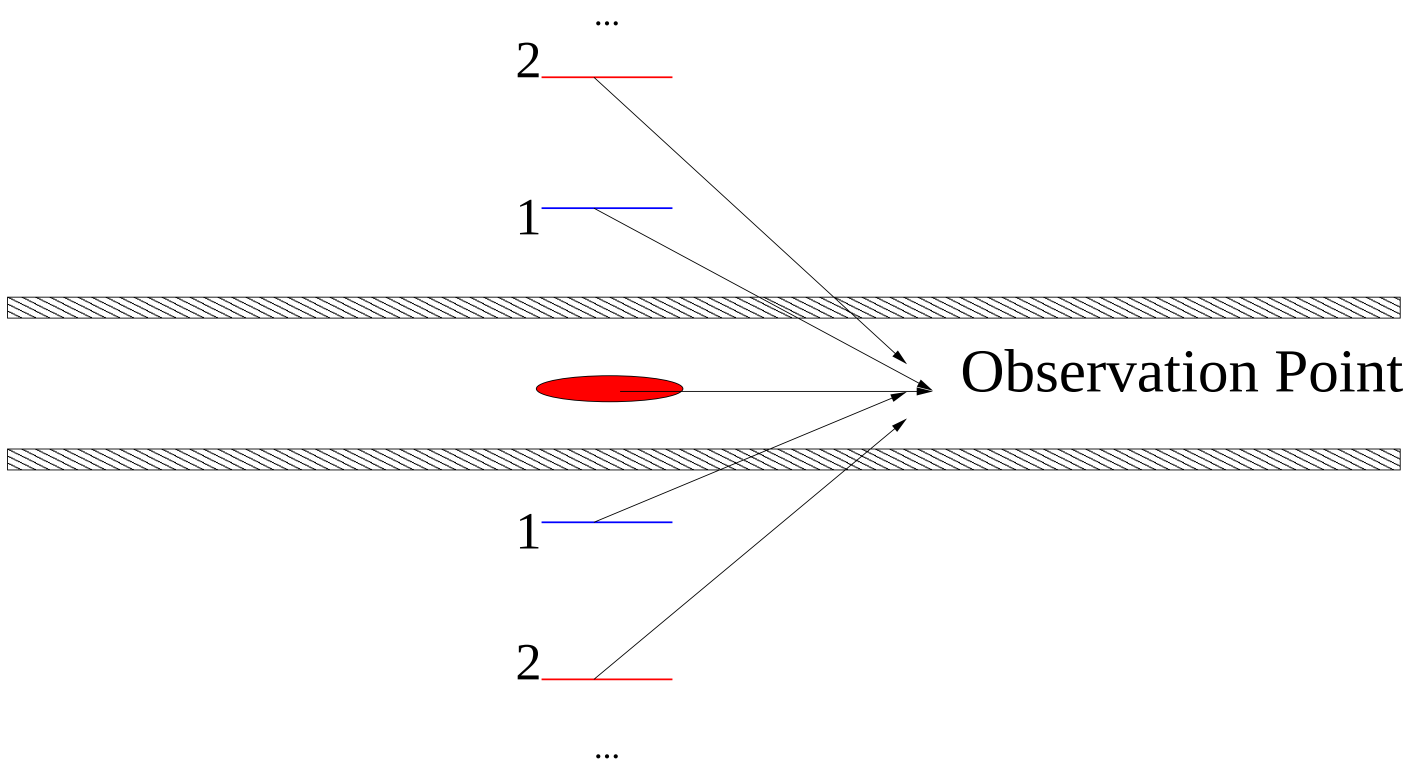
Self-consistent Tracking



So far, only free space was considered. Real-world situations impose boundary conditions on the problem.

TraFiC^4 handles a limited case of shielding: infinitely extended, perfect conducting horizontal parallel plates.

This is done by use of image charges.



As we can't encounter the singularity of the field-generating bunch, one-dimensional charges are used.

Benefits of shielding: Power radiated longitudinally (and thus induced energy spread) is reduced.

- 3D multi-particle tracking code
- calculates fields from first principles
- (almost) arbitrary beamlines
- valid for any energy range
- tons of output: phasespace, correlations,. . .
- highly configurable:
  - macro language for beamline setup
  - bunch setup: dimensionality, optics, smearing, . . .
  - fast/exact calculation
  - control output
  - slice/projective emittance
  - fields at given sampling points only, no tracking

TraFiC^4 Features

- it's slow:
  - $\approx$ hours for production-quality calculation for free space
  - $\approx$ days for shielded calculations
- MP to the rescue $\longrightarrow$ other talk
- huge output files ($\approx 10^{7\ldots8}$ bytes)

Conclusion and Outlook

- Handling beam dynamics on bent trajectories requires special care
- Conventional simulation codes can't be used
- TraFiC^{4}was written to handle the specific effects of bent trajectories
- Used as a design tool for the bunch compression sections at TTF
- Agreement with analytical (steady-state) results
- Reasonable agreement with experimental data (still sparse data)
- Inconclusive results for newest CLIC experiments